Temperature dependence of the Aharonov-Bohm oscillations and the energy spectrum in a single-mode ballistic ring

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The temperature dependence of the amplitude of the Aharonov-Bohm (AB) oscillations in a single mode ballistic ring has been measured. The experimental data is analyzed using the exact energy spectrum of an ideal ring with a finite width and the Landauer-Büttiker formula to calculate the conductance of the ring. We show that the temperature dependence of the AB oscillations can be explained in terms of the thermally averaged transmission probability through the energy levels of the ring provided the additional charging energy is taken into account. This demonstrates the effect of Coulomb repulsion on the AB oscillations in a ring interferometer.

I. INTRODUCTION

The electron interference phenomena in solid state structures have been extensively studied since the beginning of 1980s (for a review see Ref. 1). Due to the progress in technology in the past few years, it has become possible to fabricate ballistic ring interferometers.[2,3] At low temperatures the electron phase coherence in these structures is preserved on a length scale that is comparable to or larger than the ring dimensions. This opens up the way for sophisticated experiments related to different aspects of the coherent transport such as the recent ''which-path'' detector study carried out in Ref. 4. On the other hand, there still remain several basic issues, which, while expected to be strongly dependent on the type of the transport involved (diffusive or ballistic), have so far been investigated only in diffusive rings. Thus, for example, the temperature dependence of the Aharonov-Bohm (AB) oscillations in a single mode ballistic ring has yet to be studied.

It has been shown that in dirty rings where the transport is diffusive there are, in general, two main mechanisms responsible for the temperature damping of the amplitude of AB oscillations. The first is associated with the loss of the phase coherence, which at low temperatures is destroyed by electron-electron or electron-phonon scattering. The electron phase coherence length L_ϕ determines the length scale at which the interference effects may be observed. If, with decreasing temperature, L_ϕ becomes less than the circumference of the ring, the AB oscillations start to decay exponentially. This has been demonstrated in earlier experiments on metal rings.[5] The other mechanism leading to a reduction of the AB amplitude with temperature is ensemble energy averaging,[6] which dominates when the phase coherence length is larger than the circumference of the ring. In clean ballistic rings these two mechanisms are still valid but the behavior of AB oscillations with temperature is expected to be different. The phase coherence length in the ballistic regime is much longer than that in diffusive samples and may be comparable to or exceed the half circumference of the ring. Under these circumstances the temperature dependence of AB oscillations is largely determined by the ensemble energy averaging, which in a ballistic ring is very different from the ensemble averaging in diffusive samples.

In this paper we report on measurements of the temperature dependence of the amplitude of AB oscillations in a single mode $Al_xGa_{1-x}As/GaAs$ ballistic ring. We calculate the exact energy spectrum of an ideal ring with a finite width. This, combined with the Landauer-Büttiker formalism, allows us to calculate the conductance of the ring at finite temperatures with no adjustable parameters. We find, however, that the experimental temperature dependence is not entirely consistent with this model. Reasonable agreement is obtained only if we include the additional electrostatic energy associated with the capacitance of the ring. The influence of the Coulomb repulsion on the energy spectra and transport has been extensively studied in quantum dots and is well known as the Coulomb blockade effect (see, for example, Ref. 7). Here, we demonstrate the importance of the Coulomb blockade for the AB oscillations in a ring interferometer.

II. EXPERIMENTAL RESULTS AND ANALYSIS

A. Experiment

The samples were fabricated on $Al_xGa_{1-x}As/GaAs$ modulation doped layers grown by molecular beam epitaxy (MBE). The ring geometry was defined by electron beam lithography and plasma etching (Fig. 1). All active portions of the device were covered by an Au/Ti gate. Devices of two different types have been used. The devices of the first type were fabricated on a heterolayer with a carrier density $n_s = (1.5-2)\times 10^{11}$ cm^{-2} and a mobility $\mu \approx 10^6$ cm^2/V s.

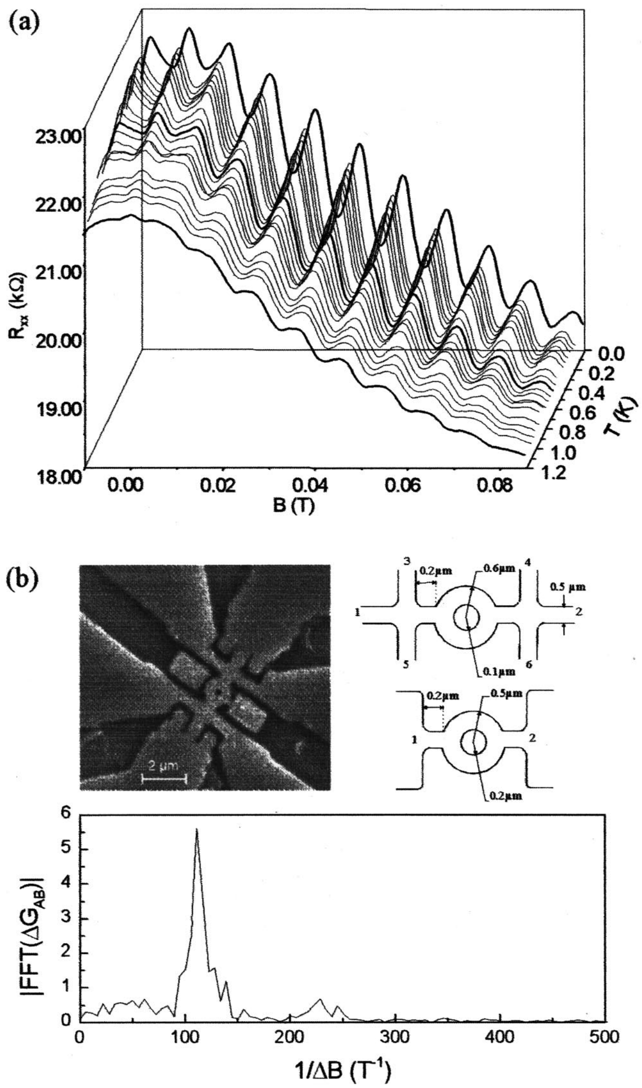

FIG. 1. (a) Set of magnetoresistance curves $R_{xx}(B)$ for different temperatures, from 40 mK to 1.2 K; (b) Fourier transform of the 40 mK curve. Insert: Schematic view of the samples and microphotograph of the sample.

These devices had two pairs of voltage probes and were designed for four-point measurements. The average radius of the rings was $r_0 = 0.35$ μm and the lithographical width of the channels $W_{lith} = 0.5$ μm. The devices of the second type had only the current probes. They were defined on a heterolayer with $n_s = (5-6)\times 10^{11}$ cm^{-2} and $\mu \approx 5 \times 10^5$ cm^2/V s. The radius of the rings was the same, $r_0 = 0.35$ μm, but the lithographical width of the ring channels was less $W_{lith} = 0.3$ μm. The samples were mounted in the mixing chamber of a dilution refrigerator. The resistance was measured using conventional phase sensitive detection with an ac current of 1 nA.

Figure 1(a) shows a typical series of magnetoresistance traces measured at different temperatures and used to calculate the temperature dependence of the amplitude of the AB oscillations. These oscillations have a period corresponding to the threading of one additional flux quantum h/e through the area of the ring. The regularity of AB oscillations in Fig. 1(a) indicates that only a single one-dimensional subband in the interferometer channels is occupied. The samples were very stable and the magnetoresistance curves reproduced perfectly during several hours of measurements. It was, however, possible to change the state of the sample or, equivalently, the waveguide characteristics of the ring by thermal cycling, illumination, or gate voltage and to evaluate the temperature dependence in different states. The experimental data were then processed in the following way. First, the resistance $R(B)$ dependence was transformed into a conductance $G(B)$. Then the background, obtained by averaging the $G(B)$ over 0.016 T wide bins, was subtracted, leaving only the AB component $\Delta G_{AB}(B)$. After this a Fourier transform was performed [Fig. 1(b)]. To quantify the size of the spectral peaks we integrated the spectral weights over the peak's breadth. This integral is proportional to the average amplitude $\langle \Delta G_{AB}(B) \rangle$ of the AB oscillations. Figure 2 shows a typical temperature dependence of the amplitude of the AB oscillations for the two samples investigated.

B. Analysis

In the following analysis of our experiment we assume that the phase coherence length is larger than the circumference of the ring and the temperature dependence of the AB oscillations can be explained by the temperature averaging alone. This assumption is based on the fact that in the original two-dimensional electron gases (2DEG's), from which the experimental structures have been made, the electron mean free path is about 5 μm, and so the phase coherence length L_ϕ is expected to be at least as large. In a patterned sample these values may be smaller than those in a bulk 2DEG. Nevertheless, in our structures we observe a number of features that are associated with classical ballistic effects (the quenching of the quantum Hall effect at low magnetic fields and classical Hall plateau). This indicates that the mean free path is greater than the width of the voltage probes. In magnetic field up to 1 T we observe a gradual decrease of about 30% of the longitudinal resistance, which we attribute to the suppression of backscattering at the entrance of the ring structure. This fact supports our assumption that the transport in our ring is ballistic or, at least, quasiballistic (the mean free path is comparable to the half-circumference of the ring). Since in ballistic structures the phase coherence length is comparable with the mean free path, we expect that $L_\phi \geqslant 2\pi r_0$, where r_0 is the average radius of the ring. It will be shown further that this assumption is also supported by other experimental evidence.

In order to analyze our experimental data we calculate the conductance of the ring at a finite temperature using the Landauer-Büttiker formalism:

$$G(E_F,B) = -\frac{e^2}{\hbar}\int T(\varepsilon,B)\frac{df(\varepsilon,E_F,T)}{d\varepsilon}d\varepsilon. \qquad (1)$$

Here $T(\varepsilon,B)$ is the magnetic field and energy dependent transmission from the left reservoir to the right, $f(\varepsilon,E_F,T)$ is the Fermi-Dirac distribution function, and E_F is the Fermi energy. The transmission coefficient $T(\varepsilon,B)$ of the ring can be evaluated using the Breit-Wigner formula,[8] whose validity has been demonstrated in the experiment on the electron transport through a ballistic cavity in a strong magnetic field.[7] In our case the Breit-Wigner formalism should be valid if the ring is considered as weakly coupled to the leads

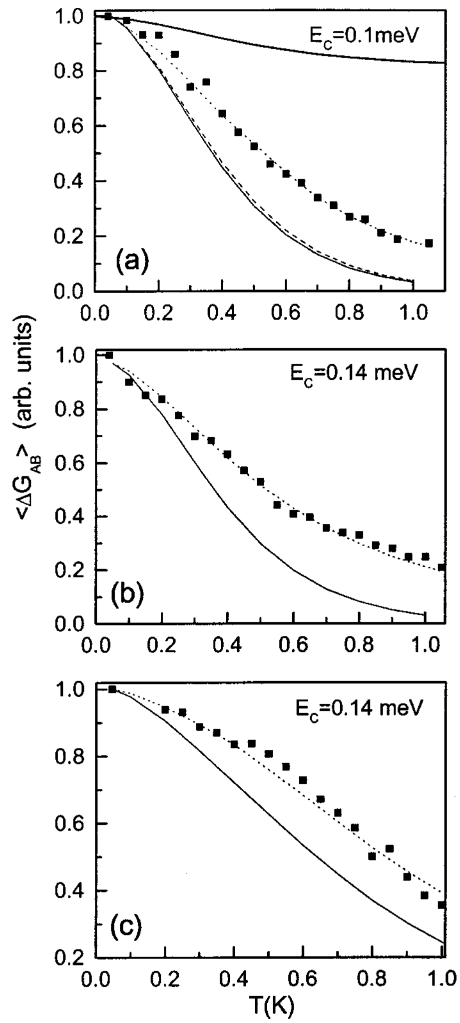

FIG. 2. Experimental temperature dependence of the amplitude of Aharonov-Bohm oscillations (black squares) for the four-lead interferometer (a, b) and the two-lead interferometer (c). The solid line in each case correspond to the temperature dependence calculated using the exact energy spectrum of the rings (Ref. 9). The dashed line in (a) is the temperature dependence calculated for the case when the Fermi energy in the four-lead ring is just below the bottom of the second subband (maximal energy level spacing). The thick solid line corresponds to the temperature dependence calculated using Eq. (5) for $\epsilon=0.45$. The dotted lines are fits to experimental curves with the Coulomb energy E_C as an adjustable parameter.

or, equivalently, if the ring resistance $R \gg h/2e^2$. The resistance of the rings used in the experiment is about 13–15 kΩ or higher, indicating that our use of the Breit-Wigner formula is justified. The transport through the ring can then be viewed as a tunneling of electrons from one lead to the other via a quasibound state in the ring.

The energy spectrum in the ring can be calculated using the model proposed by Tan and Inkson,[9] which gives an exact analytical solution for the electron states in an ideal ring with a finite width. In this model the 2D ring is defined by the following radial potential:

$$V(r)=a_1 r^{-2}+a_2 r^2-V_0, \tag{2}$$

where $V_0=2\sqrt{a_1 a_2}$. For r close to $r_0=(a_1/a_2)^{1/4}$ (the average radius of the ring) this potential has a parabolic form $V(r)\approx\frac{1}{2}\omega_0^2(r-r_0)^2$, where μ is the electron effective mass and $\omega_0^2=(8a_2/\mu)$. We assume that in our case a parabolic potential is a good approximation of the potential of the ring conducting channels. The energy levels in magnetic field are given by

$$E_{n,m}=\left(n+\frac{1}{2}+\frac{M}{2}\right)\hbar\omega-\frac{m\hbar\omega_c}{2}-\frac{\mu\omega_0^2 r_0^2}{4}, \tag{3}$$

where $n=0,1,2,3,\ldots$, $m=\ldots,-2,-1,0,1,2,\ldots$, $M=\sqrt{m^2+2a_1\mu/\hbar^2}$, $\omega=(\omega_c^2+\omega_0^2)^{1/2}$, and $\omega_c=eB/\mu$. Thus, the energy spectrum is fully determined by the value of the average radius r_0 and the harmonic frequency ω_0. The average radius of the rings is known precisely from the period of AB oscillations. The harmonic frequency ω_0 can be evaluated from the 2D electron density in the channels of the ring using the method proposed in Ref. 10. The 2D electron density in the channels is given by the positions of the R_{xx} minima in the quantum Hall effect (QHE) regime and is always lower than the 2D electron density in the bulk of the sample.

Using this approach we calculate the energy spectrum for both the four-lead and the two-lead rings as a function of magnetic field. We find that only the first radial subband in the interferometer channels is occupied in the four-lead ring, which is consistent with the observed regularity of the AB oscillations. In the two-lead ring the electron density is higher and so the Fermi energy is closer to the bottom of the second subband. We can now proceed to calculate the temperature dependence of the amplitude of the AB oscillations in our rings. Following the Breit-Wigner formula the transmission coefficient of the ring is given by

$$T(\varepsilon,B)=\sum_{n,m}\frac{\Gamma^2}{[\varepsilon-E_{n,m}(B)]^2-\Gamma^2}, \tag{4}$$

where $E_{n,m}$ are the energy levels in the ring determined by Eq. (3) and Γ is the broadening of the energy levels due to the overlap between the bound states in the ring and the extended states in the leads. The broadening Γ of the levels determines the amplitude of the AB oscillations and in this way can be found from the experiment. However, we find that the choice of Γ has no effect on the temperature dependence of the amplitude of the oscillations.

The solid lines in Fig. 2 are the calculated T dependence of the amplitude of the AB oscillations. We see that the theoretical model predicts a systematically stronger decrease of the amplitude of AB oscillations with temperature than found experimentally. This discrepancy is found to be present in all our devices and also in different states of one device. It cannot be explained by the phase coherence length L_ϕ being smaller than the circumference of the ring. On the contrary, when $L_\phi<\pi r_0$ the amplitude of AB oscillations decreases even more rapidly with the increasing temperature by the additional factor $\exp(-\pi r_0/L_\phi)$. Under these circumstances the only factor that influences the temperature dependence of the amplitude of AB oscillations is the spacing between the energy levels near the Fermi energy in the ring.

We find that the theoretical curves fit well the experimental dependences if we assume that the energy level spacing is 0.5 meV for sample 1 and 0.65 meV for sample 2. However,

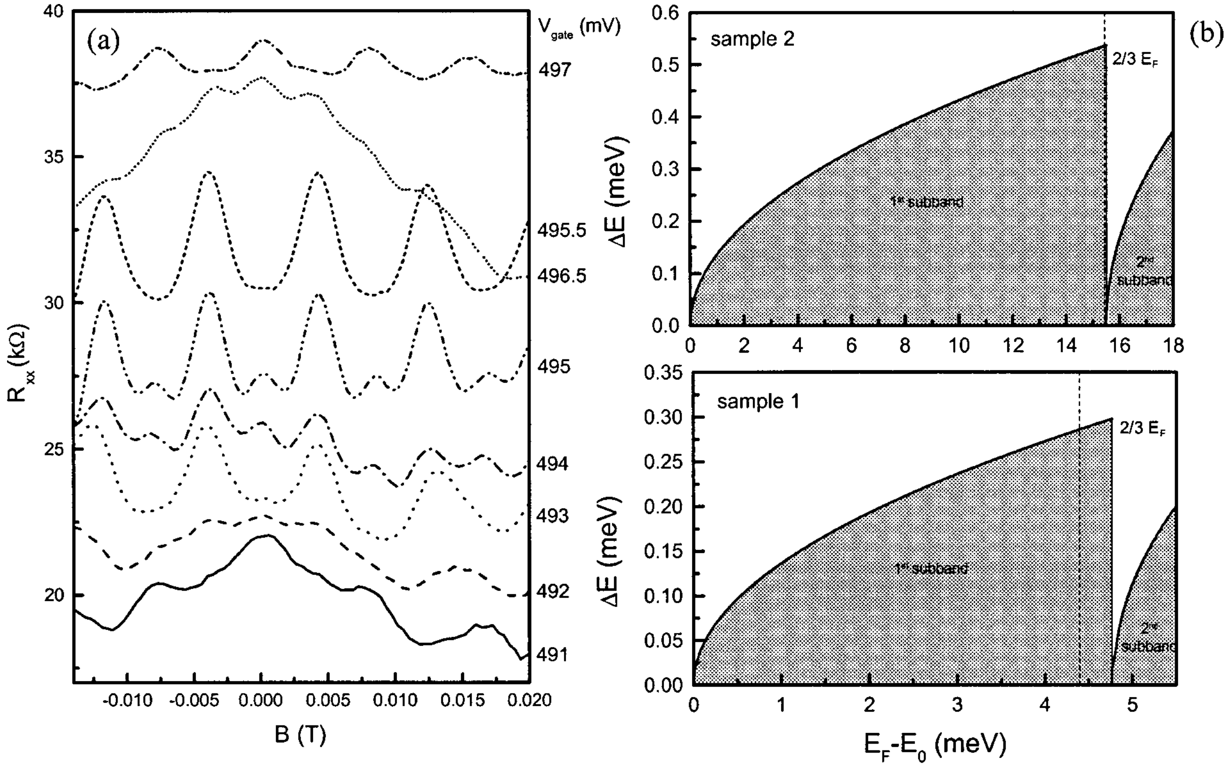

FIG. 3. (a) The variation of the AB oscillations with the gate voltage in symmetric ballistic ring uniformly covered by a gate; (b) dependence of the energy level spacing in the two samples versus (E_F-E_0), where E_0 is the bottom of the first subband, and E_F is constant, determined from experiment (the dashed line is the value of E_F-E_0 used in our calculation).

these values are nearly twice as large as those obtained from the energy spectra of the rings. As shown in Fig. 3(b), the energy level spacing increases with E_F-E_0 as $\hbar\sqrt{2(E_F-E_0)/\mu}/r_0$, where $E_0=\frac{1}{2}\hbar\omega_0$ is the bottom of the first subband in the ring. The level spacing reaches its maximum when the Fermi level is just below the bottom of the second subband ($\frac{3}{2}\hbar\omega_0$) and then goes sharply down when the Fermi level moves into the second subband and the levels of the two subbands become intermixed. The Fermi energy in our samples is fixed and can be determined exactly from the period of the Shubnikov–de Haas (SdH) oscillations in the macroscopic part of the sample. From our calculation of the energy spectrum we find that in both our rings E_F is close to the bottom of the second subband and hence the energy level spacing is near its maximum.

Moreover, we find that the predicted temperature dependence corresponding to the largest possible energy spacing in our rings ($\hbar\omega_0=\frac{2}{3}E_F$) still falls below the experimental curve [Fig. 2(a), the dashed line]. This proves that the discrepancy cannot be explained by a possible insufficient accuracy of our estimation of ω_0 and all the other parameters in the model are determined independently and are well known.

Fluctuations of the energy level spacing associated with quantum chaos have been reported for single quantum dots.[7] However, in our case we expect the level spacing to be regular as it is determined solely by the radius of the ring and not by the shape of the structure as in quantum dots. The regularity of the AB oscillations is also an indication that there are no fluctuations of level spacing in our rings. In addition, the averaging over magnetic field, which is part of our evaluation procedure to determine the temperature dependence of the AB amplitude, would cancel the effect of energy level fluctuations if such exists. Since the resistance of our rings is higher than $R_T=h/2e^2$, the charging energy e^2/C associated with the capacitance of the ring may become important.[11]

At low temperature the conductance of a small sample is suppressed because of the charging energy. This phenomenon known as the Coulomb blockade has been widely studied in quantum dots.[7] The Coulomb blockade strongly depends on the tunneling conductance G_T from the leads to the dots and is suppressed when $G_T>2e^2/h$.[12] When the Fermi energy of 2DEG in the source and drain is aligned with an energy level in the dot there is a peak in the conductance. If the charging energy is the dominant energy in the system the conductance oscillates as a function of the gate voltage with a period corresponding to a change by one in the number of electrons in the dot. For a quantum dot of radius 200 nm the Coulomb energy $E_C\approx 1$ meV. However, when the contact resistance becomes lower than $h/2e^2$, E_C decreases and goes to zero. Since the spacing between the single particle energy levels in our rings is too small to explain the temperature dependence of the AB oscillations, we suggest that some additional energy factor should be considered. It could be the charging energy e^2/C, where C is the mutual capacitance of the ring, which strongly depends on the environment. As the conductance of our samples is only slightly smaller than G_T, we can estimate the charging energy as being less than 1 meV.

With this in mind, we calculate the temperature dependence of the amplitude of the AB oscillations in the ring using as an adjustable parameter the charging energy, which we add to the energy level spacing just above the Fermi energy. Figure 2 shows the result of these calculations (the dotted lines). We find that the experimental results are consistent with the model for $E_C \approx 0.1$ meV. Considering that we are in a weak ''Coulomb blockade regime,'' this value seems quite plausible. The effect of Coulomb repulsion on the Aharonov-Bohm effect in quantum dots and rings has been investigated theoretically in Ref. 13.

Figure 3(a) shows the modification of the AB oscillations with gate voltage in our symmetrical rings covered by a top gate. We see that at certain gate voltages the amplitude of AB oscillations is indeed strongly suppressed. This is accompanied by a change in the period of the oscillations from h/e to $h/2e$. Then, when the h/e oscillations reappear, their phase is shifted by π.

Several successive modifications like this have been observed in our samples in the operational gate voltage range. Outside this range the amplitude of the AB oscillations begins to deteriorate until the oscillations die out completely. We explain this by the depletion of one of the ring channels for large negative voltages or by the smearing out of the ring structure for large positive voltages. A detailed analysis of the above dependence of the oscillations on the Fermi energy based on the single particle spectrum of the ring [Eq. (3)] is published elsewhere.[14] Since the Coulomb gap is smaller than the single electron level spacing, the Coulomb effect is not directly observed in our samples. The reduction of the amplitude of the AB oscillations for intermediate states ($h/2e$ oscillations) is mostly due to the reduction of the energy level spacing. Also, because of the deterioration of AB oscillations at strong negative biases we cannot explore our ring in a regime where the Coulomb blockade would be more pronounced. Nevertheless, when we add the Coulomb energy to the single particle energy spectrum we obtain a weaker temperature dependence which is consistent with our experimental results.

We have to complete this work by comparison of our experimental results with another theoretical approach of the one-dimensional ring: the scattering matrix theory. Within this approximation electrons are scattered by junctions between leads and ring, and the conductance of the ring is determined by the junction transmission amplitude, which forms the scattering matrix. The expression of the transmission probability for the single mode ring connected to current leads has been derived firstly in Ref. 15. The temperature dependence of the Aharonov-Bohm conductance oscillations by using scattering matrix theory has been studied in Refs. 16 and 17. For the single channel case the conductance at $T=0$ K can be written in the form:

$$G(B)=\frac{2e^2}{h}\left|\frac{\epsilon Q}{1-b^2Q^2}+\frac{\epsilon P}{1-b^2P^2}\right|^2, \tag{5}$$

where $b=\frac{1}{2}(\sqrt{1-2\epsilon}+1)$, $P=e^{i\pi q_+}$, $Q=e^{-i\pi q_-}$, $q_\pm = Br_0^2/2 \pm r_0\sqrt{k_F^2W^2/\pi^2-\pi^2}$, k_F is the Fermi wave vector, ϵ is the coupling parameter, which is equal to 0 when the current leads and the ring are decoupled, and $\epsilon=0.5$ for strong coupling limit. We calculate the temperature dependence using the *S*-matrix approach for $\epsilon=0.45$. The results are shown in Fig. 2(a). We see that in accordance with the theoretical predictions[16,17] the oscillation amplitude no longer decreases with increasing temperature when the temperature is higher than 0.6 K. We see also that the calculated T dependence is much weaker than found experimentally. In the limit of the small ϵ, as we can expect, the *S*-matrix approach gives an exponential decrease with temperature, which, however, is much stronger than that found experimentally. We have to note also that the *S*-matrix theory predicts saturation of the T dependence of the AB oscillation amplitude at $T>300-400$ mK,[16,17] which is not observed in our experiments.

Therefore, we can conclude that the Breit-Wigner formula is more appropriate to model the T dependence of the amplitude of AB. The *S*-matrix approach predicts a very weak T dependence in the single mode ring with relatively strong coupling to the leads.[16,17] The reason why the *S*-matrix theory is not appropriate for modeling of an AB ring behavior is not clear to us at present. However, we should note that other authors experienced similar difficulties. For example, the irregular behavior of the AB amplitude with magnetic field observed in Ref. 18 can be described using the Breit-Wigner formula together with the exact energy spectrum of the ring with a relatively strong coupling to the outside.[9]

III. CONCLUSION

In conclusion, we show that the temperature dependence of the AB oscillations in a single mode ballistic interferometer can be explained by the temperature averaging of the ring transmission coefficient. The only parameter that governs the temperature dependence of AB oscillations in this case is the spacing between the energy levels in the ring near the Fermi energy. We find, however, that our experimental results imply a larger energy spacing than follows from the exact single particle energy spectrum of the ring. This additional energy may be due to the charging energy associated with the capacitance of the ring. We thus demonstrate that in a ring with a resistance larger than $h/2e^2$ the Coulomb energy can play an important role and modify the behavior of Aharonov-Bohm oscillations.

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