Transport through double quantum dots

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We study transport through two quantum dots in series. Electron-electron interactions are taken into account in the capacitive model with an additional interdot capacitance. The tunneling rates between the dots and the outside reservoirs are assumed to be weak, therefore we treat it perturbatively and derive a master equation. The interdot tunneling is treated in two limits: weak interdot tunneling is included perturbatively, whereas in the opposite limit we assume that there is only one level in the dot in the relevant energy range such that the Hamiltonian of the dots can be diagonalized exactly. We calculate the current through the structure as a function of the two gate voltages. The well-known Coulomb oscillations of a single dot are changed into a characteristic structure of boomerang-like shape. The transport and gate voltages can be time-dependent, and in this case we find that the dependence of the Coulomb oscillations on the two gate voltages allows us to identify which level dominates the transport.

I. INTRODUCTION

Semiconductor heterostructures have proven to be not just the basis for very powerful and versatile devices, but they have also stimulated condensed-matter physics by providing us with a model system with unique properties: the two-dimensional electron gas (2DEG). In GaAs:$Al_xGa_{1-x}As$ heterostructures, the difference in the band gaps of GaAs and $Al_xGa_{1-x}As$ is used to create a one-dimensional potential well (say, in the z direction), which leads to quantization of the z component of the wave vector. If the density of charge carriers and the temperature are arranged such that only the lowest of these quantized levels is occupied, there are no degrees of freedom connected to motion in the z direction any more, and in that sense we have a truly two-dimensional situation (notwithstanding the fact that the electron layer has a finite extension in the z direction). Many important applications have been made possible by structuring the 2DEG by gate electrodes. Applying a negative potential to a gate electrode put on top of a GaAs:$Al_xGa_{1-x}As$ heterostructure will deplete the 2DEG below, which leads to the possibility to form narrow constrictions, quantum dots, and leads in which the number of channels (modes) that contribute to transport can be tailored.[1]

Transport through ultrasmall islands of a 2DEG (quantum dots) has created a lot of interest in the past decade because of interaction (Coulomb-blockade) effects. Recently, transport through systems of double or multiple quantum dots under Coulomb-blockade conditions has been studied both theoretically[2–9] and experimentally.[10–13] Related experiments are described in Ref. 14, which investigates transport through one dot coupled to a second transverse dot (which is not connected to the two reservoirs).

In two early theoretical papers,[3] transport through a double dot was studied for the case in which both dots are coupled to a single gate voltage and the charging energies of the two dots are incommensurate. Since transport is only possible if rungs of both ''Coulomb ladders'' are situated at the Fermi energy, transport will occur for pseudorandom values of the gate voltage. This phenomenon was called *stochastic Coulomb blockade*.[3] Matveev *et al.*[7] investigated the case in which tunneling between the dots is much stronger than tunneling to the outside reservoirs. They were able to map the double dot onto an effective single dot and to apply earlier work on the electron box to this case.

In this work, we will study transport through double quantum dots under Coulomb-blockade conditions for both low and high transparency between the dots. The transparency of the junctions coupling the dots to the left and right reservoirs is always assumed to be low. Consequently, we perform an expansion in the tunneling rate and derive a master equation for the occupation probabilities of the levels in the two dots. We also take into account time-dependent gate and transport voltages leading to photon-assisted tunneling. This has become a very active area recently (see Refs. 15–32 and Refs. 33 and 34 for reviews).

The paper is organized as follows. In Sec. II we introduce the Hamiltonian of two quantum dots connected by a tunnel junction and coupled capacitively. We discuss the model and the new features produced by the second dot. In the following section, we assume the dots to be weakly coupled such that higher-order tunneling processes can be neglected. Both the tunnel junctions to the reservoirs and the junction between the dots are treated in lowest-order perturbation theory in the tunneling, i.e., we consider sequential tunneling. The master equation derived in this section allows us to treat arbitrarily many levels in each of the dots. In Sec. IV we treat the case of strong coupling between the two dots. For the case of a single level within each dot, we can diagonalize the resulting 2×2 matrix which amounts to including tunneling processes between the dots of all orders. We discuss the width of the resonances at high transport voltage and show that it is independent of temperature (as already discussed in van der Vaart *et al.*[11]). We also include time-dependent gate voltages, which allows us to discuss new spectroscopic features of the double dot, viz., the shape of

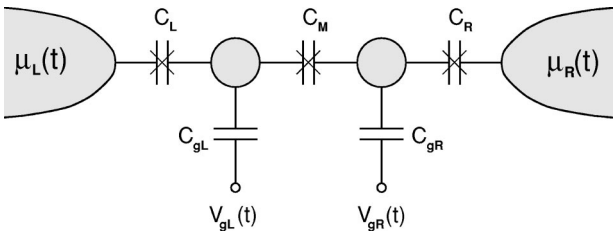

FIG. 1. System of a double quantum dot coupled to left and right reservoirs by tunnel junctions.

the resonance peaks as a functions of both gate voltages.

Some of the algebraic details of Secs. III and IV have been shifted to the appendixes.

II. MODEL

The system we have in mind is shown in Fig. 1. As usual, we describe the reservoirs by free fermions at temperature T and chemical potential μ_α, $\alpha=L,R$. The quantum dots are modeled by (zero-dimensional) sites coupled to the reservoirs and to each other by tunnel junctions. The Coulomb interaction is taken into account using the capacitive model, i.e., the tunnel junctions of the left, right, and middle junction are supposed to have capacitances C_L, C_R, and C_M. The chemical potential of the two dots can be modified by two gate voltage sources V_{gL} and V_{gR} coupled to the dots by gate capacitances C_{gL} and C_{gR}. Our formalism can also be used for time-dependent voltages, i.e., we start with a general Hamiltonian of the form $H(t)=H_{\rm res}(t)+H_D(t)+H_T$, where

$$H_{res}(t)=\sum_{\alpha=L,R}\sum_{k}\epsilon_{k\alpha}(t)a^\dagger_{k\alpha}a_{k\alpha} \tag{1}$$

describes the two reservoirs. Here, $\epsilon_{k\alpha}(t)=\epsilon^0_{k\alpha}+\Delta_\alpha(t)$.

The interaction effects on the two dots are described by

$$H_D(t)=\sum_{\alpha=L,R}\sum_{l_\alpha}\overline{\epsilon}_{l_\alpha}(t)c^\dagger_{l_\alpha}c_{l_\alpha}+H_{\rm ch}(\hat{N}_{DL},\hat{N}_{DR}), \tag{2}$$

where

$$H_{\rm ch}(\hat{N}_{DL},\hat{N}_{DR})=\gamma_L\hat{N}_{DL}(\hat{N}_{DL}-1)+\gamma_R\hat{N}_{DR}(\hat{N}_{DR}-1)+\gamma_M\hat{N}_{DL}\hat{N}_{DR}. \tag{3}$$

Here, $\hat{N}_{D\alpha}$ are the particle number operators of the left and right dot. The parameters γ_α are functions of the capacitances and are derived in Appendix A. The one-particle levels ϵ_{l_α} can be shifted by a time-dependent gate voltage leading to effective levels $\overline{\epsilon}_{l_\alpha}(t)=\epsilon_{l_\alpha}+\Delta_{D\alpha}(t)$, see Appendix A.

The dots are coupled to the reservoirs and to each other by the tunneling Hamiltonian

$$H_T=\sum_\alpha\sum_{k,l_\alpha}(T_{kl_\alpha}a^\dagger_{k\alpha}c_{l_\alpha}+\text{H.c.})+\sum_{l_L,l_R}(T_{l_Ll_R}c^\dagger_{l_L}c_{l_R}+\text{H.c.}). \tag{4}$$

The basic phenomenon that we are interested in is the Coulomb blockade. As in the case of one dot, transport through the system is possible only (to lowest order in the tunneling) if the gate voltages are adjusted appropriately, that is, if the following three conditions are satisfied: at the left junction,

$$\mu_L\geqslant\overline{\epsilon}_{N_{DL}+1}+E_{\rm ch}(N_{DL}+1,N_{DR})-E_{\rm ch}(N_{DL},N_{DR}); \tag{5}$$

at the middle junction,

$$\overline{\epsilon}_{l_L}+E_{\rm ch}(N_{DL}+1,N_{DR})=\overline{\epsilon}_{l_R}+E_{\rm ch}(N_{DL},N_{DR}+1); \tag{6}$$

and at the right junction,

$$\mu_R\leqslant\overline{\epsilon}_{l_R}+E_{\rm ch}(N_{DL},N_{DR}+1)-E_{\rm ch}(N_{DL},N_{DR}), \tag{7}$$

see Fig. 2.

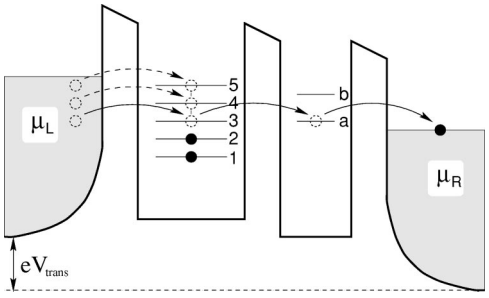

FIG. 2. Potential landscape of a double quantum dot at finite transport voltage (nonlinear case).

Since the levels in the two dots are discrete, energy conservation leads to the equality sign in Eq. (6). This is an important difference from the case of a single dot for which tunneling out of the dot is always into a continuum of states. Higher-order tunneling processes will lead to a broadening of these discrete states. We assume the coupling to the reservoirs to be weak, i.e., the lifetime of the dot states is determined mostly by the transmission through the barrier between the dots. The tunneling matrix element of this barrier will therefore provide a new energy scale; if the transport voltage $\mu_L-\mu_R$ is large, $eV_{\rm trans}\gg|T_{l_Ll_R}|$, the width of the resonance peaks will be determined not by the temperature, but by the matrix element $|T_{l_Ll_R}|$ itself.[11]

The ground state of the two dots as a function of the gate voltages is illustrated in Fig. 3. Inside each hexagon, the

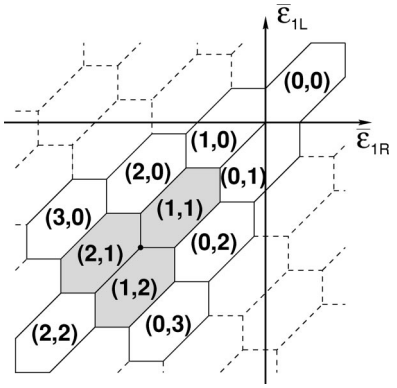

FIG. 3. Ground-state occupation numbers (N_{DL},N_{DR}) of the system as a function of the two gate voltages. Transport is only possible in points where three regions meet, whereas within each hexagon electron numbers are fixed.

particle numbers on each dot are well-defined and fixed, i.e., no transport is possible in linear response (and to lowest order in the tunneling). At the corners of each hexagon, three states are energetically degenerate, e.g., the system can go from state (1,1) via (2,1) and (1,2) to state (1,1), which means that one electron has passed through both dots.

In the following two sections, we will calculate this current first in the sequential (incoherent) limit which is realized if the tunneling between the dots is weak. We will also treat the coherent case if there is only one level in each dot in the relevant energy range. In this case, we can diagonalize the effective (2×2) matrix and take into account tunnel processes to all orders.

III. SEQUENTIAL TUNNELING

In order to examine the transport properties of the system for sequential tunneling, we calculate the current through the system as a function of the transport voltage $V_{\rm trans}$ and the gate voltages V_{gL}, V_{gR} or alternatively $\overline{\epsilon}_{1L}, \overline{\epsilon}_{1R}$ (which are linear functions of the gate voltages). The current from reservoir α to dot α ($\alpha = L,R$) is given by

$$I_\alpha^{\rm tun}(t) = -e\frac{d}{dt}\langle \hat{N}_\alpha \rangle_{\varrho(t)}. \tag{8}$$

The tunneling part of the Hamiltonian introduced in Sec. II is treated as a small perturbation. After changing to the interaction picture $[H_T \to H_{TW}(t)]$ we perform an expansion in $H_{TW}(t)$ and obtain for the current

$$I_\alpha^{\rm tun}(t) = -e\sum_{s,s'}\int_{-\infty}^{t}\Gamma_{s-s'}(t,t')[N_\alpha(s')-N_\alpha(s)]P_s(t')dt', \tag{9}$$

where $|s\rangle$ are eigenstates of the unperturbed part $H_0(t) = H_{\rm res}(t)+H_D(t)$ of the Hamiltonian and where $P_s(t) = \langle s|\varrho(t)|s\rangle$ indicates diagonal elements of the density matrix. The number of particles in the reservoir α is given by $N_\alpha = \langle s|\hat{N}_\alpha|s\rangle$ and the transition rate from $|s\rangle$ to $|s'\rangle$ is denoted by $\Gamma_{s-s'}(t,t')$:

$$\Gamma_{s-s'}(t,t') = \frac{2}{\hbar^2}{\rm Re}\{\langle s|H_{TW}(t)|s'\rangle\langle s'|H_{TW}(t')|s\rangle\}. \tag{10}$$

The occupation probabilities $P_s(t)$ are calculated by using the master equation with the rates given in Eq. (10):

$$\dot{P}_s(t) = \sum_{s'}\int_{-\infty}^{t}\Gamma_{s-s'}(t,t')[P_{s'}(t')-P_s(t')]dt'. \tag{11}$$

The states $|s\rangle$ can be factorized in a part $|\phi\rangle$ describing the reservoirs and a part $|r\rangle$ for the two dots.

Since we assume weak coupling between reservoirs and dots, we can factorize

$$P_s(t) = \langle s|\varrho(t)|s\rangle = \langle \phi, r|\varrho(t)|r,\phi\rangle = P_{\phi,\rm eq}P_r(t) \tag{12}$$

in an equilibrium part $P_{\phi,\rm eq}$ for the reservoirs and a part $P_r(t)$ for the system of the two dots. The derivation of the Fourier-transformed master equation and the Fourier coefficients of the current is given in Appendix B.

The result is in qualitatively good agreement with the experiments of van der Vaart *et al.*[11]

IV. COHERENT TUNNELING

Now we would like to address strong tunneling through the barrier between the dots. In this case, it is not sufficient to consider only lowest-order processes. For the simplest situation of only one level in each dot, we obtain an effective 2×2 problem and a significantly simplified charging part of the Hamiltonian:

$$H_{\rm ch}(\hat{N}_{DL},\hat{N}_{DR}) = \gamma_M c_L^\dagger c_L c_R^\dagger c_R. \tag{13}$$

We shift the part of the tunneling operator describing the barrier between the dots in the operator $H_D(t)$. Writing $H_D(t)$ in the occupation number basis $|r\rangle = |n_L, n_R\rangle$, i.e., $\{|1\rangle = |0,0\rangle,\ |2\rangle = |1,1\rangle,\ |3\rangle = |0,1\rangle,\ |4\rangle = |1,0\rangle\}$, we obtain

$$H_D(t) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \overline{\epsilon}_{LR} & 0 & 0 \\ 0 & 0 & \overline{\epsilon}_R & T_{LR}^* \\ 0 & 0 & T_{LR} & \overline{\epsilon}_L \end{pmatrix}, \tag{14}$$

where $\overline{\epsilon}_{LR} \equiv \overline{\epsilon}_L + \overline{\epsilon}_R + \gamma_M$. Only the states $|1,0\rangle$ and $|0,1\rangle$ are coupled by the middle barrier, therefore Eq. (14) will split up in a diagonal part and a 2×2 matrix $H_{D,\rm red}(t)$, which has to be diagonalized.

We assume the time dependence to be of the form $\Delta_D(t) \equiv \Delta_{DL}(t) = \Delta_{DR}(t)$ leading to

$$H_{D,\rm red}(t) = H_{D,\rm red}^0 + \Delta_D(t) = \begin{pmatrix} \epsilon_R & T_{LR}^* \\ T_{LR} & \epsilon_L \end{pmatrix} + \Delta_D(t), \tag{15}$$

where the time-dependent part is proportional to the unit matrix.

The time-independent matrix is then trivially diagonalized, leading to the eigenvalues

$$\epsilon_+ = \epsilon_L + \hat{\epsilon} \quad \text{and} \quad \epsilon_- = \epsilon_R - \hat{\epsilon}, \tag{16}$$

where

$$\hat{\epsilon} = \frac{1}{2}[(\epsilon_R - \epsilon_L) - \sqrt{(\epsilon_R-\epsilon_L)^2 + 4|T_{LR}|^2}]. \tag{17}$$

With this convention, $\epsilon_+ < \epsilon_-$. The (orthonormal) eigenstates have the form

$$\begin{aligned} |n_+ = 1, n_- = 0\rangle &= \frac{T_{LR}}{\sqrt{\hat{\epsilon}^2 + |T_{LR}|^2}}|n_L = 1, n_R = 0\rangle \\ &\quad + \frac{\hat{\epsilon}}{\sqrt{\hat{\epsilon}^2 + |T_{LR}|^2}}|n_L = 0, n_R = 1\rangle, \\ |n_+ = 0, n_- = 1\rangle &= \frac{\hat{\epsilon}}{\sqrt{\hat{\epsilon}^2 + |T_{LR}|^2}}|n_L = 1, n_R = 0\rangle \\ &\quad - \frac{T_{LR}^*}{\sqrt{\hat{\epsilon}^2 + |T_{LR}|^2}}|n_L = 0, n_R = 1\rangle, \end{aligned} \tag{18}$$

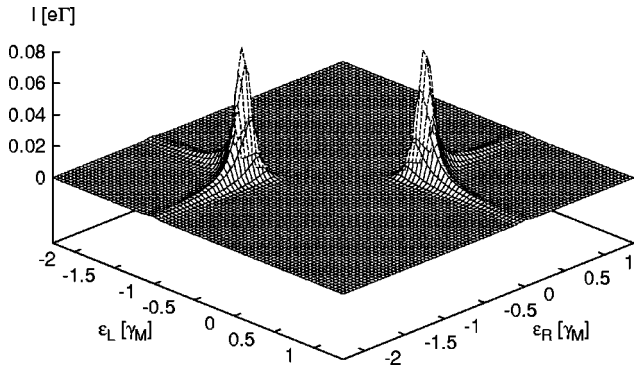

FIG. 4. Current (in units of $e\Gamma$) at intermediate coupling ($|T_{LR}|=0.072\gamma_M$) between the two dots. Here, gate and transport voltages are time-independent [$k_BT=0.035\gamma_M, eV_{\rm trans}=0.023\gamma_M$, with $\Gamma\equiv\Gamma_L\Gamma_R/(\Gamma_L+\Gamma_R)$].

where n_+ (n_-) are the occupation numbers of the effective levels with the energy ϵ_+ (ϵ_-). In order to write the Hamiltonian in the new basis, we define creation and annihilation operators ($c_+^\dagger, c_+, c_-^\dagger, c_-$) for the eigenstates in Eq. (18),

$$c_+^\dagger=\frac{T_{LR}}{\sqrt{\hat\epsilon^2+|T_{LR}|^2}}\,c_L^\dagger+\frac{\hat\epsilon}{\sqrt{\hat\epsilon^2+|T_{LR}|^2}}\,c_R^\dagger, \tag{19}$$

$$c_+=\frac{T_{LR}^*}{\sqrt{\hat\epsilon^2+|T_{LR}|^2}}\,c_L+\frac{\hat\epsilon}{\sqrt{\hat\epsilon^2+|T_{LR}|^2}}\,c_R,$$

$$c_-^\dagger=\frac{\hat\epsilon}{\sqrt{\hat\epsilon^2+|T_{LR}|^2}}\,c_L^\dagger-\frac{T_{LR}^*}{\sqrt{\hat\epsilon^2+|T_{LR}|^2}}\,c_R^\dagger,$$

$$c_-=\frac{\hat\epsilon}{\sqrt{\hat\epsilon^2+|T_{LR}|^2}}\,c_L-\frac{T_{LR}}{\sqrt{\hat\epsilon^2+|T_{LR}|^2}}\,c_R.$$

This leads to the following Hamiltonian (which is diagonal in the occupation number basis) describing the two dots and the barrier between them:

$$H_D(t)=[\epsilon_++\Delta_D(t)]c_+^\dagger c_++[\epsilon_-+\Delta_D(t)]c_-^\dagger c_- +\gamma_M c_+^\dagger c_+ c_-^\dagger c_-. \tag{20}$$

Whereas the reservoir Hamiltonian is unchanged, the tunneling part in the new basis reads

$$H_T=\frac{1}{\sqrt{\hat\epsilon^2+|T_{LR}|^2}}\sum_k\{T_{kL}(T_{LR}a_{kL}^\dagger c_++\hat\epsilon a_{kL}^\dagger c_-) +T_{kR}(\hat\epsilon a_{kR}^\dagger c_+-T_{LR}^* a_{kR}^\dagger c_-)+\text{H.c.}\}. \tag{21}$$

Now we perform an expansion in H_T and write down a master equation for the effective states $|n_+,n_-\rangle$ as in Sec. III. Further details of the calculation and the system of linear equations for the occupation probabilities are given in Appendix C.

In the following, we want to discuss the results first for the simple case (time-independent voltages) and then for the case with time-dependent gate or transport voltages. In both cases we assume the transport voltage and the temperature to be small compared with the charging energy and the matrix element $|T_{LR}|$. As shown in Fig. 3, transport through the system (in the lowest order) is only possible in those points where three hexagons meet, i.e., (0,0), (1,0), and (0,1) or (0,1), (1,1), and (1,0). Varying $\overline{\epsilon_L}$ and $\overline{\epsilon_R}$ [which are linear functions of the gate voltages, see Eq. (A5)] we expect to observe two peaks along the diagonal of Fig. 4. One of them occurs when the level ϵ_+ is equal to the Fermi energy, see Fig. 5. The other one shows up when $\overline{\epsilon_L}$ and $\overline{\epsilon_R}$ is shifted down by $2|T_{LR}|+\gamma_M$. Now, an electron at the Fermi energy can occupy the level ϵ_- because its energy is high enough to provide the charging energy necessary to bring in the second electron.

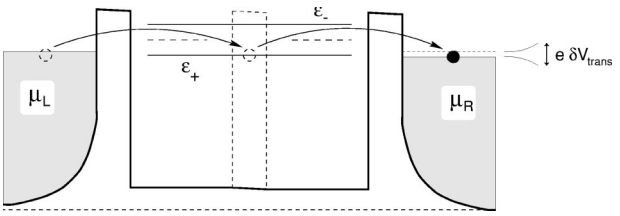

FIG. 5. Transport via the effective state ϵ_+ obtained by diagonalizing the matrix given in Eq. (14).

Having diagonalized the system of the two levels in two dots, we have taken into account tunnel processes of all orders between them. One important consequence of the higher-order terms is the shoulders of the peaks shown in Fig. 4 leading to a boomerang-like shape. This can be easily understood considering transport through level ϵ_+ and noting that the energy of this level will not change substantially when we start with $\overline{\epsilon_L}=\overline{\epsilon_R}$ and shift either $\overline{\epsilon_L}$ or $\overline{\epsilon_R}$ to higher energies, see Eq. (16). The shoulders get less pronounced on increasing either $\overline{\epsilon_L}$ or $\overline{\epsilon_R}$ because the coupling of the level ϵ_+ to the reservoirs gets weaker. If we consider transport through the level ϵ_-, we get the same situation on lowering one of the levels $\overline{\epsilon_\alpha}$. This means that the boomerang-like structures in Fig. 4 are opened towards higher (lower) values of $\overline{\epsilon_\alpha}$ for the level at ϵ_+ (ϵ_-). Increasing the coupling $|T_{LR}|$ of the dots to each other produces more pronounced shoulders and enlarges the distance of the peaks.

Time-dependent transport voltages or gate voltages lead to side bands of the levels ϵ_+ and ϵ_- at distance $n\hbar\omega$. Now transport is also possible through these side bands, which leads to satellite peaks (Fig. 6), i.e., copies of the boomerang-like structures shifted by $\pm\hbar\omega$. In Fig. 7 we show a contour plot of this situation and in Fig. 8 the dc current for the same parameters.

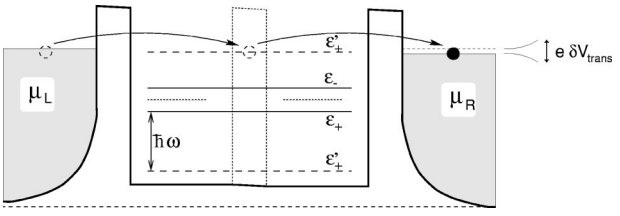

FIG. 6. Transport through a side band of ϵ_+ giving rise to the satellite peaks shown in the following two figures.

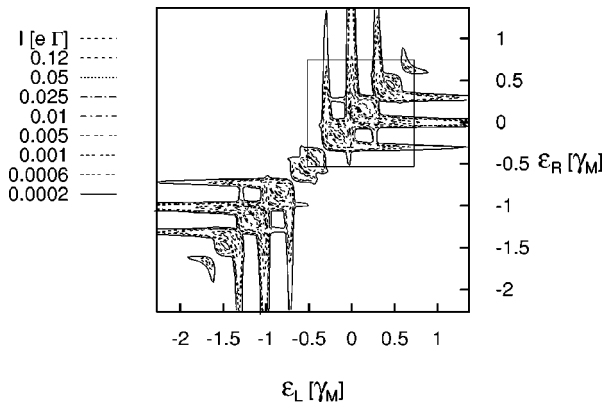

FIG. 7. Contour plot of the dc current (in units of $e\Gamma$) in the linear case at intermediate coupling ($|T_{LR}|=0.072\gamma_M$) and with periodic time dependence of the gate and transport voltages [$k_BT=0.011\gamma_M$, $eV_{\rm trans}=0.023\gamma_M$, $\hbar\omega=0.3\gamma_M$, $\Delta_0^\alpha=0.23\gamma_M$, where $\Gamma\equiv\Gamma_L\Gamma_R/(\Gamma_L+\Gamma_R)$].

Another effect of time-dependent voltages is that the occupation probability is finite for the state in which the level ϵ_- is occupied and ϵ_+ is empty. Therefore, it is possible to observe peaks which are hidden in the time-independent case,[19] see Fig. 9

The height of these peaks depends strongly on the ratio between $\hbar\omega$ and coupling $|T_{LR}|$ (remember that the difference $\epsilon_--\epsilon_+$ is given by $2|T_{LR}|$). The contribution of the side bands is modulated by Bessel functions, i.e., only those side bands with energies $\epsilon_\pm\pm\hbar\omega$ contribute significantly. For $2|T_{LR}|>\hbar\omega$, the peaks hidden in the time-independent case are small, because the first side band of ϵ_+ is at lower energy than ϵ_- and therefore the probability for the state in which ϵ_- is occupied and ϵ_+ is empty becomes nearly negligible. For the opposite case ($2|T_{LR}|<\hbar\omega$) these peaks are pronounced. The boomerang-like shoulders are much shorter for peaks that are hidden in the time-independent case and for satellite peaks which belong to them for the same reason (see Fig. 7). They disappear when either $\overline{\epsilon_L}$ or $\overline{\epsilon_R}$ is shifted far enough that ϵ_+ (ϵ_-) passes the first side band of ϵ_- (ϵ_+).

Another important effect visible in transport through two coupled dots is that the peaks are not necessarily broadened by temperature.[11] When we fix the levels in one dot and shift the levels in the other one up (down), the energy scale of $|T_{LR}|$ becomes important. For small temperatures (at which

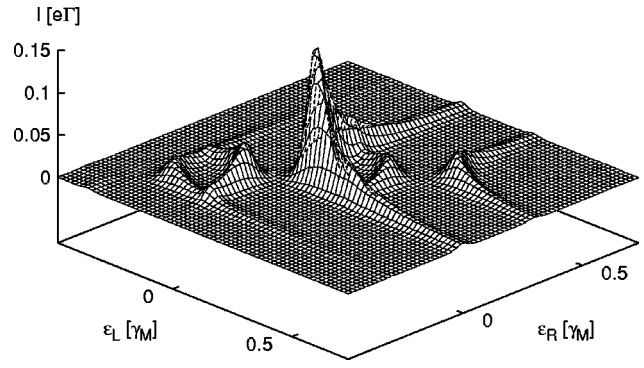

FIG. 8. dc current (in units of $e\Gamma$) as a function of the two gate voltages in the square region marked in Fig. 7.

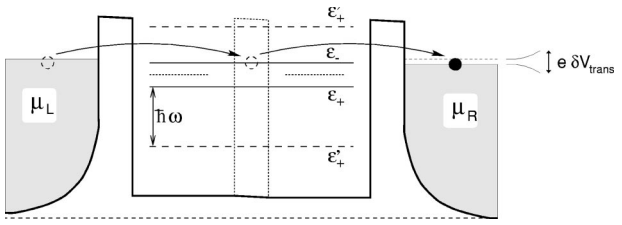

FIG. 9. Although the state ϵ_- is occupied, there is a finite probability for the state ϵ_+ to be empty because of tunneling out of the side band ϵ'_+ above the Fermi energy. Therefore, peaks are observable which are hidden when gate and transport voltages are time-independent.

the assumption of no coupling to an external bath is justified) and for $eV_{\rm trans}\gg|T_{LR}|$, the width of the peaks is determined by $|T_{LR}|$ even for $k_BT\gg|T_{LR}|$ (see Fig. 10).

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APPENDIX A: HAMILTONIAN

In this section we will describe the charging part $H_{\rm ch}(\hat{N}_{DL},\hat{N}_{DR})$ of the Hamiltonian more explicitly. We use the well-known capacitive model to calculate the charging energy. We will take into account the on-site interaction in each of the two dots and also the interaction between the two dots. The relevant capacities and voltages are shown in Fig. 11.

After a systematic and straightforward calculation, we obtain the following expression for the Hamiltonian describing the two dots (charging plus one-particle energies; tunneling is excluded):

$$H_D(t)=\sum_{\alpha=L,R}\sum_{l_\alpha}\overline{\epsilon}_{l_\alpha}(t)c^\dagger_{l_\alpha}c_{l_\alpha}+H_{\rm ch}(\hat{N}_{DL},\hat{N}_{DR}) \quad \text{(A1)}$$

with

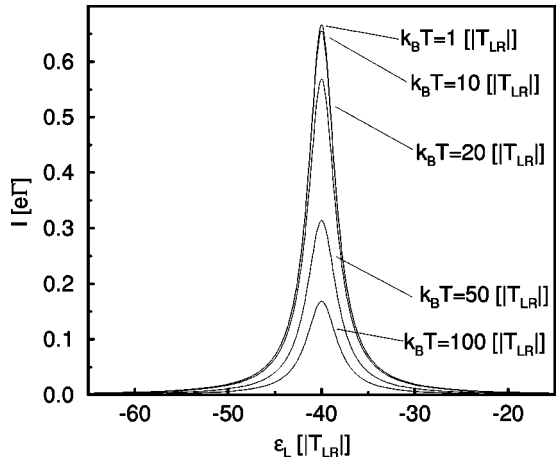

FIG. 10. For $|T_{LR}|<k_BT$ and $|T_{LR}|\ll eV_{\rm trans}$ the width of the peaks becomes independent of temperature ($eV_{\rm trans}=80|T_{LR}|$).

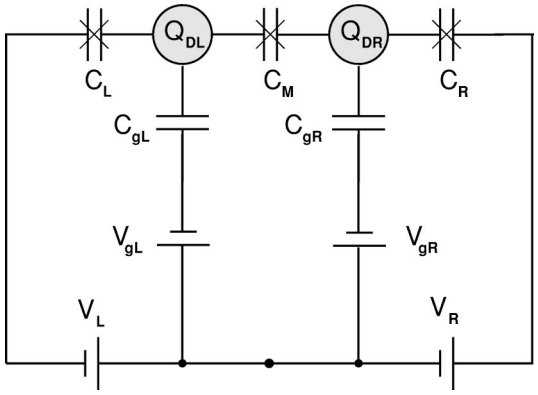

FIG. 11. Circuit diagram of the double quantum dot.

$$H_{\rm ch}(\hat{N}_{DL},\hat{N}_{DR})=\gamma_L\hat{N}_{DL}(\hat{N}_{DL}-1)+\gamma_R\hat{N}_{DR}(\hat{N}_{DR}-1)+\gamma_M\hat{N}_{DL}\hat{N}_{DR}. \tag{A2}$$

Here, we have introduced the constants $\gamma_L,\gamma_R,\gamma_M$ which depend on the capacitances as

$$\gamma_\alpha=\frac{C_{L,\Sigma}C_{R,\Sigma}}{C_{L,\Sigma}C_{R,\Sigma}-C_M^2}\frac{e^2}{2C_{\alpha,\Sigma}} \tag{A3}$$

for $\alpha=L,R$, and

$$\gamma_M\equiv\frac{2e^2C_M}{C_{L,\Sigma}C_{R,\Sigma}-C_M^2}, \tag{A4}$$

where $C_{\alpha,\Sigma}\equiv C_\alpha+C_{g\alpha}+C_M$ and $q_\alpha\equiv C_\alpha V_\alpha+C_{g\alpha}V_{g\alpha}$.

The time-dependent part of the charging energy is caused by time-dependent gate voltages. It is linear in the particle number operators and therefore leads to a shift of the one-particle energies,

$$\begin{aligned}\overline{\epsilon}_{l_L}(t)&=\epsilon^0_{l_L}-\frac{2}{e}\left[\gamma_L\left(q_L(t)-\frac{e}{2}\right)+\frac{1}{4}\gamma_Mq_R(t)\right],\\ \overline{\epsilon}_{l_R}(t)&=\epsilon^0_{l_R}-\frac{2}{e}\left[\gamma_R\left(q_R(t)-\frac{e}{2}\right)+\frac{1}{4}\gamma_Mq_L(t)\right].\end{aligned} \tag{A5}$$

The shift of the level energy consists of a constant and a time-dependent part; since we are not interested in the detailed dependences, we write the level energy as $\overline{\epsilon_{l_\alpha}}(t)=\epsilon_{l_\alpha}+\Delta_{D\alpha}(t)$.

APPENDIX B: MASTER EQUATION

In this section we show some details of the calculation of the Fourier-transformed master equation and the expression for the current. The unitary transformation of the Hamiltonian

$$U(t,t_0)=\exp\left(-\frac{i}{\hbar}\int_{t_0}^t[H_{\rm res}(\tau)+H_D(\tau)]d\tau\right) \tag{B1}$$

produces the part of the Hamiltonian describing the tunneling in the interaction picture. To calculate the matrix elements, we factorize the states $|s\rangle=|\phi\rangle|r\rangle$ into a part $|\phi\rangle$ describing the reservoirs which are assumed to be in thermal equilibrium and a part $|r\rangle$ describing the system of the two dots. As a consequence, we obtain for the rates in the master equation

$$\begin{aligned}\Gamma_{s-s'}(t,t')=&\frac{2\pi}{\hbar}\sum_{\alpha=L,R}\sum_{k,l_\alpha}|T_{kl_\alpha}|^2\delta_{\phi'\phi_{k\alpha}}\delta_{r'r_{l_\alpha}}\\ &\times\{A_{\Delta_\alpha,\Delta_{D\alpha}}(E_{k\alpha,l_\alpha},t,t')n_{k\alpha}(\phi)[1-n_{l_\alpha}(r)]\\ &+A_{\Delta_{D\alpha},\Delta_\alpha}(E_{l_\alpha,k\alpha},t,t')[1-n_{k\alpha}(\phi)]n_{l_\alpha}(r)\}\\ &+\frac{2\pi}{\hbar}\sum_{l_L,l_R}|T_{l_Ll_R}|^2\delta_{\phi'\phi}\delta_{r'r_{l_L,l_R}}\\ &\times\{A_{\Delta_{DL},\Delta_{DR}}(E_{l_L,l_R},t,t')n_{l_L}(r)[1-n_{l_R}(r)]\\ &+A_{\Delta_{DR},\Delta_{DL}}(E_{l_R,l_L},t,t')[1-n_{l_L}(r)]n_{l_R}(r)\}\end{aligned} \tag{B2}$$

with

$$\langle\phi|\phi'\rangle=\prod_{k\alpha}\delta_{n_{k\alpha}(\phi)n_{k\alpha}(\phi')}\equiv\delta_{\phi\phi'}. \tag{B3}$$

The states $\phi_{k\alpha}$ differ from the ϕ in that the occupation of the level $k\alpha$ is inverted, i.e.,

$$\phi_{k\alpha}:\begin{cases}n_{k'\alpha'}(\phi_{k\alpha})=n_{k'\alpha'}(\phi) & \text{for } k'\alpha'\neq k\alpha\\ n_{k'\alpha'}(\phi_{k\alpha})=1-n_{k'\alpha'}(\phi) & \text{for } k'\alpha'=k\alpha.\end{cases} \tag{B4}$$

The states r_{l_α} and $r_{l_Ll_R}$ are defined in the same way, i.e., the occupation of the level given in the index is inverted. In order to write the rates more clearly, we also defined the function $A_{x,y}(E,t,t')$ by

$$A_{x,y}(E,t,t')=\frac{1}{\pi\hbar}\mathrm{Re}\left\{\exp\left(\frac{i}{\hbar}\int_{t'}^t[x(\tau)-y(\tau)]d\tau+\frac{i}{\hbar}E(t-t')\right)\right\}. \tag{B5}$$

We divide the energy of the level $k\alpha$ and l_α into a time-independent part and a time-dependent part $\epsilon_{k\alpha}(t)=\epsilon^0_{k\alpha}+\Delta_\alpha(t)$, $\overline{\epsilon}_{l_\alpha}(t)=\epsilon_{l_\alpha}+\Delta_{D\alpha}(t)$, and denote the time-independent energy differences for the various processes by

$$\begin{aligned}E_{k\alpha,l_\alpha}&=\epsilon^0_{k\alpha}-\epsilon_{l_\alpha}-U_\alpha(N_{D\alpha}(r),N_{D\bar{\alpha}}(r))\\ E_{l_\alpha,k\alpha}&=\epsilon_{l_\alpha}-\epsilon^0_{k\alpha}+U_\alpha(N_{D\alpha}(r)-1,N_{D\bar{\alpha}}(r))\\ E_{l_L,l_R}&=\epsilon_{l_L}-\epsilon_{l_R}+U_L(N_{DL}(r)-1,N_{DR}(r))-U_R(N_{DR}(r),N_{DL}(r)-1)\\ E_{l_R,l_L}&=\epsilon_{l_R}-\epsilon_{l_L}+U_R(N_{DR}(r)-1,N_{DL}(r))-U_L(N_{DL}(r),N_{DR}(r)-1).\end{aligned} \tag{B6}$$

Here

$$U_\alpha(N_{D\alpha},N_{D\bar{\alpha}})=E_{ch}(N_{D\alpha}+1,N_{D\bar{\alpha}})-E_{ch}(N_{D\alpha},N_{D\bar{\alpha}})$$
$$=2\gamma_\alpha N_{D\alpha}(r)+\gamma_M N_{D\bar{\alpha}} \quad \text{(B7)}$$

is the charging energy which is needed to bring another electron to the dot α when there are already $N_{D\alpha}$ electrons on the dot α and $N_{D\bar{\alpha}}$ electrons on the dot $\bar{\alpha}$ ($\bar{\alpha}=R$ when $\alpha=L$ and vice versa). Using these rates we can write the master equation in the following form:

$$\dot{P}_r(t)=-\frac{2\pi}{\hbar}\sum_\alpha\sum_{k,l_\alpha}|T_{kl_\alpha}|^2$$
$$\times\Bigg[\int_{-\infty}^{t}dt' A_{\Delta_\alpha,\Delta_{D\alpha}}(E_{k\alpha,l_\alpha},t,t')\{P_r(t')f_\alpha(\epsilon^0_{k\alpha})$$
$$-P_{r_{l_\alpha}}(t')[1-f_\alpha(\epsilon^0_{k\alpha})]\}(1-n_{l_\alpha})$$
$$+\int_{-\infty}^{t}dt' A_{\Delta_{D\alpha},\Delta_\alpha}(E_{l_\alpha,k\alpha},t,t')$$
$$\times\{P_r(t')[1-f_\alpha(\epsilon^0_{k\alpha})]-P_{r_{l_\alpha}}(t')f_\alpha(\epsilon^0_{k\alpha})\}n_{l_\alpha}\Bigg]$$
$$-\frac{2\pi}{\hbar}\sum_{l_L,l_R}|T_{l_Ll_R}|^2\Bigg[\int_{-\infty}^{t}dt' A_{\Delta_{DL},\Delta_{DR}}(E_{l_L,l_R},t,t')$$
$$\times\{P_r(t')-P_{r_{l_Ll_R}}(t')\}n_{l_L}(1-n_{l_R})+L\leftrightarrow R\Bigg], \quad \text{(B8)}$$

taking into account

$$\sum_\phi n_{k\alpha}(\phi)P_{\phi,\mathrm{eq}}=f_\alpha(\epsilon^0_{k\alpha}), \quad \text{(B9)}$$

where f is the Fermi function. To get an expression for the current, we insert the rates Eq. (B2) into Eq. (9) taking into account Eq. (B9) and obtain

$$I^{\mathrm{tun}}_\alpha(t)=\frac{2\pi e}{\hbar}\sum_r\sum_{k,l_\alpha}|T_{kl_\alpha}|^2\Bigg[\int_{-\infty}^{t}dt' A_{\Delta_\alpha,\Delta_{D\alpha}}(E_{k\alpha,l_\alpha},t,t')$$
$$\times P_r(t')f_\alpha(\epsilon^0_{k\alpha})[1-n_{l_\alpha}(r)]-\int_{-\infty}^{t}dt' A_{\Delta_{D\alpha},\Delta_\alpha}$$
$$\times(E_{l_\alpha,k\alpha},t,t')P_r(t')[1-f_\alpha(\epsilon^0_{k\alpha})]n_{l_\alpha}(r)\Bigg]. \quad \text{(B10)}$$

In the following we Fourier-transform the master equation first for the sequential case and then for the coherent case. We apply time-dependent gate or transport voltages with frequency ω,

$$\Delta^\alpha(t)=\Delta_\alpha(t)-\Delta_{D\alpha}(t)=\Delta^\alpha_0\sin(\omega t+\delta_\alpha),$$
$$\Delta^M(t)=\Delta_{DL}(t)-\Delta_{DR}(t)=\Delta^M_0\sin(\omega t+\delta_M), \quad \text{(B11)}$$

leading to probabilities and currents that are periodic with the same frequency, i.e., $P_r(t+T)=P_r(t)$ with $T=2\pi/\omega$. Therefore, we can expand, e.g., the probabilities in a Fourier series:

$$P_r(t)=\sum_{n=-\infty}^{\infty}P_r(n)e^{-in\omega t} \quad \text{(B12)}$$

and

$$P_r(n)=\frac{1}{T}\int_0^T P_r(t)e^{in\omega t}dt. \quad \text{(B13)}$$

Inserting Eq. (B11) and using the following identity for Bessel functions $J_k(z)$

$$\exp(iz\cos\phi)=\sum_{m=-\infty}^{\infty}i^m J_m(z)\exp(im\phi) \quad \text{(B14)}$$

and

$$J_m(-z)=J_{-m}(z), \quad \text{(B15)}$$

we can write the function $A_{x,y}(E,t,t')$ as

$$A_{\Delta_\alpha,\Delta_{D\alpha}}(E_{k\alpha,l_\alpha},t,t')=\frac{1}{\pi\hbar}\mathrm{Re}\left\{\exp\left(\frac{i}{\hbar}\int_{t'}^{t}[\Delta^\alpha_0\sin(\omega\tau+\delta_\alpha)]d\tau+\frac{i}{\hbar}E_{k\alpha,l_\alpha}(t-t')\right)\right\}$$
$$=\frac{1}{2\pi\hbar}\left\{\exp\left(i\frac{\Delta^\alpha_0}{\hbar\omega}[\cos(\omega t'+\delta_\alpha)-\cos(\omega t+\delta_\alpha)]+\frac{i}{\hbar}E_{k\alpha,l_\alpha}(t-t')\right)+\mathrm{c.c.}\right\}$$
$$=\frac{1}{2\pi\hbar}\sum_{m,m'=-\infty}^{\infty}i^{m-m'}J_m\left(\frac{\Delta^\alpha_0}{\hbar\omega}\right)J_{m'}\left(\frac{\Delta^\alpha_0}{\hbar\omega}\right)e^{i(m-m')\delta_\alpha}$$
$$\times[e^{i\omega(mt'-m't)}e^{(i/\hbar)E_{k\alpha,l_\alpha}(t-t')}+e^{i\omega(mt-m't')}e^{-(i/\hbar)E_{k\alpha,l_\alpha}(t-t')}]. \quad \text{(B16)}$$

We include a finite lifetime for the quantum-mechanical levels in the dot described by an intrinsic width Γ_{br}. Now we are able to perform the integration over t' and obtain

$$\int_{-\infty}^{t} dt' A_{\Delta_\alpha,\Delta_{D\alpha}}(E_{k\alpha,l_\alpha},t,t')P_r(t') = \sum_{n,m,m'=-\infty}^{\infty} \frac{P_r(n)}{2\pi} i^{m-m'} e^{i(m-m')\delta_\alpha} J_m \times \left(\frac{\Delta_0^\alpha}{\hbar\omega}\right) J_{m'}\left(\frac{\Delta_0^\alpha}{\hbar\omega}\right) e^{i(m-m'-n)\omega t} \times h(E_{k\alpha,l_\alpha})_{m'+n,n-m}. \quad \text{(B17)}$$

Here we have introduced the function

$$h(E)_{r,s} = \frac{1}{\hbar\Gamma_{\rm br} - i(\hbar\omega r - E)} + \frac{1}{\hbar\Gamma_{\rm br} - i(\hbar\omega s + E)}. \quad \text{(B18)}$$

We insert $\int dE\,\delta(E-\epsilon^0_{k\alpha})$ in each contribution of the master equation describing transitions between the discrete states of the dots and the continuum of states of the reservoirs and substitute

$$\left(\int dE\right) 2\pi \sum_k |T_{kl_\alpha}|^2 \delta(E-\epsilon^0_{k\alpha}) = \Gamma^\alpha_{l_\alpha}\left(\int dE\right). \quad \text{(B19)}$$

This substitution is correct only if the tunneling matrix elements are independent of energy and if the density of states is constant, which is the case for energies close to the Fermi energy.

These results lead to the following expression for the first term S_1 of the master equation:

$$S_1 = -\frac{2\pi}{\hbar}\sum_\alpha \sum_{k,l_\alpha} |T_{kl_\alpha}|^2 \int_{-\infty}^{t} dt' A_{\Delta_\alpha,\Delta_{D\alpha}}(E_{k\alpha,l_\alpha},t,t') \times P_r(t') f_\alpha(\epsilon^0_{k\alpha})[1-n_{l_\alpha}(r)]$$
$$= -\sum_{\alpha,l_\alpha} \Gamma^\alpha_{l_\alpha} \sum_{n,m,m'} \frac{P_r(n)}{\hbar} i^{m-m'} e^{i(m-m')\delta_\alpha} J_m \times \left(\frac{\Delta_0^\alpha}{\hbar\omega}\right) J_{m'}\left(\frac{\Delta_0^\alpha}{\hbar\omega}\right) e^{i(m-m'-n)\omega t}[1-n_{l_\alpha}(r)] \times \frac{1}{2\pi}\int dE\, h(E_{k\alpha,l_\alpha})_{m'+n,n-m} f_\alpha(E), \quad \text{(B20)}$$

where we used the delta function $\delta(E-\epsilon^0_{k\alpha})$ to exchange E and $\epsilon^0_{k\alpha}$ in the energy difference $E_{k\alpha,l_\alpha}$.

Now it is possible to perform the integration over the continuum of energy states in the terms describing transitions between a dot and a reservoir. We obtain

$$\frac{1}{2\pi}\int dE\, h(E_{k\alpha,l_\alpha})_{m'+n,n-m} f_\alpha(E) = f_\alpha[i\hbar\Gamma_{\rm br} + \hbar\omega(m'+n) + E_{l_\alpha}(N_{D\alpha},N_{D\bar\alpha}) + \mu_\alpha] - \frac{i}{2\pi}\left\{\psi\left(\frac{1}{2} + i\frac{\beta}{2\pi}[\hbar\omega(m'+n) + E_{l_\alpha}(N_{D\alpha},N_{D\bar\alpha}) + i\hbar\Gamma_{\rm br}]\right) - \psi\left(\frac{1}{2} - i\frac{\beta}{2\pi}[\hbar\omega(n-m) - E_{l_\alpha}(N_{D\alpha},N_{D\bar\alpha}) + i\hbar\Gamma_{\rm br}]\right)\right\}, \quad \text{(B21)}$$

with the digamma function $\psi(x) = d\ln\Gamma(x)/dx$ and the energy expression $E_{l_\alpha}(N_{D\alpha},N_{D\bar\alpha}) = \epsilon_{l_\alpha} + U_\alpha(N_{D\alpha}(r),N_{D\bar\alpha}(r)) - \mu_\alpha$.

The definition of the function Y,

$$Y^\alpha_{l_\alpha}(N_{D\alpha},N_{D\bar\alpha})_{r,s} = f_\alpha[i\hbar\Gamma_{\rm br} + \hbar\omega r + E_{l_\alpha}(N_{D\alpha},N_{D\bar\alpha}) + \mu_\alpha] - \frac{i}{2\pi}\left\{\psi\left(\frac{1}{2} + i\frac{\beta}{2\pi}[\hbar\omega r + E_{l_\alpha}(N_{D\alpha},N_{D\bar\alpha}) + i\hbar\Gamma_{\rm br}]\right) - \psi\left(\frac{1}{2} - i\frac{\beta}{2\pi}[\hbar\omega s - E_{l_\alpha}(N_{D\alpha},N_{D\bar\alpha}) + i\hbar\Gamma_{\rm br}]\right)\right\}, \quad \text{(B22)}$$

allows us to simplify the Fourier-transformed master equation. We find a system of linear equations for the Fourier coefficients of the occupation probabilities (all particle numbers refer to the state $|r\rangle$ of the two dots):

$$\left(in'\hbar\omega - \sum_{\alpha,l_\alpha}\Gamma^\alpha_{l_\alpha} n_{l_\alpha}\right) P_r(n') = -\sum_{\alpha,l_\alpha}\Gamma^\alpha_{l_\alpha}(1-n_{l_\alpha})P_{r_{l_\alpha}}(n') + \sum_{\alpha,l_\alpha}\Gamma^\alpha_{l_\alpha}\sum_{n,m} i^{n-n'} e^{i(n-n')\delta_\alpha} J_m\left(\frac{\Delta_0^\alpha}{\hbar\omega}\right) J_{m+n'-n} \times \left(\frac{\Delta_0^\alpha}{\hbar\omega}\right) Y^\alpha_{l_\alpha}(N_{D\alpha}-n_{l_\alpha},N_{D\bar\alpha})_{m+n',n-m}(1-2n_{l_\alpha})[P_r(n)+P_{r_{l_\alpha}}(n)] + \sum_{l_L,l_R}|T_{l_Ll_R}|^2 \times \sum_{n,m} i^{n-n'} e^{i(n-n')\delta_M} J_m\left(\frac{\Delta_0^M}{\hbar\omega}\right) J_{m+n'-n}\left(\frac{\Delta_0^M}{\hbar\omega}\right)\{h(E_{l_L,l_R})_{m+n',n-m}[P_r(n)n_{l_L}(1-n_{l_R}) - P_{r_{l_L,l_R}}(n)(1-n_{l_L})n_{l_R}] + h(E_{l_R,l_L})_{n-m,n'+m}[P_r(n)(1-n_{l_L})n_{l_R} - P_{r_{l_L,l_R}}(n)n_{l_L}(1-n_{l_R})]\}. \quad \text{(B23)}$$

In the same way we Fourier-transform the expression for the current Eq. (B10) and obtain

$$I_\alpha^{\rm tun}(n')=-\frac{e}{\hbar}\sum_r\sum_{l_\alpha}\Gamma_{l_\alpha}^\alpha P_r(n')n_{l_\alpha}+\frac{e}{\hbar}\sum_r\sum_{l_\alpha}\Gamma_{l_\alpha}^\alpha\sum_{n,m}P_r(n)i^{n-n'}e^{i(n-n')\delta_\alpha}J_m$$
$$\times\left(\frac{\Delta_0^\alpha}{\hbar\omega}\right)J_{m+n'-n}\left(\frac{\Delta_0^\alpha}{\hbar\omega}\right)Y_{l_\alpha}^\alpha(N_{D\alpha}-n_{l_\alpha},\overline{N_{D\alpha}})_{m+n',n-m}. \tag{B24}$$

The intrinsic width $\Gamma_{\rm br}$ which we have included means that detailed balance may be violated leading to unphysical consequences such as regimes in which the conductance is negative. In order to restore detailed balance at the barrier between the dots, we assume the presence of an external bath coupled to the dots.

Consequently, we change the rates at the barrier between the dots such that transitions which absorb energy E out of the external bath are exponentially suppressed by a factor $\exp(-\beta\Delta E)$. This is equivalent to modifying the master equation Eq. (B23) by replacing the functions $h(E)_{r,s}$ by $\tilde{h}(E)_{r,s}$ defined in the following way:

$$\tilde{h}(E)_{r,s}=\frac{1}{\hbar\Gamma_{\rm br}-i(\hbar\omega r-E)}[1-\Theta(\hbar\omega r-E)(1-e^{\beta(E-\hbar\omega r)})]+\frac{1}{\hbar\Gamma_{\rm br}-i(\hbar\omega s+E)}$$
$$\times[1-\Theta(-\hbar\omega s-E)(1-e^{\beta(E+\hbar\omega s)})]. \tag{B25}$$

APPENDIX C: COHERENT TUNNELING BETWEEN THE DOTS

After the system of the two dots containing one level each has been exactly diagonalized, the calculation of the occupation probabilities and the current in the coherent case is performed as in Appendix B. We obtain the following system of linear equations for the Fourier coefficients of the occupation probabilities (note that all of the particle numbers refer to the state $|r\rangle$):

$$\{in'\hbar\omega[\hat{\epsilon}^2+|T_{LR}|^2]-[|T_{LR}|^2\Gamma_L+\hat{\epsilon}^2\Gamma_R]n_+-[\hat{\epsilon}^2\Gamma_L+|T_{LR}|^2\Gamma_R]n_-\}P_r(n')$$
$$=-[|T_{LR}|^2\Gamma_L+\hat{\epsilon}^2\Gamma_R](1-n_+)P_{r_+}(n')-[\hat{\epsilon}^2\Gamma_L+|T_{LR}|^2\Gamma_R](1-n_-)P_{r_-}(n')$$
$$+\Gamma_L\sum_{n,m}i^{n-n'}e^{i(n-n')\delta_L}J_m\left(\frac{\Delta_0^L}{\hbar\omega}\right)J_{m+n'-n}\left(\frac{\Delta_0^L}{\hbar\omega}\right)\{|T_{LR}|^2Y_+^L(n_-)_{m+n',n-m}(1-2n_+)[P_r(n)+P_{r_+}(n)]$$
$$+\hat{\epsilon}^2Y_-^L(n_+)_{m+n',n-m}(1-2n_-)[P_r(n)+P_{r_-}(n)]\}+\Gamma_R\sum_{n,m}i^{n-n'}e^{i(n-n')\delta_R}J_m\left(\frac{\Delta_0^R}{\hbar\omega}\right)J_{m+n'-n}\left(\frac{\Delta_0^R}{\hbar\omega}\right)$$
$$\times\{\hat{\epsilon}^2Y_+^R(n_-)_{m+n',n-m}(1-2n_+)[P_r(n)+P_{r_+}(n)]+|T_{LR}|^2Y_-^R(n_+)_{m+n',n-m}(1-2n_-)[P_r(n)+P_{r_-}(n)]\}. \tag{C1}$$

Here, we define

$$Y_\pm^\alpha(n_\mp)_{r,s}=f_\alpha(i\hbar\Gamma_{\rm br}+\hbar\omega r+E_{\alpha,\pm}+\mu_\alpha)-\frac{i}{2\pi}\left\{\psi\left(\frac{1}{2}+i\frac{\beta}{2\pi}(\hbar\omega r+E_{\alpha,\pm}+i\hbar\Gamma_{\rm br})\right)-\psi\left(\frac{1}{2}-i\frac{\beta}{2\pi}(\hbar\omega s-E_{\alpha,\pm}+i\hbar\Gamma_{\rm br})\right)\right\} \tag{C2}$$

where $E_{\alpha,\pm}=\epsilon_\pm+\gamma_M n_\mp-\mu_\alpha$.

The Fourier coefficients of the tunneling current are given in the coherent case by (we only give $I_L^{\rm tun}$ since current conservation leads to $I_R^{\rm tun}=-I_L^{\rm tun}$)

$$I_L^{\rm tun}(n')=-\frac{e}{\hbar}\sum_r\frac{\Gamma_L}{\hat{\epsilon}^2+|T_{LR}|^2}P_r(n')[|T_{LR}|^2n_++\hat{\epsilon}^2n_-]+\frac{e}{\hbar}\sum_r\frac{\Gamma_L}{\hat{\epsilon}^2+|T_{LR}|^2}\sum_{n,m}P_r(n)i^{n-n'}e^{i(n-n')\delta_L}J_m$$
$$\times\left(\frac{\Delta_0^L}{\hbar\omega}\right)J_{m+n'-n}\left(\frac{\Delta_0^L}{\hbar\omega}\right)\{|T_{LR}|^2Y_+^L(n_-)_{m+n',n-m}+\hat{\epsilon}^2Y_-^L(n_+)_{m+n',n-m}\}. \tag{C3}$$

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