Supercurrent quantization in narrow-channel superconductor–normal-metal–superconductor junctions

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We study the opening of transport channels in mesoscopic ballistic superconductor–normal-metal–superconductor Josephson contacts. Determining the quasiparticle excitation spectrum in the normal contact region as a function of gate voltage, we discuss the transformation of the electronic levels into phase-sensitive Andreev levels with increasing chemical potential. The opening of these superchannels leads to a supercurrent quantization that exhibits nonuniversal behavior in general, and we discuss its dependence on the junction parameters.

I. INTRODUCTION

Over the past decade, the miniaturization of electronic structures has reached the regime where the transport proceeds via few or even a single conducting channel.[1–3] Such devices exhibit distinct steps in the conductance G as the number of channels is modified, providing microscopic information on the junction itself, as demonstrated in recent experiments on break junctions.[4] In superconducting junctions,[5] it is the maximal (critical) supercurrent I_c that is expected to exhibit a similar quantization. E.g., using gated structures,[6] superconducting junctions can be smoothly transformed from insulating [superconductor–insulator–superconductor (SIS)] to superconducting [superconductor–normal-metal–superconductor (SNS)]. The onset of superflow then proceeds in steps associated with the opening of transverse channels: with the conductance G quantized in units of $2e^2/h$ in metallic contacts,[1,7] a critical supercurrent quantization in units of $e\Delta/\hbar$ can be inferred from the relation[8] $I_c=(\pi/e)\Delta G$ (Δ is the superconducting gap in the banks). In fact, this result applies to short junctions[2,9,10] of length $L\ll\xi_0$ ($\xi_0=\hbar v_{\rm F}/\pi\Delta$ is the coherence length), however, contrary to the universality of the quantization in a normal contact, the quantization of the critical supercurrent is nonuniversal in general;[10] while experiments on superconducting quantum point contacts do show steps in the critical current I_c, their values are nonuniversal and depend on the junction parameters.[2,6] In this paper, we study the opening of superconducting channels in the metallic link of narrow-gated ballistic SNS Josephson contacts and determine the dependence of the (nonuniversal) supercurrent quantization on the junction parameters.

While the behavior of macroscopic SNS Josephson junctions is well understood,[11] the present interest concentrates on structures of mesoscopic size. Such quantum point contacts are realized in heterostructures,[1,6] with the help of break junctions,[2–4] or via manipulations with a scanning tunneling microscope.[12] Recently, an SNS junction with few conducting channels had been constructed by connecting two superconducting banks via a carbon nanotube.[13] Theoretically, the supercurrent-phase relation in mesoscopic SNS junctions with a δ scatterer has been analyzed[14] and the phenomenon of supercurrent quantization has been studied in short junctions.[9] Nonuniversal features of supercurrent quantization had first been observed by Furusaki *et al.*[10] — unfortunately, these numerical results provide limited insight into the physical origin and the parametric dependence of these effects. Here, we present a detailed discussion of the opening of superchannels in gated mesoscopic SNS junctions [see Fig. 1(a)] using quasiclassical and scattering matrix techniques. Given the relation $I_\nu=(2e/\hbar)\partial_\varphi\varepsilon_\nu$ between the supercurrent I_ν and the phase (φ) sensitivity of the energy levels ε_ν in the link, we discuss the nontrivial evolution of the excitation spectrum as the chemical potential drops below the superconducting gap: we analyze in detail the transformation of the ballistic SNS structure characterized by phase-sensitive Andreev states carrying large supercurrents into a SIS tunnel junction involving only phase-insensitive electronic states. Given the phase dependence of the quasiparticle spectrum in the metallic link, we then find the supercurrent quantization in short and long junctions, where the contribution from the continuous spectrum can be ignored.

II. GATED BALLISTIC SNS CONTACTS

A. Setup

Consider a narrow metallic lead (with few transverse channels) connecting two superconducting contacts as

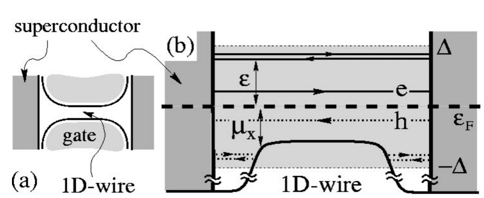

FIG. 1. Narrow-channel SNS contact: (a) geometrical setup showing the adiabatic joining of the wire to the superconductor, (b) potential landscape with a flat potential in the wire center and smoothly dropping to the band bottom in the superconducting banks. While electrons and holes with small excitation energies $\varepsilon<\mu_x$ form current carrying Andreev states, the hole propagation is quenched at large energies $\varepsilon>\mu_x$, and the Andreev levels transform into localized electronic states.

sketched in Fig. 1(a) [we assume piecewise constant gap parameters $\Delta(x<-L/2)=\Delta\exp(i\varphi_{\rm L})$, $\Delta(|x|<L/2)=0$, and $\Delta(L/2<x)=\Delta\exp(i\varphi_{\rm R})$]. Joining the wire smoothly to the superconducting banks, we adopt the adiabatic description for the longitudinal motion of the quasiparticles:[7,15] separating the wave function into longitudial and transverse components $\Psi(x,x_\perp)=\exp[(i/\hbar)\int^x dx' p_x(x')]\phi(x_\perp)$ and imposing the transverse boundary condition $\phi(x_\perp=\pm d(x)/2)=0$, the longitudinal problem reduces to one describing motion in the effective (chemical) potential $\mu_x(x)=\varepsilon_{\rm F}-\varepsilon_\perp(x)$ [see Fig. 1(b)], where $\varepsilon_\perp(x)$ denotes the transverse energy of the channel as obtained from the solution of the eigenvalue problem $-\nabla_\perp^2\phi(x_\perp)=(2m/\hbar^2)\varepsilon_\perp\phi(x_\perp)$, $\phi(x_\perp=\pm d(x)/2)=0$. Provided the curvature R of the wire's throat is small compared to its width, $R\ll d$, channel mixing between different transverse modes is exponentially suppressed, leaving backscattering within the same channel as the most relevant process. For each transverse channel in the metallic wire, the quasiparticle spectrum ε_ν then is determined through the one-dimensional Bogoliubov–de Gennes equation[16] (we choose states with $\varepsilon_\nu\geqslant 0$)

$$\begin{bmatrix} -\dfrac{\hbar^2\partial_x^2}{2m}-\mu_x(x) & \Delta(x) \\ \Delta^*(x) & \dfrac{\hbar^2\partial_x^2}{2m}+\mu_x(x)\end{bmatrix}\begin{bmatrix} u_\nu(x)\\ v_\nu(x)\end{bmatrix}=\varepsilon_\nu\begin{bmatrix} u_\nu(x)\\ v_\nu(x)\end{bmatrix},$$

where u_ν and v_ν denote the electronlike and holelike components of the wave function Ψ_ν. The spectrum splits into continuous and discrete parts, and we concentrate on the latter part in the following, $\varepsilon_\nu<\Delta$.

B. Quasiclassics

We begin with a brief description of the central physical idea: Within a quasiclassical formulation, we describe the quasiparticles in terms of their kinetic energies $K_\pm=\hbar^2k_\pm^2/2m=\mu_x(x)\pm\varepsilon$ and assume transmission and reflection to be ideal (the excitation energies $\varepsilon=E-\varepsilon_{\rm F}>0$ are measured with respect to the Fermi energy $\varepsilon_{\rm F}$). An electron with energy $\varepsilon<\Delta$ below the gap is reflected back from the superconductor as a hole with kinetic energy $K_-=\mu_x-\varepsilon$, injecting a Cooper pair into the superconducting contact, a process known as Andreev reflection.[17] A second reflection at the opposite NS boundary transforms the hole state back into the original electron state, thus producing a phase-sensitive Andreev level carrying the supercurrent across the normal-metal lead. The hole part associated with the Andreev level can propagate only if its kinetic energy is positive, $K_->0$ [see Fig. 1(b)]. Otherwise, the hole is back-reflected from the normal potential in the junction and transformed into an electron at the NS boundary—the incident electron is effectively reflected back as an electron and a phase-insensitive electronic level is formed. Hence, the superchannel starts being modified when the chemical potential μ_x drops below the gap Δ and is quenched completely with all Andreev levels transformed into electronic ones when μ_x becomes negative.

C. Scattering matrix formalism

Going beyond quasiclassics, the above physics is conveniently described through the scattering matrix formalism.[18,19] We define scattering states in the normal region and characterize them through the energy-dependent transmission and reflection amplitudes $t\exp(i\chi^t)$ and $r\exp(i\chi^r)$ describing the propagation of quasiparticles incident from the left through the junction [given the smooth geometry of the wire, adiabaticity of the levels allows us to ignore the mixing between different transverse channels see the discussion above and Ref. 15]. Matching these states with the evanescent modes in the superconductors, we obtain [within the Andreev approximation,[17] $(K_+-K_-)/(K_++K_-)\ll 1$ at the NS interface] the quantization condition[20]

$$\cos(S_+-S_--\alpha)=r_+r_-\cos\beta+t_+t_-\cos\varphi, \qquad (1)$$

where the $+$ $(-)$ signs refer to the positive and negative energies $\pm\varepsilon$ of the electronlike (holelike) quasiparticles and $S_\pm(\varepsilon)=\chi_\pm^t+k_{0,\pm}L$, with $k_{0,\pm}L=\sqrt{2m(\varepsilon_{\rm F}\pm\varepsilon)}L/\hbar$ the phase for free propagation (while the phase S refers to the propagation from $-L/2$ to $L/2$, the scattering phases χ^t and χ^r refer to the origin). Andreev scattering at the NS boundaries introduces the phase $\alpha=2\arccos(\varepsilon/\Delta)$ decreasing from π at $\varepsilon=0$ to 0 at the gap $\varepsilon=\Delta$, as well as the phase difference $\varphi=\varphi_{\rm L}-\varphi_{\rm R}$ between the two superconducting banks. The phase $\beta=(\chi_+^t-\chi_+^r)-(\chi_-^t-\chi_-^r)$ reduces to $\beta=0$ for a symmetric barrier in the absence of perfect resonances [as follows from the unitarity of the scattering matrix; for a symmetric potential shifted by a from the center we have $\beta=2(k_{0,+}-k_{0,-})a$]. The secular equation (1) involves two main energy dependencies originating from the propagation through the wire and from scattering at the NS boundaries, e.g., due to potential steps or barriers. Here, we concentrate on the case where the transport through the junction is dominated by the normal metallic wire—we will comment on the effect of resonances introduced by an additional scattering at the NS boundaries below.

D. Quasiparticle spectrum: Flat potential

A rough understanding of the transformation from a metallic to an insulating junction is obtained in the quasiclassical approximation using a flat potential [see Fig. 1(b)]: For a large chemical potential $\mu_x>\varepsilon$, we have $r_\pm=0$, $t_\pm=1$ in Eq. (1), and we obtain the Bohr-Sommerfeld quantization condition for the (phase-sensitive) Andreev levels

$$S_+-S_--\alpha\pm\varphi=2n\pi. \qquad (2)$$

On the other hand, for $-\varepsilon<\mu_x<\varepsilon$ only the electron gets transmitted $(t_+=1\rightarrow r_+=0)$, while the hole is always reflected $(r_-=1\rightarrow t_-=0)$ and the right-hand side of Eq. (1) vanishes. Using $S_-=3\pi/2$ as appropriate for a hard-wall potential, we then find the quantization condition

$$2S_+-2\alpha=2n\pi, \qquad (3)$$

for the electronic levels.

Evaluating these conditions for a flat potential (ignoring contributions to $S_\pm=k_{{\rm F},x}L\sqrt{1\pm\varepsilon/\mu_x}$ originating from the

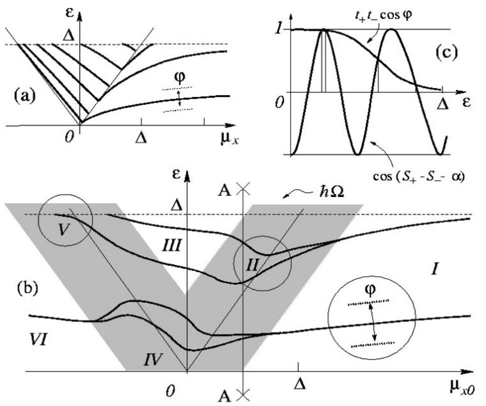

FIG. 2. Discrete energy spectrum [(a) qualitative sketch for a flat potential, (b) smooth parabolic potential]. For $\mu_{x0}>\varepsilon$ (region I) both electrons and holes propagate, forming phase-(φ) sensitive Andreev levels carrying supercurrent. The double degeneracy of the Andreev states is lifted by a finite phase drop φ across the junction as well as a finite reflection in the wire [see (c)], the latter becoming relevant upon decreasing μ_{x0}. As μ_{x0} drops below ε, the Andreev levels first transform into electronic states (regions II and III) and finally turn into boundary states trapped at the NS interfaces when $\mu_{x0}\lesssim-\varepsilon$ (regions V and VI). Within the shaded regions around $\varepsilon=\pm\mu_{x0}$, the transmission drops from unity to zero for holes and electrons. (c) Graphical solution of Eq. (1) along the cut A-A in (b).

adiabatic joints), we obtain the level scheme shown in Fig. 2(a): For a large potential $\mu_x\gg\Delta(k_{\rm F}L)^{1/3}$, we can expand $S_\pm\approx k_{{\rm F},x}L(1\pm\varepsilon/2\mu_x)$ and obtain the usual Andreev levels at $\varepsilon_n(\varphi)=(\hbar v_{{\rm F},x}/2L)[2n\pi+\alpha(\varepsilon_n)\mp\varphi]$. As the effective chemical potential becomes small (but still $\mu_x>\varepsilon$), we cannot expand any longer and find the energy levels at

$$\varepsilon_n(\varphi)=\mu_x\sqrt{1-\left[1-\frac{[2n+(\alpha\mp\varphi)/\pi]^2\varepsilon_L}{2\mu_x}\right]^2},\qquad(4)$$

where $[\cdots]>0$ and $\varepsilon_L\equiv\hbar^2\pi^2/2mL^2$.

Let us follow the evolution of the spectrum as a function of chemical potential μ_x: For $\varphi=0$ the nth level appears as μ_x is increased beyond $\varepsilon_n=\varepsilon_L(2n+\alpha/\pi)^2/2$ and rises toward the gap Δ with increasing μ_x [see Fig. 2(a)]. These levels are phasesensitive with a finite phase difference φ producing a level splitting. In the regime $-\varepsilon<\mu_x<\varepsilon$, electronic states are trapped at $\epsilon_n=\varepsilon_L(n+\alpha/\pi)^2-\mu_x$; they first appear as μ_x drops below $\epsilon_n=\varepsilon_L(n+\alpha/\pi)^2/2$, rise toward the gap with decreasing μ_x, and are phase insensitive. Note that we have twice as many electronic than Andreev levels as the latter are doubly degenerate at $\varphi=0$—the exact transformation of the Andreev levels into electronic ones at $\varepsilon\approx\mu_x$ requires a more careful analysis accounting for the nonideal transmission and reflection through the normal channel, see below. Finally, as μ_x drops below $-\varepsilon$, both the electronlike and holelike trajectories are quenched. Note the evolution of the right-hand side of Eq. (1), going from $\cos\varphi$ at large positive $\mu_x>\varepsilon$, to a small value in the intermediate region $-\varepsilon<\mu_x<\varepsilon$, and back to unity at large negative $\mu_x<-\varepsilon$. The replacement of $\cos\varphi$ by unity with decreasing chemical potential μ_x reflects the crossover from phase-sensitive (Andreev) levels to phase-insensitive (electronic) levels. This provides us with a first rough understanding of the SNS to SIS transformation.

E. Quasiparticle spectrum: Smooth potential

In a more accurate study of the evolution of the bound-state spectrum from a SNS to a SIS junction, we assume a smooth potential $\mu_x(x)$ with a small curvature $m\Omega^2=\partial_x^2\mu_x$, $\hbar\Omega<\Delta$, producing a sharp switching between transmission and reflection within the energy interval $\hbar\Omega$ (a δ-function scatterer[14] does not describe a pronounced transformation from a SNS to a SIS junction). Adiabatic joining of the wire to the superconducting banks requires that $m\Omega^2(L/2)^2/2\sim\varepsilon_{\rm F}$ and allows us to make use of the Andreev approximation while avoiding the appearance of resonances [this condition can be relaxed as the Andreev approximation requires $m\Omega^2(L/2)^2/2\gg\Delta$, while a step in the potential $\Delta V<0.9\varepsilon_{\rm F}$ produces only weak resonances (see Ref. 21)]. In summary, a smooth contact without resonances requires the parameter setting $\sqrt{\varepsilon_{\rm F}\varepsilon_{\rm L}}<\hbar\Omega<\Delta$; this condition implies that the junction has to be long, $L>\xi_0$, and hence a relatively large number $n\sim\sqrt{\Delta/\varepsilon_{\rm L}}$ of levels is trapped. For such a smooth potential, the Kemble formula is valid and the transmission probabilities take the form[7]

$$T_\pm=t_\pm^2=\frac{1}{1+\exp\{-2\pi[\mu_x(0)\pm\varepsilon]/\hbar\Omega\}}.\qquad(5)$$

Figure 2(b) shows the refined results for the SNS to SIS transformation using the quadratic potential $\mu_x(x)=\mu_{x0}+m\Omega^2x^2/2$ with the parameter $\hbar\Omega(\mu_{x0})=(4/\pi)\sqrt{\varepsilon_L(\varepsilon_{\rm F}-\mu_{x0})}$ and $\mu_{x0}=\mu_x(0)$, joining the band bottom of the superconductors at the two NS boundaries. For this case, the quasiclassical dimensionless action S takes the form

$$S(E)=\frac{2E}{\hbar\Omega}\{\kappa^2\sqrt{1+\kappa^{-2}}+\ln[|\kappa|(1+\sqrt{1+\kappa^{-2}})]\},\quad(6)$$

with

$$\kappa^2=Q\frac{\hbar\Omega}{E}=\frac{\pi^2\hbar^2\Omega^2}{16E\varepsilon_L},\qquad Q=\frac{\pi^2}{16}\frac{\hbar\Omega}{\varepsilon_L}\gg1$$

a large parameter [$S_\pm=S(E=\mu_{x0}\pm\varepsilon)$; an additional phase π, which cannot be obtained within the quasiclassical scheme, is picked up over the energy interval $\hbar\Omega$ as E goes through zero;[22] this additional phase shift has been included in the determination of the spectrum in Fig. 2(b)]. As for the flat potential, the Andreev levels at large chemical potential $\mu_{x0}>\varepsilon+\hbar\Omega$ (region I) are converted in steps (regions II–V) to the electronic states at negative potential $\mu_{x0}<-\varepsilon-\hbar\Omega$: We begin with the crossover from region I to III at $\varepsilon\sim\mu_{x0}$ and analyze Eq. (1) in more detail. The argument $S_+-S_--\alpha$ starts from $-\alpha(0)=-\pi$ and increases with energy ε. The first level (or pair of levels) appears as $S_+-S_--\alpha$ goes through zero; in region I, the right-hand side of Eq. (1) is (close to) unity and the solutions describe Andreev levels that split at finite values of φ. Upon approaching the line ε

$=\mu_{x0}$ (region II), the transmission coefficient t_- for the hole drops below unity. For $\varepsilon \geqslant \hbar\Omega$, the electron quasiparticle is still transmitted perfectly $[r_+ < \exp(-4\pi)]$ and the condition (1) reads $\cos(S_+ - S_- - \alpha) = t_+ t_- \cos\varphi < 1$, hence a level splitting occurs either as a consequence of a finite phase difference $\varphi \neq 0$ or even at $\varphi = 0$ due to the nonperfect transmission of the holelike quasiparticle, (see Fig. 2). For $\varphi = 0$, the product $t_+ t_-$ is unity for small energies $\varepsilon < \mu_{x0} - \hbar\Omega$ and drops to zero over the range $\hbar\Omega$ with increasing energies—correspondingly, the Andreev levels at small energies turn into electronic states at large energies, [see region II of Fig. 2(b) and the construction in Fig. 2(c)]. The splitting of the levels with increasing hole reflection starts with an exponential increase $\delta\varepsilon \sim 2\hbar r_- /(\tau_- + \tau_+)$ and becomes large, of order $\pi\hbar/(\tau_- + \tau_+)$ at $\varepsilon = \mu_{x0}$. Here, $\tau_\pm = \hbar \partial_\varepsilon S_\pm$ denotes the propagation times through the normal region N; for the smooth quadratic potential, we find

$$\tau(E) = \frac{2}{\Omega} \ln[|\kappa|(1 + \sqrt{1+\kappa^{-2}})]. \tag{7}$$

For small energies, the travel time increases logarithmically,

$$\tau(E) \approx \Omega^{-1} \ln(4Q\hbar\Omega/E) \tag{8}$$

within the interval $\hbar\Omega < E < Q\hbar\Omega$, and saturates at

$$\tau_0 \approx \Omega^{-1} \ln(4Q) \tag{9}$$

as E drops below $\hbar\Omega$ (in order to obtain this saturation, one has to go beyond the quasi-classical approximation).

Within region III, the hole propagation is quenched while the electronic transmission is still perfect. Across region V, where $\mu_{x0} \approx -\varepsilon$, the transformation of the SNS junction into a SIS tunneling junction is being completed. Here, the main contribution to the right-hand side in Eq. (1) is from the term $r_+ r_- \lesssim 1$, while the transmission provides only an exponentially small correction leading to a correspondingly weak dependence of the levels on the phase φ (and hence to an exponentially small supercurrent). For a wide barrier with only a small interval left for free propagation, i.e., $S_+ - S_- \ll \pi$, the levels appear close to Δ as the condition (1) requires the (Andreev) scattering phase to vanish, $\alpha(\varepsilon \to \Delta) \to 0$ [see Fig. 2(a)]. On the other hand, a wider region of free propagation allows for the accumulation of sufficient phase $S_+ - S_- > \pi$ to produce electronic levels at lower energies $\varepsilon < \Delta$ [see Fig. 2(b)]. The level splitting is a consequence of the inequality $t_+ t_- + r_+ r_- < 1$ and vanishes as μ_{x0} drops below $-\varepsilon$, producing (almost) degenerate (φ-independent) electronic states within the SIS regime VI, the analog of the (almost) degenerate (φ-dependent) Andreev states in the SNS regime I, where $\mu_{x0} > \varepsilon$. Physically, the levels at $\mu_{x0} \sim -\varepsilon$ can be understood in terms of NS interface states where the degeneracy is lifted due to the finite tunneling across the potential barrier.[23]

Finally, as the quasiparticle energy ε vanishes in region IV, $\varepsilon \leqslant \hbar\Omega$, both the electronic and the hole component undergo a finite reflection probability and the distinction between Andreev and electronic levels is gone—this resembles the situation of a SNS junction with a δ-scattering potential, (see, e.g., Ref. 14). In a long junction with $Q \gg 1$, the spectrum close to $\mu_{x0} = 0$ (i.e., upon channel opening) involves levels at $\varepsilon_n \approx \hbar\{(2n+1)\pi \pm \arccos[\cos^2(\varphi/2)]\}/2\tau_0$ [we approximate $t_\pm \approx 1/\sqrt{2}$ in Eq. (1) and use $S_\pm \approx \varepsilon\tau_0/\hbar$, $\alpha \approx \pi$]—at $\varphi = 0,\pi$ the finite reflection produces the minigaps $\varepsilon_0(\varphi=0) \approx \pi\hbar/2\tau_0$ and $\varepsilon_0(\varphi=\pi) \approx \pi\hbar/4\tau_0$. As usual, the degeneracies at $\varphi = 0$ are lifted when we account for the deviation of $r_+ r_- + t_+ t_-$ from unity.

F. Critical supercurrent

Next, we concentrate on the quasiclassical region I and study the evolution of the critical supercurrent as the channel is switched on and off. The supercurrent I flowing through the junction splits into the two contributions from the discrete ($I_{\rm dis}$) and the continuous ($I_{\rm con}$) parts of the spectrum. Here, we concentrate on $I_{\rm dis}$, which dominates the expression for the critical supercurrent in the quasiclassical regime I (requiring $\hbar\Omega < \mu_{x0}$ is sufficient). The current of the νth level (including a factor 2 for spin) can be obtained from the derivative

$$I_\nu = \frac{2e}{\hbar} \frac{d\varepsilon_\nu}{d\varphi} = \frac{2e}{\mathcal{T}} t_+ t_- \sin\varphi, \tag{10}$$

with the generalized traveling time

$$\frac{\mathcal{T}}{\hbar} = \sin(\delta S - \alpha)\partial_\varepsilon[\delta S - \alpha] + \partial_\varepsilon[(t_+ t_-)\cos\varphi + (r_+ r_-)\cos\beta]. \tag{11}$$

Within the quasiclassical region I, each Andreev level carries a finite supercurrent of amplitude $2e/[\tau_+ + \tau_- + 2\hbar/\sqrt{\Delta^2 - \varepsilon^2}]$, with $\tau_\pm = \hbar \partial_\varepsilon S_\pm$ the propagation times defined above.

Adding up the contributions from levels at $\varphi = 0$, the pairwise degenerate levels $\varepsilon_{n\pm}$ produce equal currents of opposite sign, and the sum over the discrete spectrum gives no current. Increasing φ, the degeneracy is lifted, with the levels $\varepsilon_{n,+}(\varepsilon_{n,-})$ going up (down) in energy as the phase φ increases. The currents of the individual pairs no longer cancel but produce a monotonously growing current of the same sign, hence, the largest current is reached at $\varphi = \pi^-$. On the other hand, at $\varphi = \pi^-$ the levels become degenerate again, but this time the mutual cancellation of currents occurs between the levels $\varepsilon_{n,+}$ and $\varepsilon_{n+1,-}$. The lowest level $\varepsilon_{0,-}$ then remains unpaired and thus carries all the supercurrent from the discrete part of the spectrum. The continuous part of the spectrum vanishes at $\varphi = \pi$, however, this is *a priori* not sufficient to guarantee that the critical current I_c is the current I_0 carried by the lowest level—we have to show, in addition, that the maximum of $I = I_{\rm dis} + I_{\rm con}$ is reached at $\varphi = \pi^-$ [indeed, using the usual Green's-function technique,[10] we could prove that this condition is fulfilled within a regime of the L-μ_{x0} plane away from $(L \sim (\xi_0/k_{\rm F})^{1/2}, \mu_{x0} \sim \Delta)$]. In the end, we arrive at a particularly simple expression for the critical current density in the quasiclassical region I,

$$I_c = \frac{e}{\tau_0 + \hbar/\Delta}. \tag{12}$$

The travel time τ_0 (i.e., the time a quasiparticle requires to traverse the normal region of length L) is constant $(\Omega^{-1}\ln 4Q)$ at the opening of the channel, decreases as $\Omega^{-1}\ln 4Q\hbar\Omega/\mu_{x0}$ for $\mu_{x0}>\hbar\Omega$, and transforms to the free travel time $L/v_{\mathrm{F},x}$ for $\mu_{x0}>Q\hbar\Omega$. As the channel becomes wide open at high energies, the critical current saturates at the *nonuniversal* value $I_c=ev_{\mathrm{F}}/(L+\pi\xi_0)$.

The particular (exponential) cutoff of the critical current throughout the regions II–V depends on the details of the junction potential. The quenching involves two steps, with an exponential reduction $I\propto t_-$ in region II (while the electron still propagates with $t_+\approx 1$, the hole undergoes tunneling) and a further reduction $I\propto t_-t_+$ in region V (both, electron and holetunneling). Note that for the ballistic case (region I) the factor $\sin(\varphi\to 0)\sim\varphi$ is compensated by the time $\mathcal{T}\propto\varphi$, while the finite reflection in regions II–V leads to a vanishing supercurrent as $\varphi\to 0$ for each level separately.

The above discussion dealt with the parameter settings $\sqrt{\varepsilon_{\mathrm{F}}\varepsilon_{\mathrm{L}}}<\hbar\Omega<\Delta$ requiring a junction with $L>\xi_0$. Releasing the condition of small curvature and assuming $\Delta<\hbar\Omega$, the SNS to SIS transformation is smeared and the region IV occupies all of the interesting crossover regime. For a flat potential $\hbar\Omega<\sqrt{\varepsilon_{\mathrm{F}}\varepsilon_L}$, the situation is complicated by the appearance of resonances due to reflection from the potential step at the NS boundary.[21] The situation simplifies for a very short junction with $L\ll\xi_0$, where we can again make use of Eq. (1) to reproduce a simple and universal result for the current-phase relation (see Refs. 24–26): with $\delta S=S_+-S_-\approx 0$ and $t_-\approx t_+\approx\sqrt{T}$, $r_-\approx r_+\approx\sqrt{R}$, one finds that only one level remains trapped in the junction at $\varepsilon_0=\Delta[1-T\sin^2(\varphi/2)]^{1/2}$ (here, we require a width $\hbar\Omega>\Delta$ in order to avoid a strong energy dependence in the transmission probability T). Determining the current $I_0(\varphi)$ from ε_0 and maximizing, we obtain

$$I_c=\frac{e\Delta}{\hbar}(1-\sqrt{R}), \tag{13}$$

in marked difference from the result for the conductance quantization $G=(2e^2/h)(1-R)$: a finite reflection $0<R\ll 1$ will affect the supercurrent quantization much more strongly than the conductance quantization.

G. Supercurrent quantization

Finally, we discuss the supercurrent quantization ''steps'' appearing as the gate potential is decreased to open the conducting channel. We concentrate on the quasiclassical regime I, assuming a parabolic potential $\mu_x(x)$ in the junction which matches the band bottom of the superconductors at the NS boundary and ignore a possible change in the effective mass. The quantized transverse energy of a channel of width d is given by $\varepsilon_{\perp;l}\approx\hbar^2\pi^2l^2/2md^2$; these levels match up with the Fermi energy when $d=d_k=k\pi/k_{\mathrm{F}}$. As we open the kth channel, the other open channels have already dropped by $\mu_{x0;l,k}=\varepsilon_{\mathrm{F}}-\varepsilon_{\perp;l,k}=\varepsilon_{\mathrm{F}}(1-l^2/k^2)$, where $\varepsilon_{\perp;l,k}=\hbar^2\pi^2l^2/2md_k^2$, e.g., the first channel is wide open when the second channel appears, $\mu_{x0;1,2}=(3/4)\varepsilon_{\mathrm{F}}$. Increasing the channel width d, the first channel opens [i.e., $\mu_{x0}=\varepsilon_{\mathrm{F}}(1-\pi^2/k_{\mathrm{F}}^2d^2)$ turns positive] as we reach $d_1=\pi/k_{\mathrm{F}}$, the critical current increases sharply $I_c\approx e\Omega/\ln[4/(k_{\mathrm{F}}^2d^2/\pi^2-1)]$ (the logarithmic singularity is cut off at $\hbar\Omega$) and saturates at $I_c\approx ev_{\mathrm{F}}/(L+\pi\xi_0)$ (see Fig. 3).

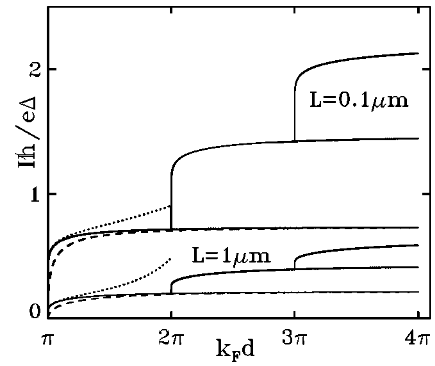

FIG. 3. Supercurrent quantization: with increasing width d of the normal channel the supercurrent increases in steps of $e/(\tau_0+\hbar/\Delta)$. The dotted and dashed lines give the approximations $\tau_0\approx\Omega^{-1}\ln(4Q\hbar\Omega/E)$ and $\tau_0\approx L/v_{\mathrm{F}}$ at small and large energies (parameters: $\varepsilon_{\mathrm{F}}=1$ eV, $\Delta/\varepsilon_{\mathrm{F}}=10^{-3}$, $L/\xi_0\sim 1{,}10$; with the curvature $\hbar\Omega/\Delta<5{,}0.5$ no smearing is visible at the supercurrent onset).

Here, we have ignored the smearing near the onset within the range $\hbar\Omega$ due to a finite reflection—while the interesting evolution of the quasiparticle spectrum is washed out as $\hbar\Omega$ increases beyond the gap Δ (see Fig. 2), the steps in the onset of the supercurrent are much more robust. On the other hand, the absence of sharp steps of universal height in the critical current is an intrinsic feature of the superconducting junction: the steps are rounded at the top and their height is limited by the junction length, $\delta I_c\sim ev_{\mathrm{F}}/L$ for junctions longer than the coherence length ξ_0 of the superconductor. For short junctions with $L\ll\xi$, the sharpness of the steps in I_c is dictated by the reflection probability R of the junction and, thus, is more similar to the steps in the conductance G. However, with $I_c\propto(1-\sqrt{R})$, the steps in I_c are always ''smoother'' than those in the conductance $G\propto(1-R)$.

III. CONCLUSION

In the end, universal supercurrent quantization first seems to require short junctions, but the gate needed to switch the channels will produce backscattering and spoil the quantization. While going over to longer contacts helps to produce sharp and universal conductance steps $\delta G\approx 2e^2/\hbar$, the steps in the supercurrent exhibit flat tops and assume the nonuniversal value $\delta I_c\approx ev_{\mathrm{F}}/L$ due to the long traveling time for the Andreev states.

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