Magneto-optical effects in quantum wells irradiated with light pulses

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A method to detect and investigate the magnetopolaron effect in semiconductor quantum wells in a strong magnetic field, based on pulse light irradiation, and measuring the reflected and transmitted pulses, is proposed. It is shown that a beating amplitude on the frequencies, corresponding to the magnetopolaron energy level splitting, depends strongly on the exciting pulse width. The existence of time points of the total reflection and total transparency is predicted. High orders of the perturbation theory on electron-electromagnetic field interaction are taken into account.

Time-resolved scattering (TRS) investigations of excitons in semiconductor bulk crystals and quantum wells (QW's) are often discussed in the current literature.[1,2] The most interesting results are due to the discrete energy levels and pulse irradiation of the physical subjects. It is also well known that a pair of energy levels close to each other results in an interesting effect: sinusoidal beatings on the frequency $\Delta E/\hbar$, corresponding to the energy distance ΔE between the energy levels, appear in the reflected pulses.

In this paper we theoretically investigate TRS from a semiconductor QW in a strong magnetic field (SMF). When a SMF is directed perpendicularly to the QW surface, discrete energy levels of the electron excitations exist there. These discrete energy levels are characterized by the Landau and size-quantization quantum numbers of electrons and holes. In an ideal situation the Landau levels (LL's) are equidistant. It is clear that one cannot restrict the consideration to some pair of the LL's, and has to take into account either one LL or a large number of LL's. The last variant was used in Ref. 3, where a ladder-type structure of the reflected and transmitted pulses was predicted for a strongly nonsymmetrical exciting light pulse. The ladder-type structure is characterized by the period $2\pi/\omega_\mu$, where $\omega_\mu=|e|H/\mu c$ is the cyclotron frequency, $\mu=\mu_e\mu_h/(\mu_e+\mu_h), m_e(m_h)$ is the electron (hole) effective mass, and H is the magnetic-field intensity. In the case of arbitrary exciting pulses a beating structure with a frequency ω_μ depends on the pulse form, but generally speaking is not a sinusoidal one. In the case of a symmetrical exciting pulse, the duration of which is much longer than the period $2\pi/\omega_\mu$, the beatings on the frequency ω_μ have a very small amplitude. Then one can restrict the consideration by the only LL, with which the exciting pulse is in the resonance.

In Ref. 4 the shape of the reflected and transmitted pulses was determined in the vicinity of the resonance of the exciting pulse carrier frequency ω_l, with the only energy level in the QW. Maybe this level is either an excitonic level at $H=0$ or a discrete energy level in the SMF. It has been shown that under the condition

$$\gamma_r \geqslant \gamma, \tag{1}$$

where $\gamma_r(\gamma)$ is the inverse radiative (nonradiative) lifetime of the electronic excitation, a strong change of the transmitted and reflected pulses, comparatively with the shape of the exciting pulse, has to occur.

The above-mentioned change occurs in the case when the interaction of the electronic excitation with other excitations in the QW (in particular with phonons) does not essentially influence the energy spectrum of the electronic excitations. The role of the electron (hole)–LO-phonon interaction grows sharply at resonant values of H when the magnetopolaron effect appears,[5] i.e., electron (hole) energy-level splitting occurs.

The polaron state formation takes place in both three-dimensional (3D) and 2D semiconductor systems. In the 2D case the splitting value is about $\alpha^{1/2}\hbar\omega_{LO}$,[6] where α is the Fröhlich electron–LO-phonon coupling constant.[7] The usual, combined and weak, magnetopolarons[8] exist there. The resonant magnetic-field values for the usual magnetopolarons are determined as

$$\omega_{LO}=j\Omega_{e(h)}, \quad j=1,2,3,\ldots, \tag{2}$$

where ω_{LO} is the LO-phonon frequency, and $\Omega_{e(h)}=|e|H/m_{e(h)}c$ is the electron (hole) cyclotron frequency. The fractional j corresponds to the weak polarons.

The values ΔE of the polaron splittings for the weak polarons are smaller than those for the usual polarons: They are of higher order in α than $\alpha^{1/2}$. The values γ_{ra} and γ_{rb} for both upper (a) and lower (b) energy levels were obtained in Ref. 9, along with estimations of γ_a and γ_b.

Let us show that the values ΔE of the polaron splittings can be measured in experiments with the pulse irradiation of a QW in a SMF. We suppose that a light pulse incidents on a single QW from the left perpendicularly to the QW surface. The pulse electric field intensity is

$$\mathbf{E}_0(z,t)=E_0\mathbf{e}_1 e^{-i\omega_l(t-zn/c)}\{\Theta(t-zn/c)e^{-\gamma_l(t-zn/c)/2} +[1-\Theta(t-zn/c)]e^{\gamma_l(t-zn/c)/2}\}+\text{c.c.}, \tag{3}$$

where E_0 is the real amplitude, $\mathbf{e}_l$ is the polarization vector, ω_l is the pulse carrier frequency, n is the refraction index outside of the QW, γ_l determines the pulse duration, and $\Theta(x)$ is the Haeviside step function. After the Fourier transformation, we obtain

$$\mathbf{E}_0(z,t)=E_0\mathbf{e}_1\int_{-\infty}^{\infty}d\omega e^{-i\omega(t-zn/c)}D_0(\omega)+\text{c.c.}, \tag{4}$$

$$D_0(\omega)=\frac{i}{2\pi}[(\omega-\omega_l+i\gamma_l/2)^{-1}-(\omega-\omega_l-i\gamma_l/2)^{-1}]. \tag{5}$$

When $\gamma_l\to 0$ the pulse [Eq. (3)] transits into a monochromatic wave. Let us suppose that the incident wave has a circular polarization. We imply that both waves of the circular polarization correspond to the excitations of two types of the EHP, the energies of which are equal. Let us consider a QW, the width of which is much smaller than the light wavelength value $c/\omega_l n$. Then the electric fields on the left (on the right) of the QW are determined by the expressions[4]

$$\mathbf{E}_{left(right)}(z,t)=\mathbf{E}_0(z,t)+\Delta\mathbf{E}_{left(right)}, \tag{6}$$

$$\Delta\mathbf{E}_{left(right)}(z,t)=E_0\mathbf{e}_l\times\int_{-\infty}^{\infty}d\omega e^{-i\omega(t\pm zn/c)}D(\omega)+\text{c.c.}, \tag{7}$$

where the upper (lower) sign corresponds to the indexes $\langle$left$\rangle$($\langle$right$\rangle$)

$$D(\omega)=-4\pi\chi(\omega)D_0(\omega)/[1+4\pi\chi(\omega)], \tag{8}$$

$$\chi(\omega)=(i/4\pi)\sum_{\rho}(\gamma_{r\rho}/2)[(\omega-\omega_\rho+i\gamma_\rho/2)^{-1}+(\omega+\omega_\rho+i\gamma_\rho/2)^{-1}], \tag{9}$$

where ρ is the index of the excited state, and $\hbar\omega_\rho$ is the excitation energy measured from the ground-state energy. Applying Eq. (8) means that the theory is constructed taking into account the highest orders of the perturbation theory on the electron–electromagnetic-field interaction.[3,4,9–11] The second term in the square brackets of Eq. (9) is a nonresonant one, and is omitted below.

Below we consider the case of two excited energy levels when the index ρ takes two values: $\rho=1$ and 2. These levels are the lower and upper magnetopolaron energy levels. With the help of Eqs. (6)–(9), we obtain

$$\begin{aligned}\Delta\mathbf{E}_{left}(z,t)=-iE_0\mathbf{e}_l\Bigg\{&\Theta(s)\Bigg[e^{-i\omega_l s-\gamma_l s/2}\left(\frac{\bar{\gamma}_{r1}/2}{\omega_l-\Omega_1+i(G_1-\gamma_l)/2}+\frac{\bar{\gamma}_{r2}/2}{\omega_l-\Omega_2+i(G_2-\gamma_l)/2}\right)\\&-e^{-i\Omega_1 s-G_1 s/2}(\bar{\gamma}_{r1}/2)\left(\frac{1}{\omega_l-\Omega_1+i(G_1-\gamma_l)/2}-\frac{1}{\omega_l-\Omega_1+i(G_1+\gamma_l)/2}\right)\\&-e^{-i\Omega_2 s-G_2 s/2}(\bar{\gamma}_{r2}/2)\left(\frac{1}{\omega_l-\Omega_2+i(G_2-\gamma_l)/2}-\frac{1}{\omega_l-\Omega_2+i(G_2+\gamma_l)/2}\right)\Bigg]\\&+[1-\Theta(s)]e^{-i\omega_l s+\gamma_l s/2}\left(\frac{\bar{\gamma}_{r1}/2}{\omega_l-\Omega_1+i(G_1+\gamma_l)/2}+\frac{\bar{\gamma}_{r2}/2}{\omega_l-\Omega_2+i(G_2+\gamma_l)/2}\right)\Bigg\},\end{aligned} \tag{10}$$

where $s=t+zn/c$. The expression for $\Delta E_{right}(z,t)$ differs from Eq. (10) by the substitution $p=t-zn/c$ instead of s. The following designations are used:

$$\begin{aligned}&(\Omega-iG/2)_{1(2)}=\frac{1}{2}[\tilde{\omega}_1+\tilde{\omega}_2\pm\sqrt{(\tilde{\omega}_1-\tilde{\omega}_2)^2-\gamma_{r1}\gamma_{r2}}],\\&\tilde{\omega}_{1(2)}=\omega_{1(2)}-i\Gamma_{1(2)}/2,\quad \Gamma_{1(2)}=\gamma_{1(2)}+\gamma_{r1(2)};\\&\bar{\gamma}_{r1}=\gamma_{r1}+\Delta\gamma,\quad \bar{\gamma}_{r2}=\gamma_{r2}-\Delta\gamma,\\&\Delta\gamma=\{\gamma_{r1}[\Omega_2-\omega_2-i(G_2-\gamma_2)/2]\\&\quad+\gamma_{r2}[\Omega_1-\omega_1-i(G_1-\gamma_1)/2]\}\\&\quad\times[\Omega_1-\Omega_2+i(G_2-G_1)/2]^{-1}.\end{aligned} \tag{11}$$

The upper (lower) sign in Eq. (11) corresponds to 1(2). The values $\Omega_{1(2)}$ and $G_{1(2)}$ are real by their definition. At $\gamma_l=0$ Eq. (10) transits into the formula for the induced field at a monochromatic irradiation.[9] At $\gamma_{r2}=0$, from Eq. (10) we obtain

$$\begin{aligned}\Delta\mathbf{E}_{left}(z,t)=-iE_0\mathbf{e}_l(\gamma_{r1}/2)\Bigg\{&\Theta(s)\Bigg[\frac{e^{-i\omega_l s-\gamma_l s/2}}{\omega_l-\omega_1+i(\Gamma_1-\gamma_l)/2}\\&-e^{-i\omega_l s-\Gamma_1 s/2}\left(\frac{1}{\omega_l-\omega_1+i(\Gamma_1-\gamma_l)/2}-\frac{1}{\omega_l-\omega_1+i(\Gamma_1+\gamma_l)/2}\right)\Bigg]\\&+[1-\Theta(s)]\frac{e^{-i\omega_l s+\gamma_l s/2}}{\omega_l-\omega_1+i(\Gamma_1+\gamma_l)/2}\Bigg\}+\text{c.c.},\end{aligned} \tag{12}$$

which corresponds to the case of the single excited energy level. Comparing Eqs. (10) and (12), one finds that the fields

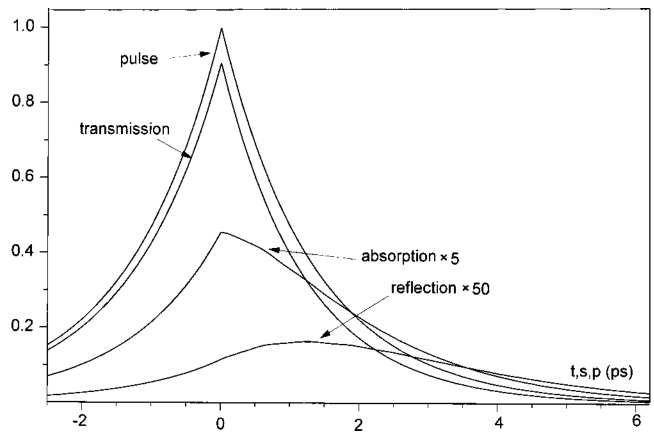

FIG. 1. The modulus of the exciting, transmitted, reflected, and absorbed energy fluxes as time functions: $\omega_l=\Omega_1$, $\Delta\omega=0.00665$, $\gamma_r=0.00005$, $\gamma=0.0005$, and $\gamma_l=0.0005$. All the parameters are given in eV.

from two levels not only add, but that a renormalization of the frequencies $\omega_{1(2)}$, broadenings $\Gamma_{1(2)}$, and factors $\gamma_{r1(2)}$ has occurred. These are substituted for by $\Omega_{1(2)}$, $G_{1(2)}$, and $\bar{\gamma}_{r1(2)}$, respectively. In the case of two merging levels, i.e., under conditions $\gamma_{r1}=\gamma_{r2}$, $\gamma_1=\gamma_2$, and $\omega_1=\omega_2$, from Eq. (10) we obtain an expression like Eq. (12), in which the value γ_{r1} has to be substituted for by $2\gamma_{r1}$. This means that in the case of a twice degenerated level, the twofold value of the inverse radiative lifetime figures into all the formulas.

In Figs. 1–5 the moduli $\mathcal{P}$ and $\mathcal{T}$ of the exciting and transiting pulses as functions of $p=t-zn/c$, the module $\mathcal{R}$ of the reflected pulse flux as function of $s=t+zn/c$, and the absorption, which is defined at $z=0$ as

$$\mathcal{A}=\mathcal{P}-\mathcal{T}-\mathcal{R}, \tag{13}$$

are represented. The modulus of the energy fluxes are represented in units $(c/2\pi n)E_0^2$.

It follows from Eq. (10) that the results for the energy fluxes are dependent on the parameters $\omega_l-\omega_1$ and γ_l, characterizing the exciting pulse, and the parameters $\Delta\omega=\omega_1-\omega_2$, γ_{r1}, γ_{r2}, γ_1, and γ_2, characterizing the system with two excited levels. It follows from Eq. (10) that there are resonances on the frequencies $\omega_l=\Omega_1$, $\omega_l=\Omega_2$, and that beatings on the three frequencies $\omega_l-\Omega_1$, $\omega_l-\Omega_2$, $\Omega_1-\Omega_2$ exist.

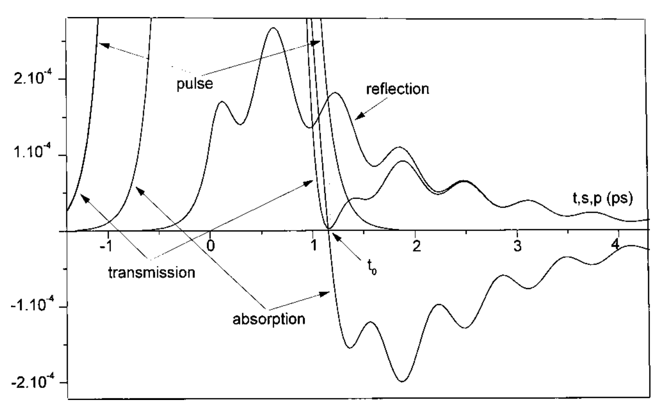

FIG. 2. Same as Fig. 1 for $\gamma_l=0.005$.

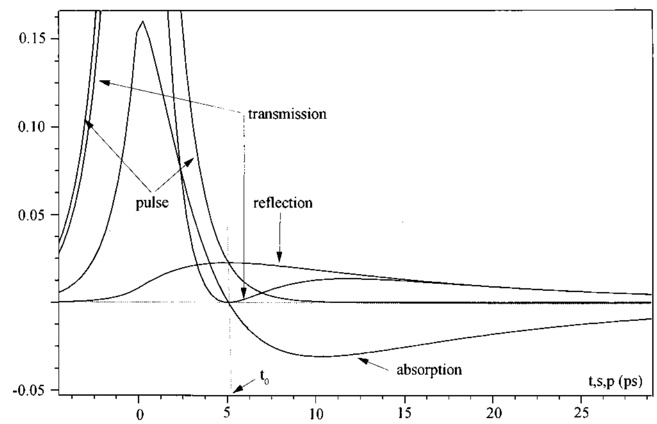

FIG. 3. Same as Fig. 1 for $\gamma=0$ and $\gamma_l=0.0005$.

In the cases

$$\omega_l=\Omega_1,\quad \gamma_{r1}=\gamma_{r2}=\gamma_r,\quad \gamma_1=\gamma_2=\gamma, \tag{14}$$

the results for the fluxes are represented in Figs. 1–5. Thus only the parameters γ_l, $\Delta\omega$, γ_r, and γ are variable. Obviously, under Eqs. (14) beatings are possible only at the frequency $\Delta\Omega=\Omega_1-\Omega_2$, if the value $\Delta\Omega\neq 0$, which is not always fulfilled, as we will see below. Figures 1 and 2 correspond to the parameters

$$\begin{gathered}\Delta\omega=0.00665,\quad \gamma_r=0.00005,\quad \gamma=0.0005,\\ \gamma_l=0.0005\ \text{(Fig. 1)},\quad \gamma_l=0.005\ \text{(Fig. 2)}.\end{gathered} \tag{15}$$

We use arbitrary units because the results for the fluxes depend on the parameter interrelations only. If eV's are used, then $\Delta\omega$ from Eq. (15) corresponds[8,9] to the usual polaron at $j=1$ [see Eq. (2)] at the QW width $d=300$ Å for GaAs. The relation $\gamma/\gamma_r=10$ is chosen arbitrarily. $\Omega_1\simeq\omega_1$ and $\Omega_2\simeq\omega_2$ when the parameters of Eq. (15) are used.

Figures 1 and 2 differ from each other sharply. The beatings are almost invisible in Fig. 1. This means that these results correspond to the induced electric field [Eq. (12)] under the condition $\omega_l=\omega_1$. In Fig. 2 the beatings at the frequency $\Delta\omega=\omega_1-\omega_2$ are seen brightly in the reflected pulse. Reflection and absorption are much smaller than unity in Figs. 1 and 2, and reflection is much smaller than absorption. These features characterize the case $\gamma_r\ll\gamma$. As for beat-

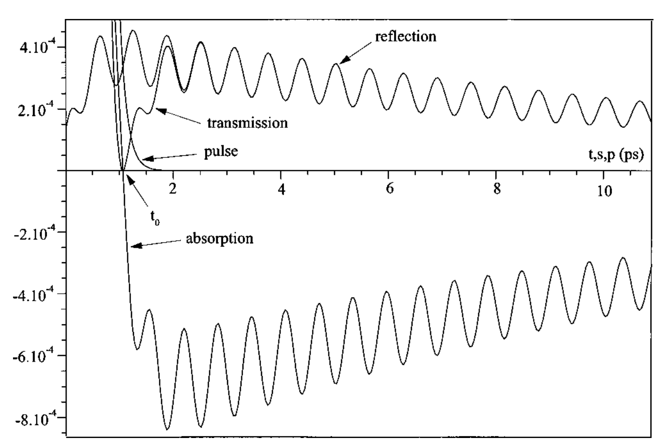

FIG. 4. Same as Fig. 1 for $\gamma=0$ and $\gamma_l=0.005$.

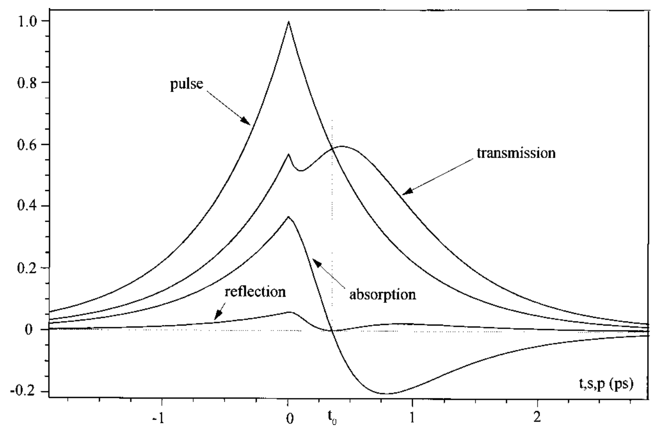

FIG. 5. Same as Fig. 1 for $\gamma_r = 0.00666$, $\gamma = 0.0001$, and $\gamma_l = 0.001$.

ings, their appearance depends sharply on the pulse duration γ_l^{-1}. Beatings are clearly visible for short pulses when $\gamma_l \simeq \Delta\omega$. This is clear physically: The frequencies, which are close to ω_1 and ω_2, are essential in the frequency spectrum [Eq. (5)] of the short pulse.

The former parameters and $\Delta\omega = 0.00665$ and $\gamma_r = 0.00005$, but $\gamma = 0$ corresponds to Figs. 3 and 4. The parameter γ_l in Figs. 3 and 4 coincides with γ_l in Figs. 1 and 2, respectively. The beatings in the reflected and absorbed pulses are expressed only in Fig. 4 when $\gamma_l \simeq \Delta\omega$. In Figs. 3 and 4 reflection and absorption are prolonged more than in Figs. 1 and 2, because in the case $\gamma = 0$ the prolongation is determined by the value γ_r^{-1}.

In Figs. 2, 3, and 4 there is a time point t_0, where the absorption is equal to zero and changes its sign. At $s_0 = t_0$ the reflected flux is equal to the exciting flux, and the transmitted flux is equal to zero at $p_0 = t_0$. This is the time point at which the field from the right of the QW is equal to zero (the total reflection point). Analysis shows that in the case of a single excited level at $\omega_l = \omega_1$, the especial point exists only at $t_0 > 0$, which corresponds to the condition $\gamma_l > \gamma$:

$$t_0 = \frac{2}{\Gamma_1 - \gamma_l} \ln \frac{2\gamma_{r1}\gamma_l}{(\gamma_l - \gamma_1)(\Gamma_1 + \gamma_l)}. \tag{16}$$

In Figs. 1 and 3 curves are presented at which level 2 almost has no influence. The condition $\gamma_l > \gamma$ is satisfied for Fig. 3, but not for Fig. 1. Therefore, the point of total reflection is presented in Fig. 3 and is absent in Fig. 1. In Ref. 4 it was shown that total reflection points also exist for other forms of pulses including nonsymmetrical ones.

The relations

$$\mathcal{P} \simeq 0, \quad \mathcal{T} \simeq \mathcal{R}, \quad \mathcal{A} \simeq -2\mathcal{T}$$

are fulfilled for times very much larger than γ_l^{-1} in Figs. 2, 3, and 4. The sense of them is clear: If $\gamma_l \gg \gamma + \gamma_r$, then the fields created by the exciting pulse are very small at quite large values $t \pm zn/c$, and the fluxes of $\mathcal{T}$ and $\mathcal{R}$ are determined only by the induced fields $\Delta E_{left(right)}$. The picture becomes a symmetrical one relative to the plane $z = 0$ where the QW is placed; the fluxes from the left and right of the plane are equal in absolute values. The absorbed flux is negative, and is equal to the sum of fluxes going to the left and the right. That means that the QW gives back the accumulated energy.

In Fig. 5 the parameters

$$\Delta\omega = 0.00665, \tag{17}$$

$$\gamma_r = 0.00666, \quad \gamma = 0.0001, \quad \gamma_l = 0.001$$

are used, i.e., the special case $\Delta\omega \simeq \gamma_r$ is represented. The beating absence is due to $\Delta\Omega = 0$, but not to the small influence of one of the levels. Indeed, as follows from Eqs. (11) and (14), $\Omega_1 = \Omega_2 = (\omega_1 + \omega_2)/2$ at $|\Delta\omega| \leqslant \gamma_r$. Thus, if $\omega_l = \Omega_1$, the pulse carrier frequency is situated exactly between the two levels in the case of Fig. 5.

The beatings are absent in Fig. 5, but the figure structure differs radically from the structure for the case of two energy levels. In Fig. 5 there is a time point t_x of the total transparency in which reflection and absorption are equal to 0, and the transmitted flux is equal to the exciting flux. An analysis shows that under Eqs. (14) and $|\Delta\omega| = \gamma_r$,

$$t_x = \frac{2}{\gamma_l - \Gamma} \ln \frac{(\gamma_l - \gamma)(\Gamma + \gamma_l)^2}{2\gamma_l(\gamma_l^2 + \gamma_r^2 - \gamma^2)}, \tag{18}$$

and the total transparency point exists in the case $\gamma_l < \Gamma$, which is fulfilled for Fig. 5. The electric field to the left of the QW is equal to zero in the time point t_x.

In all the figures, except for Fig. 1, it is seen that the absorbed flux is equal to zero at the total reflection or total transparency points and changes its sign, i.e., becomes a negative flux. This means that the electronic system at first accumulates the energy of the created electron-hole pairs and then gives it back. In the case $\gamma_1 = \gamma_2 = 0$ all the accumulated energy comes back as radiation, i.e., the integral absorption is equal to zero. This result is true for the cases of Figs. 3 and 4.

In Figs. 1–5 there are only a few examples of the shapes of the reflected, absorbed, and transmitted pulses, corresponding to some combinations from the seven parameters. It is worth stressing that the experimental results for the pulse irradiation of the QW's in a magnetic field allow us in principle to detect and investigate the magnetopolaron states. Measuring the time lags of the reflected and transmitted pulses allows us to determine the lifetimes of the polaron states. A special subject of interest is the investigation of the points of the total reflection and total transition; the first appear in the case of short pulses. For typical polarons the case of Figs. 1 and 2 is the most real. With the help of pulse irradiation of the QW's one can investigate not only typical polarons, but weak polarons also. The smaller $\Delta\omega$ for the weak polarons are even preferable because, first, the beating frequency diminishes and, second, one can use longer pulses than in the case of typical polarons.

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[1] H. Stolz, *Time-Resolved Light Scattering from Excitons*, Springer Tracts in Modern Physics (Springer, Berlin, 1994).

[2] J. Shah, *Ultrafast Spectroscopy of Semiconductors and Semiconductor Nanostructures* (Springer, Berlin, 1996).

[3] I.G. Lang and V.I. Belitsky, Phys. Lett. A **245**, 329 (1998).

[4] I.G. Lang and V.I. Belitsky, Solid State Commun. **107**, 577 (1998).

[5] E.J. Johnson and D.M. Larsen, Phys. Rev. Lett. **16**, 655 (1966).

[6] L.I. Korovin, S.T. Pavlov, and B.E. Eshpulatov, Fiz. Tverd. Tela (Leningrad) **20**, 3594 (1978) [Sov. Phys. Solid State **20**, 2077 (1978)].

[7] H. Fröhlich, Adv. Phys. **3**, 325 (1954).

[8] L.I. Korovin, I.G. Lang, and S.T. Pavlov, Zh. Éksp. Teor. Fiz. **115**, 187 (1999) [JETP **88**, 105 (1999)].

[9] I.G. Lang, L.I. Korovin, A. Contreras Solorio, and S.T. Pavlov, cond-mat/0001248 (unpublished).

[10] L.C. Andreani, F. Tassone, and F. Bassani, Solid State Commun. **77**, 641 (1991).

[11] L.C. Andreani, G. Panzarini, A.V. Kavokin, and M.R. Vladimirova, Phys. Rev. B **57**, 4670 (1998).