Theory of quasi-one-dimensional electron liquids with spin-orbit coupling

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(Received 3 June 2000; revised manuscript received 5 September 2000)

We present a model for the study of spin-orbit coupling in interacting quasi-one-dimensional electron systems (Q1DES's) and solve it exactly to find both single- and two-particle characteristics. We show that the combination of spin-orbit coupling and electron-electron interactions results in the replacement of separate spin and charge excitations with two kinds of bosonic mixed-spin-charge excitation, and a characteristic modification of the spectral function and single-particle density of states. Analysis of the two-particle characteristics indicates that as the strength of the spin-orbit coupling is increased it is likely that a Q1DES would undergo a magnetic ordering phase transition. We also study the transport properties of a Q1DES and prove, in particular, that its ballistic conductance is not renormalized by the spin-orbit interaction.

I. INTRODUCTION

In contemporary condensed matter physics there are a great variety of electron systems that can be considered as quasi-one-dimensional (Q1D). Among them are semiconductor heterostructures,[1] carbon nanotubes,[2] and conducting polymers.[3] Q1D systems combine the richness of observable physical properties with the possibility that exact solutions to nontrivial interacting problems may be found. The major theoretical advance in this field was the formulation and solution of the Tomonaga-Luttinger model,[4] which revealed the generic physical behavior expected in interacting Q1D systems: the separation of charge and spin degrees of freedom, and the anomalous scaling of correlation functions.[5]

It is known that an electron moving in an electric field experiences not only an electrostatic force but also a relativistic influence that couples the spin and orbital degrees of freedom of the electron and is referred to as spin-orbit (SO) interaction or spin-orbit coupling. Despite its purely relativistic nature, the SO coupling appears to be important in Q1D systems because it can result, e.g., in a significant modification to the band structure (see, e.g., experimental[6,7] and theoretical[8,9] papers).

In most Q1D systems, there are two distinct sources of electrical field that give rise to SO coupling: (i) microscopic periodic modulation arising from the crystal potential and producing an observable SO coupling in materials lacking inversion symmetry (GaAs and many III-V and II-VI compounds); (ii) macroscopic electric fields induced by nearby unbalanced charge and potentials applied to surface gates. The latter fields confine lateral motion of electrons and are responsible for the very existence of Q1D systems (or quantum wires). The electric field at a heterojunction or metal-oxide-semiconductor (MOS) interface produces a particularly strong intrinsic SO coupling in 2D electron and hole systems. Owing to the different density dependences of SO couplings from these two sources of electric field they can be distinguished experimentally.[10,7] In contrast to the crystal-field SO coupling, the strength of the quantum-well-induced coupling can be changed and controlled reliably by simultaneous adjustment of two surface gates, one above the 2D gas and one below.[11] In a variety of experimental systems including 2D electron and hole gases in Si-MOS field-effect transistors[10] and GaAs-AlGaAs heterostructures[7] the strength of the quantum-well SO coupling is dominant. It is therefore reasonable to think of SO coupling in Q1D systems as resulting mainly from macroscopic electric fields and being controllable by applying potentials to surface gates.

In strictly 1D and 2D clean systems the single-particle wave functions are plane waves. Therefore the quantum-well SO coupling, which is proportional to the electron momentum and to the electron spin, has a simple effect on the band structure: the energy branches corresponding to spin-up and spin-down electron states split ''horizontally'' in momentum space by a momentum-independent value.[9,12] In Q1D systems in the presence of a confining potential the transverse single-particle wave functions are more complicated (e.g., the Hermite functions for a parabolic confinement). As a result, along with the horizontal splitting of the spin branches, SO coupling leads to a deformation of each branch of the single-particle dispersion relations (see Fig. 1 and Ref. 13). The most important feature of this deformation is that each branch loses its vertical symmetry axis and the electron Fermi velocities become different for different directions of motion. In other words, a breakdown of chiral symmetry occurs. Since this effect was predicted only recently, an experimental measurement of its strength is not available yet. Nevertheless, our calculations[13] indicate that the difference of the Fermi velocities monotonically increases as the SO coupling is enhanced, and in some Q1D semiconductor sys-

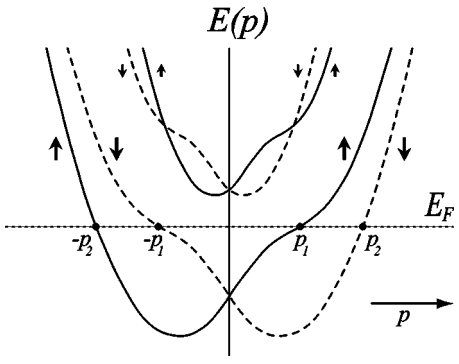

FIG. 1. Single-electron energy spectrum in a quantum wire with spin-orbit interaction. The dotted line is the Fermi energy level $E = E_F$ and $\pm p_{1,2}$ are the Fermi momenta of respective groups of electrons.

tems it may reach 10–20 %. This is well within the range for possible experimental observation.[14]

Semiconductor Q1D electron systems offer a unique opportunity of varying and controlling their properties in a wide range via chemical composition, band engineering, external fields, etc. In particular, by varying carrier concentration and effective width of the system one can change the strength of electron-electron (EE) interactions by orders of magnitude. Apparently, a very interesting physical situation occurs as the carrier concentration and the system width are made sufficiently small, such that EE interactions become strong (up to a few meV) and may not be neglected. In this case one can expect that the properties of the system might be drastically modified (for a relevant review see, e.g., Ref. 5). Since the strength of the SO coupling can be changed independently,[11] it appears possible to create a strongly interacting Q1D electron system whose single-particle energy spectrum lacks chiral symmetry as in Fig. 1. In this case one faces the following fundamental question: how does such an interacting system respond to the asymmetry of the single-electron spectrum? In this paper we answer this question theoretically and describe possible experimental manifestations of the results we find. In Sec. II we formulate a model for the SO coupling in interacting Q1D systems and apply the bosonization technique[15] to recast it in a solvable form. The destruction of the spin-charge separation by the SO coupling is demonstrated and discussed. In Sec. III we calculate the retarded single-electron Green's function, the spectral function, and the density of electron states. We analyze the dependence of the spectral function and the density of states on the energy of elementary excitations for varying strength of the SO coupling and EE interactions. Some of the results of Sec. III have been recently published in Ref. 16. In Sec. IV we study the effect of the SO coupling on two-particle characteristics, such as the charge compressibility, the spin susceptibility, and the ballistic conductance of a finite system. Section V is a conclusion.

II. MODEL FORMULATION AND FORMALISM

A. Fermionic Hamiltonian

In constructing a Hamiltonian for a quasi-one-dimensional electron system (Q1DES) with SO coupling, we consider the case where the Fermi energy E_F is sufficiently small such that only the lowest-energy subband in a quantum wire is partly filled, while all the others are empty (see Fig. 1). Also we assume that EE interactions are not too strong to lead to noticeable hybridization of the two lowest subbands. This regime has proved to be the richest in nontrivial experimental results,[2,17] and the SO effects are expected[13] to be most pronounced here. As a natural way of capturing the essential physics in a quantum wire, we suggest the use of a modified Tomonaga-Luttinger model that takes into account the asymmetric single-particle spectrum in Fig. 1. That is, we consider the model Hamiltonian of a Q1DES in the form $H=H_0+H_{int}$, where

$$H_0=-iv_1\int dx(\psi^\dagger_{R,\uparrow}\partial_x\psi_{R,\uparrow}-\psi^\dagger_{L,\downarrow}\partial_x\psi_{L,\downarrow}) -iv_2\int dx(\psi^\dagger_{R,\downarrow}\partial_x\psi_{R,\downarrow}-\psi^\dagger_{L,\uparrow}\partial_x\psi_{L,\uparrow}) \tag{1}$$

and

$$H_{int}=\int dx\int dx' g_1(x-x'):\psi^\dagger_{R,\uparrow}(x)\psi_{L,\uparrow}(x)\psi^\dagger_{L,\downarrow}(x')\psi_{R,\downarrow}(x'):+\frac{1}{L}\sum_{q,s,s'}[g_{2\parallel}(q)\delta_{s,s'}+g_{2\perp}(q)\delta_{s,-s'}]\varrho_{R,s}(q)\varrho_{L,s'}(-q) +\frac{1}{2L}\sum_{r,q,s,s'}[g_{4\parallel}(q)\delta_{s,s'}+g_{4\perp}(q)\delta_{s,-s'}]:\varrho_{r,s}(q)\varrho_{r,s'}(-q):+(R\leftrightarrow L). \tag{2}$$

Here H_0 is the kinetic part of the Hamiltonian (we use units where $\hbar=1$). The operators $\psi_{r,s}(x)$ $(r=R,L;s=\uparrow,\downarrow)$ annihilate spin-up ($\uparrow$) and spin-down ($\downarrow$) electrons near the right (R) and left (L) Fermi points. In what follows, the indices r and s will also be used as factors in equations and take the values $+1$ (-1) for $R(L)$ and $\uparrow(\downarrow)$, respectively. The term H_{int} in the Hamiltonian describes the EE interactions and has the standard form:[5] $g_1(x)$ is the backward scattering coefficient, while the momentum-dependent functions $g_{2\parallel}(q)$, $g_{2\perp}(q)$, $g_{4\parallel}(q)$, and $g_{4\perp}(q)$ correspond to forward scattering. The symbols $:\cdots:$ in Eq. (2) denote normal ordering. The Fourier component $\varrho_{r,s}(q)$ of the chiral charge density operator is defined as

$$\varrho_{r,s}(q)=\int_0^L dx\exp(-iqx)\psi^\dagger_{r,s}(x)\psi_{r,s}(x), \tag{3}$$

where L is the length of the Q1DES. In our Hamiltonian we neglected the Umklapp scattering, which is not relevant[5] in quantum wires patterned in semiconductor heterostructures[1] where energy bands are far from being half filled. In this paper we assume repulsive EE interactions.

The Hamiltonian (1) and (2) is reminiscent of that for the multicomponent Tomonaga-Luttinger model,[5,18] which consists of mutually interacting Luttinger liquids with different Fermi velocities. However, in contrast to our case, the model[5,18] assumes that each liquid has a *symmetric* single-electron spectrum (as occurs, for example, with Zeeman splitting[19]) and therefore, as we will show, it describes qualitatively different physical behavior.

B. Bosonization

We study the Hamiltonian (1) and (2) with the help of the bosonization technique.[15,20] This technique is based on the

following representation of the fermion operators in terms of boson operators:

$$\psi_{r,s}(x) \to \frac{i^{(1-r)/2}}{\sqrt{2\pi\Lambda}} :\exp[-i\Phi_{r,s}(x)]:, \tag{4}$$

where Λ is an infinitesimal ultraviolet cutoff parameter and

$$\Phi_{r,s}(x) = r\varphi_s(x) - \pi \int_{-\infty}^{x} \Pi_s(x')dx'. \tag{5}$$

The spin-up and spin-down phase operators φ_s and Π_s satisfy the canonical commutation relations $[\varphi_s(x), \Pi_{s'}(x')] = i\delta_{ss'}\delta(x-x')$.

The bosonized form of bilinear combinations of the fermion operators and their derivatives can be obtained[15] by expanding $\psi^{\dagger}_{r,s}(x)\psi_{r',s}(x+a)$ in powers of a as $a \to 0$. Proceeding in this way, we find

$$\psi^{\dagger}_{r,s}(x)\psi_{r,s}(x) = -(r/2\pi)\partial_x\Phi_{r,s}(x),$$

$$\psi^{\dagger}_{r,s}(x)\partial_x\psi_{r,s}(x) = (ir/4\pi)[\partial_x\Phi_{r,s}(x)]^2, \tag{6}$$

$$\psi^{\dagger}_{R(L),s}(x)\psi_{L(R),s}(x) = (2\pi\Lambda)^{-1} \times \exp\{\pm i[\Phi_{R,s}(x) - \Phi_{L,s}(x)]\}.$$

Substituting these expressions into Eq. (1) we obtain

$$H_0 = \frac{v_0}{2\pi}\int dx[(\partial_x\varphi_{\uparrow})^2 + (\pi\Pi_{\uparrow})^2 + (\partial_x\varphi_{\downarrow})^2 + (\pi\Pi_{\downarrow})^2] + \frac{\delta v}{2\pi}\int dx[(\pi\Pi_{\uparrow})(\partial_x\varphi_{\uparrow}) - (\pi\Pi_{\downarrow})(\partial_x\varphi_{\downarrow})], \tag{7}$$

$$v_0 = (v_1+v_2)/2, \quad \delta v = v_2 - v_1. \tag{8}$$

Without losing the essential features of SO effects in Q1DES's, from now on we assume that the EE interactions are of the density-density type only. Microscopically this assumption means[5] that in the model Hamiltonian (2) one should put $g_1 = g_2 = g_4$, where $g_{2,4}$ are taken in the coordinate representation. In addition, we consider only pointlike interactions (e.g., well-screened Coulomb repulsion), when all the g's are constants. Then all EE interaction processes involving only one spin orientation become forbidden by the Pauli principle. As applied to the Hamiltonian (2), this means that the terms proportional to $g_{2\parallel}$ and $g_{4\parallel}$ cannot exist and we must put $g_{2\parallel} = g_{4\parallel} = 0$. Eventually, for pointlike density-density interactions with $g_1 = g_{2\perp} = g_{4\perp} \equiv g$ the Hamiltonian (2) reduces to

$$H_{int} = \frac{2g_1}{(2\pi\Lambda)^2}\int dx \cos[2(\varphi_{\uparrow} - \varphi_{\downarrow})] + \beta\frac{v_0}{\pi}\int dx(\partial_x\varphi_{\uparrow})(\partial_x\varphi_{\downarrow}), \tag{9}$$

$$\beta = \frac{2g}{\pi v_0}.$$

Alternatively, in terms of charge (φ_ρ and Π_ρ) and spin (φ_σ and Π_σ) phase fields defined as

$$\varphi_{\rho(\sigma)} = (\varphi_{\uparrow} \pm \varphi_{\downarrow})/\sqrt{2}, \quad \Pi_{\rho(\sigma)} = (\Pi_{\uparrow} \pm \Pi_{\downarrow})/\sqrt{2}, \tag{10}$$

the Hamiltonian (7) and (9) has the form

$$H = \frac{1}{2\pi}\int dx\Big\{ v_\rho K_\rho(\pi\Pi_\rho)^2 + \frac{v_\rho}{K_\rho}(\partial_x\varphi_\rho)^2 + v_\sigma K_\sigma(\pi\Pi_\sigma)^2 + \frac{v_\sigma}{K_\sigma}(\partial_x\varphi_\sigma)^2\Big\} + \frac{\delta v}{2\pi}\int dx\{(\partial_x\varphi_\rho)(\pi\Pi_\sigma) + (\partial_x\varphi_\sigma)(\pi\Pi_\rho)\} + \frac{2g_1}{(2\pi\Lambda)^2}\int dx\cos(2\sqrt{2}\varphi_\sigma), \tag{11}$$

where

$$v_{\rho(\sigma)} = v_0(1\pm\beta)^{1/2}, \quad K_{\rho(\sigma)} = (1\pm\beta)^{-1/2}. \tag{12}$$

The second integral in Eq. (11) is proportional to the velocity difference $\delta v = v_2 - v_1$ and therefore represents the strength of the SO interaction. In the standard Tomonaga-Luttinger model, where $v_1 = v_2$, this term is absent and this results in a decoupling of the charge and spin degrees of freedom: the so-called *spin-charge separation*.[5] This decoupling inhibits the existence of quasiparticles with spin 1/2 and charge $-e$, the basic excitations of a Fermi liquid, and gives rise to a different state of matter, the Luttinger liquid, which has bosonic excitations in the form of independent spin and charge density waves.

As the SO interaction is switched on, i.e., at $\delta v \neq 0$, the second integral in Eq. (11) starts to affect the dynamics of the system. It couples together the ρ and σ fields and thereby *destroys* the spin-charge separation. A similar effect is also found in a Luttinger model with spin in a magnetic field.[19] In our case, the asymmetry of the single-electron spectrum in Fig. 1 results in a mechanism of violation of spin-charge separation different from that of Ref. 19.

The physical properties of our model can be deduced from various correlation functions that can be calculated using functional integration formalism in imaginary (Matsubara) time.[21] In this formalism, observables are expressed as averages $\langle(\cdots)\rangle$ over the boson fields $\varphi(x,\tau)$ and $\Pi(x,\tau)$:

$$\langle(\cdots)\rangle \equiv \frac{1}{Z_0}\int \mathcal{D}\Pi_{\uparrow}\mathcal{D}\Pi_{\downarrow}\mathcal{D}\varphi_{\uparrow}\mathcal{D}\varphi_{\downarrow}(\cdots)\exp\{iS[\Pi_s;\varphi_s]\}. \tag{13}$$

Here Z_0 is the partition function and the action S is

$$iS = \int_0^{\beta} d\tau \int_0^{L} dx\{i\Pi_{\uparrow}(\partial_\tau\varphi_{\uparrow}) + i\Pi_{\downarrow}(\partial_\tau\varphi_{\downarrow}) - H[\Pi_s;\varphi_s]\}, \tag{14}$$

where $\beta = T^{-1}$ is the inverse temperature and H is given by Eqs. (7) and (9). The real-time characteristics are then obtained by analytical continuation $\tau \to it$.

III. SINGLE-PARTICLE CHARACTERISTICS

To find single-particle characteristics, such as the spectral function and the density of states, we first calculate the fermion Green's function.

A. Green's function

The retarded single-particle Green's function is defined as[21]

$$G^{(ret)}_{r,s}(x,t) = -i\theta(t)\langle\{\psi_{r,s}(x,t),\psi^{\dagger}_{r,s}(0,0)\}\rangle = -i\theta(t)[G_{r,s}(x,t)+G_{r,s}(-x,-t)], \quad (15)$$

where the curly brackets $\{\cdots\}$ denote anticommutators, $\theta(t)$ is the step function, and

$$G_{r,s}(x,t) = G_{r,s}(x,\tau)\big|_{\tau\to it}, \quad (16)$$

$$G_{r,s}(x,\tau) = \langle \psi_{r,s}(x,\tau)\psi^{\dagger}_{r,s}(0,0)\rangle. \quad (17)$$

To express the Green's function in terms of the boson fields, we apply the representation (4) to Eq. (17), and make use of the Campbell-Baker-Hausdorff formula $\exp(A)\exp(B) = \exp(A+B)\exp([A,B]/2)$ and the commutation relation for $\Phi_{r,s}$. As a result, we have

$$G_{r,s}(x,\tau) = \frac{1}{2\pi\Lambda}\langle\exp\{-i[\Phi_{r,s}(x,\tau)-\Phi_{r,s}(0,0)]\}\rangle. \quad (18)$$

The average in this equation can be rewritten in a more convenient form with the help of the formula $\langle\exp(A)\rangle = \exp(\langle A^2\rangle/2)$ that is valid for a linear functional A in boson operators averaged with a quadratic action:

$$G_{r,s}(x,\tau) = (2\pi\Lambda)^{-1}\exp[I_{r,s}(x,\tau)], \quad (19)$$

$$I_{r,s}(x,\tau) = \langle\Phi_{r,s}(x,\tau)\Phi_{r,s}(0,0)\rangle - \langle\Phi^2_{r,s}(0,0)\rangle.$$

According to the definition (5),

$$\langle\Phi_{r,s}(x,\tau)\Phi_{r,s}(0,0)\rangle = \langle\varphi_s(x,\tau)\varphi_s(0,0)\rangle + \pi^2\int_{-\infty}^{x}\int_{-\infty}^{0}\langle\Pi_s(x',\tau)\Pi_s(x'',0)\rangle dx'\,dx'' - \pi r\int_{-\infty}^{0}\langle\varphi_s(x,\tau)\Pi_s(x',0)\rangle dx' - \pi r\int_{-\infty}^{x}\langle\Pi_s(x',\tau)\varphi_s(0,0)\rangle dx'. \quad (20)$$

From Eqs. (15)–(20) it follows that the calculation of the Green's function $G^{(ret)}_{r,s}(x,t)$ reduces to the calculation of binary correlators of the phase boson fields φ and Π. However, the presence of the backscattering term (the first integral) in the Hamiltonian (9) prevents the action (14) from being quadratic in φ and Π, and therefore the correlators in Eq. (20) cannot be found explicitly. In Sec. III C we show by renormalization analysis that for repulsive EE interactions the backscattering contribution to the action can be neglected in an infinitely long system. Therefore hereafter we assume the limit $L\to\infty$ (if not stated otherwise) and drop the backscattering term in the Hamiltonian (9). The details of calculation of the correlators (20) with the resulting quadratic action are given in the Appendix.

By substituting Eqs. (A7)–(A9) for the boson correlators into Eq. (20) we have

$$I_{r,s}(x,\tau) = 2\pi v_0\int_{\omega_n,q}[\exp(iqx - i\omega_n\tau)-1] \times\exp(-\Lambda|q|)\frac{b_{r,s}(q,\omega_n)}{\omega_n^2\Delta(q,\omega_n)}, \quad (21)$$

where

$$b_{r,s}(q,\omega_n) = F_0(r,s)\left(\frac{v_0 q}{\omega_n}\right)^2 + iF_1(r,s)\left(\frac{v_0 q}{\omega_n}\right) + F_2(r,s) + ir\left(\frac{\omega_n}{v_0 q}\right), \quad (22)$$

$$F_0(r,s) = 1-\frac{\beta^2}{2}+rs\frac{\epsilon}{2}-\frac{\epsilon^2}{4}-rs\frac{\epsilon^3}{8}, \quad F_1(r,s) = r+s\epsilon+r\frac{\epsilon^2}{4}, \quad F_2(r,s) = 1-rs\frac{\epsilon}{2}. \quad (23)$$

In Eq. (21) we used the standard[5] ultraviolet cutoff at $q\sim\Lambda^{-1}$ to avoid divergence in our long-wavelength theory.

To perform the Fourier transformation in Eq. (21) we first differentiate $I_{r,s}(x,\tau)$ with respect to x. We restrict ourselves to the case of zero temperature, so that the sum over the discrete Matsubara frequencies ω_n transforms into an integral that can be taken explicitly and gives the result

$$\frac{\partial}{\partial x}I_{r,s}(x,\tau) = \frac{i}{2}\frac{1}{\eta_2^2-\eta_1^2} \times\left\{\frac{\xi_1^+(r,s)\eta_1}{\Lambda-ix+\eta_1 v_0\tau}-\frac{\xi_1^-(r,s)\eta_1}{\Lambda+ix+\eta_1 v_0\tau} - \frac{\xi_2^+(r,s)\eta_2}{\Lambda-ix+\eta_2 v_0\tau}+\frac{\xi_2^-(r,s)\eta_2}{\Lambda+ix+\eta_2 v_0\tau}\right\}, \quad (24)$$

where

$$\xi_i^{\pm}(r,s) = F_0(r,s)\eta_i^{-2}\pm F_1(r,s)\eta_i^{-1}-F_2(r,s)\mp r\eta_i \quad (25)$$

and

$$\eta_{1,2}^2 = 1 + \epsilon^2/4 \mp \sqrt{\beta^2 + \epsilon^2}. \tag{26}$$

We next integrate Eq. (24) over x and take into account the boundary condition $I_{r,s}(x=0,\tau=0)=0$ to obtain

$$\exp[I_{r,s}(x,\tau)] = \left(\frac{\Lambda}{\Lambda - ix + \eta_1 v_0 \tau}\right)^{\theta_1^+} \left(\frac{\Lambda}{\Lambda + ix + \eta_1 v_0 \tau}\right)^{\theta_1^-} \times \left(\frac{\Lambda}{\Lambda - ix + \eta_2 v_0 \tau}\right)^{\theta_2^+} \left(\frac{\Lambda}{\Lambda + ix + \eta_2 v_0 \tau}\right)^{\theta_2^-}. \tag{27}$$

Here

$$\theta_i^{\pm}(r,s) = (-1)^i \xi_i^{\pm}(r,s) \frac{\eta_i}{2(\eta_1^2 - \eta_2^2)}. \tag{28}$$

Finally, we substitute Eq. (27) into Eq. (19), replace the complex time τ with it, and find the Green's function $G_{r,s}(x,t)$:

$$G_{r,s}(x,t) = \frac{1}{2\pi\Lambda}\left[\frac{\Lambda}{\Lambda + i(u_1 t - x)}\right]^{\theta_1^+}\left[\frac{\Lambda}{\Lambda + i(u_1 t + x)}\right]^{\theta_1^-} \times \left[\frac{\Lambda}{\Lambda + i(u_2 t - x)}\right]^{\theta_2^+}\left[\frac{\Lambda}{\Lambda + i(u_2 t + x)}\right]^{\theta_2^-}. \tag{29}$$

The total retarded Green's function $G_{r,s}^{(ret)}(x,t)$ can easily be reconstructed from this formula using the definition (15). From Eq. (29) it follows that, although the SO coupling destroys the spin-charge separation, it nevertheless preserves the anomalous scaling of correlation functions. As the SO coupling is switched off, i.e., when $\epsilon = 0$, Eq. (29) reduces to the known expression that can be found in the review Ref. 5.

The quantities $u_{1,2} \equiv u_{1,2}(\epsilon)$ in Eq. (29) defined by

$$u_{1,2} = \eta_{1,2} v_0, \tag{30}$$

are velocities of the *independent* bosonic excitations that replace the spin and charge density waves, respectively, as the single-electron spectrum becomes asymmetric ($\epsilon \neq 0$). Each of these excitations is a superposition of the previous ones; they carry *both* charge and spin, and have a soundlike spectrum $\omega = u_{1,2} q$. For $\epsilon = 0$, we have $u_{1,2} = v_{\sigma(\rho)}$ and return to the spin-charge separation. Equation (26) and Fig. 2 demonstrate that increasing ϵ at constant β pushes u_1 and u_2 away from v_σ and v_ρ as well as from each other: one of the excitations monotonically accelerates (u_2 grows) while the other monotonically slows down (u_1 decreases). This effect becomes more pronounced in systems with stronger EE interaction, i.e., with larger β (see Fig. 2).

Since the parameter ϵ is controlled by the SO interaction and therefore by the lateral electrical confinement, the strength of this confinement must drastically affect the dynamics of elementary excitations in quantum wires. Increasing ϵ eventually results in the velocity u_1 vanishing at a ''critical'' point $\epsilon_0 = 2(v_\sigma / v_0)$. As we will see in Sec. IV A, the spin susceptibility tends to diverge as the critical point is approached, thereby indicating a possible phase transition (see, e.g., the review Ref. 5 and references therein).

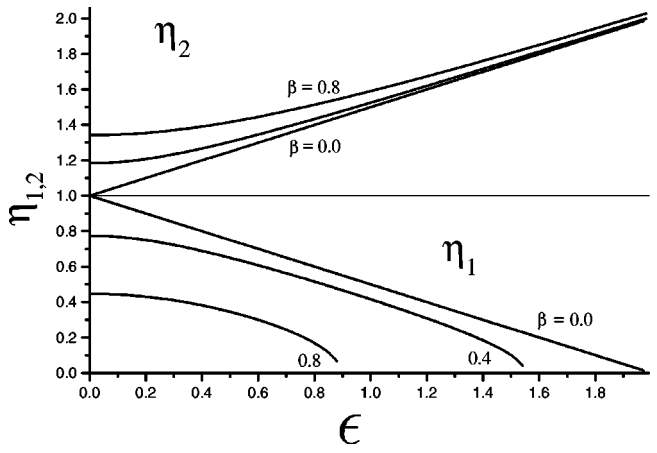

FIG. 2. The dimensionless velocities $\eta_{1,2}(\epsilon)$ of the bosonic excitations vs ϵ for three fixed $\beta = 0.0, 0.4, 0.8$. The curves below (above) the horizontal line correspond to η_1 (η_2).

B. Spectral function and density of states

The most profound effect of the SO coupling on interacting Q1D systems can be seen in the behavior of single-particle characteristics, such as the spectral function $\rho_r(q,\omega)$,

$$\rho_r(q,\omega) = -\frac{1}{\pi}\sum_s \mathrm{Im}\, G_{r,s}^{(ret)}(q,\omega), \quad r = R,L, \tag{31}$$

and the density of electron states $N(\omega)$,

$$N(\omega) = \sum_r \int_{-\infty}^{\infty} \frac{dq}{2\pi} \rho_r(q,\omega) = -\frac{1}{\pi}\mathrm{Im}\sum_{r,s} \int_{-\infty}^{\infty} dt \exp(i\omega t) G_{r,s}^{(ret)}(x=0,t), \tag{32}$$

where $G_{r,s}^{(ret)}(q,\omega)$ is the Fourier transform of the retarded Green's function (15) and (29) and ω is the energy of an elementary excitation. Both quantities $\rho_r(q,\omega)$ and $N(\omega)$ are meaningful experimental observables and can be obtained in different tunneling experiments. For example, in measurements of magnetotunneling between 1D and 2D systems[14,22] the tunneling current is proportional to the overlap between the spectral functions of the 1D and 2D systems and can be used to reconstruct $\rho_r(q,\omega)$ for the 1D system since the 2D spectral function is accurately known. The density of states can be extracted from, e.g., tunneling current measurements between a wide (3D) metal and an interacting quantum wire, because in this case the tunneling current is proportional to $N(eV)$, where V is an applied voltage (see Ref. 23). It can also be determined in angle-integrated photoemission experiments.[24]

Since the exponents $\theta_{1,2}^{\pm}$ in Eq. (29) are, in general, noninteger for finite EE interactions, the calculation of $G_{r,s}^{(ret)}(q,\omega)$ and hence $\rho_r(q,\omega)$ is a rather cumbersome mathematical problem. In the case of zero SO coupling the detailed solution was presented in Ref. 25. The presence of a finite SO coupling does not affect qualitative properties of the spectral function but modifies its quantitative character-

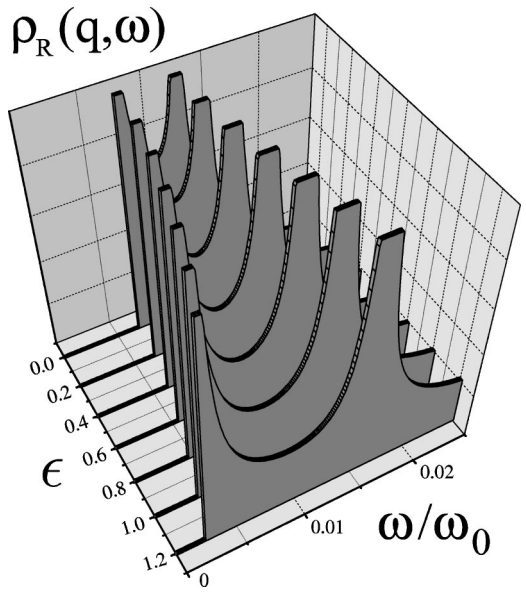

FIG. 3. Spectral function $\rho_R(q,\omega)$ (in arbitrary units) for a fixed $q=0.01\Lambda^{-1}$ and several values of ϵ and for $\beta=0.5$ ($\omega_0\equiv v_0/\Lambda$). Only the interval $\omega>0$ is shown, where all the singularities relevant to magnetotunneling experiments are located. Note that $\rho_L(q,\omega)=\rho_R(q,-\omega)$.

istics. Therefore we can use the results of Ref. 25 for $\rho_r(q,\omega)$, into which we substitute the SO-dependent exponents $\theta^{\pm}_{1,2}$ [Eq. (28)]. In doing so, we see that the most remarkable feature of $\rho_r(q,\omega)$ as a function of ω is the presence of two singular points corresponding to the bosonic excitations with velocities u_1 and u_2 (see Fig. 3). The behavior of $\rho_r(q,\omega)$ in the vicinity of these points is governed by the power laws

$$\rho_r(q,\omega)\sim\sum_s\,(\omega-qu_{1,2})^{\delta_{1,2}(r,s)-1/2}, \tag{33}$$

$$\delta_{1,2}(r,s)=-1/2-\theta^-_{1,2}(r,s)-\theta^+_{2,1}(r,s)-\theta^-_{2,1}(r,s)$$

for the left and right point, respectively. Since both $\delta_{1,2}(r,s)$ and the velocities $u_{1,2}$ depend on the parameter ϵ [see Eqs. (26) and (28)], the positions and the sharpness of the singularities of $\rho_r(q,\omega)$ should be sensitive to the strength of the SO coupling. Indeed, Fig. 3 demonstrates that the distance between the singular points grows with ϵ. This fact is in full accordance with the behavior of the dimensionless velocities $\eta_{1,2}=\eta_{1,2}(\epsilon)$ shown in Fig. 2. Increasing ϵ has a different effect on the sharpness of each singularity. It turns out that the left singularity becomes sharper while the right one becomes smoother as ϵ grows. As ϵ approaches ϵ_0, the divergence exponent for the left singularity tends to -2, while for the right singularity it tends to -1.

Figure 3 shows how the spectral function $\rho_r(q,\omega)$ depends on ω and ϵ at fixed q. It is worth noting that the dependence of ρ_R on q at fixed ω is very similar to that shown in Fig. 3 and exhibits the same tendencies as a function of ϵ. We mention that for $\omega<0$ the presence of SO coupling does not bring about any peculiarities in $\rho_r(q,\omega)$.

The calculation of the single-particle density of states $N(\omega)$ [Eq. (32)] is much simpler than that of $\rho_r(q,\omega)$ be-

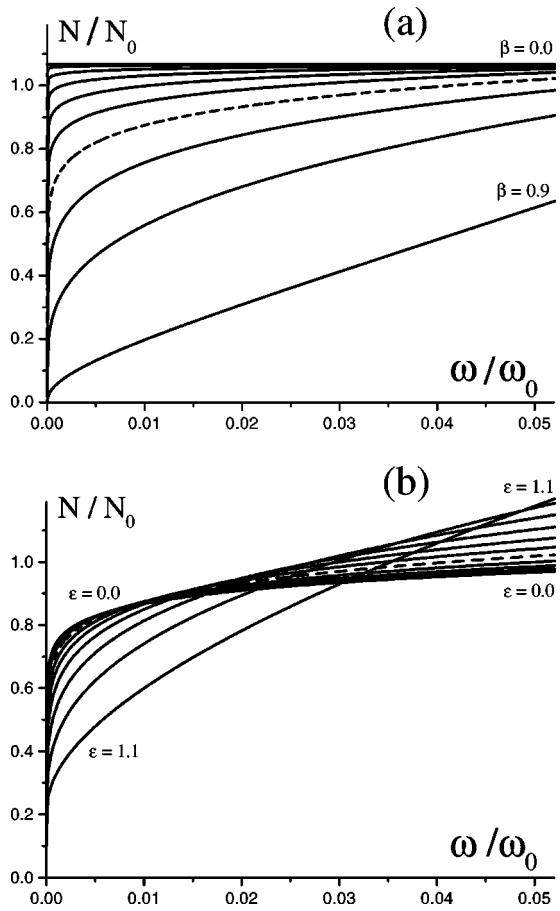

FIG. 4. Normalized single-electron density of states: (a) $\epsilon=0.5$ and $\beta=0.0,0.1,\ldots,0.9$; (b) $\beta=0.6$ and $\epsilon=0.0,0.1,\ldots,1.1$. The dashed curves correspond to $\epsilon=0.5$ and $\beta=0.6$.

cause at $x=0$ the Green's function (29) reduces to the product of only two factors and the integral over t can be taken with the use of Ref. 26. The first nonvanishing term of the expansion of $N(\omega)$ in $\omega\to 0$ (the limit of low-lying excitations) has the following form:

$$\frac{N(\omega)}{N_0}=\frac{1}{4}\sum_{r,s}\frac{(\omega/\omega_0)^{\theta_1+\theta_2-1}}{\eta_1^{\theta_1}\eta_2^{\theta_2}\Gamma(\theta_1+\theta_2)},\qquad \theta_i=\theta_i^+ +\theta_i^- . \tag{34}$$

Here $\omega_0=v_0/\Lambda$ is the natural energy unit of the order of the Fermi energy, $N_0=2/\pi v_0$ is the density of states at the Fermi level for $\beta=\epsilon=0$, and $\Gamma(x)$ is the Gamma function.

Figure 4(a) demonstrates the effect of the EE interaction on the density of states $N(\omega)$ at a fixed strength of the SO coupling. At zero EE interaction ($\beta=0$), the function $N(\omega)$ is a constant, $N(\omega)/N_0=1/(1-\epsilon^2/4)$. As the EE interaction is switched on ($\beta\neq 0$), it starts to affect the formation of the lowest-lying excitations and $N(\omega)$ takes on a power-law behavior in the vicinity of $\omega=0$. The width of this power-law interval becomes progressively larger as the strength of the EE interaction grows and leads to a monotonic suppression of the density of states. This effect is also present for zero SO coupling.[5]

Figure 4(b) shows the evolution of the function $N(\omega)$ for fixed β as the SO coupling is varied. For large values of ω, where the role of EE interactions is not significant, the mag-

nitude of $N(\omega)$ increases as ϵ grows. However, at small values of ω, where the nature of the elementary excitations is essentially dictated by EE interactions, the effect of the SO coupling on $N(\omega)$ is qualitatively the same as that of the EE interaction, that is, increasing ϵ leads to a suppression of the density of states. We emphasize that in the standard multicomponent Tomonaga-Luttinger model[5,18] as well as in spin-polarized Luttinger liquids,[19] there is no interval of ω where the density of states is suppressed by increasing the velocity difference between spectral branches. The existence of such an interval is a unique manifestation of the SO coupling in quantum wires.

The explicit dependence of the dimensionless velocities $\eta_{1,2}$ [see Eq. (26)] on the microscopic parameters ϵ and β should enable one to extract ϵ and β from experiment and thus determine how strong the EE and SO interactions are. For example, one can use magnetotunneling measurements[14,22] to find the values of $\eta_{1,2}$ for a given Q1D system. Knowing these two quantities should be sufficient to deduce the constants ϵ and β from Eq. (26). The results of such experiments can be substantiated and verified by performing measurements of the density of states $N(\omega)$ (e.g., by angle-integrated photoemission[24]) on the same system with different strengths of the SO coupling. This can be achieved by changing the SO coupling directly by varying an electric field perpendicular to a quantum wire (quantum-well field) as was done, e.g., in Ref. 11.

Our calculations[13] of the electron band structure modified by SO coupling indicate that the value of ϵ in typical Q1D semiconductor systems should be ~ 0.1–0.2 and appears sufficiently large to observe the principal tendencies in the behavior of $\rho(q,\omega)$ and $N(\omega)$ caused by the SO coupling.

C. Role of backscattering

We now discuss conditions under which the backscattering term in the Hamiltonian (9) can be neglected so that the action (14) becomes quadratic in the boson fields. In the absence of SO coupling (i.e., when the spin-charge separation holds), a consistent treatment of backscattering based on renormalization group analysis of the action can be found, e.g., in Ref. 27. There it was shown that in infinitely long systems with attractive EE interactions the backscattering contribution to the action renormalizes to infinity, whereas for repulsive interactions it vanishes. In other words, for repulsive (e.g., Coulomb) interactions the backscattering may be safely neglected. In our situation, when the SO coupling destroys the spin-charge separation, the validity of this result[27] is not obvious and requires additional analysis. With the backscattering taken into account, our action has the form [see Eqs. (9), (14), and (A3)]

$$iS[\varphi_\uparrow,\varphi_\downarrow]=iS_0[\varphi_\uparrow,\varphi_\downarrow]-\gamma\int dx d\tau \cos[2(\varphi_\uparrow-\varphi_\downarrow)], \tag{35}$$

$$\gamma=2g_1/(2\pi\Lambda)^2.$$

We now apply a uniform boost in the space-time $r \to \lambda r$, where $r\equiv(x,\tau)$. Under this boost the initial backscattering constant γ transforms into $\gamma'=\lambda^{2+\Delta_{\cos}}\gamma$, where we assume that the field $\cos[2(\varphi_\uparrow-\varphi_\downarrow)]$ acquires the prefactor $\lambda^{\Delta_{\cos}}$ with $\Delta_{\cos}$ being a scaling exponent. Therefore the scaling dimensionality Δ_γ of γ is

$$\Delta_\gamma=2+\Delta_{\cos}. \tag{36}$$

To define the scaling dimensionality $\Delta_{\cos}$ we exploit the fact that in Luttinger liquids correlators like that in Eq. (18) exhibit power-law scaling in r [see Eq. (29)]. Therefore we first calculate the binary correlator $\langle\cos\{2[\varphi_\uparrow(r)-\varphi_\downarrow(r)]\}\cos\{2[\varphi_\uparrow(0)-\varphi_\downarrow(0)]\}\rangle$ and find how it scales with r. Then, following reasoning suggested in Ref. 28, we simply define $\Delta_{\cos}$ as one-half of the scaling exponent of this binary correlator.

From the definitions (A2) and (A5) it follows that

$$\begin{aligned}K(r)&\equiv\langle\cos\{2[\varphi_\uparrow(r)-\varphi_\downarrow(r)]\}\cos\{2[\varphi_\uparrow(0)-\varphi_\downarrow(0)]\}\rangle\\&=\frac{1}{2}\exp\Bigg\{-4\int_{\omega,q}[\exp(iqx-i\omega\tau)-1]\exp(-\Lambda|q|)\\&\quad\times[B_{11}(q,\omega)+B_{22}(q,\omega)-B_{12}(q,\omega)-B_{21}(q,\omega)]\Bigg\}.\end{aligned} \tag{37}$$

The integrals in this equation can be calculated similarly to those in Eq. (21) and give a power-law dependence of the correlator (37) on r and therefore the scaling $K(\lambda r)=\lambda^{2\Delta_{\cos}}K(r)$, where $\Delta_{\cos}<0$. Extracting from this dependence the scaling dimensionality $\Delta_{\cos}$ and substituting it into Eq. (36), we have

$$\Delta_\gamma(\epsilon,\beta)=2\frac{\eta_1\eta_2(\eta_1+\eta_2-1)-1-\beta+\epsilon^2/4}{\eta_1\eta_2(\eta_1+\eta_2)}, \tag{38}$$

where $\eta_{1,2}$ are defined in Eq. (26). This expression is consistent with the result[5] $\Delta_\gamma=2(1-K_\sigma)$ obtained for $\epsilon=0$.

The analysis of the function Δ_γ proves that for $\beta<0$ the scaling exponent Δ_γ remains positive for all $\epsilon<\epsilon_0$. This means that under the boost $r\to\lambda r$ with $\lambda\to\infty$ (thermodynamic limit, or infinitely long system $L\to\infty$) the backscattering constant γ scales up to infinity and we fall into the so-called strong-coupling limit where the effect of the backscattering becomes dominant and essentially nonperturbative.[27]

On the other hand, in systems with repulsive EE interactions ($\beta>0$) the quantity Δ_γ always remains negative. Therefore the constant γ vanishes in the limit $L\to\infty$ and the backscattering processes become negligible. Since in this paper we always assume repulsive EE interactions, the backscattering term in the action (35) can be left out.

IV. TWO-PARTICLE CHARACTERISTICS AND TRANSPORT

We now discuss the influence of the SO coupling on some two-particle characteristics of Q1DES's, such as the charge compressibility, the spin susceptibility, and the transport properties.

A. Charge compressibility and spin susceptibility

The charge compressibility $\mathcal{K}_\rho(q,\omega)$ is defined as

$$\mathcal{K}_\rho(q,\omega)=\mathcal{K}_\rho(q,\omega_n)|_{i\omega_n\to\omega+i0}, \tag{39}$$

where $\mathcal{K}_\rho(q,\omega_n)$ is the charge density correlator in the Matsubara representation. The charge density operator in the bosonized form is given by

$$\hat{\varrho}(x,\tau)=-\frac{\sqrt{2}}{\pi}\partial_x\hat{\varphi}_\rho. \tag{40}$$

Then

$$\mathcal{K}_\rho(q,\omega_n)=\frac{2}{\pi^2}q^2\langle\varphi_\rho(-q,-\omega_n)\varphi_\rho(q,\omega_n)\rangle. \tag{41}$$

To calculate this correlator we make use of the definitions (10), (A2), and (A5) to replace $\varphi_\rho\to(\varphi_\uparrow+\varphi_\downarrow)/\sqrt{2}$ and express $\mathcal{K}_\rho(q,\omega_n)$ via the functions $B_{ij}(q,\omega_n)$:

$$\mathcal{K}_\rho(q,\omega_n)=\frac{q^2}{\pi^2}[B_{11}(q,\omega_n)+B_{22}(q,\omega_n)+B_{12}(q,\omega_n)+B_{21}(q,\omega_n)]. \tag{42}$$

Given the explicit form of $B_{ij}(q,\omega_n)$ [Eq. (A6)], it is easy to find that

$$\mathcal{K}_\rho(q,\omega_n)=\frac{2v_0q^2}{\pi\omega_n^2\Delta(q,\omega_n)}[\mathrm{Re}\nu(q,\omega_n)-\zeta(q,\omega_n)]. \tag{43}$$

By taking the limits $i\omega_n\to\omega+i0$ and $\omega\to 0$, we obtain the static charge compressibility:

$$\kappa\equiv\mathcal{K}_\rho(q,\omega=0)=\kappa_0[1-(\epsilon/\epsilon_\rho)^2]^{-1}, \tag{44}$$

$$\kappa_0=N_0K_\rho\frac{v_0}{v_\rho},\quad \epsilon_\rho=2\frac{v_\rho}{v_0},$$

where κ_0 is the charge compressibility in the absence of SO coupling. We see that κ increases as ϵ grows and diverges as $\epsilon\to\epsilon_\rho$. However, under the assumption of repulsive EE interactions when $g>0$, ϵ_ρ turns out to be larger than the value ϵ_0 at which $\eta_1(\epsilon)$ vanishes. This means that it is unlikely that a possible phase transition occurring at the point $\epsilon=\epsilon_0$ is caused by critical behavior of the charge subsystem.

The spin susceptibility $\mathcal{K}_\sigma(q,\omega)$ is

$$\mathcal{K}_\sigma(q,\omega)=\mathcal{K}_\sigma(q,\omega_n)|_{i\omega_n\to\omega+i0}, \tag{45}$$

where $\mathcal{K}_\sigma(q,\omega_n)$ is the Matsubara spin density correlator. The spin density operator is defined as

$$\hat{\sigma}(x,\tau)=-\frac{\sqrt{2}}{\pi}\partial_x\hat{\varphi}_\sigma. \tag{46}$$

It is easy to verify that $\mathcal{K}_\sigma(q,\omega_n)$ is given by Eq. (42) with $B_{12}(q,\omega_n)\to-B_{12}(q,\omega_n)$ and $B_{21}(q,\omega_n)\to-B_{21}(q,\omega_n)$. Using the definitions (A6) and taking the limits $i\omega_n\to\omega+i0$ and $\omega\to 0$, we obtain the static spin susceptibility:

$$\chi\equiv\mathcal{K}_\sigma(q,\omega=0)=\chi_0[1-(\epsilon/\epsilon_\sigma)^2]^{-1}. \tag{47}$$

$$\chi_0=N_0K_\sigma\frac{v_0}{v_\sigma},\quad \epsilon_\sigma=2\frac{v_\sigma}{v_0},$$

where χ_0 is the spin susceptibility for zero SO coupling. Like κ [Eq. (44)], the quantity χ is an increasing function of ϵ and diverges at the critical value of $\epsilon=\epsilon_\sigma$. Remarkably, ϵ_σ coincides with ϵ_0. This fact suggests that as ϵ approaches the point ϵ_0 the spin subsystem exhibits critical behavior which manifests itself, in particular, in ''freezing'' the slower boson excitation ($u_1\to 0$) that is mainly formed by spin waves. In other words, it is very probable that the alleged phase transition at $\epsilon=\epsilon_0$ has a spin nature.

B. Ballistic conductance

It is well known[1] that the conductance of a clean Q1D mesoscopic system is quantized in multiples of e^2/h (per spin) as a function of the Fermi energy. This property is so robust that even arbitrarily strong repulsive EE interactions are not capable of renormalizing the conductance quantization steps.[29] As was shown in Sec. II B, a finite SO coupling destroys the spin-charge separation in an interacting Q1DES and thereby makes the processes of charge and spin transfer through the system correlated. To understand whether this can affect the conductance, let us define the conductance G in a static external electric field as[29]

$$G=\frac{1}{L^2}\int_0^L dx\int_0^L dx'\,\sigma_\omega(x,x')|_{\omega\to 0}, \tag{48}$$

where

$$\sigma_\omega(x,x')=-\frac{e^2}{\omega_n}\int_0^\infty d\tau\exp(i\omega_n\tau)\times\langle T_\tau\hat{j}_\rho(x,\tau)\hat{j}_\rho(x',0)\rangle|_{i\omega_n\to\omega+i0} \tag{49}$$

is the nonlocal ac conductivity. The current operator $\hat{j}_\rho(x,\tau)$ is related to the charge density operator $\hat{\varrho}(x,\tau)$ by the charge continuity equation in the Matsubara representation: $i(\partial\hat{\varrho}/\partial\tau)+\partial\hat{j}_\rho/\partial x=0$. We integrate this equation over x, substitute $\hat{j}_\rho(x,\tau)$ obtained into the current-current correlator in Eq. (49), and take into account the boson representation (40) for $\hat{\varrho}(x,\tau)$. As a result, Eq. (49) transforms into

$$\sigma_\omega(x,x')=\frac{2e^2}{\pi^2}\omega_n\mathcal{G}^{\rho\rho}_{\omega_n}(x,x')\Big|_{i\omega_n\to\omega+i0}, \tag{50}$$

$$\mathcal{G}^{\rho\rho}_{\omega_n}(x,x')=\frac{1}{\omega_n^2}\int_0^\infty d\tau\exp(i\omega_n\tau)\times\langle T_\tau\partial_\tau\varphi_\rho(x,\tau)\partial_\tau\varphi_\rho(x',0)\rangle \tag{51}$$

with $\mathcal{G}^{\rho\rho}_{\omega_n}(x,x')$ being the propagator of the boson field φ_ρ. We note that the conductivity (50) and (51) has exactly the same appearance as it has for zero SO coupling.[29] The presence of the SO coupling in Eq. (51) is hidden within the thermodynamic average $\langle\cdots\rangle$ which should be calculated with the SO-dependent action (A3).

From the experimental point of view the most realistic situation is one with a Q1DES occupying a finite range $0 \leqslant x \leqslant L$ and connected adiabatically to two- or three-dimensional Fermi leads where EE and SO effects are negligible. In this case we follow the procedure suggested in Ref. 29, namely, we exploit the following differential equation for the propagator $\hat{\mathcal{G}}_{\omega_n}(x,x')$ of the boson fields φ_ρ and φ_σ:

$$\hat{\mathcal{G}}^{-1}_{\omega_n}\mathcal{G}_{\omega_n}(x,x') = \hat{I}\,\delta(x-x'), \tag{52}$$

where $\hat{I}$ is the identity operator in the $\varphi_{\rho(\sigma)}$ space. The differential operator $\hat{\mathcal{G}}^{-1}_{\omega_n}$ is obtained from the matrix (A4) by rotating it to the basis $(\varphi_\rho,\varphi_\sigma)$ and replacing $q \to i\partial_x$. The function $\mathcal{G}^{\rho\rho}_{\omega_n}(x,x')$ is the left uppermost element of the matrix $\mathcal{G}_{\omega_n}(x,x')$. In contrast to Ref. 29, the charge and spin degrees of freedom are coupled in our case and therefore the equation for $\mathcal{G}^{\rho\rho}_{\omega_n}(x,x')$ turns out to be coupled to the equation for $\mathcal{G}^{\sigma\rho}_{\omega_n}(x,x')$:

$$\left\{-v_0\frac{d}{dx}\left[f_+(x)\frac{d}{dx}\right]+\frac{\omega_n^2}{v_0}\right\}\mathcal{G}^{\rho\rho}_{\omega_n}(x,x')+\epsilon\,\omega_n\frac{d}{dx}\mathcal{G}^{\sigma\rho}_{\omega_n}(x,x') = \pi\,\delta(x-x'), \tag{53}$$

$$\left\{-v_0\frac{d}{dx}\left[f_-(x)\frac{d}{dx}\right]+\frac{\omega_n^2}{v_0}\right\}\mathcal{G}^{\sigma\rho}_{\omega_n}(x,x')+\epsilon\,\omega_n\frac{d}{dx}\mathcal{G}^{\rho\rho}_{\omega_n}(x,x') = 0. \tag{54}$$

Here

$$f_\pm(x) = 1 \pm \beta(x) - \epsilon^2(x)/4,$$

and the parameters β and ϵ are assumed to be coordinate dependent according to the model of a finite Q1DES, i.e., $\beta(x)=\beta$ and $\epsilon(x)=\epsilon$ for $0 \leqslant x \leqslant L$ and zero otherwise. Equations (53) and (54) lead to the following boundary conditions: the functions $\mathcal{G}^{\rho\rho}_{\omega_n}(x,x')$, $\mathcal{G}^{\sigma\rho}_{\omega_n}(x,x')$, and $f_-(x)(d\mathcal{G}^{\sigma\rho}_{\omega_n}/dx)$ are continuous at $x=0,L$ and $x=x'$; the function $f_+(x)(d\mathcal{G}^{\rho\rho}_{\omega_n}/dx)$ is continuous at $x=0,L$ but undergoes a jump at $x=x'$, i.e.,

$$-v_0 f_+\frac{d}{dx}\mathcal{G}^{\rho\rho}_{\omega_n}(x,x')\Big|^{x=x'+0}_{x=x'-0} = \pi.$$

The function $\mathcal{G}^{\rho\rho}_{\omega_n}(x,x')$ that satisfies Eqs. (53) and (54) has a simple exponential form

$$\begin{aligned}\mathcal{G}^{\rho\rho}_{\omega_n}(x,x') = {} & A_1(x')\exp(|\omega_n|x/u_1)+A_2(x')\\ & \times\exp(-|\omega_n|x/u_1)+A_3(x')\exp(|\omega_n|x/u_2)\\ & +A_4(x')\exp(-|\omega_n|x/u_2)\end{aligned}$$

in all regions of x, where $A_{1-4}(x')$ are deduced from the boundary conditions. As a result, $\mathcal{G}^{\rho\rho}_{\omega_n}(x,x')$ within the interacting region $0 \leqslant x \leqslant L$ is described by the asymptotic $\mathcal{G}^{\rho\rho}_{\omega_n}(x,x') \simeq \pi/2\omega_n$ at $\omega_n \to 0$. Substituting this result into Eq. (50) and then into Eq. (48) gives the dc conductance of a finite Q1DES:

$$G = \frac{e^2}{\pi} \quad (\hbar = 1). \tag{55}$$

This formula coincides with the usual conductance quantization step in ballistic Q1DES's. Thus, we conclude that the SO coupling does *not* renormalize the conductance, which is still formed solely by the contact resistance at the ends of the interacting region. We stress that this conclusion applies only to the static situation with $\omega=0$, whereas the ac conductivity and the ac conductance *do* depend on the strength of the SO coupling. This dependence is expected to be quadratic in ϵ as $\epsilon \to 0$. Also we should mention that the length L of the interacting region should be sufficiently large to ensure negligibility of the EE backscattering.

C. Single-impurity scattering

We now consider the effect of a single δ-function impurity on current passing through a Q1D system in the presence of SO coupling. For the case of zero SO coupling this problem was originally solved in Ref. 30.

For an impurity with the potential $V\delta(x)$ the action in terms of the boson fields has the usual form from Refs. 5 and 30:

$$iS_{\rm imp} = -\frac{2V}{\pi\Lambda}\int d\tau \cos[\sqrt{2}\varphi_\rho(x=0,\tau)]\cos[\sqrt{2}\varphi_\sigma(x=0,\tau)]. \tag{56}$$

Since $S_{\rm imp}$ depends on both charge and spin degrees of freedom, one might expect the SO coupling to be important in determining the effect of impurity scattering on the system.

To figure out the scaling properties of the impurity potential V we follow the same method as we used in analyzing the EE backscattering in Sec. III C. In doing so, we define the scaling dimensionality $\Delta_{\rm imp}$ of V/Λ as

$$\Delta_{\rm imp} = 1 + \tilde{\Delta}_{\cos}, \tag{57}$$

where $\tilde{\Delta}_{\cos}$ is the scaling dimensionality of the field

$$\tilde{\varphi}(\tau) \equiv \cos[\sqrt{2}\varphi_\rho(x=0,\tau)]\cos[\sqrt{2}\varphi_\sigma(x=0,\tau)],$$

which can be deduced[28] from its binary correlator.

Obviously, the variable x is dummy for the local action $S_{\rm imp}$ [Eq. (56)]. Therefore, for the calculation of $\tilde{\Delta}_{\cos}$ it appears natural to integrate out the variable x in the action (A3), so that the resulting effective action includes the fields $\varphi_{\rho(\sigma)}$ at the point $x=0$ only:

$$iS_0 \to -\frac{1}{2}T\sum_{n=-\infty}^{\infty}(\varphi_\rho\varphi_\sigma)(-\omega_n)\hat{g}^{-1}(\omega_n)\begin{pmatrix}\varphi_\rho\\ \varphi_\sigma\end{pmatrix}(\omega_n), \tag{58}$$

where $\varphi_{\rho(\sigma)}(\omega_n)$ are Fourier transforms of the local fields $\varphi_{\rho(\sigma)}(x=0,\tau)$ and $\hat{g}(\omega_n)$ is given by

$$\hat{g}(\omega_n) = \int_{-\infty}^{\infty}\frac{dq}{2\pi}\hat{\mathcal{G}}_0(q,\omega_n).$$

The Green's function $\hat{\mathcal{G}}_0(q,\omega_n)$ is obtained from Eq. (A4) by matrix inversion and subsequent transformation from the basis $(\varphi_\uparrow,\varphi_\downarrow)$ to $(\varphi_\rho,\varphi_\sigma)$ according to Eq. (10). It is easy to verify that diagonal elements of $\hat{\mathcal{G}}_0(q,\omega_n)$ are even functions of both q and ω_n, whereas its off-diagonal elements are odd in q and ω_n. As a result, off-diagonal elements of the matrix $\hat{g}(\omega_n)$ are zeros, while diagonal ones have the form

$$\hat{g}_{ii}=\frac{1+[1+(-1)^i\beta-\epsilon^2/4]/\eta_1\eta_2}{\eta_1\eta_2}\frac{\pi}{2|\omega_n|},\quad i=1,2. \tag{59}$$

We now calculate the binary correlator $\langle\tilde{\varphi}(\tau)\tilde{\varphi}(0)\rangle$ as a product of two independent binary correlators of the ρ and σ fields, extract the scaling dimensionality $\tilde{\Delta}_{\cos}$ from it, and substitute $\tilde{\Delta}_{\cos}$ into the definition (57):

$$\Delta_{\rm imp}(\epsilon,\beta)=1-\frac{\eta_1\eta_2+1-\epsilon^2/4}{\eta_1\eta_2(\eta_1+\eta_2)}. \tag{60}$$

For $\epsilon=0$ the result[30] $\Delta_{\rm imp}=1-(K_\rho+K_\sigma)/2$ is recovered.

The analysis of Eq. (60) indicates that $\Delta_{\rm imp}$ remains negative for all positive and negative β and for all $\epsilon<\epsilon_0$. Thus, we conclude that the impurity potential V in Eq. (56) is irrelevant in the thermodynamic limit and the backscattering from a single impurity can always be neglected. We note that this is true for both zero and finite values of ϵ, i.e., after all, the SO coupling turns out to have no essential effect on the interaction of bosons with an impurity scatterer.

It is worth noting that a conclusion about scaling properties of the impurity potential crucially depends on the approximation used to describe EE interactions. Indeed, if one starts with the traditional model Hamiltonian (1) and does not assume any specific microscopic mechanism for EE interactions, then one can use all four parameters $v_{\rho(\sigma)}$ and $K_{\rho(\sigma)}$ (or equivalently $g_{2\|}$, $g_{2\perp}$, $g_{4\|}$, and $g_{4\perp}$) as *independent*. Then in analyzing the result[5,30] $\Delta_{\rm imp}=1-(K_\rho+K_\sigma)/2$ one can consider the spin-symmetric case $K_\sigma=1$ and conclude that $\Delta_{\rm imp}<0$ for $K_\rho>1$ and $\Delta_{\rm imp}>0$ for $K_\rho<1$. In this paper we consider only pointlike density-density interactions and therefore end up with the single parameter β describing EE interactions. The stiffness constants K_ρ and K_σ are expressed in terms of β [see Eq. (12)] and are not independent of each other. In this case there is *no* range of values of β and ϵ where $\Delta_{\rm imp}>0$. In other words, for pointlike density-density EE interactions the backscattering from a single impurity is unlikely to lock a quantum wire, no matter whether the SO coupling is present or not.

We should mention that in solving the single-impurity problem we assumed zero EE-induced backscattering in the action, i.e., we put $g_1=0$. It is unclear *a priori* what the region of applicability of this assumption is. The corresponding analysis is beyond the scope of this paper.

V. CONCLUSIONS

In conclusion, we have formulated and solved analytically the problem of the interplay between electron-electron and spin-orbit interactions in Q1D electron systems and found the following. (i) The spin-charge separation of the traditional Luttinger liquid is destroyed by the SO coupling. (ii) The independent bosonic spin and charge excitations of the Tomonaga-Luttinger model are replaced by two independent bosonic *mixed* spin and charge excitations in our model. As the strength of the SO coupling increases, the velocity of one of these excitations decreases to zero where it becomes predominantly a spin excitation. This indicates the possibility for the occurrence of a phase transition involving some kind of magnetic ordering. We support this view with calculations of the charge compressibility and spin succeptibility. (iii) The single-particle characteristics, such as the spectral function and the density of states, are essentially modified and controlled by the strength of the SO coupling so that experimental varying of the SO coupling with the external electric field could be used to extract the microscopic parameters of quantum wires. The magnitude of the predicted effects is well within the reach of existing experimental techniques. (iv) The electron backscattering is irrelevant in the thermodynamic limit for repulsive EE interactions and relevant for attractive EE interactions. (v) The properties of a Q1D system in the presence of a single impurity are not affected by SO coupling: the impurity potential renormalizes to zero for all strengths of EE and SO interactions. (vi) The conductance of a finite-length ballistic quasi-one-dimensional channel in the presence of both SO coupling and EE interactions is not renormalized from the noninteracting value.

ACKNOWLEDGMENTS

A.V.M. thanks the ORS, COT, and Corpus Christi College for financial support. K.V.S. and C.H.W.B. thank the EPSRC for financial support.

APPENDIX: BOSON CORRELATORS

To find the correlators in Eq. (20), we introduce the generating functional $Z[\mathbf{U},\mathbf{V}]$,

$$Z[\mathbf{U},\mathbf{V}]\equiv\left\langle \exp\left\{ \int_0^\beta d\tau\int_0^L dx \times[U_\uparrow\varphi_\uparrow+U_\downarrow\varphi_\downarrow+V_\uparrow\Pi_\uparrow+V_\downarrow\Pi_\downarrow]\right\}\right\rangle, \tag{A1}$$

with $\mathbf{U}=\{U_\uparrow,U_\downarrow\}$ and $\mathbf{V}=\{V_\uparrow,V_\downarrow\}$ being arbitrary functions of x and τ. Obviously, all the correlators in Eq. (20) can be expressed as variational derivatives of $Z[\mathbf{U},\mathbf{V}]$ with respect to the functions $U_{\uparrow\downarrow}$ and/or $V_{\uparrow\downarrow}$, e.g.,

$$\langle\varphi_s(x,\tau)\varphi_s(0,0)\rangle=\left.\frac{\delta^2 Z[\mathbf{U},\mathbf{V}]}{\delta U_s(x,\tau)\,\delta U_s(0,0)}\right|_{\mathbf{U}=\mathbf{V}=\mathbf{0}}. \tag{A2}$$

The averaging here is assumed to be taken over the quadratic part of the action (14). To calculate this average we first take Gaussian integrals over $\Pi_\uparrow$ and $\Pi_\downarrow$ [see the definitions (13) and (14)]. After this integration the action takes the following form:

$$iS_0[\varphi_\uparrow,\varphi_\downarrow]=-\frac{1}{2}T\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{dq}{2\pi}(\varphi_\uparrow\varphi_\downarrow)\times(-q,-\omega_n)\hat{G}_0^{-1}(q,\omega_n)\begin{pmatrix}\varphi_\uparrow\\ \varphi_\downarrow\end{pmatrix}(q,\omega_n), \tag{A3}$$

where $\varphi_s(q,\omega_n)$ are Fourier transforms of the fields $\varphi_s(x,\tau)$. In Eq. (A3) $\omega_n=2\pi nT$ is the boson Matsubara frequency and $\hat{G}_0$ is the Green's function given by

$$\hat{G}_0^{-1}(q,\omega_n)=\frac{\omega_n^2}{\pi v_0}\begin{pmatrix}\nu(q,\omega_n) & \zeta(q,\omega_n)\\ \zeta(q,\omega_n) & \nu^*(q,\omega_n)\end{pmatrix}, \tag{A4}$$

where

$$\nu(q,\omega_n)=1+\left(1-\frac{\epsilon^2}{4}\right)\left(\frac{v_0q}{\omega_n}\right)^2-i\epsilon\left(\frac{v_0q}{\omega_n}\right),$$

$$\zeta(q,\omega_n)=\beta\left(\frac{v_0q}{\omega_n}\right)^2,\quad \epsilon=\delta v/v_0.$$

The functional integrals over $\varphi_{\uparrow\downarrow}$ in Eq. (A1) can be simply taken to give the result

$$Z[\mathbf{U},\mathbf{V}]=\exp\left\{\frac{1}{2}\int_{\omega_n,q}(\mathbf{UV})(-q,-\omega_n)\times\hat{B}(q,\omega_n)\begin{pmatrix}\mathbf{U}\\ \mathbf{V}\end{pmatrix}(q,\omega_n)\right\}. \tag{A5}$$

The integral here stands for summation over ω_n and integration over q. The elements of the matrix $\hat{B}(q,\omega)$ are given by [the arguments (q,ω_n) are assumed where necessary]

$$B_{11}=\frac{\pi v_0\nu^*}{\omega_n^2\Delta},\quad B_{12}=-\frac{\pi v_0\zeta}{\omega_n^2\Delta},$$

$$B_{13}=\frac{\nu^*\xi}{\omega_n\Delta},\quad B_{14}=-\frac{\zeta\xi^*}{\omega_n\Delta},$$

$$B_{23}=-\frac{\zeta\xi}{\omega_n\Delta},\quad B_{24}=\frac{\nu\xi^*}{\omega_n\Delta},$$

$$B_{33}=\frac{\Delta-\nu^*\xi^2}{\pi v_0\Delta},\quad B_{34}=\frac{\zeta|\xi|^2}{\pi v_0\Delta}, \tag{A6}$$

$$B_{22}(q,\omega_n)=B_{11}^*(q,\omega_n),\quad B_{44}(q,\omega_n)=B_{33}^*(q,\omega_n),$$

$$B_{ij}(q,\omega_n)=B_{ji}(-q,-\omega_n).$$

Here

$$\xi(q,\omega_n)=1-i\epsilon(v_0q/\omega_n)/2,$$

$$\Delta(q,\omega_n)=|\nu(q,\omega_n)|^2-\zeta^2(q,\omega_n).$$

We now return to the coordinate-time representation in Eq. (A5) and obtain for the correlators in Eq. (20)

$$\langle\varphi_\uparrow(x,\tau)\varphi_\uparrow(0,0)\rangle=\int_{\omega_n,q}\exp(iqx-i\omega_n\tau)B_{11}(q,\omega_n), \tag{A7}$$

$$\int_{-\infty}^{0}dx'\langle\varphi_\uparrow(x,\tau)\Pi_\uparrow(x',0)\rangle=\int_{-\infty}^{x}dx'\langle\Pi_\uparrow(x',\tau)\varphi_\uparrow(0,0)\rangle=i\int_{\omega_n,q}\exp(iqx-i\omega_n\tau)q^{-1}B_{13}(q,\omega_n), \tag{A8}$$

$$\int_{-\infty}^{x}dx'\int_{-\infty}^{0}dx''\langle\Pi_\uparrow(x',\tau)\Pi_\uparrow(x'',0)\rangle=\int_{\omega_n,q}\exp(iqx-i\omega_n\tau)q^{-2}B_{33}(q,\omega_n). \tag{A9}$$

The correlators for $s=\downarrow$ can be deduced from Eqs. (A7)–(A9) by replacing $B_{11}\to B_{22}$, $B_{13}\to B_{24}$, and $B_{33}\to B_{44}$.

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