Conductance fluctuations in a double-barrier resonant tunneling device

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We have investigated conductance fluctuations due to tunneling through impurity states in the quantum well of a double-barrier resonant tunneling device. The impurity states are donor-related and are associated with a low-density Si δ-doping layer incorporated into the center plane of the quantum well. At constant temperature, the relative amplitude of the conductance fluctuations is determined by the absolute number of donor impurities in the well and is found to scale as $(SN_d)^{-1/2}$, where N_d is the areal density of donor atoms in the well, and S is the area of the device. The typical voltage period of the fluctuations is determined by the larger of kT or the natural linewidth of the state. There is excellent quantitative agreement between the experimental results and an existing theoretical model for conductance fluctuations in this type of system.

I. INTRODUCTION

The study of conductance fluctuations has provided a great deal of information about the nature of transport in mesoscopic devices. In the linear transport regime, fluctuations are generally a result of quantum interference effects and consequently are governed by the phase coherence of the electrons.[1] In contrast, in the tunneling regime, the form of the fluctuations depends on the nature and number of the tunneling channels.[2] Although there may be contributions to the tunneling conductance from phase-coherent effects,[3] in this paper we concentrate on systems where tunnelling occurs from a continuum through individual localized states that are formed within the tunnel barrier. For this case, Larkin and Matveev[4] (see also Ref. 2) predicted conductance fluctuations (CF) which were essentially a statistical consequence of the finite number of tunneling channels. For N channels one expects relative conductance fluctuations, $\delta G/G \sim N^{-1/2}$. Despite the simplicity of this idea, the prediction has not previously been tested experimentally, to our knowledge, although similar statistical fluctuations have been seen in many analogous systems. There have been many examples of tunneling through individual states in single barrier devices[5–10] but generally the nature and number of the tunneling channels have not been well characterized. Hence, although there have been many reports of CF in tunneling systems, there has not been a quantitative comparison with theory.

In this paper, we describe a system that permits a detailed investigation of CF arising from tunneling through a finite number of localized channels. The system comprises a GaAs/(AlGa)As *double*-barrier device in which the center plane of the quantum well is δ doped with Si donors at a low density, $<10^{14}\,\mathrm{m}^{-2}$. We have previously shown[11–13] that the incorporation of impurities leads to the creation of discrete, localized electronic states in the well at energies below that of the lowest energy two-dimensional subband. Electrons may tunnel through the device via these impurity states and this gives rise to a peak in the current-voltage characteristics, $I(V)$, at biases below that of the first subband resonance. In effect, in this regime, the double-barrier device acts like a single barrier with impurity states in the center of the barrier. The advantage of this system is that the number of the impurities and their location in the growth direction are known and can be controlled. Furthermore, we are able to vary the tunneling rate into the impurity states by the application of a magnetic field perpendicular to the current direction.[11] This has the dual effect of altering, by several orders of magnitude, both the natural linewidth of the impurity state and the average current through the device. In the course of a number of recent, related experiments[11–16] we have developed a comprehensive understanding of the electrical properties of these devices so this system is an excellent test bed for the theoretical description of the CF. Generally, we find that the principal theoretical predictions for the amplitude and quasiperiodicity of the CF are well confirmed.

II. EXPERIMENT

We have fabricated double-barrier, resonant tunneling devices (RTD's) from layers grown by molecular-beam epitaxy with the substrate at 550 °C. The growth temperature is a compromise between the conflicting requirements for high-quality interfaces and for low rates of diffusion and segregation of impurities. Spacer layers of 20 nm of undoped GaAs separate the heavily doped n^+-GaAs contacts from the active region of the device, which comprises a 9-nm GaAs quantum well (QW) enclosed by two 5.7-nm-wide $(Al_{0.4}Ga_{0.6})As$ barriers. A δ layer of silicon donor impurities with concentration N_d either 4×10^{13} or $8\times10^{13}\,\mathrm{m}^{-2}$ is incorporated in the center plane of the quantum well during growth. Square mesas with side L between 7 and 100 μm were fabricated using standard optical lithographic techniques. Mesas are labeled A–L according to Table I. Further details of the devices are provided in Ref. 11.

A conduction-band diagram of a device under bias is shown in Fig. 1(a). The presence of the spacer layer ensures that tunnelling occurs from a two-dimensional electron gas

TABLE I. Details of mesas used in the experiments. SN_d is the approximate number of donors within the quantum well.

Sample	Donor concentration N_d	Mesa size	SN_d
A	$4\times10^{13}\,m^{-2}$	7 μm	2000
B	$4\times10^{13}\,m^{-2}$	9 μm	3200
C	$4\times10^{13}\,m^{-2}$	11 μm	4800
D	$4\times10^{13}\,m^{-2}$	12 μm	5800
E	$4\times10^{13}\,m^{-2}$	100 μm	4.0×10^5
F	$8\times10^{13}\,m^{-2}$	7 μm	3900
G	$8\times10^{13}\,m^{-2}$	9 μm	6500
H	$8\times10^{13}\,m^{-2}$	11 μm	9700
K	$8\times10^{13}\,m^{-2}$	12 μm	11 500
L	$8\times10^{13}\,m^{-2}$	100 μm	8.0×10^5

(2DEG) formed in an accumulation layer at the left-hand emitter barrier. The potential difference V_1 between the emitter 2DEG and the states of the quantum well is roughly proportional to the total voltage drop V across the device; the ''leverage factor'' $f=dV_1/dV$ is typically ~0.3 for our devices in the bias range of interest. For the concentrations of δ doping employed, well below the values corresponding to the metal-insulator transition, the shallow donor impurities act as independent charge centers since their mean spacing is much larger than the effective Bohr radius (~10 nm) of an electron bound to a donor. The electron states associated with the donors in the quantum well have binding energies ~6–13 meV relative to the continuum state, depending on their separation from the barriers.[17] Consequently, it is possible for electrons to tunnel through these localized states at biases below that of the first main resonance of the RTD, $R1$, which corresponds to tunnelling into the 2D continuum of states associated with the lowest subband $E1$ of the QW. A typical low-bias $I(V)$ curve, with the corresponding differential conductance $G(V)=dI/dV$, is shown in Fig. 1(b) for mesa E. The pronounced shoulder feature in $I(V)$ around 90 mV has been identified unambiguously[11,12] as a single donor resonance (SDR) due to tunnelling through the bound states of the donors.[18] The peak of the resonance $R1$ occurs at higher bias. Devices incorporating no δ-doped layer do not show the shoulder feature, as can be seen in the $I(V)$ of a control sample with no δ doping shown in Fig. 1(b). The feature occurs ~30–40 mV below the onset of the first main resonance, consistent with a donor binding energy ~9–12 meV after the leverage has been taken into account. Note that, since the electrons are tunnelling through states of lower energy than the continuum state in the well, the double-barrier device effectively blocks all tunnelling processes except those through the impurity-related states.

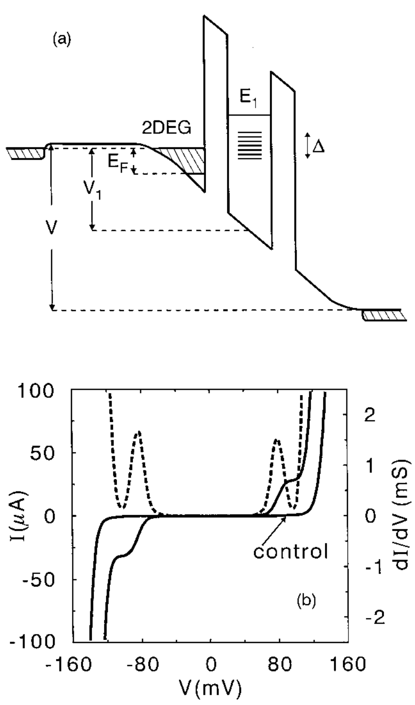

FIG. 1. (a) Schematic illustration of a typical device under bias, showing the definition of the leverage factor, $f=dV_1/dV$. Tunnelling occurs from a 2DEG, with Fermi energy E_F, through impurity-related states spread over an energy range Δ. (b) $I(V)$ (solid) and dI/dV (dashed) at 4.2 K for mesa E. Also shown is $I(V)$ for a control sample of the same mesa size but with no intentional impurities in the quantum well.

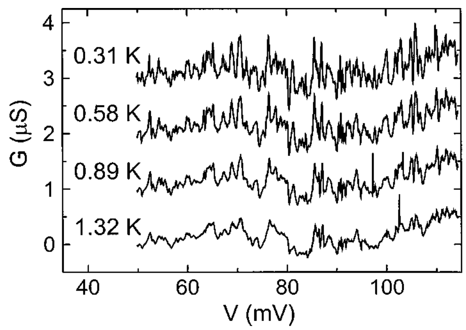

FIG. 2. G vs V for mesa A at various T in $B=10$ T. Successive curves have been offset by 1 μS.

$I(V)$ curves similar to that shown in Fig. 1(b) have been studied for all mesas over the whole range of bias. We have shown in earlier papers that, at very low bias, the onset of current is due to tunneling through individual localized states in the quantum well[13,15] and is thermally activated. At low temperatures, electron-electron interaction effects are important and a Fermi edge singularity occurs in $I(V)$ close to the onset.[14] In this paper we focus on the regime within the SDR, where the impurity states are more numerous and where we can describe the density of strongly localized states by means of a quasicontinuous density-of-states. Figure 2 shows the differential conductance, $G=dI/dV$, of mesa A within the SDR at various temperatures. The data in Fig. 2 were taken in the presence of a magnetic field, $B=10$ T, applied perpendicular to the current direction, i.e., parallel to the plane of the quantum well. The role of the magnetic field will be discussed in detail below but for all the measurements described in this paper the direction of B is always parallel to the plane of the QW. Sharp conductance fluctuations are clearly visible; these occur within the SDR and are unambiguously associated with the impurity states of the donor atoms. On the scale of Fig. 2, there is very little noise and the fluctuations are entirely reproducible, provided the device is maintained at low temperature. It is clear that while the average G is independent of temperature T, the relative amplitude of the fluctuations $\delta G/G$ increases with decreasing T. In addition, the characteristic period of the fluctuations becomes smaller with decreasing T. At low temperatures, the fluctuations are very clear in the current as well as the conductance but, as we shall see below, it is the conductance which is more relevant for comparison with theory. We emphasize that CF have been observed in all devices studied, even though they are macroscopic in size, with mesa diameter up to 100 μm.

III. THEORY

Conductance fluctuations arising from tunneling through localized states within a potential barrier were predicted by Larkin and Matveev.[4] Raikh and Ruzin[2] reviewed this model and related theoretical work. The model describes a linear tunneling device, in which the barrier states are resonant with the emitter and collector chemical potentials at zero bias, i.e., there is a finite conductance at zero bias. However, it can be readily modified to the case of our nonlinear device. For simplicity, we make the reasonable assumption that the tunnel current in our device is determined by the rate at which electrons tunnel from the emitter into the localized donor states in the well. The transmission coefficient for tunneling out of the donor states through the collector barrier is assumed to be larger than the tunneling rate from the emitter. It is justified by the form of the potential profile under bias [see Fig. 1(a)] which shows that the collector barrier is lower than the emitter barrier. In this case, the donor states have a low occupancy, i.e., the states are empty for most of the time. As the voltage V increases, the current also increases as more states come into resonance. If the Fermi energy E_F of the 2DEG is sufficiently large that all the impurity states can be on resonance at the same value of V, the peak current amplitude J is given by

$$J = eN_d S\Gamma_e/\hbar, \tag{1}$$

where $\Gamma_e/\hbar$ is the tunnelling rate through the emitter barrier and S is the area of the device. Note that J is the peak value of the absolute current due to the impurity states, not the current density. In the case where the impurity states are spread over an energy range Δ, which is larger than E_F, Eq. (1) becomes

$$J = eN_d S\Gamma_e E_F/\hbar\Delta, \tag{2}$$

where the impurity states are assumed to have a constant distribution over the energy range Δ as shown in Fig. 1(a). As it turns out, in our devices $E_F \approx \Delta$. Therefore, since we do not expect either E_F or Δ to vary between different mesas, Eq. (1) and Eq. (2) differ only by a constant of order unity; consequently, we use Eq. (1) for simplicity. To determine the conductance we note that increasing the emitter energy by eV_1 causes geV_1 and more states contribute to the current, where g is the density of localized states in the well. If we assume that the impurity states are spread uniformly over a range Δ due to disorder,[15] then $g = N_d S/\Delta$. The differential conductance $G = dI/dV$ is then given by

$$G = fe^2 N_d s\Gamma_e/\hbar\Delta. \tag{3}$$

Equation (3) is valid provided that the only contribution to G is from states coming onto resonance. The situation is more complicated for our nonlinear device because at some biases there is a negative contribution to the conductance from impurity states at the low-energy end of the distribution, which at these biases move below the band edge of the emitter 2DEG. This point is discussed below. Note that, consistent with experiment, neither the average current nor the average conductance is predicted to have a temperature dependence for $kT \ll E_F,\ \Delta$.

According to Larkin and Matveev,[4] the relative amplitude of the CF is given by

$$\delta G/G = M^{1/2}, \tag{4}$$

where M is the number of impurity channels resonant with electrons at the Fermi energy in the emitter, i.e., the number of states involved in the conduction process. Note that M is distinct from $N_d S$, the total number of tunnel channels available. This is the central idea that underpins the model to describe the CF. To analyze the data from our nonlinear device, it is more convenient to relate the CF to the peak amplitude J of the current due to the donor states, rather than the average conductance. Using Eqs. (1), (3), and (4) we obtain

$$\frac{\delta G}{J} = \left(\frac{ef}{\Delta}\right) M^{-1/2}. \tag{5}$$

At $T=0$, where there is no thermal smearing of the emitter 2DEG, only impurity-related states within a natural linewidth Γ_0 are resonant with the electrons at the emitter Fermi energy. In general, Γ_0 is larger than Γ_e, since tunneling through the collector barrier will also affect Γ_0. We obtain

$$M = N_d S\Gamma_0/\Delta. \tag{6}$$

In this case, provided Γ_0 is constant we have

$$\frac{\delta G}{J} \propto (SN_d)^{-1/2}. \tag{7}$$

For $kT \gg \Gamma_0$, the number of states resonant with the emitter Fermi energy is limited not by the linewidth of the tunneling channel but by kT, so Eq. (6) becomes

$$M = \alpha N_d SkT/\Delta, \tag{8}$$

where α is a constant of order unity. Equations (5) and (8) indicate that, in this limit, provided Γ_e is the same for all devices,

$$\frac{\delta G}{J} \propto (SN_d)^{1/2} T^{-1/2}. \tag{9}$$

We have explicitly assumed that the donor states are distributed evenly over the range Δ, which is not valid in the real devices. However, it is reasonable to assume that the distribution does not vary rapidly over a small voltage range. Provided we compare data between different devices over a similar bias range, Eqs. (7) and (9) should still be applicable.

These predictions are valid for the situation where the density of impurity states remains constant over the whole voltage range of interest. In this case, an increase in bias leads to more impurity states contributing to the current. However, in our nonlinear device, the energy range of impurity states is finite and there is a region of bias where the number of states available for tunneling with energy conservation decreases with increasing bias. In Eq. (4), M is then the number of states resonant with the band edge of the 2DEG accumulation layer. In the intermediate bias regime, there are contributions to the CF both from states coming onto resonance and those falling below resonance; these contributions will add incoherently.

In addition to their amplitude, the other parameter which characterizes the CF is their typical voltage period. We may define a correlation function[2] by

$$K(\Delta V)=\frac{\langle G(V)G(V+\Delta V)\rangle-\langle G(V)\rangle^2}{\langle G(V)^2\rangle-\langle G(V)\rangle^2}, \tag{10}$$

where the brackets indicate an average over the chosen range of voltage. The typical period of the fluctuations may be defined as ΔV_c, the value of ΔV such that $K(\Delta V)$ has fallen to one-half its value at $\Delta V=0$. Then we have[2] at $T=0$, $\Delta V_c \sim \Gamma_0/ef$ and for $kT \gg \Gamma_0$, $\Delta V_c \sim kT/ef$, where f is the leverage factor discussed earlier. This is intuitively correct since the energy scale for the variation of M, the number of states resonant with the Fermi energy of the emitter 2DEG, is Γ_0 for $kT \ll \Gamma_d$ and kT for $kT \gg \Gamma_0$.

The above discussion places a number of constraints on the measurement and analysis of the $I(V)$ curves. First, in order to resolve the CF, the voltage increments for the measurement must be less than $kT/e \sim 30\ \mu$eV at the lowest T. Second, the range of voltage chosen for averaging to define $K(\Delta V)$ must be large compared with the CF period (i.e., kT) but small compared with Δ. For our system, $\Delta \sim 7$ meV and the maximum value of kT is ~ 1 meV allowing a reasonable window for averaging. Finally, note that, even in the smallest device (mesa A), there are about 2000 donors in the well, giving an average energy spacing of 3.5 μeV between donor states. This corresponds to $T \approx 30$ mK, so that the δG and ΔV_c should be limited by kT (or Γ_0 if that is larger) over the whole range investigated. It is tempting to identify each peak with tunneling through a single state. However, this is quite incorrect. In fact, even for device A, with the smallest number of donors in the well, approximately 10 donor impurity states contribute to each peak in G.

IV. RESULTS AND DISCUSSION

The strongest prediction that follows from Eqs. (7) and (8) and that underpins the theoretical model is that $\delta G/G \propto (\mathrm{SN}_d)^{-1/2}$ when all other parameters are held constant. To test this for our nonlinear devices, we measured δG, averaged over a bias range of 10 mV, just below the maximum in the current due to the impurity states, i.e., where the conductance is due to states coming onto resonance. Figure 3 shows a plot of $\delta G/J$ vs $(N_d S)^{-1/2}$ for mesas A, D, F, and K at $T=0.3$ K in $B=0$ T. For comparison with theory, δG is defined as the rms value of the fluctuation amplitude. The agreement is excellent, particularly when we take into account that each point in the data taken in Fig. 3 corresponds to a different device and that two different wafers are involved.

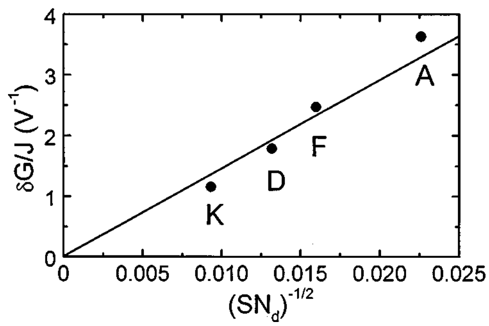

FIG. 3. Amplitude of conductance fluctuations δG, normalized to the peak current J vs $(\mathrm{SN}_d)^{-1/2}$ for mesas A, D, F, and K in $B=0$ T. The line is a guide to the eye to show that $\delta G/J$ is proportional to the total number of donors in the quantum well.

In addition, Fig. 2 shows that δG is temperature dependent whereas the average conductance is not. This confirms that, for these data at this magnetic field, the limit $kT > \Gamma_0$ applies over at least part of the temperature range. In this limit, δG should vary as $T^{-1/2}$. Figure 4 shows δG plotted vs $T^{-1/2}$ for mesas A and D in $B=5$ T. Again there is good agreement with the prediction of the model; similar behavior is found for all devices at all values of B. However, at the lowest temperatures, below 0.5 K, there is some evidence for saturation of $\delta G/J$. Furthermore, the saturation effect is more pronounced at lower values of the magnetic field. This is discussed further below.

We now turn to the typical period of the fluctuations. There is a problem in calculating $K(\Delta V)$ from the experimental data, which is related to our assumption that the impurity states are uniformly distributed in energy over the range Δ. In reality, this will not be true and we expect the distribution to be approximately Gaussian. While this does not have any qualitative effect on the theoretical model, it does mean that the amplitude of δG is voltage dependent. In calculating $K(\Delta V)$, we average over a voltage range of 10 mV (equivalent to an energy range of 3 meV) in the bias range below the maximum current. This ensures that we can ignore the contribution from states going off resonance below the band edge. The inset to Fig. 5 shows a typical correlation function for $B=0$ T and $T=0.6$ K. ΔV_c is determined from the value of ΔV where K falls to 50% of its maximum. Figure 5 shows ΔV_c vs T for mesa A in (a) $B=0$ and (b) $B=10$ T. Although there is only a limited range of T, the data for 10 T are well described by a linear relationship with $e\Delta V_c = 2.0 \pm 0.2$ kT. At $B=0$ T, as with $\delta G/J$, there is evidence for saturation below 0.5 K, but at higher T,

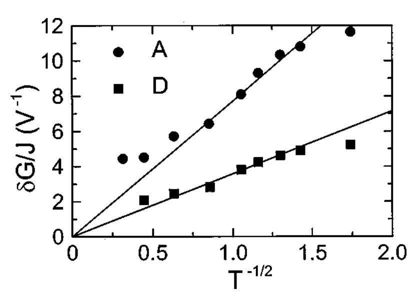

FIG. 4. $\delta G/J$ vs $T^{-1/2}$ at 0.3 K in $B=5$ T for mesas A and D. Lines are guides to the eye.

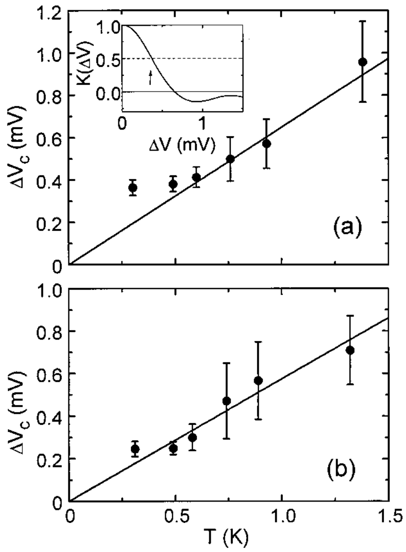

FIG. 5. Typical period of the fluctuations, or correlation voltage, ΔV_c vs T for mesa A in (a) $B=0$ T and (b) $B=10$ T. The lines are 2.2 kT and 2.0 kT, respectively for (a) and (b). Inset shows the correlation curve calculated for mesa A in $B=0$ at 0.6 K. The curve is calculated by averaging data between 55 mV and 65 mV. The arrow shows the definition of ΔV_c.

we have $e\Delta V_c=2.2\pm0.2\,$kT. The agreement between the values is remarkable when one considers that the current falls by more than two orders of magnitude between 0 and 10 T.

The saturation observed at low temperature in both the amplitude and the typical period of the fluctuations occurs more readily at low values of the applied magnetic field. The primary effect of B is that it adds an additional magnetic potential to the height of the tunnel barriers. This reduces the tunnelling rates,[11] Γ_e and Γ_0, into and out of the impurity state, leading to a reduction of current and a narrowing of the natural linewidth of the state. Consequently, there are two possible origins of the saturation effect. The first possibility is that at $B=0$, the relatively high current causes electron heating and a genuine temperature saturation. The application of B causes a rapid fall in the current and the saturation effect disappears. The second, more interesting possibility is that the saturation may be because, at $B=0$, Γ_0 may be larger than kT at the lower temperatures, leading to T-independent values of $\delta G/J$ and ΔV_c. Increasing B causes Γ_0 to fall below kT and the temperature dependence is restored. Either of these explanations is plausible but we believe the latter is the true situation. To test this, we assume that at low temperature, Eq. (6) is valid. In that case, where we allow Γ_0 to vary with magnetic field, we obtain

$$\frac{\delta G}{J}\propto(SN_d\Gamma_0)^{-1/2}. \tag{11}$$

We cannot measure simply how Γ_0 varies with B, but we are able to measure how J, and hence Γ_e, varies with B. From Eqs. (1) and (11)

$$\frac{\delta G}{J}\propto\left(\frac{\Gamma_0}{\Gamma_e}\right)^{-1/2}J^{-1/2}, \tag{12}$$

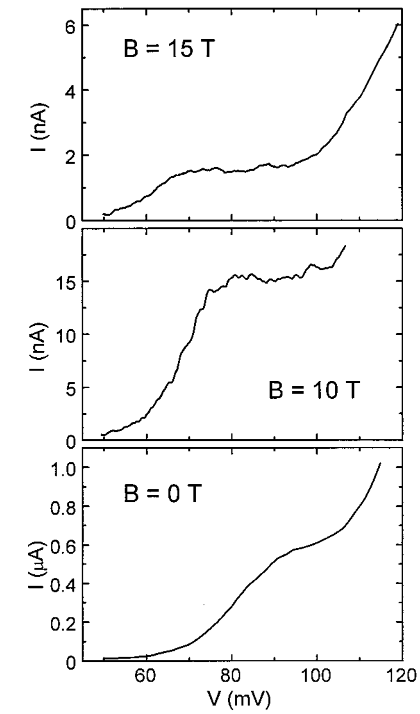

FIG. 6. $I(V)$ for mesa F at 0.3 K in $B=0$, 10, and 15 T. Note that the relative amplitude of the fluctuations is larger at higher B.

which enables us to compare measurements of the CF, not only between devices with different numbers of donor impurities, but also between different values of magnetic field. We expect that Γ_0 and Γ_e will be affected in the same way by the magnetic field. Therefore, at the lowest temperatures our simple model has the strong prediction that $\delta G/J\propto J^{-1/2}$.

To look at the magnetic field dependence of G in more detail, in Fig. 6 we plot $I(V)$ for mesa F at $T=0.3\,$K in $B=0$, 10, and 15 T. Clearly, $\delta G/J$ increases as B increases, as we would expect for Γ_0 decreasing. In Fig. 7 we plot $\delta G/J$ vs $J^{-1/2}$ for the four devices of Fig. 3 at both 0 T and 10 T. In agreement with Eq. (12), the points all lie on a universal straight line. This is a remarkable result in that the peak current is varying more than two orders of magnitude over the range of the data. Figure 7 confirms that, at low temperatures, we are indeed in the regime where the CF are limited by the natural linewidth of the state. It is also a spectacular confirmation of the basic predictions of our simple model for the CF.

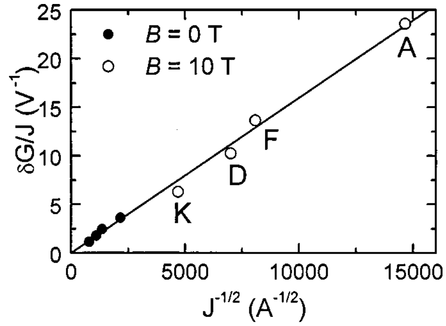

FIG. 7. $\delta G/J$ vs $J^{-1/2}$ at $T=0.3\,$K for mesas A, D, F, and K in $B=0$ T and $B=10$ T. The line is a guide to the eye, illustrating the dependence predicted by Eq. (12).

Note also from Fig. 6 that the detailed form of the fluctuations is not preserved as B increases, indicating that the impurity states that contribute to the current are themselves changing in some respect with B. We would expect this to be the case, since the magnetic length, $l=(\hbar/eB)^{1/2}$, is comparable with the effective Bohr radius of the impurity states in this field range. However, although the detailed form of the CF changes with B, our model for the amplitude of the CF depends only on the number of channels and our predictions are unaffected. Other workers have reported CF dependent on magnetic field but attributed their origin to a different effect.[20]

In conclusion, the δ-doped quantum well provides an excellent system for studying the CF due to impurity states in a tunnel barrier. Unlike earlier experiments that reported CF in tunnel devices, the nature of the impurity state is well understood in our system. Furthermore, we are able vary in a controlled manner, the number of impurities and also the linewidth of the state, using an applied magnetic field. We find that our experiments are able to confirm the predictions of Larkin and Matveev.[4] In particular, the amplitude of the CF is in proportion to the square root of the number of tunnelling channels, confirming their origin as statistical fluctuations in a finite number of tunnelling channels. The typical period of the CF is determined either by the natural linewidth of the impurity state or by the kT smearing of the Fermi level in the emitter accumulation layer; by adjusting the external field, we are able to move from one regime to the other. It is interesting to note that we measure clear CF even in devices 100 μm across, containing $\sim 10^5 - 10^6$ impurities. We also see CF in nominally undoped devices,[15] where the impurity states arise from unintentional, background doping and segregation during growth.[19] The CF are an intrinsic feature of all RTD at low temperatures.

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