Nodal liquids in extended *t-J* models and dynamical supersymmetry

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In the context of extended *t-J* models, with intersite Coulomb interactions of the form $-V\Sigma_{\langle i,j\rangle}n_i n_j$, with n_i denoting the electron number operator at site *i*, nodal liquids are discussed. We use the spin-charge separation ansatz as applied to the nodes of a *d*-wave superconducting gap. Such a situation may be of relevance to the physics of high-temperature superconductivity. We point out the possibility of existence of certain points in the parameter space of the model characterized by dynamical supersymmetries between the spinon and holon degrees of freedom, which are quite different from the symmetries in conventional supersymmetric *t-J* models. Such symmetries pertain to the continuum effective-field theory of the nodal liquid, and one's hope is that the ancestor lattice model may differ from the continuum theory only by renormalization-group irrelevant operators in the infrared. We give plausible arguments that nodal liquids at such supersymmetric points are characterized by superconductivity of Kosterlitz-Thouless type. The fact that quantum fluctuations around such points can be studied in a controlled way, probably makes such systems of special importance for an eventual nonperturbative understanding of the complex phase diagram of the associated high-temperature superconducting materials.

I. INTRODUCTION

The study of strongly correlated electron systems (SCES's) is a major enterprise in modern condensed-matter physics primarily due to high-temperature (planar) superconductors, fractional Hall conductors, and, more recently, in semiconductor quantum dots. Owing to various non-Fermi-liquid features of SCES's, many believe that the low-energy excitations of these systems are influenced by the proximity of a critical Hamiltonian in a generalized coupling-constant space. In this scenario, known as spin-charge separation,[1] these excitations are spinons, holons, and gauge fields.

Important paradigms for SCES's are the conventional Hubbard model, or its *t-j* extension, both of which have been conjectured to describe the physics of high-temperature superconducting doped antiferromagnets. Numerical simulations of such models,[2] in the presence of very low doping, have provided evidence for electron substructure (spin-charge separation) in such systems.

In Ref. 3, an extension of the spin-charge separation representation, allowing for a particle-hole symmetric formulation away from half filling, was introduced by writing

$$\chi_{\alpha\beta}\equiv\begin{pmatrix}\psi_1 & \psi_2\\ -\psi_2^\dagger & \psi_1^\dagger\end{pmatrix}_i\begin{pmatrix}z_1 & -\overline{z_2}\\ z_2 & \overline{z_1}\end{pmatrix}_i, \qquad (1)$$

where the fields $z_{\alpha,i}$ obey canonical *bosonic* commutation relations, and are associated with the *spin* degrees of freedom (''spinons''), while the fields ψ are Grassmann variables, which obey Fermi statistics, and are associated with the electric charge degrees of freedom (''holons''). There is a hidden non-Abelian gauge symmetry $SU(2)\otimes U_S(1)$ in the representation (1), which becomes a dynamical symmetry of the pertinent planar Hubbard model, studied in Ref. 3.

The representation (1) is different from that of Refs. 4 and 5, where the holons are represented as charged bosons, and the spinons as fermions. That framework, unlike ours, is not a convenient starting point for making predictions such as the behavior of the system under the influence of strong external fields. As argued in Ref. 6, a strong magnetic field induces the opening of a second superconducting gap at the nodes of the *d*-wave gap, in agreement with recent experimental findings on the behavior of the thermal conductivity of high-temperature cuprates under the influence of strong external magnetic fields.[7]

In Ref. 3 a single-band Hubbard model was used. Such a model should not be regarded as merely phenomenological for cuprate superconductors since it can be deduced from chemically realistic multiband models involving both Cu and O orbitals and it has extra nearest-neighbor interactions of the form[8]

$$H_{int}=-V\sum_{\langle ij\rangle}n_i n_j, \quad n_i\equiv\sum_{\alpha=1}^{2}c^\dagger_{\alpha,i}c_{\alpha,i}, \qquad (2)$$

as well as longer finite-range hoppings.

What we shall argue below is that the presence of interactions of the form (2) is crucial for the appearence of supersymmetric points in the parameter space of the spin-charge separated model. Such points occur for particular doping concentrations. As we shall discuss, this supersymmetry is a *dynamical symmetry* of the spin-charge separation, and occurs between the spinon and holon degrees of freedom of the ansatz (1). Its appearance may indicate the onset of unconventional superconductivity of the Kosterlitz-Thouless (KT) type[9,10] in the liquid of excitations about the nodes of the *d*-wave superconducting gap (''nodal liquid''), to which we restrict our attention for the purposes of this work.

It should be stressed that the supersymmetry characterizes the continuum *relativistic* effective- (gauge-) field theory of the nodal liquid. The progenitor lattice model is of course *not supersymmetric* in general. What one hopes, however, is that at such supersymmetric points the universality class of the

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continuum low-energy theory is the *same* as that of the lattice model, in the sense that the latter differs from the continuum effective theory only by renormalization-group *irrelevant* operators (in the infrared). This remains to be checked by detailed studies, which do not constitute the topic of this article.

In general, supersymmetry provides a much more controlled way for dealing with quantum fluctuations about the ground state of a field-theoretic system than a nonsupersymmetric theory.[11] In this sense, by working in such supersymmetric points in the parameter space of the nodal liquid one might obtain some exact results about the phase structure, which might be useful for a nonperturbative understanding of the complex phase diagrams that characterize the physics of the (superconducting) doped antiferromagnets. As we shall discuss below, to obtain supersymmetric points one needs to make specific assumptions about the regime of the parameters of the model; from an energetics point of view, such assumptions are retrospectively justified by the fact that supersymmetric ground states are characterized by *zero energy*,[11] and hence are acceptable ground states from this point of view.

Significant progress towards a nonperturbative understanding of non-Abelian gauge-field theories, in four space-time dimensions, based on supersymmetry has been made by Seiberg and Witten.[12] The fact that the spin-charge separation representation (1) of the doped antiferromagnet is known to be characterized by such non-Abelian gauge structure is an encouraging sign. However, it should be noted that in the case of Ref. 12 extended supersymmetries were necessary for yielding exact results. As we shall discuss below, under special conditions for doped antiferromagnets, the supersymmetric points are characterized by $N=1$ three-dimensional supersymmetries. Under certain circumstances the supersymmetry may be elevated to $N=2$,[13] for which it is possible to obtain some exact results concerning the phase structure.[14] In the present state of the understanding of SCES's it is a pressing need to have relevant models for which we can extract nontrivial exact information. However, for a realistic condensed-matter system such as a high-temperature superconductor, even the $N=1$ supersymmetry of the supersymmetric points is expected to be broken at finite temperatures or under the influence of external elctromagnetic fields. Nevertheless, one may hope that by viewing the case of broken supersymmetry as a perturbation about the supersymmetric point, valuable nonperturbative information may still be obtained. As we shall see, a possible example of this may be the above-mentioned KT superconducting properties[9] that characterize such points.

II. MODEL AND ITS PARAMETERS

In Ref. 8 it was argued that BCS-like scenarios for high-T_c superconductivity based on extended t-J models yield reasonable predictions for the critical temperature $T_c^{\max}$ at optimum doping. There it was argued that a pivotal role was played by next-to-nearest neighbor and third neighbor hoppings t', and t'', respectively. In particular, the combination $t_- \equiv t' - 2t''$ determines the shape of the Fermi surface and the nature of the saddle points and the associated $T_c^{\max}$.

Our aim is to use the extended t-J model studied in Ref. 8 in order to discuss the appearance of relativistic charge liquids at the nodes of the associated d-wave superconducting gap. We will argue that the nodes characterize the model in a certain range of parameters. We will demonstrate that at a certain regime of the parameters and doping concentration the nodal liquid effective-field theory of spin-charge separation exhibits supersymmetry. This supersymmetry is dynamical and should not be confused with the nondynamical symmetry under a graded supersymmetry algebra that characterizes the spectrum of doped antiferromagnets at two special points of the parameter space.[15] We shall also discuss unconventional mechanisms for superconductivity in the nodal liquid similar to the ones proposed in Refs. 9 and 10.

To start with, let us describe briefly the extended t-J model used in Ref. 8. The Hamiltonian is given by

$$H = P(H_{hop} + H_J + H_V)P + PH_\mu P, \tag{3}$$

where

(i)

$$H_{hop} = -\sum_{\langle ij\rangle} t_{ij} c_{i\alpha}^{+} c_{j\alpha} - \sum_{[ij]} t'_{ij} c_{i\alpha}^{+} c_{j\alpha} - \sum_{\{ij\}} t''_{ij} c_{i\alpha}^{+} c_{j\alpha}, \tag{4}$$

and $\langle \cdots \rangle$ denotes nearest-neighbor (NN) sites, $[\cdots]$ next-to-nearest neighbor (NNN), and $\{\cdots\}$ third nearest neighbor. Here repeated spin (or ''color'') indices are summed over. The Latin indices i,j denote lattice sites and the Greek indices $\alpha=1,2$ are spin components.

(ii)

$$H_J = J\sum_{\langle ij\rangle} \left(T_{i,\alpha\beta} T_{j,\beta\alpha} - \frac{1}{4} n_i n_j \right) + J' \sum_{[ij]} T_{i,\alpha\beta} T_{j,\beta\alpha}, \tag{5}$$

with $n_i = \Sigma_{\alpha=1}^{2} c_{i\alpha}^{+} c_{i\alpha}$, and $T_{i,\alpha\beta} = c_{i\alpha}^{+} c_{i\beta}$. The quantities J, J' denote the couplings of the appropriate Heisenberg antiferromagnetic interactions. We shall be interested[10] in the regime where $J' \ll J$.

(iii)

$$H_\mu = \mu \sum_i c_{i\alpha}^{+} c_{i\alpha}, \tag{6}$$

and μ is the chemical potential.

(iv)

$$H_V = -V \sum_{\langle ij\rangle} n_i n_j. \tag{7}$$

This is an effective static NN interaction which, in the bare t-J model, is induced by the exchange term, because of the extra magnetic bond in the system when two polarons are on neighboring sites.[8] Notice that this term, when combined with the Coulomb interaction terms in H_J, yields in the effective action a total intersite Coulomb interaction term with coupling

$$V_{total} = V + 0.25J. \tag{8}$$

In Ref. 8 the strength of the interaction (7) is taken to be

$$V \approx 0.585J. \tag{9}$$

This is related to the regime of the parameters used in Ref. 8, for which the NN hopping element satisfies $t \ll J$. In fact, for the effective t-j-V model of Ref. 8, viewed as an appropriate reduction of a single-band Hubbard model, one has the relation

$$J = \frac{4t^2}{U_{eff} + V'} + J_{SB}, \tag{10}$$

where U_{eff} is an effective Hubbard interaction, and J_{SB} is a ferromagnetic exchange Heisenberg energy for the single-band model. We have $|V'| \neq |V|$ in general, unlike the case of the standard Hubbard model with a supplementary intersite Coulomb interaction. However, one may consider more general models, in which the above restriction is not imposed, and V is viewed as an independent parameter of the effective theory, e.g.,

$$V \approx bJ, \tag{11}$$

where b is a constant to be determined phenomenologically. Such a situation may arise, for instance, in effective models where one considers repulsive on-site Coulomb interactions[8] (e.g., between holes and/or electrons) *in addition* to the (electron-hole) attractions (7). As we shall discuss below, such more general cases turn out to be useful for the existence of supersymmetric points in the parameter space of the model.

(v) The operator P is a projector operator, expressing the absence of double occupancy at a site.

We define the doping parameter $0 < \delta < 1$ by

$$\langle n_i \rangle = 1 - \delta. \tag{12}$$

d-wave pairing, which seems to have been confirmed experimentally for high-T_c cuprates, was assumed in Ref. 8. A d-wave gap is represented by an order parameter of the form

$$\Delta(\vec{k}) = \Delta_0 (\cos k_x a - \cos k_y a), \tag{13}$$

where a is the lattice spacing. The relevant Fermi surface is characterized by the following four nodes where the gap vanishes:

$$\left(\pm \frac{\pi}{2a}, \pm \frac{\pi}{2a} \right). \tag{14}$$

We now consider the generalized dispersion relation[3,16] for the quasiparticles in the superconducting state:

$$E(\vec{k}) = \sqrt{[\varepsilon(\vec{k}) - \mu]^2 + \Delta^2(\vec{k})}. \tag{15}$$

In the vicinity of the nodes it is reasonable[3,16] to assume that $\mu \approx 0$ or equivalently we may linearize about μ, i.e., write $\varepsilon(\vec{k}) - \mu \approx v_D |\vec{q}|$ (Ref. 9) where v_D is the effective velocity at the node and q is the wave vector with respect to the nodal point.

III. NON-ABELIAN SPIN-CHARGE SEPARATION IN THE *t*-*J* MODEL

As already mentioned in the introduction, it was *proposed* in Ref. 3 that for the large-U limit of the *doped* Hubbard model the following *"particle-hole" symmetric spin-charge separation* representation occurs at *each site i*:

$$\chi_{\alpha\beta,i} = \psi_{\alpha\gamma,i} z_{\gamma\beta,i} \equiv \begin{pmatrix} c_1 & c_2 \\ c_2^\dagger & -c_1^\dagger \end{pmatrix}_i = \begin{pmatrix} \psi_1 & \psi_2 \\ -\psi_2^\dagger & \psi_1^\dagger \end{pmatrix}_i \begin{pmatrix} z_1 & -\overline{z_2} \\ z_2 & \overline{z_1} \end{pmatrix}_i, \tag{16}$$

where the fields $z_{\alpha,i}$ obey canonical *bosonic* commutation relations, and are associated with the *spin* degrees of freedom (spinons), while the fields $\psi_{a,i}$, $a = 1,2$ have *fermionic* statistics, and are assumed to *create holes* at the site i with spin index α (holons). The ansatz (16) has spin-electric-charge separation, since only the fields ψ carry *electric* charge. Generalization to the non-Abelian model allows for intersublattice hopping of holes which is observed experimentally.

It should be noticed that the anticommutation relations for the electron fields c_α, $c_\beta^\dagger$, do not follow from the ansatz (16) without additional constraints. Indeed, assuming the canonical (anti-)commutation relations for the z (ψ) fields, one obtains from the ansatz (16)

$$\{c_{1,i}, c_{2,j}\} \sim 2\psi_{1,i}\psi_{2,i}\delta_{ij},$$

$$\{c_{1,i}^\dagger, c_{2,j}^\dagger\} \sim 2\psi_{2,i}^\dagger\psi_{1,i}^\dagger\delta_{ij},$$

$$\{c_{1,i}, c_{2,j}^\dagger\} \sim \{c_{2,i}, c_{1,j}^\dagger\} \sim 0,$$

$$\{c_{\alpha,i}, c_{\alpha,j}^\dagger\} \sim \delta_{ij} \sum_{\beta=1,2} [z_{i,\beta}\overline{z}_{i,\beta} + \psi_{\beta,i}\psi_{\beta,i}^\dagger],$$

$$\alpha = 1,2 \quad \text{no sum over } i,j. \tag{17}$$

To ensure the *canonical* anticommutation relations for the c operators we must therefore *impose* at each lattice site the (slave-fermion) constraints

$$\psi_{1,i}\psi_{2,i} = \psi_{2,i}^\dagger \psi_{1,i}^\dagger = 0,$$

$$\sum_{\beta=1,2} [z_{i,\beta}\overline{z}_{i,\beta} + \psi_{\beta,i}\psi_{\beta,i}^\dagger] = 1. \tag{18}$$

Such relations are understood to be satisfied when the holon and spinon operators act on *physical* states. Both of these relations are valid in the large-U limit of the Hubbard model and encode the nontrivial physics of constraints behind the spin-charge separation ansatz (16). They express the constraint of *at most one electron or hole per site*, which characterizes the large-U Hubbard models we are considering here, and are similar to constraints used in the conventional slave-representation methods.

There is a local phase (gauge) non-Abelian symmetry hidden in the ansatz (16) (Ref. 3) $G = SU(2) \times U_S(1)$, where $SU(2)$ stems from the spin degrees of freedom, $U_S(1)$ is a statistics changing group, which is exclusive to two spatial dimensions and is responsible for transforming bosons into fermions and vice versa. As remarked in Ref. 3, the $U_S(1)$ effective interaction is responsble for the equivalence between the slave-fermion ansatz (i.e., where the holons are viewed as charged bosons and the spinons as electrically neutral fermions[4]) and the slave-boson ansatz (i.e., where the holons are viewed as charged fermions and the spinons as neutral bosons[17,3]). This is analogous (but not identical) to the bosonization approach of Ref. 18 for anyon systems.

The application of the ansatz (16) to the Hubbard (or t-j models) necessitates a ''particle-hole'' symmetric formulation of the Hamiltonian (3), which, as shown in Ref. 3, is expressible in terms of the operators χ. In this way, for instance, the NN Heisenberg interactions terms become

$$H_J = -\frac{J}{8}\sum_{\langle ij\rangle}\mathrm{Tr}[\chi_i\chi_j^\dagger\chi_j\chi_i^\dagger]. \quad (19)$$

By making an appropriate Hubbard-Stratonovich transformation on H_J with Hubbard-Stratonovich fields Δ_{ij}, we obtain the effective spin-charge separated action for the doped-antiferromagnetic model of Ref. 3:

$$H_{HF} = \sum_{\langle ij\rangle}(\mathrm{Tr}\{(8/J)\Delta_{ij}^\dagger\Delta_{ji} + |A_1|[-t_{ij}(1+\sigma_3) + \Delta_{ij}]\psi_j V_{ji} U_{ji}\psi_i^\dagger\} + \mathrm{Tr}[K\overline{z_i}V_{ij}U_{ij}z_j] + \mathrm{H.C.}) + \ \dots, \quad (20)$$

with the $\cdots$ denoting chemical potential terms and NNN hopping terms (the latter are essential for the model of Ref. 8; we shall discuss their effects below).

This form of the action describes low-energy excitations about the Fermi surface of the theory. The field Δ_{ij} is matrix valued in color space; generically it may be expanded in components in a canonical basis of 2×2 matrices, $\{1,\sigma^a\}$, $a=1,2,3$, as follows:

$$(\Delta_{ij})_{\alpha\beta} = A_0\delta_{\alpha\beta} + A_a(\sigma^a)_{\alpha\beta}, \quad (21)$$

where Greek indices denote 2×2 color indices.

The quantities V_{ij} and U_{ij} denote lattice link variables associated with elements of the $SU(2)$ and $U_S(1)$ groups, respectively. They are associated[3] with phases of vacuum expectation values of bilinears $\langle\overline{z_i}z_j\rangle$ and/or $\langle\psi_i^\dagger[-t_{ij}(1+\sigma_3)+\Delta_{ij}]\psi_j\rangle$. It is understood that, by integrating out in a path integral over z and ψ variables, fluctuations are incorporated, which go beyond a Hartree-Fock treatment.

The quantity $|A_1|$ is the amplitude of the bilinear $\langle\overline{z_i}z_j\rangle$ assumed frozen.[3] By an appropriate normalization of the respective field variables, one may set $|A_1|=1$, without loss of generality. In this normalization, one may then parametrize the quantity K, which is the amplitude of the appropriate fermionic bilinears, as[3,10]

$$K \equiv [J|\Delta_z|^2(1-\delta)^2]^{1/2}; \quad 1-\delta = \left\langle\sum_{\alpha=1}^{2}\psi_\alpha\psi_\alpha^\dagger\right\rangle, \quad (22)$$

with δ the doping concentration in the sample. The quantity $|\Delta_z|$ is considered as an arbitrary parameter of our effective theory, of dimensions $[\mathrm{energy}]^{1/2}$, whose magnitude is to be fixed by phenomenological or other considerations (see below). To a first approximation we assume that Δ_z is doping independent. However, from its definition, as a $\langle\cdots\rangle$ of a quantum model with complicated δ dependences in its couplings, the quantity Δ_z may indeed exhibit a doping dependence. For some consequences of this we refer the reader to the discussion in Sec. VI, below. The dependence on J and δ in Eq. (22) is dictated[10] by the correspondence with the conventional antiferromagnetic $CP^1\sigma$ model in the limit $\delta\to0$.

The model of Ref. 8 differs from that of Ref. 3 in the existence of NNN hopping t' and triple neighbor hopping t'', which were ignored in the analysis of Ref. 3. For the purposes of this work, which focuses on the low-energy (infrared) properties of the continuum field theory of Eq. (20), this can be taken into acount by assuming that

$$|t_{ij}| = t'_+ \equiv t + 2t_+, \quad t_+ \equiv t' + 2t'' \quad (23)$$

in the notation of Ref. 8. The relation stems from the observation that in the continuum low-energy field-theory limit such NNN and triple hopping terms can be Taylor expanded (in derivatives). It is the terms linear in derivatives that yield the shift (23) of the NN hopping element t. Higher derivatives terms, of the form $\partial_x\partial_y$ are suppressed in the low-energy (infrared) limit.

It is important to note that the model of Ref. 3, as well as its extension (20), in contrast to that discussed in Ref. 9, involves only a *single lattice* structure, with nearest-neighbor hopping ($\langle ij\rangle$) being taken into account, t_{ij}. The antiferromagnetic nature is then viewed as a property of a color degree of freedom, expressed via the non-Abelian gauge structure of the spin-charge separation ansatz (16). As we shall discuss later, this is very important in yielding the correct number of fermionic (holons Ψ) degrees of freedom in the continuum low-energy field theory to match the bosonic degrees of freedom (spinons z) at the supersymmetric point.

IV. EFFECTIVE LOW-ENERGY GAUGE THEORY

It is instructive to discuss in some detail the derivation of a conventional lattice gauge theory form of the action (20). One first shifts the Δ_{ij} field: $\Delta_{ij}\to\Delta'_{ij}=\Delta_{ij}+t_{ij}\sigma_3$, and then assumes that the fluctuations of the Δ'_{ij} field are frozen in such a way that only the $\langle A'_0\rangle$ component is nontrivial in the corresponding expansion in terms of the Pauli matrices (21). This is a variational ansatz that can be justified in the regime of the parameters of the statistical model $J\gg t'_+$, in which case the dominant Δ_{ij} configurations (in the path integral) may be taken to be of order J, and thus any effect of the σ_3 color structure in the action (20) is safely negligible. As we shall discuss in what follows, the elimination of the σ_3 terms from the action (20) results in canonical Dirac kinetic terms for the fermionic parts of the nodal liquid effective (low-energy) action.

However, in view of Eq. (23), in the model of Ref. 8, such an assumption is not valid, given that the renormalized hopping parameter, due to NNN and triple neighbor hoppings, is of similar order as J. Nevertheless, for our generic purposes in this work we shall work in a model where $J\gg t'_+$. Alternatively, we can assume that the effects of the σ_3 color structures can be safely neglected even for the case of the model of Ref. 8. Such assumptions are retrospectively justified by the fact that the model of Ref. 8 cannot yield supersymmetric points even under the above assumption, for other reasons to be discussed below. Thus our approach in this paper is to identify the circumstances under which deformations of the model presented in Ref. 8 can yield such points in the parameter space.

Notably, the situation $J \gg t'_{+}$ may be met in the models of Dagotto *et al.*,[8] where NNN hopping t' is neglected, but where the Coulomb attraction (7) is present, in order to guarantee the existence of *d*-wave superconducting gaps.[19] Moreover, in the context of generalizations of the *t*-*V*-*J* models of Feiner *et al.*,[8] such a situation [cf. Eq. (10)] is met if one assumes an appropriate attractive V', of opposite sign to the repulsion U_{eff}, but close to it in magntitude [notice that, on account of Eq. (23), in our generalization fo the *t*-*V*-*j* model, one should replace t in Eq. (10) by t'_{+}]. In such a case one has an additional large dimensionful scale U_{eff}, like in the case of the conventional Hubbard model of Ref. 3.

We next remark that in conventional non-Abelian gauge theories the fermionic fields are usually spinors in the fundamental representation of the gauge group. Let us examine under what condition this is feasible in our case. To this end we assemble the fermionic degrees of freedom into two two-component Dirac spinors:[3]

$$\tilde{\Psi}^{\dagger}_{1,i} = (\psi_1 \quad -\psi_2^{\dagger})_i, \quad \tilde{\Psi}^{\dagger}_{2,i} = (\psi_2 \quad \psi_1^{\dagger})_i, \tag{24}$$

where α in $\tilde{\Psi}^{\dagger}_{\alpha,i}$ is the color index. We also consider very weakly coupled $SU(2)$ gauge groups, with couplings $g_{\mathrm{SU}(2)} \equiv g_2 \ll 1$. In the weak gauge-field approximation, where the gauge group element (link) along the μ space-time direction is $U_{ij;\mu} \sim 1 + g_s \int_i^{j;\mu} B^a_\mu \sigma^a + \mathcal{O}(g_2^2)$ (with $\sigma^a, a = 1,2,3$ the Pauli matrices), one observes the following mathematical identities:

$$\mathrm{Tr}(\psi_i \psi^{\dagger}_{i+\mu}) = \tilde{\Psi}^{\dagger}_i \tilde{\Psi}_{i+\mu}, \quad \mathrm{Tr}(\psi_i \sigma_1 \psi^{\dagger}_{i+\mu}) = \tilde{\Psi}^{\dagger}_i \tau_1 \tilde{\Psi}_{i+\mu},$$

$$\mathrm{Tr}(\psi_i \sigma_3 \psi^{\dagger}_{i+\mu}) = \tilde{\Psi}^{\dagger}_i \tau_3 \tilde{\Psi}_{i+\mu},$$

$$\mathrm{Tr}(\psi_i \sigma_2 \psi^{\dagger}_{i+\mu}) = i\left(-\tilde{\Psi}^{\dagger}_i \sigma_3 \frac{1}{2}(\tau_1 + i\tau_2)\tilde{\Psi}_{i+\mu} + \tilde{\Psi}^{\dagger}_i \frac{1}{2}(1+\tau_3)\tilde{\Psi}_{i+\mu}\right), \tag{25}$$

where the Pauli matrices $\tau^a, a = 1,2,3$ refer to color space, and should be distinguished from the σ_3 matrices, which although are color matrices, they refer to the action (20), in which the fermionic degrees of freedom consist of Grassmann variables assembled in 2×2 matrices. From the last of Eqs. (25), therefore, it becomes evident that the action (20) may be mapped to a conventional lattice action, with spinors (24) in the fundamental representation of the color group, provided that the coupling $g_2 \ll 1$ is *weak*, and in addition there is a *gauge fixing* [Note that the requirement for weak g_2 coupling is essential, given the fact that due to the non-Abelian nature of the gauge field, the local gauge fixing $B^2_\mu = 0$ alone is not sufficient to eliminate dangerous terms proportional to σ^2; this can be easily seen from the Baker-Hausdorff identity:

$$e^{ig_2 \Sigma_{a=1,3}\sigma^a B^a_\mu} = (\Pi_{a=1,3} e^{ig_2 \int_i^j \sigma^a B^a_\mu}) e^{1/2 (g_2)^2 [\sigma^1, \sigma^3] B^1_\mu B^3_\mu + \cdots},$$

with the commutator being proportional to σ_2; however, such terms are of higher order in g_2, and hence restriction to weak couplings suffices to yield the conventional relativistic gauge form of the effective action upon the appropriate gauge fixing.]: $\int_i^j dx^\mu B^2_\mu = 0$.

The weakness of the $SU(2)$ coupling guarantees that a mass gap in the problem is only generated by the $U_S(1)$ group.[3] In the context of the Hubbard model of Ref. 3, the coupling g_2 of the gauged $SU(2)$ interactions, pertaining to the spin degrees of freedom in the problem, is naturally weak, since it is related to the Heisenberg exchange energy *J*. Given that in three space-time dimensions the gauge couplings are dimensionful, with dimensions of energy, one may define dimensionless couplings by dividing them with the ultraviolet scale of the low-energy theory, which in the model of Ref. 3 is the (strong) Hubbard interaction $U \gg J$. Thus a dimensionless coupling $g_2 \sim J/U \ll 1$ is naturally small in this context. A similar situation arises in the context of the effective single-band *t*-*V*-*j* model of Ref. 8, in the large $U_{eff} \gg J$ limit [cf. Eq. (10)]. On the other hand, the strong $U_S(1)$ coupling g_1, responsible for mass gap generation for the holons, may be assumed to be of order U_{eff}, since this is the highest energy scale. However, in general for *t*-*j* models that we consider these relations may not be valid. Still as we shall see below, the ultraviolet cutoff of the effective theory, in the regime relevant for supersymmetric points, we are interested here, may be up to two orders of magnitude higher than *J*, thereby allowing the $U_S(1)$ interactions to be considerably stronger than the $SU(2)$ ones, if one wishes so.

To generate the conventional Dirac γ-matrix structure for the fermionic action one may redefine the spinors in the path integral $\tilde{\Psi} \to \Psi$, where Ψ are *two-component* colored spinors, related to the spinors in Eq. (24) via a Kawamoto-Smit transformation,[20]

$$\Psi_\alpha(r) = \gamma_0^{r_0} \cdots \gamma_2^{r_2} \tilde{\Psi}_\alpha(r),$$

$$\overline{\Psi}_\alpha(r) = \overline{\tilde{\Psi}}_\alpha(r)(\gamma_2^{\dagger})^{r_2} \cdots (\gamma_0^{\dagger})^{r_0}, \tag{26}$$

where r is a point on the Euclidean lattice, and $\alpha = 1,2$ is a color index, expressing the initial antiferromagnetic nature of the system. We notice that as a result of the γ-matrix algebra fermion bilinears of the form $\overline{\Psi}_{i,\alpha}\Psi_{i,\beta}$ (i=lattice index) satisfy

$$\overline{\Psi}_{i,\alpha}\Psi_{i,\beta} = \overline{\tilde{\Psi}}_{i,\alpha}\tilde{\Psi}_{i,\beta} \tag{27}$$

on a Euclidean lattice. As we shall see later on, this last identity will be crucial in yielding a relativistic form of the effective action for the interacting nodal liquid of excitations in generalized Hubbard models.

We next notice that on a lattice, in the path integral over the fermionic degrees of freedom in a quantum theory, the variables $\overline{\Psi}$ and Ψ are viewed as *independent*. In view of this, the spinors $\Psi^{\dagger}_\alpha$ in Eq. (24) may be replaced by $\overline{\Psi}_\alpha$, as being path-integral variables on a Euclidean lattice appropriate for the Hamiltonian system (3). This should be kept in mind when discussing the microscopic structure of the theory in terms of the holon creation and annihilation operators $\psi^{\dagger}_\alpha, \psi_\alpha, \alpha = 1,2$.

The fermionic part of the long-wavelegth Lattice Lagrangian, then, reads

$$S=\frac{1}{2}K'\sum_{i,\mu}\left[\bar{\Psi}_i(-\gamma_\mu)U_{i,\mu}V_{i,\mu}\Psi_{i+\mu}+\bar{\Psi}_{i+\mu}(\gamma_\mu)U^{\dagger}_{i,\mu}V^{\dagger}_{i,\mu}\Psi_i\right]+\text{Bosonic}\quad CP^1\quad\text{parts},\tag{28}$$

where the Bosonic CP^1 parts denote magnon-field z-dependent terms, and are given in Eq. (20). The coefficient K' is a constant which stems from the t_{ij}- and Δ_{ij}- dependent coefficients in front of the fermion terms in Eq. (20). An order of magnitude estimate of the modulus of (the shifted) Δ'_{ij} then, which determines the strength of the coefficient K', may be provided by its equations of motion. Assuming that the modulus of (the dimensionless) fermionic bilinears is of order unity, then, we have as an order of magnitude

$$K'\sim\left(t'_{+}+\frac{J}{8}\right).\tag{29}$$

Note that in the regime of the parameters of Ref. 8 $t\ll t_{+}$ and $t_{+}\simeq\frac{3}{2}J$ for momenta close to a node in the Fermi surface, of interest to us here. Thus

$$K'\simeq 25J/8.\tag{30}$$

However, one may even consider more general models, in which K' and the Coulomb intersite interaction V are treated as independent phenomenological parameters.

At this stage we would like to make a remark concerning the relativistic form of the action (28). Although in Eq. (28) we did not give explicitly the CP^1 parts, however, we have tacitly assumed the *equality of the effective velocities* for spin v_S and charge v_F (Fermi velocity of holes) degrees of freedom. If such an assumption is not made, then the relativistic invariant form of the effective Lagrangian is spoiled.[9] However, at the supersymmetric points of the nodal liquid, where, as we shall discuss later on, the dynamically generated mass gaps between spinons and holons must be equal, the equality $v_S=v_F$ is essential, otherwise there would be different dispersion relations, leading to a difference in mass gaps. These comments should be understood in what follows. From now on we shall work in units of the Fermi velocity v_F.

Moreover, it should be stressed once again that the relativistic form of the action (28) is derived for a weakly coupled $SU(2)$ gauge group, and under a specific gauge fixing. However, in view of the gauge invariance characterizing Eqs. (20) and (28), the physical results based on the above effective actions, in particular the existence of supersymmetric points in the parameter space, which is of interest to us here, are independent of the gauge chosen.

As discussed in Ref. 13, supersymmetrization of CP^1-type models, like the ones considered here, requires that the CP^1 constraint be of the form $\Sigma^2_{\alpha=1}|z_\alpha|^2=1$. In our case, however, the no-double occupancy constraint, when expressed in terms of the z and $\tilde{\Psi}_\alpha$, $\alpha=1,2$, (spinor) fields, with α a color index, is written as

$$\sum_{\alpha=1}^{2}\left[\overline{z^\alpha}z_\alpha+\beta\bar{\tilde{\Psi}}^\alpha\sigma_3\tilde{\Psi}_\alpha\right]=1,\tag{31}$$

where $\beta=1/K'^2$, K' is given by Eq. (30), the 2×2 matrix σ_3 acts in spinor space, and the fermions $\tilde{\Psi}$ are the *two-component* spinors (24). Equivalently the fermion bilinear terms in Eq. (31) can be expressed in terms of the spinors Ψ (26), which have conventional Dirac kinetic terms. It is understood that appropriate rescalings can be made in the definition of the spinors so as to ensure the canonical kinetic (Dirac) term. We have also taken into account that in a Euclidean path integral the variables $\Psi^\dagger$ and Ψ are viewed as independent, which implies that one may redefine $\Psi^\dagger\to\bar{\Psi}$ where $\bar{\Psi}$ in later analysis will nevertheless be considered in the conventional way, i.e., as $\Psi^\dagger\gamma_0$. Consequently, we can interpret the fermion term in the constraint (31) as $\Psi^\dagger\Psi$ the fermion number term.

The presence of the $\Psi^\dagger\Psi$ (nonrelativistic) fermion number term in the constraint (31) appears at first sight to complicate things, since the conventional CP^1 constraint $|z|^2=1$ is no longer valid. In fact, supersymmetry is compatible with the following form of the constraints:

$$|z_\alpha|^2=1,\quad \overline{z_\alpha}\Psi_\alpha=0,\tag{32}$$

arising from the superfield version of the CP^1 constraint.[13,23] The fermionic counterpart of Eq. (32) can be solved by means of a colorless fermion field $\mathcal{X}$ that satisfies [on account of the bosonic CP^1 parts of Eq. (32)]:

$$\Psi_\alpha=\epsilon_{\alpha\beta}\overline{z_\beta}\mathcal{X},\quad \mathcal{X}=\epsilon_{\alpha\beta}z_\alpha\Psi_\beta,\tag{33}$$

where Ψ_α are the Dirac spinors defined above. To ensure the conventional CP^1 form of the bosonic part of the supersymmetric constraints (32) from (31) we should demand $\beta<<1$, which is satisfied in a regime of the parameters of the theory for which

$$K'\gg K=\sqrt{J}|\Delta_z|(1-\delta),\quad 0<\delta<1.\tag{34}$$

For the model of Ref. 8, for instance, on account of Eq. (30), this condition implies that

$$\sqrt{J}/|\Delta_z|\geq 0.32\ \ (1-\delta),\quad 0<\delta<1.\tag{35}$$

By appropriately rescaling the fermion fields Ψ to Ψ', so that in the continuum they have a canonical Dirac term, we may effectively constrain the z fields to satisfy the CP^1 constraint:

$$|z_\alpha|^2+\frac{1}{K'}(\Psi'-\text{bilinear terms})=1,$$

where now the fields Ψ are dimensionful, with dimensions of [energy]. A natural order of magnitude of these dimensionful fermion bilinear terms is of the order of K^2, which plays the role of the characteristic scale in the theory, being related directly to the Heisenberg exchange energy J. In the limit $K'\gg K$ [Eq. (34)] therefore the fermionic terms in the constraint can be ignored, and the constraint assumes the standard CP^1 form involving only the z fields (this being also the case for the model of Refs. 9 and 10, in a specific regime of the microscopic parameters).

As we shall see later, however, the condition (35) alone, although necessary, is not sufficient to guarantee the existence of supersymmetric points. Supersymmetry imposes additional restrictions, which in fact rule out the existence of supersymmetric points for the model of Ref. 8 compatible with superconductivity. We note in passing that in realistic materials superconductivity occurs for doping concentrations above 3%, and is destroyed for doping concentrations larger than $\delta_{\rm max}\sim 10\%$. However, this does not prevent one from considering more general models in which K' is viewed as a phenomenological parameter, not constrained by Eq. (30). In that case, supersymmetric points may occur for a certain regime of the respective parameters.

However, as a result of the spin-charge separation formalism, there is a different way to treat the constraints in a path integral, which however takes into account the coupling of the system to an external elelctromagnetic field, and as such is not *a priori* relevant to the supersymmetric regime. Nevertheless, as we shall discuss in Sec. VI, this will be relevant for electric charge transport in the model for which supersymmetry (in the absence of external fields) will be argued to play a rather crucial but subtle role.

Indeed, the fermion number terms in Eq. (31) may be absorbed in a rescaling of the (quantum fluctuations of the) temporal component of the electromagnetic field $A_0(\vec{x},t)$, which couples (relativistically) only to the spinors Ψ (see Sec. VI B below). Explicitly, by implementing the constraint (31) in a path integral via the introduction of a Lagrange multiplier field $\lambda(x)$

$$\delta(|z_\alpha|^2+\beta\overline{\Psi}_\alpha\sigma_3\Psi_\alpha-1) = \int D\lambda(x)e^{i\lambda(x)(z_\alpha\bar{z}_\alpha+\beta\overline{\Psi}_\alpha\sigma_3\Psi_\alpha-1)}, \tag{36}$$

and on absorbing $\lambda(x)$ in a shift of $A_0(\vec{x},t)$, one obtains from the Maxwell terms in the electromagnetic part of the effective action the following combination:

$$\mathcal{L}_{em}\ni -\frac{1}{4(e^2/c^2)}[2\partial_i\lambda F_{0i}+(\partial_i\lambda)^2] + \text{standard Maxwell terms}, \tag{37}$$

where F_{0i} is the appropriate components of the Maxwell tensor of the (redefined) electromagnetic field, the index i is a spatial index, and repeated indices denote summation, The equations of motion for λ in the effective action obtained after integrating out, say, the z degrees of fredom yield the standard CP^1 model terms,[21] but also terms of the form $\nabla_i^2\lambda+2\nabla^i F_{0i}$. One therefore may consider a phase in which $\langle\lambda(x)\rangle=\text{const}\neq 0$, provided that the electromagnetic field is chosen as an external one, satisfying Maxwell's equations, which is our case.

The bosonic part of the constraint, then, implies a mass for the spinons $m_z\propto\langle\lambda(x)\rangle$.[21] The fermionic part, on the other hand, has the form of a temporal component of the electric current (see Sec. VI B below). The coefficient $\beta\langle\lambda(x)\rangle$ may be absorbed in a shift of the quantum fluctuations of $A_0(\vec{x},t)$. Quantum fluctuations of the electromagnetic field will not be of further interest to us here, given that we shall treat it only as external background.

From the above discussion it becomes clear, then, that in either case one maps the double occupancy constraint (31) into the standard CP^1 constraint:

$$\sum_{\alpha=1}^{2}|z_\alpha|^2=1. \tag{38}$$

However, as we have explained above, the restriction given in Eq. (35) cannot be avoided if supersymmetric points are to exist. Any alternative treatment would require coupling the system to (supersymmetry-breaking) external electromagnetic fields, since otherwise the fermionic parts of Eq. (31) would be present. As we shall see in Sec. VI, though, the alternative treatment of the constraint leads to interesting phases of the theory characterized by superconducting electric-charge transport. Any supersymmetry that might have existed before coupling to electromagnetism would then play an important (but subtle) role in ensuring the existence of superconductivity.

In addition to the CP^1 constraint, one also encounters the remaining constraints (18). These may be treated in a similar way, using appropriate Lagrange multiplier fields $\lambda_2(x),\lambda_3(x)$. It can be easily seen that such a treatment leads to structures in the effective low-energy action which involve "electric current" operators $J_i=\overline{\Psi}\gamma_i\Psi,\ i=1,2$ (see Sec. VI B), and as such can be absorbed in the quantum fluctuations of the spatial components of the electromagnetic field $\vec{A}(\vec{x},t)$.

It should be stressed again that the situation in which the Lagrange multiplier fields acquire nonzero vacuum expectation values (VEV's), $\langle\lambda(x)\rangle$, $\langle\lambda_i(x)\rangle\neq 0$, $i=2,3$, corresponds to the selection of a specific ground state of the system (phase), about which one considers quantum fluctuations. There is always the phase in which such VEV's are zero, in which case one implements the constraints directly on the path-integral correlators, e.g., correlation functions proportional to $\psi_1\psi_2$ are set to zero in this phase. In what follows, we shall first resolve the constraints in this latter phase, and later on (Sec. VI) we shall discuss the other phases of the model. As we shall discuss later, this phase is characterized by spin transport but not electric-charge transport, a situation that should be compared with the case of the nodal liquids of Ref. 5, where the electrically neutral fermion representation for spinons is used. On the contrary, as we shall show in Sec. VI, the phase in which the Lagrange multiplier VEV's are nontrivial may yield unconventional superconductivity of Kosterlitz-Thouless type.[9,3]

We will consider from now on the standard CP^1 constraint involving only z fields. By an appropriate normalization of z to $z'=z/\sqrt{1-\delta}$ the constraint then acquires the familiar normalized CP^1 form $|z_\alpha|^2=1$ form. This implies a rescaling of the normalization coefficient K in Eq. (20):

$$K\rightarrow\frac{1}{\gamma}\equiv K(1-\delta)\simeq\sqrt{J}|\Delta_z|(1-\delta)^2. \tag{39}$$

In the naive continuum limit, then, the effective Lagrangian of spin and charge degrees of freedom describing the low-energy dynamics of the Hubbard (or t-j) model (20) of Ref. 3 is then

$$\mathcal{L}_2 \equiv \frac{1}{\gamma}\,\mathrm{Tr}|(\partial_\mu + ig_2\tau^a B^a_\mu + ig_1 a_\mu)z|^2 + \bar{\Psi} D_\mu \gamma_\mu \Psi, \tag{40}$$

where z denotes the 2×2 CP^1 matrix appearing in Eq. (1), and the (complex) fields z_α, $\alpha=1,2$ satisfy the constraint (38). The trace Tr is over group indices, $D_\mu = \partial_\mu - ig_1 a^S_\mu - ig_2\tau^a B_{a,\mu} - (e/c)A_\mu$, B^a_μ is the gauge potential of the local (''spin'') $SU(2)$ group, and a_μ is the potential of the $U_S(1)$ group. It should be remarked that we are working in units of the Fermi velocity $v_F(=v_D)$ of holes, which plays the role of the limiting velocity for the nodal liquid.

V. NN INTERACTION TERMS H_V

We will now discuss the Coulomb-interaction (attractive) terms

$$H_V = -V_{total}\sum_{\langle ij\rangle} n_i n_j \tag{41}$$

introduced in Ref. 8, where V_{total} is given in Eq. (8). Using the ansatz (16) at a site i, the electron number operator n_i may be expressed through the determinant (det) of the χ matrix in Eq. (16), and consequently in terms of the spin operator, z_α, $\alpha=1,2$, and charge operator ψ_α, $\alpha=1,2$, as

$$\begin{aligned} n_i &\equiv \sum_{\alpha=1}^{2} c^\dagger_{\alpha,i}c_{\alpha,i} \\ &= \det\, \chi_{\alpha\beta,i} \\ &= \det\, \hat{z}_{\alpha\beta,i} + \det\, \hat{\psi}_{\alpha\beta,i} \\ &= \sum_{\alpha=1}^{2} (\psi_\alpha\psi^\dagger_\alpha + |z_\alpha|^2). \end{aligned} \tag{42}$$

We may express the quantum fluctuations for the Grassmann fields ψ_α (which now carry a color index $\alpha=1,2$, unlike in abelian spin-charge separation models) via

$$\psi_{\alpha,i}\psi^+_{\alpha,i} = \langle\psi_{\alpha,i}\psi^+_{\alpha,i}\rangle + :\psi_{\alpha,i}\psi^+_{\alpha,i}: \quad \text{no sum over } i, \tag{43}$$

where $:\cdots:$ denotes normal ordering of quantum operators, and repeated indices are summed over, from now on unless explicitly stated otherwise. Since

$$\langle\psi_{\alpha,i}\psi^+_{\alpha,i}\rangle \equiv 1-\delta, \quad \text{no sum over } i,$$

δ the doping concentration in the sample (12), we may rewrite n_i as

$$n_i = [|z_\alpha|^2 + (1-\delta) + :\psi_\alpha\psi^\dagger_\alpha:]_i$$

which in terms of the spinors $\tilde{\Psi}$ is given by [cf. Eq. (24)]:

$$n_i = 2-\delta + \frac{1}{2}(\tilde{\Psi}^\dagger_\alpha \sigma_3 \tilde{\Psi}_\alpha)_i \tag{44}$$

where

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

acts in (space-time) spinor space, and the CP^1 constraint (38) has been taken into account.

Consider now the attractive interaction term H_V (41), introduced in Ref. 8. We then observe than the terms linear in $(2-\delta)$ in the expression for H_V can be absorbed by an appropriate shift in the chemical potential, about which we linearize to obtain the low-energy theory. We can therefore ignore such terms from now on.

Next, we make use of the fact, mentioned earlier, that in a lattice path integral the spinors $\tilde{\Psi}^\dagger_\alpha$ may be replaced by $\bar{\tilde{\Psi}}_\alpha$. From the structure of the spinors (24), then, we observe that we may rewrite the H_V term *effectively* as a Thirring vector-vector interaction among the spinors $\tilde{\Psi}$

$$H_V = +\frac{V_{total}}{4}\sum_{\langle ij\rangle} (\bar{\tilde{\Psi}}_\alpha \gamma_\mu \tilde{\Psi}_\alpha)_i (\bar{\tilde{\Psi}}_\beta \gamma^\mu \tilde{\Psi}_\beta)_j, \tag{45}$$

where $\mu=0,1,2$, with $\mu=0$ a temporal index. To arrive at Eq. (45) we have expressed σ_3 as $-i\gamma_0$, and used the Clifford algebra and the off-diagonal nature of the $\gamma_{1,2}=i\sigma_{1,2}$ matrices, as well as the constraints (18). In particular the latter imply that any scalar product between Grassmann variables ψ_α (or $\psi^\dagger_\beta$) with different color indices *vanish*.

Taking the continuum limit of Eq. (45), and ignoring higher derivative terms involving four-fermion interactions, which by power counting are irrelevant operators in the infrared, we obtain after passing to a Lagrangian formalism

$$\mathcal{L}_V = -\frac{V_{total}}{4K'^2}(\bar{\tilde{\Psi}}_\alpha \gamma_\mu \tilde{\Psi}_\alpha)^2, \tag{46}$$

where we have used rescaled spinors, with the canonical Dirac kinetic term with unit coefficient, for which the canonical form of the CP^1 constraint (38) is satisfied. For notational convenience we use the same notation $\tilde{\Psi}$ for these spinors as the unscaled ones. Although this is called the naive continuum limit, it actually captures correctly the leading infrared behavior of the model.

We then use a Fierz rearrangement formula for the γ matrices:

$$\gamma^\mu_{ab}\gamma_{\mu,cd} = 2\,\delta_{ad}\delta_{bc} - \delta_{ab}\delta_{cd},$$

where Latin letters indicate spinor indices, and Greek letters space-time indices. The Thirring (four-fermion) interactions (45) then become

$$(\bar{\tilde{\Psi}}_\alpha \gamma_\mu \tilde{\Psi}_\alpha)^2 = -3(\bar{\tilde{\Psi}}_\alpha \tilde{\Psi}_\alpha)^2 - 4\sum_{\alpha<\beta}(\bar{\tilde{\Psi}}_\alpha \tilde{\Psi}_\beta \bar{\tilde{\Psi}}_\beta \tilde{\Psi}_\alpha). \tag{47}$$

Notice that this form permits us to use, on account of the identity (27), either of the forms (26) or (24) for the spinors Ψ or $\tilde{\Psi}$ in the expression of H_V. It should be noted, though, that the canonical Dirac form of the kinetic terms for the spinors is valid only in the form (26), which we stick to from now on.

As mentioned above, in the model of Ref. 3, due to the first of the constraints (18), the mixed color terms vanish, thereby leaving us with pure Gross-Neveu *attractive* interaction terms of the form

$$\mathcal{L}_V = + \frac{3V_{total}}{4K'^2}(\overline{\Psi}_\alpha \Psi_\alpha)^2 \tag{48}$$

which describe the low-energy dynamics of the interaction (41) in the context of the non-Abelian spin-charge separation (16). It should be stressed that Eq. (48) is specific to our spin-charge separation model.

Moreover, in the context of the spinors (24), a condensate of the form $\langle \overline{\Psi}_\alpha \Psi_\alpha \rangle$ on the lattice *vanishes* because of the constraints (18). Such condensates would violate parity (reflection) operation on the planar spatial lattice, which on the spinors $\tilde{\Psi}$ is defined to act as follows:

$$\tilde{\Psi}_1(x) \to \sigma_1 \tilde{\Psi}_2(x), \quad \tilde{\Psi}_2(x) \to \sigma_1 \tilde{\Psi}_1(x),$$

or equivalently, in terms of the (microscopic) holon operxtors ψ_α, $\alpha = 1,2$:

$$\psi_1(x) \to \psi_2^\dagger(x), \quad \psi_2(x) \to -\psi_1^\dagger(x).$$

To capture correctly this fact in the context of our effective continuum Gross-Neveu interaction (48) the coupling strength *must* be subcritical, i.e., weaker than the critical coupling for mass generation. The critical coupling of the Gross-Neveu interaction is expressed in terms of a high-energy cut off scale Λ as:[22]

$$1 = 4g_c^2 \int_{S_\Lambda} \frac{d^3q}{8\pi^3 q^2} = \frac{2g_c^2 \Lambda}{\pi^2}, \tag{49}$$

where q is a momentum variable and S_Λ is a sphere of radius Λ. The divergent q integral is cut off at a momentum scale Λ which defines the low-energy theory of interest. For the case of interest $g^2 = 3V_{total}/4K'^2$; on using Eq. (30), then, the condition of subcriticality requires that

$$\Lambda \lesssim 77 \ J. \tag{50}$$

which is in agreement with the fact that in all effective models for doped antiferromagnets used in the literature the Heisenberg exchange energy $J \sim 1000$ K serves as an upper bound for the energies of the excitations of the effective (continuum) theory. However, as mentioned above, to obtain a relativistic gauge theory from the lattice action (20) one needs the $SU(2)$ interactions to be considerably weaker than the $U_S(1)$ interactions, responsible for mass generation: the above condition (50) is also compatible with this, provided one identifies the (dimensionful) coupling of the $U_S(1)$ interactions with a (high-energy) cutoff scale $\Lambda \sim 77 \ J$. In the context of the effective single-band t-V-j models (10), for instance, Λ may be identified with a $U_{eff} \gg J$.

VI. DYNAMICAL SPINON-HOLON SYMMETRY (SUPERSYMMETRY) IN THE NODAL LIQUID AND POTENTIAL PHENOMENOLOGICAL IMPLICATIONS

A. Conditions for $N=1$ supersymmetry in the nodal liquid

We turn now to conditions for supersymmetrization of the above continuum theory, i.e., conditions for dynamical symmetries between the spinon (boson) and holon (fermion) degrees of freedom. Below we shall only outline the main results. Technical details of the formalism are given in Refs. 13 and 23 where we refer the interested reader. Since it has been argued that $U_S(1)$ is responsible for dynamical mass generation (and superconductivity) in the model of Ref. 3 we shall ignore the non-Abelian $SU(2)$ interactions, keeping only the Abelian. However, since the latter argument is not rigorous, it would be desirable to supersymmetrize the full group in order to check the phenomenon of dynamical mass generation. The extension to supersymmetrizing the full gauge multiplet $SU(2) \times U_S(1)$ will be the topic of a forthcoming work. However we shall still maintain the color structure in the spinors, which is important for the ansatz (16). Ignoring the $SU(2)$ interactions implies, of course, that the color structure becomes a ''flavor'' index; however, this is essential for keeping track of the correct degrees of freedom required by supersymmetry in the problem at hand.[13].

As discussed in detail in Refs. 13 and 23, the conditions for $N=1$ supersymmetric extensions of a CP^1 σ model is that the constraint is of the standard CP^1 form (38), supplemented by *attractive* four-fermion interactions of the Gross-Neveu type (48), whose coupling is related to the coupling constant of the kinetic z-magnon terms of the σ model in a way such as to guarantee the balance between bosonic and fermionic degrees of freedom. Specifically, in terms of component fields, the pertinent Lagrangian reads

$$L = g_1^2 [D_\mu \overline{z^\alpha} D^\mu z^\alpha + i \overline{\Psi} \not{D} \Psi + \overline{F}^\alpha F^\alpha + 2i(\overline{\eta} \Psi^\alpha \overline{z^\alpha} - \overline{\Psi}^\alpha \eta z^\alpha)], \tag{51}$$

where D_μ here denotes the gauge covariant derivative with respect to the $U_S(1)$ field. The analysis of Refs. 13 and 23 shows that, upon using the equations of motion,

$$\overline{F}^\alpha F_\alpha = \sum_{\alpha=1}^{2} \frac{1}{4}(\overline{\Psi}^\alpha \Psi_\alpha)^2. \tag{52}$$

We thus observe that the $N=1$ supersymmetric extension of the $CP^1 \sigma$ model *necessitates* the presence of *attractive* Gross-Neveu type interactions among the Dirac fermions of *each sublattice*, in addition to the gauge interactions.

In the context of the effective theory (40) and (46), discussed in this article, the $N=1$ supersymmetric effective Lagrangian (51) is obtained under the following restrictions among the coupling constants of the statistical model:

$$g_1^2 = \frac{3V_{total}}{K'^2} = \gamma = \frac{1}{\sqrt{J}|\Delta_z|(1-\delta)^2}, \quad 0 < \delta < 1. \tag{53}$$

Note that in the context of the model of Ref. 8, for which Eqs. (9) and (30) are valid, the relation (53) gives the supersymmetric point in the parameter space of the model at the particular doping concentration $\delta = \delta_s$:

$$(1-\delta_s)^2 \simeq 3.89 \sqrt{\frac{J}{|\Delta_z|}}, \quad 0 < \delta_s < 1. \tag{54}$$

As discussed in Sec. IV, unbroken supersymmetry (which is valid only in the absence of external electromagnetic fields) imposes an additional restriction (35). Then we observe that compatibility of Eq. (54) with Eqs. (34) and (35) requires: $1-\delta_s \gg 1.25$, which implies that the model of Ref. 8 does not have supersymmetric points. However, one may consider more general models in which V and $K'\sim t'_{+}+J/8$ are treated as independent phenomenological parameters [cf. (11)]; in such a case one can obtain regions of parameters that characterize the supersymmetric points (53) and (54) compatible with superconductivity.

It is quite important to remark that in the model of Ref. 3, where the antiferromagnetic structure of the theory is encoded in a color (non-Abelian) degree of freedom of the spin-charge separated composite electron operator (1) without the need for sublattice structure, there is a matching between the bosonic (z spinon fields) and fermionic (Ψ holon fields) physical degrees of freedom, as required by supersymmetry, without the need for duplicating them by introducing ''unphysical'' degrees of freedom.[13]

The gauge multiplet of the CP^1 σ model also needs a supersymmetric partner which is a Majorana fermion called the gaugino. As shown in Ref. 13, such terms lead to effective electric-charge violating interactions on the spatial planes, given that the Majorana gaugino is a real field, and as such cannot carry electric charge (which couples as a phase to a Dirac field). These terms can be interpreted as the removal or addition of electrons due to interlayer hopping, which, in fact, can be shown to be suppressed by terms of order $1/\sqrt{J}$.

Another important point we wish to make concerns the four-fermion attractive Gross-Neveu interactions in Eqs. (51) and (52). As discussed in detail in Ref. 24, if the coupling of such terms is supercritical, then a parity-violating fermion (holon) mass would be generated in the model. However, the condition (50), which is valid in the statistical model of interest to us here, implies that the respective coupling is always subcritical, and thus there is no parity-violating dynamical mass gap for the holons, induced by the contact Gross-Neveu interactions. This leaves one with the possibility of *parity conserving* dynamical mass generation, due to the statistical gauge interactions in the model.[3,24]

A detailed analysis of such phenomena in the context of our CP^1 model is left for future work. For the present, however, we note that in $N=1$ supersymmetric gauge models, supersymmetry-preserving dynamical mass is possible.[13,25,26] In fact, as discussed in Ref. 26, although by supersymmetry the potential is zero, and thus there would naively seem that there is no obvious way of selecting the nonzero mass ground state over the zero mass one, however, there appear to be instabilities in the *quantum effective action* in the massless phase, which manifest themselves through instabilities of the pertinent running coupling. The opening of such a fermion mass gap has been associated with the existence of a nontrivial infrared fixed point of the renormalization-group flow, which implies non-Fermi-liquid behavior.[27]

From a physical point of view, such a phenomenon would imply that, for sufficiently strong gauge couplings, the zero-temperature liquid of excitations at the nodes of a d-wave superconducting gap would be characterized by the dynamical opening of mass gaps for the holons. At zero temperature, and for the specific doping concentrations corresponding to the supersymmetric points, as advocated above, the nodal gaps between spinon and holons would be equal, in agreement with the assumed equality of the respective propagation velocities $v_F=v_S$, which yielded the relativistic form of the effective continuum action (40) of the nodal excitations at the supersymmetric points. The opening of a nodal mass gap, due to the $U_S(1)$ gauge interactions, would imply a breaking of the fermion number [global $U(1)$] symmetry, and thus superconductivity upon coupling the system to external electromagnetic fields, according to the scenario of Refs. 9 and 3, which is reviewed briefly below for the benefit of the nonexpert reader.

B. Kosterlitz-Thouless realization of superconductivity in the $SU(2)\otimes U_S(1)$ model

This section is mainly a review of results that appear in the literature regarding the model.[3,9,24] It mainly serves as a comprehensive account of the various delicate issues involved, which play a very crucial role in the underlying physics. It is primarily addressed to the nonexperts in the area. Only the basic results will be presented; the interested reader may then find the relevant details in the published literature.

An important issue in the effective gauge theory $SU(2)\otimes U_S(1)$ model is the existence of a *global conserved symmetry*, namely the fermion number, which is due to the electric charge of the fermions Ψ. The corresponding current is given by

$$J_\mu=\sum_{\alpha=1}^{2}\overline{\Psi}^\alpha\gamma_\mu\Psi_\alpha,\quad \mu=0,1,2. \tag{55}$$

This current generates a global $U_E(1)$ symmetry, which after coupling with external electromagnetic fields is *gauged*. In this sense the holon current (55) coincides with the charge transport properties of the system.

Some discussion is in order at this point. The association of the current J_μ (55) with an electric current for holons comes about due to the similarity of the form of the spinors (24) with the conventional Nambu spinors appearing in the BCS Hamiltonian for superconductivity. Indeed, for the benefit of the reader we remind that in such a case the electron operators c_σ are assemblied, in a particle-hole formalism, into two component spinors $(c_\uparrow,c_\downarrow^\dagger)$, and the resulting Hamiltonian couples in a gauge invariant way to an external electromagnetic potential $\vec{A}$ by making the standard substitution of the momentum operator $\vec{p}\rightarrow\vec{p}-(e/c)\vec{A}$. The only difference in our nodal liquid case is that the holon spinors (24) come in two colors and, as contrasted to the generic BCS case, the problem is relativistic due to the restriction in the nodal excitations. Thus, at the level of the continuum effective action of the nodal excitations, the coupling to electromagnetic potentials is straightforward by extending the (statistical) gauge covariant derivatives in the Dirac kinetic terms (40) to incorporate the electromagnetic potential coupling terms

$$\int d^3x\frac{e}{c}\sum_{\alpha=1}^{2}\overline{\Psi}^\alpha\gamma_\mu A_\mu\Psi_\alpha, \tag{56}$$

where c is the light velocity and e is the absolute value of the electron charge (for holon excitations the charge is $+e$, for electron $-e$; in our problem here we concentrate in the holon current). The resulting nodal holon electric current is given by differentiation with respect to A_μ, i.e., by the expression (55).

Before discussing superconducting properties of the system we should remark that, as a result of the constraints (18) and the nondiagonal nature of the $\gamma_i, i=1,2$ matrices, the *spatial* components of the current (55) *vanish*, but the temporal component (charge density) is nontrivial. Moreover, given that the constraints (18) do not concern the spinons z, this means that there is a phase of the nodal liquid in which there is *no charge transport*, but only *spin transport*. The nontrivial ''spin current'' may be thought of as given by $J_\mu^{spin} \sim \overline{z}\partial_\mu z$. This situation should be compared with the corresponding phase in nodal liquids in the approach of Ref. 5, where the spinons are represented as electrically neutral fermions.

However, in our model there are other possibilities, leading to more complicated phases, as we shall discuss now. These possibilities are realized by implementing the constraints (18) via appropriate Lagrange multipliers in the path integral over the fermionic variables $\psi^\dagger, \psi$, as we discussed in Sec. IV [cf. Eq. (36)]. Expressing the products $\psi_1\psi_2$ (and their conjugates) as spatial components of the current (55), then, one may assume a specific ground state in which the appropriate Lagrange multipliers for the constraint $\psi_1\psi_2 \sim 0$ (and Hermitean conjugate) acquire nonzero vacuum expectation values that may be absorbed by appropriate shifts of the corresponding spatial components of the electromagnetic potential $\vec{A}(\vec{x},t)$ coupled to the current $\vec{J}$. As we have already discussed in Sec. IV, a nontrivial vacuum expectation value for the Lagrange multiplier $\lambda(x)$ of the last of the constraint (18) will yield mass terms for the z magnons, while the fermionic part of the constraint may be absorbed by an appropriate shift of the temporal component of the electromagnetic potential. This procedure breaks supersymmetry explicitly but, as we shall argue now, the existence of supersymmetry before coupling to external electromagnetism is crucial in implying superconducting properties after coupling to external fields.

In this framework, the constraints (18) no longer apply in the path integral, and nonvanishing spatial compontents of the electric current $\vec{J}$ appear. It should be remarked that in such a case the mixed color terms in Eq. (47) do not vanish, and hence the resulting effective Lagrangian breaks supersymmetry explicitly. This was to be expected, anyhow, from the very presence of external (nonsupersymmetric) electromagnetic fields. However, given that the coupling of such contact four fermion interactions is *subcritical* [cf. Eqs. (48) and (50)], such interactions are irrelevant operators in a renormalization-group sense, and hence the universality class of the theory (in the infrared) can still be determined using the supersymmetric version of the theory in the absence of any external fields [which also satisfies the additional restriction (35)]. As we shall argue below, this more general phase is important in that it yields unconventional superconductivity for the nodal liquid.

To this end, we remark that in the absence of external electromagnetic potentials, the symmetry $U_E(1)$ is *broken spontaneously* in the massive phase for the fermions Ψ. This can be readily seen by considering the following matrix element (see Fig. 1):

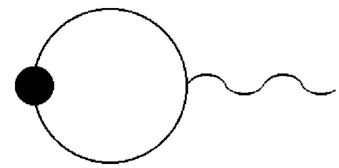

FIG. 1. Anomalous one-loop Feynman matrix element, leading to a Kosterlitz-Thouless-like breaking of the electromagnetic $U_E(1)$ symmetry, and thus superconductivity, once a fermion mass gap opens up. The wavy line represents the $SU(2)$ gauge boson B_μ^3, which remains massless, while the blob denotes an insertion of the fermion-number current $J_\mu = \overline{\Psi}\gamma_\mu\Psi$. Continuous lines represent fermions.

$$\mathcal{S}^a = \langle B_\mu^a | J_\nu | 0 \rangle, \quad a=1,2,3; \quad J_\mu = \overline{\Psi}\gamma_\mu\Psi. \tag{57}$$

As a result of the color group structure only the massless B_μ^3 gauge boson of the $SU(2)$ group, corresponding to the σ_3 generator in two-component notation, contributes to the graph. The result is[9,28]

$$\mathcal{S} = \langle B_\mu^3 | J_\nu | 0 \rangle = (\text{sgn } M)\epsilon_{\mu\nu\rho}\frac{p_\rho}{\sqrt{p_0}}, \tag{58}$$

where M is the parity-conserving fermion mass (or the holon condensate in the context of the doped antiferromagnet). In our case this mass is generated *dynamically* by means of the $U_S(1)$ interactions, as we discussed above, provided its coupling constant is sufficiently strong. The result (58) is *exact* in perturbation theory, in the sense that the only modifications coming from higher loops would be a multpilicative factor $1/[1-\Pi(p)]$ on the right-hand side, with $\Pi(p)$ the B_μ^3-gauge-boson vacuum polarization function.[28]

As discussed in Refs. 9 and 28, the B_μ^3 color component plays the role of the *Goldstone boson* of the spontaneously broken fermion-number symmetry. If this symmetry is exact, then the gauge boson B_μ^3 remains *massless*. This is crucial for the superconducting properties,[9] given that this leads to the appearance of a *massless pole* in the electric-current two-point correlators, the relevant graph being depicted in Fig. 2. This is the standard Landau criterion for superconductivity.

It can be shown[9] that in the massive-fermion [broken $SU(2)$] phase, the effective low-energy theory obtained after integrating out the massive fermionic degrees of freedom assumes the standard London action for superconductivity, the massless excitation ϕ being defined to be the *dual* of B_μ^3:

$$\partial_\mu\phi \equiv \epsilon_{\mu\nu\rho}\partial_\nu B_\rho^3. \tag{59}$$

All the standard properties of superconductivity, Meissner effect [strongly type II (Ref. 9)], flux quantization and infi-

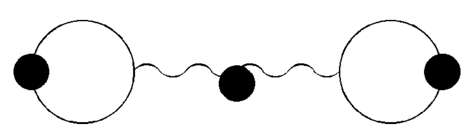

FIG. 2. The lowest-order contribution to the electric current-current correlator $\langle 0|J_\mu(p)J_\nu(-p)|0\rangle$. The blob in the propagator for the gauge boson B_μ^3 indicates fermion loop (resummed) corrections. The blob in each fermion loop indicates an insertion of the current J_μ.

nite conductivity, follow then in a standard way after coupling to external elelctromagnetic potentials, provided the excitation ϕ (and hence B^3_μ) is exactly massless.

However, it is known[9,10,3] that superconductivity is of a Kosterlitz-Thouless (KT)-type superconductivity, not characterized by a local order parameter. Let us briefly review the arguments leading to this.[9] The neutral parity-invariant condensate $\langle \overline{\Psi}_1\Psi_1 - \overline{\Psi}_2\Psi_2\rangle$, generated by the strong $U_S(1)$ interaction, is *invariant* under the $U(1)\otimes U_E(1)$, as a result of the τ_3 coupling of B^3_μ in the action, and hence does not constitute an order parameter for the spontaneous breaking of any of these symmetries [the group $U(1)$ denotes the $SU(2)$ subgroup associated with the B^3_μ gauge boson]. This is a characteristic feature of our gauge interactions. Putative charge $2e$ or $-2e$ order parameters, like the pairing interactions among opposite spins in the statistical model of Refs. 9 and 3, e.g., $\langle \Psi_1\Psi_2\rangle, \langle \overline{\Psi}_1\overline{\Psi}_2\rangle$ [in four-component notation, such fermionic bilinears correspond to $\langle \Psi\gamma_5\Psi\rangle, \langle \overline{\Psi}\gamma_5\overline{\Psi}\rangle$, considered in Ref. 9] will vanish at any finite temperature, in the sense that strong phase fluctuations will destroy the vacuum expectation values of the respective operators, due to the Mermin-Wagner theorem. Even at zero temperatures, however, such VEV's yield zero result to any order in perturbation theory trivially, due to the fact that in the context of the effective B^3_μ gauge theory of the broken $SU(2)$ phase, the gauge interactions preserve flavor. For a more detailed discussion on the symmetry breaking patterns of $(2+1)$-dimensional gauge theories, and the proper definition of order parameter fields, we refer the reader to the literature.[28,9] Thus, from the above analysis, it becomes clear that gap formation, pairing and superconductivity can occur in the above model without implying any phase coherence.

C. Instantons and the fate of superconductivity in the $SU(2)\otimes U_S(1)$ model

An important feature of the non-Abelian model is that, due to the non-Abelian symmetry breaking pattern $SU(2)\to U(1)$, the Abelian subgroup $U(1)\in SU(2)$, generated by the σ^3 Pauli generator of $SU(2)$, is *compact*, and may contain *instantons*,[29] which in three space-time dimensions are like monopoles, and are known to be responsible for giving a *small* but *nonzero mass* to the gauge boson B^3_μ,

$$m_{B^3}\sim e^{-(1/2)S_0}, \tag{60}$$

where S_0 is the one-instanton action, in a dilute gas approximation. Its dependence on the coupling constant $g_2\equiv g_{SU(2)}$ is well known:[29]

$$S_0\sim \frac{\text{const}}{g_2^2}. \tag{61}$$

For weak coupling g_2 the induced gauge-boson mass can be very small. However, even such a small mass is sufficient to destroy superconductivity, since in that case there is no massless pole in the electric current-current correlator. In Ref. 24 a breakdown of superconductivity due to instanton effects has been interpreted as implying a ''pseudogap'' phase: a phase in which there is dynamical generation of a mass gap for the nodal holons, which, however, is not characterized by superconducting properties.

The presence of *massless* fermions, with zero modes around the instanton configuration, is known[29] to suppress the instanton effects on the mass of the photon, and under certain circumstances, to be specified below, the Abelian gauge boson may remain exactly massless *even in the presence of nonperturbative effects*, thus leading to superconductivity, in the context of our model. This may happen[29] if there are extra global symmetries in the theory, whose currents connect the vacuum to the one-gauge-boson state, and thus they break spontaneously. This is precisely the case of the fermion number symmetry considered above.[29,28] In such a case, the massless gauge boson is the Goldstone boson of the (nonperturbatively) spontaneously broken symmetry. However, in our $SU(2)\otimes U_S(1)$ model,[3,24] as a result of the (strong) $U_S(1)$ interaction, a mass for the fermions is generated, and hence there is no issue of fermion zero modes in this case. The analysis of the low-energy effective theory presented in Refs. 3 and 24 is based on a Wilsonian treatment, where massive degrees of freedom are integrated out in the path integral. This includes the gapful fermions and the massive $SU(2)$ gauge bosons. The resulting effective theory, then, which encodes the dynamics of the gapped phase, is a pure gauge theory $U(1)\in SU(2)$, and the instanton contributions to the mass of B^3_μ are present, given by Eq. (60), in the one-instanton case. Thus, it seems that, generically, in the context of the $SU(2)\otimes U_S(1)$ of Ref. 3, the nodal gap is actually a pseudogap.

D. Instantons and supersymmetry

We now remark that supersymmetry is known[29] to suppress instanton contributions. For instance, in certain $N=1$ supersymmetric models with massless fermions, considered in Ref. 29 the instanton-induced mass of the Abelian gauge boson is given by

$$m_{gauge\ \ boson}\sim e^{-S_0} \tag{62}$$

which is suppressed compared to the nonsupersymmetric case (60).

$N=2$ supersymmetric theories in three space-time dimensions constitute additional examples of theories where the Abelian gauge boson remains exactly massless, in the presence of instantons.[29,30] Such theories have complex representation for fermions, and hence are characterized by extra global symmetries (like fermion number). In view of our discussion above, such models will then lead to Kosterlitz-Thouless superconductivity upon gauging the fermion number symmetry.

In this respect, the supersymmetric points (53) and (35) for which such instanton effects are argued[24] to be strongly suppressed in favor of KT superconductivity, as reviewed above, would constitute ''superconducting stripes'' in the temperature-doping phase diagram of the nodal liquid (see Fig. 3) It should be stressed that the term ''stripe'' here is meant to denote a certain region of the temperature-doping phase diagram of the nodal liquid and should not be confused with the stripe structures in real space which characterizes the cuprates at special doping concentrations. Theoretically, the stripes should have zero thickness, given that

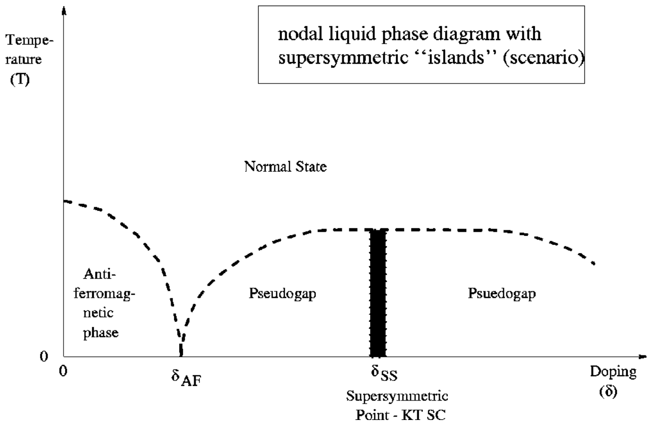

FIG. 3. A possible scenario for the temperature-doping phase diagram of a charged, relativistic, nodal liquid in the context of spin-charge separation. At certain doping concentrations (δ_{SS}) there are dynamical supersymmetries among the spinon and holon degrees of freedom, responsible for yielding thin ''stripes'' in the phase diagram (shaded region) characterized by Kosterlitz-Thouless (KT) superconductivity without a local order parameter. The diagram is conjectural at present. It pertains strictly to the nodal liquid excitations about the *d*-wave nodes of a superconducting gap, and hence should not be confused with the phase diagram of the entire (high-temperature) superconductor.

they occur for specific doping concentrations (53) and (35). However, in practice, there may be uncertainties (due to doping dependences) in the precise value for the parameter Δ_z entering Eqs. (53) and (35) which might be responsible for giving the superconducting stripe a certain (small) thickness. A detailed analysis of such important issues is still pending. It is hoped that due to supersymmetry one should be able to discuss some exact analytic results at least for zero temperatures.

We also remark that in supersymmetric theories of the type considered here and in Ref. 13, it is known[29] that supersymmetry cannot be broken, due to the fact that the Witten index $(-1)^F$, where F is the fermion number, is always nonzero. Thus in supersymmetric theories the presence of instantons should give a small mass, if at all, in *both* the gauge boson and the associated gaugino. However, in three-dimensional supersymmetric gauge theories it is possible that supersymmetry is broken by having the system in a ''false'' vacuum, where the gauge boson remains massless, even in the presence of nonperturbative configurations, while the gaugino acquires a small mass, through nonperturbative effects. The lifetime, though, of this false vacuum is very long,[29] and hence superconductivity can occur, in the sense that the system will remain in that false vacuum for a very long period of time, longer than any other time scale in the problem.

E. Some comments on supersymmetry breaking at finite temperatures

So far, our discussion was restricted to zero temperature. At *any* finite temperature, no matter how small, supersymmetry is explicitly broken, and thus the supersymmetric points should be viewed as *quantum critical points*. However, the breaking of supersymmetry is associated with different boundary conditions between fermionic and bosonic degrees of freedom, and, although the vacuum energy is no longer zero, however a detailed analysis should be made in order to determine whether the equality of mass gaps between the nodal spinons and holons at the supersymmetric points is lifted by temperature-dependent corrections. In the context of a supersymmetric theory this issue can be tackled by means of ''thermal superspace'' methods, which have been developed recently in the context of particle-physics models.[31] The generic result of such analyses seems to be that the mass degeneracy among the superpartners is lifted at the level of the mass of the various thermal modes, the corresponding lifting being proportional to the temperature. The thermal superspace method can be applied to the present model as well, however, this falls beyond the scope of the present article and is thereby left for a future work.

Moreover, as the crude analysis of Ref. 6 indicates, the nodal gaps would disappear at temperatures which are much lower than the critical temperature of the (bulk) *d*-wave superconducting gap. For instance, for a typical set of the parameters of the *t*-*j* model used in Ref. 6, the nodal critical temperature is of order of a few mK, which is much smaller than the 100-K bulk critical temperature of the high-temperature superconductors. The application of an external magnetic field in the strongly type-II high-temperautre superconducting oxides, which is another source for explicit breaking of the potential supersymmetry, enhances the critical temperature[6] up to 30 K, thereby providing a potential explanation for the recent findings of Ref. 7, according to which plateaux in the thermal conductivity as a function of the external magnetic field indicate the opening of a gap at the *d*-wave nodes.

We now remark that, if such situations with broken supersymmetry are viewed as cases of perturbed supersymmetric points, then one might hope of obtaining nonperturbative information on the phase structure of the liquid of nodal excitations in spin-charge separating scenaria of (gauge) high-temperature superconductors. This may also prove useful for a complete physical understanding of the entire phenomenon, including excitations away from the nodes.

VII. CONCLUSIONS

From the above discussion it is clear that supersymmetry can be achieved in the effective continuum field theories of doped antiferromagnetic systems exhibiting spin-charge separation only for *particular doping concentrations* [cf. Eqs. (54) and (35)]. One's hope is that the ancestor lattice model will lie in the same universality class (in the infrared) as the continuum model, in the sense that it differs from it only by the action of renormalization-group irrelevant operators. This remains to be checked by explicit lattice calculations. We should note at this stage that this is a very difficult problem; in the context of four-dimensional particle-physics models it is still unresolved.[32] However, in view of the apparent simpler form of the three-dimensional lattice models at hand, one may hope that these models are easier to handle.

By varying the doping concentration in the sample, one goes away from the supersymmetric point and breaks supersymmetry explicitly at zero temperatures. At finite temperatures, or under the influence of external electromagnetic fields at the nodes of the d-wave gap, supersymmetry will also be broken explicitly. Therefore realistic systems observed in nature will be characterized by explicitly broken supersymmetries even close to zero temperatures. However, there is value in deriving such supersymmetric results in that at such points in the parameter space of the condensed-matter system it is possible to obtain analytically some exact results on the phase structure of the theory. Supersymmetry may allow for a study of the quantum fluctuations about some exact ground states of the spin-charge separated systems in a controlled way. Then one may consider perturbing around such exact solutions to get useful information about the nonsupersymmetric models.

We have argued that such special points will yield new phases for the liquid of excitations about nodal points of the d-wave superconducting gaps, which include a phase in which there is only spin transport but not electric current transport, as well as a phase in which there are Kosterlitz-Thouless-type superconducting ''islands'' in a temperature doping phase diagram of the nodal liquid, upon the dynamical generation of holon-spinon mass gaps (of equal size). The latter property is due to special properties of the supersymmetry, associated with the suppression of nonperturbative effects of the (compact) gauge fields entering the spin-charge separation ansatz (1). This, of course, needs to be checked explicitly by carrying out the appropriate instanton calculations in the spirit of the nonperturbative modern framework of Ref. 12. At present, such nonperturbative effects can only be checked explicitly in three dimensions for highly extended supersymmetric models.[30] It is, however, possible that some exact results could be obtained at least for the $N=2$ supersymmetric models which may have some relevance for the effective theory of the nodal liquid at the supersymmetric points.[13] Then, one may get some useful information for the $N=1$ models studied here by viewing them as supersymmetry-breaking perturbations of the $N=2$ models. Such issues remain for future investigations, but we hope that the speculations made in the present work provide sufficient motivation to carry out research along these directions.

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