Envelope of commensurability magnetoresistance oscillation in unidirectional lateral superlattices

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The envelope of the commensurability magnetoresistance oscillation (Weiss oscillation) has been studied for lateral superlattices prepared from two-dimensional electron gas (2DEG) wafers with varying mobility μ and spacer-layer thickness d_s. When the 2DEG has a high enough μ and a large enough d_s, the envelope is well described by the formula given by Peeters and Vasilopoulos in a first-order perturbation theory [Phys. Rev. B **46**, 4667 (1992).]. For smaller μ or d_s, the oscillation diminishes faster than the formula at lower B. The damping can be accounted for by an additional factor of the form $[\pi/(\mu_W B)]/\sinh[\pi/(\mu_W B)]$. The parameter μ_W is found to be proportional to the mean free path L of the 2DEG, and the coefficient of proportionality increases with d_s. The magnitude of μ_W, as well as its dependence on d_s and the electron areal density n_s, is close to that of μ_Q, the mobility corresponding to the total scattering time.

I. INTRODUCTION

A two-dimensional electron gas (2DEG) under unidirectional potential modulation—a lateral superlattice (LSL)—is well known to show oscillatory magnetoresistance (Weiss oscillation)[1] as a consequence of commensurability between the cyclotron diameter $2R_c=2\hbar k_F/eB$ and the period a of the LSL, where $k_F=\sqrt{2\pi n_s}$ denotes the Fermi wave number, with n_s the areal density of the 2DEG. Quantum-mechanical theories treating the modulation $V(x)=V_0\cos(2\pi x/a)$ as a first-order perturbation were developed by several authors.[2–5] The theories show that the main contribution to the magnetoresistance oscillation results from the ''band conductivity''; the width of the Landau bands, lifted from degenerated Landau levels, oscillates with B, resulting in an oscillation of the y component of the group velocity $v_y=\partial E_{N,k_y}/\hbar\partial k_y$, with $E_{N,k_y}=\langle N,k_y|V(x)|N,k_y\rangle$ (calculated from the unperturbed wavefunction of the Nth Landau level), and hence of the conductivity σ_{yy}. The resistivity $\rho_{xx}\approx\sigma_{yy}/\sigma_{xy}^2$ oscillates accordingly. Resistivity minima occur at the condition when the Landau band collapses (flatband condition), given by

$$\frac{2R_c}{a}=n-\frac{1}{4}\quad(n=1,2,3,\ldots). \tag{1.1}$$

Peeters and Vasilopoulos[5] gave an asymptotic expression for the oscillatory part of the magnetoresistance, valid if the Landau quantum number N is large enough at the Fermi energy $E_F=\pi\hbar^2 n_s/m^*$, with m^* the electron effective mass (a condition fulfilled at low magnetic fields where Weiss oscillation is actually observed):

$$\frac{\Delta\rho_{xx}^{\mathrm{osc}}}{\rho_0}=\frac{\eta^2}{2}\frac{L}{a}B\mu A\left(\frac{T}{T_a(B)}\right)\sin\left(2\pi\frac{2R_c}{a}\right). \tag{1.2}$$

$\eta=V_0/E_F$, μ is the mobility, $L=\hbar k_F\mu/e$ is the mean free path, and $A(x)=x/\sinh(x)$. The thermal damping factor $A[T/T_a(B)]$ is determined by the ratio of temperature k_BT to the energy $k_BT_a=[1/(2\pi^2)](ak_F/2)\hbar\omega_c$, with $\omega_c=eB/m^*$. The latter energy represents the energy spread, multiplied by $1/(2\pi^2)$, over which the values of $2R_c$ differ by a, so that the periodic structure is smeared. This is reminiscent of the expression that appears in the thermal damping of the Shubnikov–de Haas (SdH) oscillation, $k_BT_c=[1/(2\pi^2)]\hbar\omega_c$, the only difference being the factor $ak_F/2$. A semiclassical theory aiming at the same target was also developed.[6] The drift velocity $\mathbf{v_d}=(\mathbf{E}\times\mathbf{B})/B^2$ of the electrons under a modulated electric field $E_x(x)=(1/|e|)dV(x)/dx$ was averaged over a cyclotron orbit, translated into conductivity through Einstein's relation, and further translated into resistivity. The flatband condition [Eq. (1.1)] is obtained as a condition for quenching the averaged drift velocity $\overline{v_d}$, highlighting the classical nature of the phenomena. The oscillation amplitude is also the same as Eq. (1.2), apart from the factor $A[T/T_a(B)]$. The factor is missing since the theory treats only the $T=0$ case.

Both quantum-mechanical and semiclassical theories have successfully reproduced the experimental positions of minima given by Eq. (1.1). It has been known for some time, however, that the experimental amplitude of the oscillation does not necessarily conform to formula (1.2). It often shows a more rapid damping at a higher index n, i.e., at a lower magnetic field (see, e.g., Ref. 7). It is not only a matter of interest to clarify the degree of deviation and the mechanism responsible for the deviation; this is necessary knowledge for accurately obtaining the magnitude V_0 of the modulation from the oscillation amplitude. In fact, a naive application of Eq. (1.2) leads to a smaller value of V_0 when picking up amplitude from a higher index oscillation.

In the present paper, we show that a LSL with small a and V_0 *does* follow Eq. (1.2) very well, if the mobility μ—and hence the mean free path L—and the spacer layer thickness d_s of the 2DEG wafer from which the LSL is prepared are large enough. For smaller L and/or d_s, the deviation from the formula becomes noticeable. We will show the dependence on L and d_s of the parameter μ_W characterizing the deviation.

II. EXPERIMENT

Lateral superlattices were prepared from several 2DEG wafers [conventional molecular-beam-epitaxy (MBE)-grown

TABLE I. Parameters of lateral superlattices measured.

Sample ID	$n_s^{\text{min a}}$ (10^{15} m^{-2})	$\mu^{\text{min a}}$ [m^2/(V s)]	$n_s^{\text{max b}}$ (10^{15} m^{-2})	$\mu^{\text{max b}}$ [m^2/(V s)]	d_s (nm)	d (nm)	a (nm)
H	2.0	69	2.3	79	40	90	115
$M1$	2.2	56	2.7	72	40	90	115
L	2.5	19	3.0	28	40	90	115
C	2.6	34	5.1	101	20	70	115
$M2$	2.2	66	2.7	85	40	90	105
$S1$	4.5	42	-	-	11.4	25	80
$S2$	4.3	17	-	-	11.4	25	70

[a]Measured in the dark.

[b]Maximum values measured after successive illumination.

GaAs/Al$_x$Ga$_{1-x}$As single heterostructures] with varying μ and d_s. The parameters of the LSL's measured are tabulated in Table I. The depth d of the heterointerface from the top surface includes a 10-nm GaAs cap layer, a 40-nm Al$_x$Ga$_{1-x}$As layer uniformly doped with Si [$(2–5)\times10^{24}$ m^{-3}], and an Al$_x$Ga$_{1-x}$As undoped spacer layer with thickness d_s for samples H, $M1$, $M2$, C, and L. For samples $S1$ and $S2$, a specially designed shallow 2DEG with δ-doped Si layer[8] was employed. As shown in Fig. 1, two serial Hall bars were prepared on one device, onto one of which a grating made of a high-resolution electron beam (EB) negative resist (calixarene derivative[9]) was placed to introduce potential modulation. The other Hall bar was used as a reference. With this procedure, a LSL with a period down to $a=70$ nm, that shows a clear Weiss oscillation,

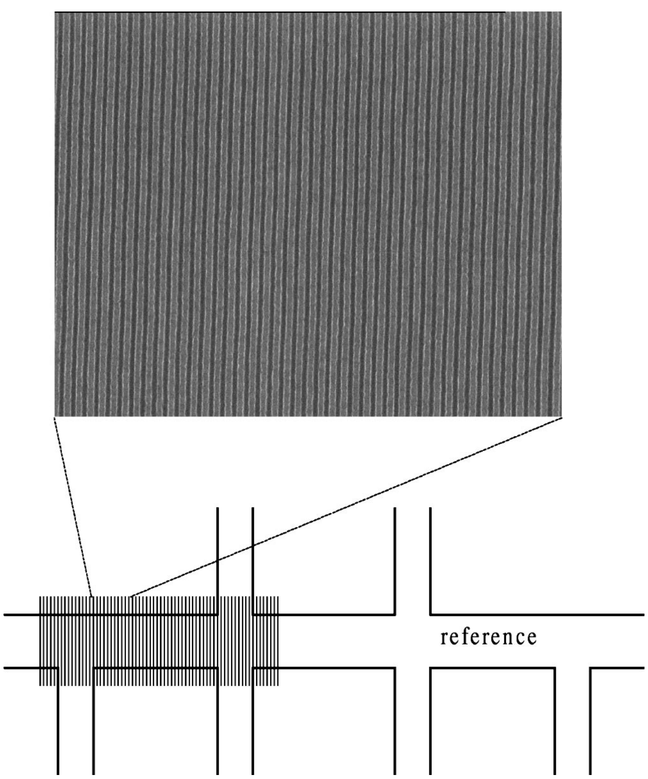

FIG. 1. Schematic diagram of a device with a scanning electron microscopy image of the grating ($a=70$ nm). Darker areas represent the resist, and brighter areas the bare GaAs surface. Bright lines are the edges of the resist.

was prepared. However, for small a, the oscillation amplitude was too small to bear a reliable quantitative analysis. Therefore we limit our analysis to the results from $a=115$ nm LSL's (samples H, $M1$, L, and C) in the following. The relatively small a allowed many oscillations up to high index n (typically $n=3–15$) to be observed. A potential modulation was brought about by differential contractions between the resist and the wafer itself when the device was cooled from room temperature down to 4.2 K, the temperature at which measurement was made. The strain, thus introduced, piezoelectrically couples to the 2DEG, and causes modulation in the 2DEG plane.[10] In order to maximize the effect, the $\langle 110\rangle$ direction was selected as the direction of modulation.[11] Even so, the modulation amplitude was very small: as will be described later, V_0 was around 0.05 meV, or less than 1% of E_F. We attribute this to the small effects, the strain and/or Fermi energy pinning, of the resist we have chosen, and also to the small a to d ratio. A small V_0 was quite favorable for validating a perturbative treatment of the modulation. By comparison with the reference Hall bar, we have verified through Hall and SdH measurements that the grating did not bring about any deterioration of μ or change in (the average) n_s, in spite of the very high dose required for the EB resist [~ 7 mC cm^{-2} (Ref. 9)].

The magnetoresistance measurement was carried out with a standard low-frequency ac technique at 4.2 K. Such a high temperature was deliberately chosen in order to kill the SdH oscillation in the field range (0.1–0.4 T) of present interest.

III. RESULTS

Figure 2(a) shows magnetoresistance of a LSL, sample H, and of its unmodulated counterpart (*control*), measured after illumination by light. From these raw data, the oscillatory part is extracted in the following procedure. As a first step to eliminate the slowly varying background, the resistivity of the control sample was subtracted [Fig. 2(b)].[12] Then the upper and lower envelope curves were found as spline curves tangential to the upper and lower bounds of the trace, respectively. The average curve of the two envelopes was subtracted from the trace as a remnant background [Fig. 2(c)]. The resultant oscillatory part is in an excellent agreement with Eq. (1.2), as shown in Fig. 2(c). For the theoretical curve, only the value of V_0 was used as a fitting parameter, and was $V_0=0.041$ meV. Similarly, traces for sample H without illumination, and for sample $M1$ with and without

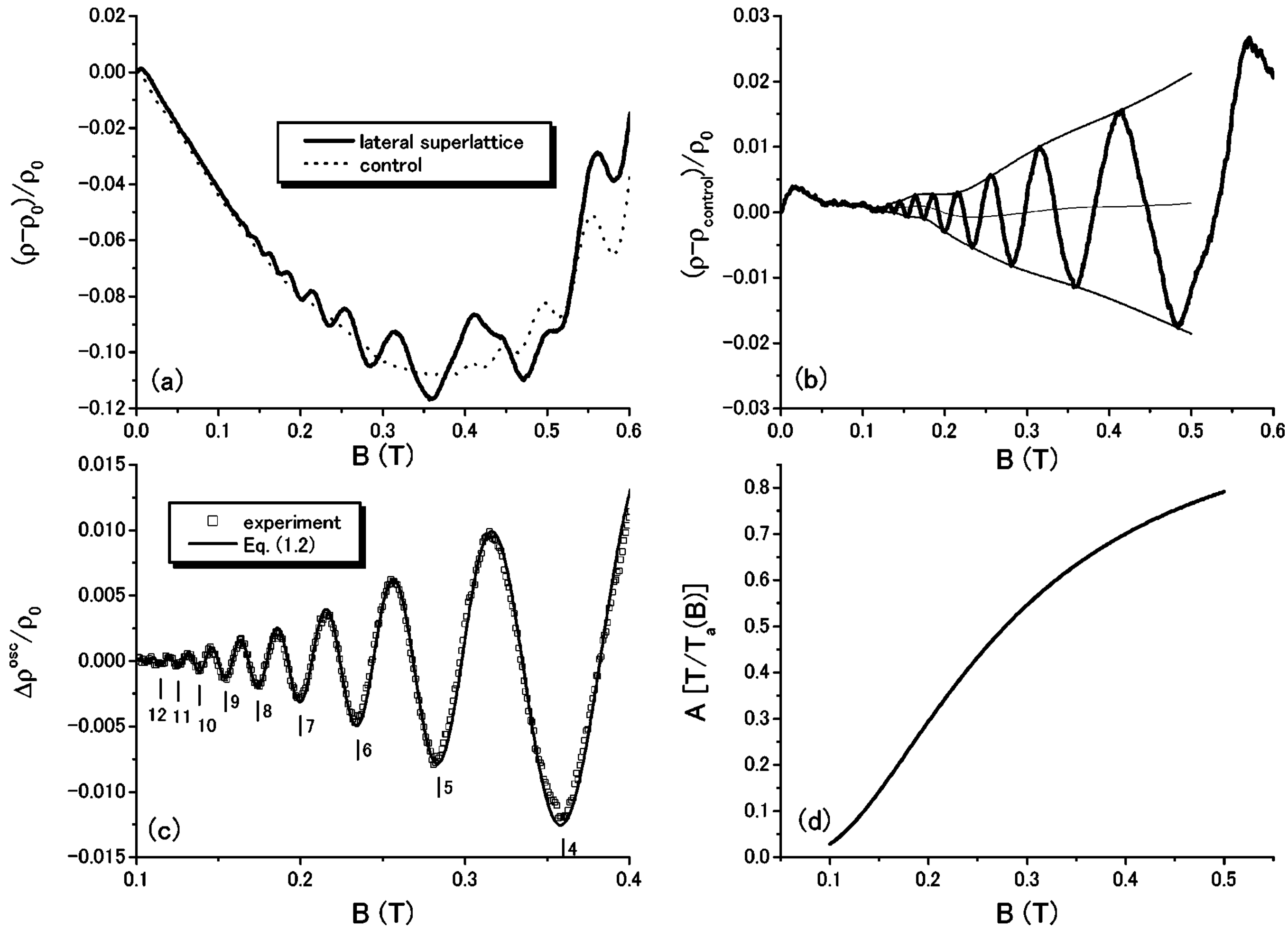

FIG. 2. (a) Magnetoresistance (MR) trace for sample H measured after slight illumination. $\mu=79\ \mathrm{m^2/(V\,s)}$ and $n_s=2.3\times10^{15}\ \mathrm{m^{-2}}$. Also shown is the MR for the adjacent unmodulated Hall bar (control). (b) Thick line: difference between the LSL and control. Thin lines: upper and lower envelopes and their average. (c) Open squares: oscillatory part of the MR obtained by subtracting the average of envelopes [thin line in the middle of (b)] from $(\rho-\rho_{\mathrm{control}})/\rho_0$ [thick line in (b)]. Line: theoretical curve [Eq. (1.2)] with $V_0=0.041$ meV. The positions for flatband condition [Eq. (1.1)] are also shown by short vertical lines with their indices n. (d) Calculated $A[T/T_a(B)]$, with $ak_F/2=6.8$ and $T=4.2$ K, the values for the present measurement.

illumination, did not display any noticeable deviation from Eq. (1.2) with $V_0=0.041$, 0.050, and 0.045 meV, respectively. (The values of V_0 shown here might possibly be underestimating the modulation amplitude by factor of roughly 2. See Sec. IV.) In contrast to what was reported so far, we have shown that Eq. (1.2) can, under a certain condition, correctly reproduce the experimental trace. It is worth pointing out here the important role played by the factor $A[T/T_a(B)]$. This factor was often neglected in semiclassical theoretical treatments,[6] including those published recently,[13,14] since they considered, at least approximately, only $T=0$. This factor is also not taken into account in experimental papers concerned with the envelope of Weiss oscillation.[15–17] The validity of the factor, nevertheless, was experimentally verified by Beton *et al.*,[7] even before the paper by Peeters and Vasilopoulos,[5] by measuring the temperature dependence of the oscillation amplitude with a *fixed B*. [Their B dependence, however, did not follow Eq. (1.2).] When $ak_F/2$ is large and (therefore) $T_a(B)$ is much larger than the measuring temperature T, $A[T/T_a(B)]\approx1$, allowing the factor to be ignored. But since our present LSL's have relatively small a and n_s, ak_F is not so large. For the measurement shown in Fig. 2, $ak_F/2=6.8$; hence $T_a(B)=6.9B$ (in T) K, which is even smaller than $T=4.2$ K at 0.1–0.4 T. As a result, $A[T/T_a(B)]$ is much smaller than unity and has a strong B dependence, resulting from the B dependence of $T_a(B)$, at the magnetic field range of interest [see Fig. 2(d)]. It is obvious that without the factor, an experimental damping of the oscillation has not been reproduced. Our measurement sheds light on the importance of the factor $A[T/T_a(B)]$, and also reconfirms the validity of the factor from a viewpoint different from that of Beton *et al.*

Although we have seen that Eq. (1.2) describes experimental traces of the Weiss oscillation very well, we cannot expect this to be true regardless of the quality of the LSL. Theories[5,6] did not take into account the collision of electrons that scatters electrons away from the cyclotron orbit before it completes a cycle. Therefore, the theories apply only for LSL's with high enough mobility so that the mean free path is much longer than the cyclotron circumference. In fact, our sample L, the LSL fabricated from a 2DEG with low mobility, showed a deviation from Eq. (1.2), as shown in Fig. 3(a). The growth profile of the 2DEG wafer, including the thickness of the spacer layer (d_s), the silicon-doped

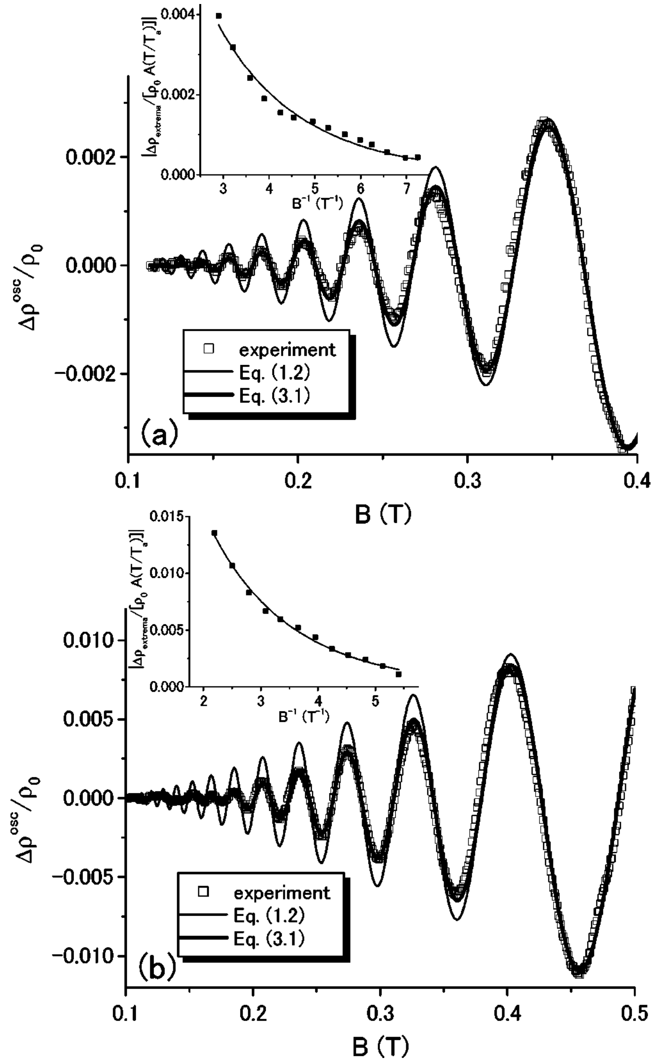

FIG. 3. MR traces for LSL's that display deviations from Eq. (1.2) (thin lines). (a) Sample L with $\mu=24$ m^2/(V s) and $n_s=2.7\times10^{15}$ m^2/(V s). (b) Sample C with $\mu=62$ m^2/(V s) and $n_s=3.3\times10^{15}$ m^2/(V s). Also shown are curves calculated from Eq. (3.1) (thick lines), with parameters that give best fits, $\mu_W=6.1$ and 4.8 m^2/(V s), respectively. The inset shows plots of the maxima and absolute values of the minima vs $1/B$, along with their fits to $P_1/\sinh(P_2/B)$.

$\mathrm{Al}_x\mathrm{Ga}_{1-x}\mathrm{As}$ layer, and the GaAs cap layer, is designed identically to samples H and $M1$.[18] Another LSL which showed a noticeable deviation was sample C [Fig. 3(b)], which has a smaller d_s than samples H, $M1$, and L (the thicknesses of the other layers are the same). According to a recent evaluation of the semiclassical Boltzmann equation,[14] a factor of the form $A[\pi/(\mu B)]=[\pi/(\mu B)]/\sinh[\pi/(\mu B)]$ should be included in Eq. (1.2) to account for the effect of scattering mentioned above (when assuming isotropic scattering). From Eq. (1.2), then, the amplitude of $\Delta\rho_{xx}^{\mathrm{osc}}/\{\rho_0 A[T/T_a(B)]\}$ is expected to be proportional to $1/\sinh[\pi/(\mu B)]$. In the insets of Figs. 3(a) and 3(b), we plot the amplitude of $\Delta\rho_{xx}^{\mathrm{osc}}/\{\rho_0 A[T/T_a(B)]\}$ (the absolute values of minima and maxima) as a function of B^{-1}. We tried fitting to the function $P_1/\sinh(P_2/B)$, with P_1 and P_2 as fitting parameters. The result of the fitting is also displayed in the insets, showing reasonable agreement with the data.[19] The value $\mu_W=\pi/P_2$ obtained from the fitting, however, is much smaller than μ calculated from the zero-field resistivity ρ_0: $\mu_W=6.1$ m^2/(V s) and 4.8 m^2/(V s), to be compared with $\mu=24$ m^2/(V s) and 62 m^2/(V s) for Figs. 3(a) and 3(b), respectively. Note the difference in the ratio μ_W/μ; the ratio for the latter is much smaller. This will be discussed later. Thus, to describe our measurement, Eq. (1.2) needs to be modified to include another damping factor $A[\pi/(\mu_W B)]$ as

$$\frac{\Delta\rho_{xx}^{\mathrm{osc}}}{\rho_0}=A\left(\frac{\pi}{\mu_W B}\right)\frac{\eta^2}{2}\frac{L}{a}B\mu A\left(\frac{T}{T_a(B)}\right)\sin\left(2\pi\frac{2R_c}{a}\right). \tag{3.1}$$

As shown in Fig. 3, Eq. (3.1) reproduces the experimental traces. When μ_W is large, $A[\pi/(\mu_W B)]$ tends to unity; for μ_W greater than about 20 m^2/(V s), Eq. (3.1) is practically indistinguishable from Eq. (1.2) at 4.2 K. Samples H and $M1$ are also described by Eq. (3.1) with large enough μ_W. Therefore Eq. (3.1) is not inconsistent with the observation of Fig. 2.

To see the dependence of our damping parameter μ_W on the mobility μ, we successively illuminated sample C with an infrared light-emitting diode, and gradually increased n_s and μ. The evolution of oscillation envelope is shown in Fig. 4. As can be seen, the experimental trace becomes progressively closer to Eq. (1.2) with the increase of μ; μ_W, that gives the best fit to Eq. (3.1), increases with μ. It is important to point out that the second and third traces, which show clear deviations from Eq. (1.2), have comparable, even higher mobilities than samples H and $M1$. Apparently μ_W is not determined solely by μ.

The values of μ_W giving the best fit are plotted in Fig. 5 for the four samples as functions of the mean free path $L=\hbar k_F\mu/e$. As can be readily seen, the plots can be divided into two groups, one with LSL's prepared from a 2DEG with $d_s=40$ nm (samples L, $M1$, and H), and the other from a 2DEG with $d_s=20$ nm (sample C). The error bars in the figure represent the uncertainty of the fitting to the function $P_1/\sinh(P_2/B)$. For large μ_W, i.e., for small P_2, the function tends to P_1B/P_2. It then becomes difficult to determine P_1 and P_2 independently, resulting in the large error bars shown. Within each group, μ_W is seen to be nearly proportional to L. The coefficient is about four times larger for the first group. The ratio is, probably fortuitously, the same as the ratio of d_s^2. Therefore, the values of μ_W replotted as a function of $k_F d_s^2\mu$ fall on a single line (numerically approximately $\mu_W=[1/(2\pi^3 a_B^*)]k_F d_s^2\mu$, with $a_B^*=10.2$ nm the effective Bohr radius in GaAs.) More significantly, the magnitude of μ_W is close to that of μ_Q, the quantum mobility, obtained from a Dingle analysis of the experimental SdH traces. This implies that small-angle scattering plays an important role in the damping of the Weiss oscillation, as will be discussed in Sec. IV.

IV. DISCUSSION

It is well known that for a $\mathrm{GaAs/Al}_x\mathrm{Ga}_{1-x}\mathrm{As}$ 2DEG, the (transport) mobility $\mu=e\tau/m^*$ is often higher than the quantum mobility $\mu_Q=e\tau_Q/m^*$ by an order of magnitude, where τ and τ_Q represent the momentum-relaxation time and the total scattering time, respectively. This is because small-

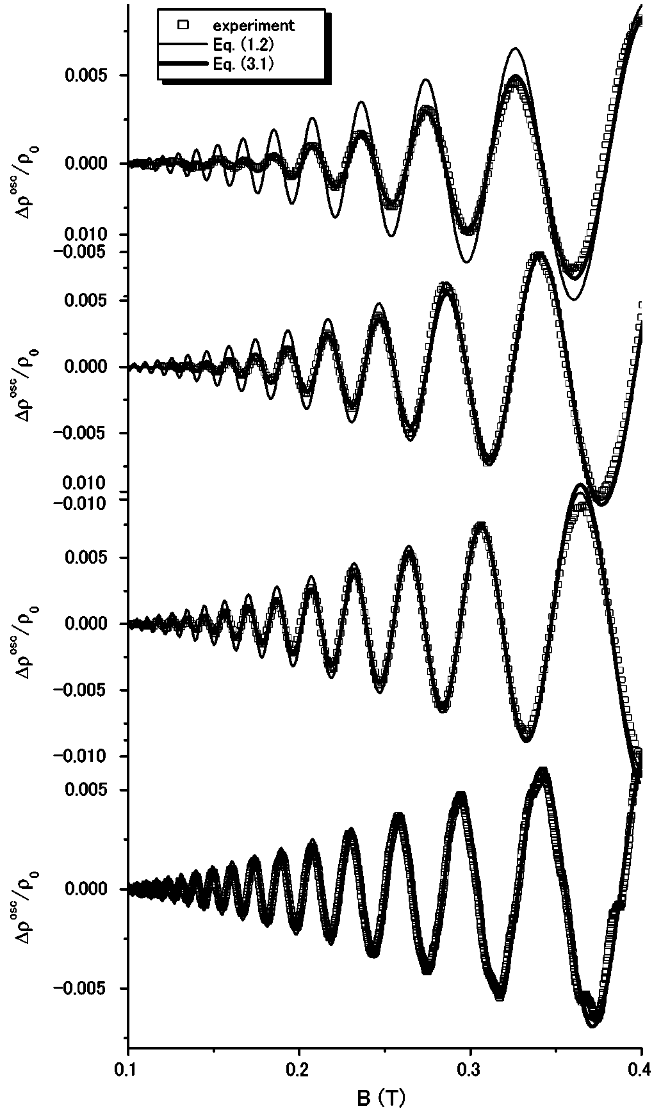

FIG. 4. Evolution (from top to bottom) of the MR trace of sample C with successive illumination with an infrared light-emitting diode. μ [in $m^2/(V\,s)$], n_s (in 10^{15} m^{-2}), and μ_W that gives the best fit to the experimental trace [in $m^2/(V\,s)$] were (from top to bottom) 62, 3.3, and 4.8 [the reproduction of Fig. 3(a)]; 72, 3.5, and 6.4; 80, 4.1, and 7.7; and 101, 5.0, and 12.

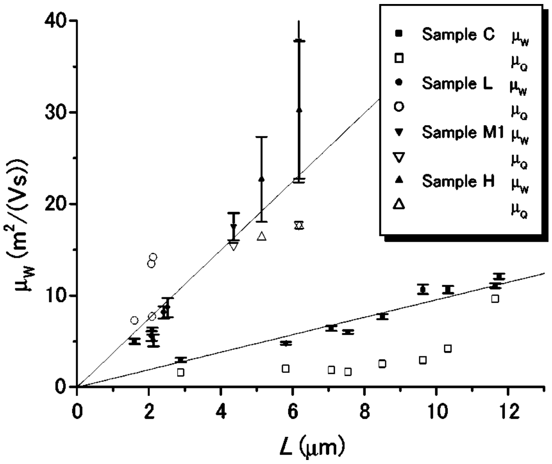

FIG. 5. Plot of μ_W for samples C, L, $M1$, and H vs $L=\hbar k_F\mu/e$ (solid symbols). The error bars represent the uncertainty of the fitting to the function $P_1/\sinh(P_2/B)$. Two straight lines that pass through the origin are the fits to samples with $d_s=40$ nm (samples L, $M1$, and H) and to sample C with $d_s=20$ nm, respectively. Also shown by open symbols are μ_Q obtained from a damping, $\exp[-\pi/(\mu_Q B)]$, of the SdH oscillation.

angle scattering by remote ionized donors, one of the main scattering processes in the system, contributes much less to the former. Nevertheless, it was pointed out by several authors[7,14–16] that small-angle scattering should be considered as a scattering that scatters electrons away from the cyclotron trajectory, and affects the amplitude of the Weiss oscillation. Without scattering, $\Delta\rho_{xx}^{osc}/\rho_0 \propto B$, apart from the B dependence of the factor $A[T/T_a(B)]$ [see Eq. (1.2)]. Beton *et al.*[7] suggested that an exponential factor should be included so that $\Delta\rho_{xx}^{osc}/\rho_0 \propto B\exp[-\pi/(\mu' B)]$, with their experimental μ' consistent with μ_Q. Paltiel *et al.*,[16] on the other hand, proposed a $B\exp(-B_0^3/B^3)$ dependence which explained their experiment (and also recent experiment by Long *et al.*[17]) well. These formulas are more or less of empirical nature, multiplying additional damping factor to B. A more rigorous treatment of Boltzmann equation by Mirlin and Wölfle,[14] however, showed that the factor B should be removed from these formulas. They showed, as mentioned earlier, that the factor that should be included is $[\pi/(\mu B)]/\sinh[\pi/(\mu B)]$ for an isotropic scattering model, a model that does not take the difference between μ and μ_Q into account. In a more realistic long-range random scattering model, they showed that the factor is modified by replacing μ with $\mu^*=\mu_Q/\{1-[1+\mu_Q/\mu(L/a)^2(2\pi)^2/(\mu B)^2]^{-1/2}\}$. Its approximate formulas can be expressed as follows: at low field $B\ll B_2\equiv\pi(L/a)\sqrt{2\mu_Q/\mu^3}$, $\Delta\rho_{xx}^{osc}/\rho_0 \propto 1/\sinh[\pi/(\mu_Q B)]\approx 2\exp[-\pi/(\mu_Q B)]$, a similar formula to that of Beton *et al.*, but without the factor B; in the middle field range $B_2\ll B\ll B_1\equiv\pi(4L/a)^{2/3}/(2\mu)$, $\Delta\rho_{xx}^{osc}/\rho_0 \propto\exp[-2\pi^3L^2/(a^2\mu^3B^3)]$, similar to Paltiel *et al.*, but again without B. The erroneous inclusion of B can lead to a factor of 2–3 overestimation of μ_Q in the low-field case.

Returning to our experiment, B_1 and B_2 (using μ_W in place of μ_Q) in the above calculation fall in between 0.7–1.2 and 0.9–2.5 T, respectively. Both fields are relatively large owing to large values of L/a, and for most of the samples B_2 is even larger than B_1. The field 0.1–0.4 T where the Weiss oscillation is observed may, therefore, be classified in the aforementioned low-field regime. Thus the oscillation amplitude is, according to the theory by Mirlin and Wölfle, expected to be proportional to $1/\sinh[\pi/(\mu_Q B)]$, implying that our μ_W equals μ_Q. This is not inconsistent with the observation in Fig. 5; μ_W and μ_Q are in reasonable agreement, although a discrepancy is seen, especially for sample C in the intermediate mean free paths. We believe the discrepancy to result mainly from the limited validity of the values of μ_Q. The Dingle analysis of the SdH oscillation has been known to be quite vulnerable to even a slight inhomogeneity of n_s. The inhomogeneity manifests itself as a curvature in the Dingle plot and/or a deviation of the $1/B\rightarrow 0$ intercept from the theoretical value ''4,'' owing to destructive interference of SdH oscillations with varying frequencies.[20] The effect usually makes the slope of the Dingle plot appear larger; hence the resulting μ_Q smaller. The degree of inhomogene-

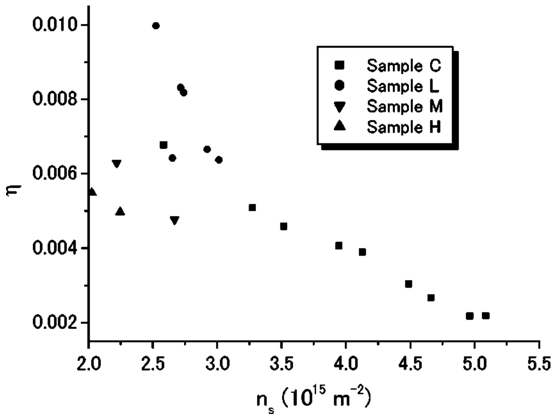

FIG. 6. Plot of η as a function of n_s. Possibly the correction factor of $\sqrt{\mu/\mu_W}$ should be included (see the text for details).

ity can vary between wafers or between different illumination conditions. It is possible that after slight illumination, before the saturation of DX-center excitation, inhomogeneities become more pronounced. However, it was not possible to quantitatively estimate these and other effects which challenge the reliability of the values of μ_Q, mainly because our experimental Dingle plot was taken from rather narrow field range of 0.5–0.9 T, where a SdH oscillation was observed at 4.2 K. Another point that suggests $\mu_W = \mu_Q$ is the dependence of μ_W on d_s. Coleridge[20] showed that both μ and μ_Q, and also the ratio μ/μ_Q, increase with d_s for small d_s, experience maximum at a certain d_s, and then decrease. The value of d_s that gives the maximum values depends on the background acceptor density N_A in the GaAs channel. For the estimated N_A for our 2DEG, $d_s = 20$–40 nm is still in the increasing regime μ_Q. Therefore, μ_Q should be larger for $d_s = 40$ nm. Conversely, we postulate, the relation $\mu_W = \mu_Q$, then the measurement of Weiss oscillation damping may provide an alternative (probably more robust) method for determining μ_Q.

Our finding of the damping factor $A[\pi/(\mu_W B)]$ is in qualitative disagreement with Paltiel *et al.*[16] and Long *et al.*[17]; none of our traces show a reasonable fit to $B\exp(-B_0^3/B^3)$. The reason for this is not clear at present.[21] At least in Ref. 16, the values of n_s, μ, μ_Q, and a are similar to ours. Therefore both B_1 and B_2 are almost the same, categorizing the sample of Ref. 16 into the low-field regime of Mirlin and Wölfle. One possible explanation for the discrepancy is the difference in the modulation amplitude V_0. The modulation amplitudes of Refs. 16 and 17 are orders of magnitude larger than ours. The potential modulation is inevitably accompanied by a position-dependent electron density, and hence by a position-dependent k_F. The effect is not taken into consideration in perturbative calculations at all, but can affect the amplitude of the Weiss oscillation, especially in the lower field, as is the case with the SdH oscillation. However, it is beyond of the scope of the present paper to evaluate this effect.

Finally we address the issue of the magnitude $\eta = V_0/E_F$ of the potential modulation that can be deduced from our present analysis. The values of η derived by fitting the experimental traces to Eq. (3.1) are plotted against the electron areal density n_s in Fig. 6; actually η was obtained by fitting the function $P_1/\sinh(P_2/B)$ to the plots of the extrema of $\Delta\rho_{xx}^{\mathrm{osc}}/\{\rho_0 A[T/T_a(B)]\}$ (see the inset of Fig. 3), with $\eta = \sqrt{2aP_1/(L\mu P_2)}$. For each sample, η decreases with n_s faster than $n_s^{-1} \propto E_F^{-1}$. This presumably reflects the increase of screening, which diminishes the efficacy of the perturbation brought about by the grating. Prior to the present study, η was usually obtained by using Eq. (1.2), often neglecting the factor $A[T/T_a(B)]$, and by picking up the oscillation amplitude of the lowest index n, i.e., the highest field (see, e.g., Ref. 17). The values of η thus obtained are identical to ours provided that the damping is completely negligible $(A[T/T_a(B)] = A[\pi/(\mu_W B)] = 1)$, the condition usually *not* fulfilled. In general, using Eq. (3.1) instead of Eq. (1.2) has the advantages of (1) taking into account the damping that has already occurred even at the lowest index, and (2) obtaining η is common to all the indices n. However, the possibility that the present treatment still underestimates the value of η cannot be completely ruled out. In obtaining Eq. (3.1), we rather arbitrarily just multiplied the Eq. (1.2) by factor $A[\pi/(\mu_W B)]$ as a natural extension of the equation. Although this procedure successfully explains the B dependence of the oscillation modified by the (small-angle) scattering, it might be argued that the scattering also reduces the amplitude by multiplying Eq. (1.2) by a factor independent of B. In fact, the theory of Mirlin and Wölfle requires the inclusion of another factor μ_Q/μ into Eq. (3.1) (with μ_W replaced by μ_Q in the equation). Identifying our μ_W with μ_Q, the resultant amplitude should be altered from η to $\eta' = \eta\sqrt{\mu/\mu_W}$. Numerically, the correction factor is roughly 2 for samples H, $M1$, and L, and around 3 for sample C. Another independent way to estimate η is desired to know which equation is the correct one. Positive magnetoresistance (PMR) at the low fields[22] is often used for this purpose. Unfortunately, owing to the smallness of the modulation amplitude, PMR was very small, sometimes totally unobservable, for our present samples, and therefore cannot be used for a reliable analysis. Our recent experiment using *magnetic* LSL with controllable modulation amplitude[23] suggests, however, that η' overestimates the amplitude by comparison with the modulation amplitude estimated from PMR.

V. CONCLUSIONS

We have shown that Eq. (3.1) reproduces the oscillatory part of the magnetoresistance very well. The parameter μ_W was found to be proportional to L, and numerically was $\mu_W \approx 1.6 \times 10^{-3} k_F d_s^2 \mu$ (with k_F in nm^{-1} and d_s in nm). For large enough μ_W, Eq. (3.1) is indistinguishable from Eq. (1.2). Comparison of the damping factor with recent theory[14] suggests $\mu_W = \mu_Q$, which is not inconsistent with our experimental μ_Q. This implies that scattering events, regardless of the scattering angle, contribute to the damping of the Weiss oscillation. To establish a more precise relation between η and the oscillation amplitude, it might be necessary to include an additional constant factor in Eq. (3.1), which is a problem that requires further study.

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