Edge magnetoplasmons in periodically modulated structures

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We present a *microscopic* treatment of edge magnetoplasmons (EMP's) within the random-phase approximation for strong magnetic fields, low temperatures, and filling factor $\nu=1(2)$, when a weak short-period superlattice potential is imposed along the Hall bar. The modulation potential modifies both the spatial structure and the dispersion relation of the fundamental EMP and leads, when the modulation potential is not too weak, to the appearance of a novel gapless mode of the fundamental EMP. For sufficiently weak modulation strengths the phase velocity of this novel mode is almost the same as the group velocity of the edge states but it should be quite smaller for stronger modulation. We discuss in detail the spatial structure of the charge density of the renormalized and the novel fundamental EMP's.

I. INTRODUCTION

The theory of edge magnetoplasmons (EMP's) in the quantum Hall regime has benefitted substantially from results of time-resolved experiments.[1] Classical[2] and quantum[3] models have been used to describe EMP modes through different wave mechanisms at the edges of the two-dimensional electron system (2DES). Recently a quasimicroscopic description for EMP's in the quantum Hall regime, embracing the edge-wave mechanisms mentioned above, has been proposed[4] that takes into account the lateral confining potential, the structure of the Landau levels (LL's) for integer values of the filling factor ν, and the dissipation that conditions the propagation of the modes. The lateral confining potential is flat in the interior of the channel and smooth on the scale of the magnetic length l_0 but sufficiently steep that the LL flattening can be neglected. The theoretical framework was also extended to take into account nonlocal contributions to the current density within the random-phase approximation.[5]

There has been a great number of studies of magnetotransport properties of 2DES's modulated by one-dimensional (1D) lateral superlattices with large period $a \gtrsim 100$ nm.[6,7] The spectrum of magnetoplasmon excitations has been studied in such systems as well.[8] Also, many works have been devoted to magnetotransport and related phenomena in the case of two-dimensional (2D) lateral superlattices both in the regime of relatively weak modulation[9,10] and for antidot arrays,[11] where the 2DES cannot penetrate into the antidot region with higher potential. Quite recently a superlattice field-effect transistor was designed in which the 2DES in a GaAs-based sample is subjected to a atomically precise 1D potential with period of 15 nm (Ref. 12) and vicinal superlattices were produced with $a\approx 16$ nm.[13] Recently attention has been focused on the transport of commensurate composite fermions in weak periodic electrostatic potentials at the half-filled LL.[7]

In this work we employ the self-consistent-field formalism, or random-phase approximation (RPA), to study the effect of a 1D weak periodic modulation, with period a, on the fundamental EMP's in the quantum Hall system for LL filling factors $\nu=1$ (2) and low temperatures, $k_B T \ll \hbar v_g / l_0$, where v_g is the group velocity of edge states. Motivated by recent results on the 2DES subjected to lateral superlattice potentials with short period,[12,13] we restrict our study, starting from Sec. III, to the short-period regime $a \ll 2\pi l_0$. More precisely we will assume that $\exp[-(\pi l_0/a)^2] \ll 1$. As in Refs. 4 and 5, our effective one-electron confining potential contains the Hartree and exchange-correlation contributions of the 2DES in addition to the bare confinement potential, which is assumed to be sufficiently steep, such that, at the channel edges, the LL flattening and the formation of compressible and incompressible strips[14] can be neglected.[15]

In Sec. II we derive the integral equations for the wave charge density and the electrostatic potential at the edge of a periodically modulated channel. In Sec. III we present our result for the dispersion relations and spatial structures of the fundamental EMP's. Finally, in Sec. IV we summarize our major conclusions.

II. INTEGRAL EQUATIONS FOR EMP'S

The noninteracting zero-thickness 2DEG, of width W and length $L_x = L$, in the presence of a strong magnetic field B along the z axis and under a 1D periodic modulation, is described by the Hamiltonian $\hat{H}_0 = \hat{h}^0 + V_s(x)$, where $\hat{h}^0 = [(\hat{p}_x + eBy/c)^2 + \hat{p}_y^2]/2m^* + V_y$. The confining potential is flat in the interior of the 2DES, ($V_y=0$) and is parabolic at its edges, and $V_y = m^*\Omega^2 (y-y_r)^2/2$, $y \geq y_r$. We assume that V_y is smooth on the scale of l_0 such that $\Omega \ll \omega_c$, where $\omega_c = |e|B/m^*c$ is the cyclotron frequency. The 1D modulation $V_s(x) = V_s \cos(Gx)$ is a weak periodic potential with $G = 2\pi/a$; so it is assumed that $V_s/2\hbar\omega_c \ll 1$. Further, it will be seen that a novel gapless mode of the fundamental EMP appears only if V_s is not too small. The pertinent conditions for the weakness of V_s will be detailed below. Within the RPA framework, the corresponding one-electron density matrix $\hat{\rho}$ obeys the equation of motion

$$i\hbar\frac{\partial\hat{\rho}}{\partial t}=[\hat{H}(t),\hat{\rho}]-\frac{i\hbar}{\tau}(\hat{\rho}-\hat{\rho}^{(0)}),\tag{1}$$

where $\hat{H}(t)=\hat{H}_0+V(x,y,t)$ with

$$V(x,y,t)=e^{-i(\omega_0 t-q_x x)}\sum_{l=-\infty}^{\infty}V_l(\omega_0,q_x,y)e^{iGlx}+\text{c.c.}\tag{2}$$

Without interaction the one-electron density matrix $\hat{\rho}^{(0)}$ is diagonal, i.e., $\langle\alpha|\hat{\rho}^{(0)}|\beta\rangle=f_\alpha\delta_{\alpha\beta}$, where $f_\alpha=\{1+\exp[(E_\alpha-E_F)/k_BT]\}^{-1}$ is the Fermi-Dirac function; $\hat{H}_0|\alpha\rangle=E_\alpha|\alpha\rangle$. Notice that $\tau\rightarrow\infty$ corresponds to the collisionless case while a finite τ provides the possibility of estimating roughly the influence of collisions. Both $\hat{H}_0$ and the sum $\Sigma_{l=-\infty}^{\infty}V_l(\omega_0,q_x,y)e^{iGlx}$ are periodic along x with period a.

Equation (1) can be solved by Laplace transforms.[5] Taking the trace of $\hat{\rho}$ with the electron density operator $e\delta(\mathbf{r}-\hat{\mathbf{r}})$ gives the wave charge density in the form

$$\rho(\omega_0,x,y)=e^{iq_x x}\sum_{l=-\infty}^{\infty}\rho_l(\omega_0,q_x,y)e^{iGlx}.\tag{3}$$

The charge density $\rho_l(\omega,q_x,y)\exp[i(q_x+Gl)x]$ induces a wave electric potential $\phi_l(\omega,q_x,y)\exp[i(q_x+Gl)x]$. From Poisson's equation this is given as

$$\phi_l(\omega,q_x,y)=\frac{2}{\epsilon}\int_{-\infty}^{\infty}dy'K_0(|q_x+Gl||y-y'|)\times\rho_l(\omega,q_x,y'),\tag{4}$$

where ϵ is the background dielectric constant, assumed spatially homogeneous, and $K_0(x)$ is the modified Bessel function; ϕ and ρ pertain to the 2D plane. Taking $|q_x|W\gg 1$, we can consider an EMP along the right edge of the channel of the form $A(\omega,q_x,x,y)\exp[-i(\omega t-q_x x)]$ totally independent of the left edge, where $A(\omega,q_x,x,y)$ is periodic along x with period a.

In the absence of an external potential $V_l(\omega,q_x,y)=e\phi_l(\omega,q_x,y)$. As a result, considering large time responses, $t\gg\tau$, after some straightforward calculations we obtain the following integral equation for $\rho_m(\omega,q_x,y)$:

$$\rho_m(\omega,q_x,y)=\frac{2e^2}{\epsilon L}\sum_{\alpha,\beta}\sum_{l=-\infty}^{\infty}\int_0^L dx\,e^{-i(q_x+Gm)x}\psi^*_\beta(\mathbf{r})\psi_\alpha(\mathbf{r})\times\frac{f_\beta-f_\alpha}{E_\beta-E_\alpha+\hbar\omega+i\hbar/\tau}\int d\tilde{\mathbf{r}}e^{i(q_x+Gl)\tilde{x}}\psi^*_\alpha(\tilde{\mathbf{r}})\psi_\beta(\tilde{\mathbf{r}})\times\int_{-\infty}^{\infty}dy'\,K_0(|q_x+Gl||\tilde{y}-y'|)\rho_l(\omega,q_x,y'),\tag{5}$$

where $\psi_\alpha=\langle\mathbf{r}|\alpha\rangle$ and we dropped the subscript 0 from ω_0. For definiteness, we take $\omega>0$.

We consider low temperatures T satisfying $\hbar v_{gn}\gg l_0k_BT$, where v_{gn} is the group velocity of the edge states of nth LL. Furthermore, we will assume the long-wavelength limit $q_xl_0\ll 1$, which is well satisfied, e.g., for the fundamental EMP in the low-frequency regime, $\omega\ll\omega_c$.[4] Then, assuming that the condition $Gv_{gn}\ll\omega_c$ is satisfied and comparing the terms proportional to $f_{\beta*}$, for a given $n_{\beta*}$, of the right-hand side (RHS) of Eq. (5), we conclude that the contribution to the summation over n_α with $n_\alpha=n_{\beta*}$ is much larger than any other term of this sum or the sum of all terms with $n_\alpha\neq n_{\beta*}$. The small parameter is $|\omega-q_xv_{gn_{\beta*}}(k_{x\beta})|/\omega_c\ll 1$, where $v_{gn_{\beta*}}(k_{x\beta})$ is the group velocity of an occupied state $|n_{\beta*},k_{x\beta}\rangle$ of the $n_{\beta*}$ LL. The inequality above also implies that $q_xv_{g0}/\omega_c\ll 1$, since v_{g0} has typically the largest value among v_{gn}. Similar results follow from an analysis of the terms proportional to $f_{\alpha*}$ in the summation over n_β on the RHS of Eq. (5). Hence, for $\omega\ll\omega_c$, $q_xv_{g0}\ll\omega_c$, and $Gv_{g0}\ll\omega_c$, the terms with $n_\alpha\neq n_\beta$ can be neglected. Then the integral equation for the electron charge density $\rho_m(\omega,q_x,y)$ becomes

$$\rho_m(\omega,q_x,y)=\frac{e^2}{L}\sum_{n_\alpha=0}^{\bar{n}}\sum_{k_{x\alpha}}\sum_{k_{x\beta}}\sum_{l=-\infty}^{\infty}\int_0^L dx\,e^{-i(q_x+Gm)x}\times\psi^*_{n_\alpha k_{x\beta}}(\mathbf{r})\psi_{n_\alpha k_{x\alpha}}(\mathbf{r})\times\frac{f_{n_\alpha k_{x\beta}}-f_{n_\alpha k_{x\alpha}}}{E_{n_\alpha k_{x\beta}}-E_{n_\alpha k_{x\alpha}}+\hbar\omega+i\hbar/\tau}\times\int d\tilde{\mathbf{r}}e^{i(q_x+Gl)\tilde{x}}\times\psi^*_{n_\alpha k_{x\alpha}}(\tilde{\mathbf{r}})\psi_{n_\alpha k_{x\beta}}(\tilde{\mathbf{r}})\phi_l(\omega,q_x,\tilde{y}),\tag{6}$$

where $\bar{n}$ denotes the highest occupied LL. For even ν, the RHS of Eq. (6) should be multiplied by 2, the spin degeneracy factor; for ν even the spin-splitting is neglected. Equation (6), for $m=0,\pm 1,\pm 2,\ldots$, gives a system of integral equations, whose solution determines $\rho_m(\omega,q_x,y)$ in the RHS of Eq. (3).

Because $V_s(x)$ is assumed weak, the eigenfunctions $\psi_{n_\alpha k_{x\alpha}}=\langle\mathbf{r}|n_\alpha,k_{x\alpha}\rangle$ and the eigenvalues $E_{n_\alpha k_{x\alpha}}$ of $\hat{H}_0$ can be evaluated by second-order perturbation theory. A straightforward calculation leads to[16]

$$|n_\alpha,k_{x\alpha}\rangle=[1-\tilde{V}^2_{sn_\alpha}(k_{x\alpha})]|n_\alpha,k_{x\alpha}\rangle^{(0)}+\tilde{V}_{sn_\alpha}(k_{x\alpha})\times[|n_\alpha,k_{x\alpha}-G\rangle^{(0)}-|n_\alpha,k_{x\alpha}+G\rangle^{(0)}]+\tilde{V}^2_{sn_\alpha}(k_{x\alpha})[|n_\alpha,k_{x\alpha}+2G\rangle^{(0)}+|n_\alpha,k_{x\alpha}-2G\rangle^{(0)}].\tag{7}$$

Here we have introduced a dimensionless parameter $\tilde{V}_{sn}(k_x)$ characterizing the strength of the periodic potential for the nth LL, near its edge, and given by

$$\tilde{V}_{sn}(k_x)=\frac{V_s}{2\hbar Gv_{gn}(k_x)}e^{-(Gl_0/2)^2}L_n[(Gl_0)^2/2],\tag{8}$$

where $v_{gn}(k_x)=\hbar^{-1}\partial E_n(k_x)/\partial k_x$ is the group velocity of a state in the edge region of the nth LL and $L_n(x)$ is the Laguerre polynomial. Due to the smoothness of the confining potential on the l_0 scale, the unperturbed eigenfunctions are well approximated by $\psi^{(0)}_{n_\alpha k_{x\alpha}}\equiv\langle\mathbf{r}|n_\alpha,k_{x\alpha}\rangle^{(0)}\equiv\psi^{(0)}_\alpha(\mathbf{r})$

$\approx e^{ik_x x}\Psi_n(y-y_0)/\sqrt{L}$, where $\Psi_n(y)$ is the harmonic oscillator function. Because we have used the condition $\omega_c \gg G v_{gn}$ to obtain Eq. (7), the small ''nonresonance'' contributions with $n_\beta \neq n_\alpha$ can be neglected. In the edge region the evaluation of the eigenvalue $E_{n_\alpha k_{x\alpha}} \equiv E_{n_\alpha}(k_{x\alpha})$, by perturbation theory, shows that the first-order correction $E^{(1)}_{n_\alpha}(k_{x\alpha})$ vanishes. As for the second-order correction $E^{(2)}_{n_\alpha}(k_{x\alpha})$, the main ''resonance'' contributions to it, with $n_\beta = n_\alpha$, are mutually canceled due to imposed conditions. Then it can be shown that $E_{n_\alpha}(k_{x\alpha})$ in the edge region can be well approximated by the zero-order term, i.e., $E_{n_\alpha}(k_{x\alpha}) \approx E^{(0)}_{n_\alpha}(k_{x\alpha})$. The energy spectrum of the nth LL, $E^{(0)}_\alpha \approx (n+1/2)\hbar\omega_c + m^*\Omega^2(y_0-y_r)^2/2$, leads to the group velocity of the edge states $v_{gn} = \partial E_n(k_r+k_e^{(n)})/\hbar\partial k_x = \hbar\Omega^2 k_e^{(n)}/m^*\omega_c^2$ with characteristic wave vector $k_e^{(n)} = (\omega_c/\hbar\Omega)\sqrt{2m^*\Delta_{Fn}}$, $\Delta_{Fn} = E_F - (n+1/2)\hbar\omega_c$, where E_F is the Fermi energy. The edge of the nth LL is denoted by $y_{rn} = y_r + l_0^2 k_e^{(n)} = l_0^2 k_{rn}$, where $k_{rn} = k_r + k_e^{(n)}$, and $W = 2y_{r0}$. We can also write $v_{gn} = cE_{en}/B$, where $E_{en} = \Omega\sqrt{2m^*\Delta_{Fn}}/|e|$ is the electric field associated with the confining potential V_y at y_{rn}. We have also introduced the wave vector $k_r = y_r/l_0^2$. The typical width of the edge region for the nth LL can be estimated here as $\eta l_0^2 k_e^{(n)} \gg l_0$, where $\eta \ll 1$. For all occupied LL's, we assume that $k_e^{(n)} \gg G \gg 1/l_0$. Since in Eqs. (5) and (6) the significant eigenstates are localized along the y direction near the right edge of the channel, i.e., with $y_0(k_x) > y_r$, we have considered only these eigenstates in Eqs. (7) and (8). Moreover, it follows from Eq. (6) that the main contributions come from $k_x \approx k_{rn}$ and $k_x \approx (k_{rn} \pm G)$; thus for the applicability of the perturbation theory here it is sufficient to assume that $\tilde{V}_{sn}(k_{rn}) \equiv \tilde{V}_{sn} \ll 1$.

III. FUNDAMENTAL EMP'S FOR $\nu = 1(2)$

We first consider the case $\nu=1$ and then indicate how the results change for $\nu=2$. For $\nu=1$, we have $\bar{n}=0$ in Eq. (6). We will look for gapless edge modes, with $\omega\to 0$ for $q_x \to 0$, and assume that $1 \gg \tilde{V}_{s0} \gtrsim \exp[-(Gl_0/2)^2]$, where $\tilde{V}_{s0} = (V_s/2\hbar G v_{g0})\exp[-(Gl_0/2)^2]$, which implies the short-period regime, i.e., $\exp[-(Gl_0/2)^2] \ll 1$. Furthermore, it follows that now we assume that the periodic potential is not too weak such that the condition $V_s/2\hbar v_{g0} G \gtrsim 1$ is fulfilled. From Eq. (6) for $m=0$, we can write the integral equation for $\rho_0(\omega,q_x,y)$ in the form

$$\rho_0(\omega,q_x,y) = [\hat{F}_1+\hat{F}_2] \times \int_{-\infty}^{\infty} dy' K_0(|q_x||\tilde{y}-y'|)\rho_0(\omega,q_x,y'), \tag{9}$$

where the integral functional $\hat{F}_1$ is given as

$$\hat{F}_1 = \frac{e^2}{\pi\hbar\epsilon}\int_{-\infty}^{\infty} dk_{x\alpha}\Pi(y,k_{x\alpha})\frac{f_{0,k_{x\alpha}-q_x}-f_{0,k_{x\alpha}}}{\omega - v_{g0}(k_{x\alpha})q_x + i/\tau} \times \int_{-\infty}^{\infty} d\tilde{y}\{\Pi(\tilde{y},k_{x\alpha}) + \tilde{V}_{s0}^2[\Pi(\tilde{y},k_{x\alpha}-G) + \Pi(\tilde{y},k_{x\alpha}+G)]\}, \tag{10}$$

with $\Pi(y,k_{x\alpha}) = |\Psi_0[y-y_0(k_{x\alpha})]|^2$. The integral functional $\hat{F}_2$ is given as

$$\hat{F}_2 = \frac{e^2}{\pi\hbar\epsilon}\int_{-\infty}^{\infty} dk_{x\alpha}\tilde{V}_{s0}^2[\Pi(y,k_{x\alpha}-G)+\Pi(y,k_{x\alpha}+G)] \times \frac{f_{0,k_{x\alpha}-q_x}-f_{0,k_{x\alpha}}}{\omega - v_{g0}(k_{x\alpha})q_x + i/\tau}\int_{-\infty}^{\infty} d\tilde{y}\,\Pi(\tilde{y},k_{x\alpha}). \tag{11}$$

In Eqs. (9)–(11), for $q_x \to 0$, we can make the approximation $(f_{0,k_{x\alpha}-q_x} - f_{0,k_{x\alpha}}) \approx q_x\delta(k_{x\alpha}-k_{r0})$. After integration over $k_{x\alpha}$, we obtain

$$\begin{aligned}\rho_0(\omega,q_x,\bar{y}) = \frac{e^2}{\pi\hbar\epsilon}\frac{q_x}{\tilde{\omega}}\Bigg\{&\Psi_0^2(\bar{y})\int_{-\infty}^{\infty} d\bar{y}'[\Psi_0^2(\bar{y}') \\ &+ \tilde{V}_{s0}^2[\Psi_0^2(\bar{y}'+Gl_0^2)+\Psi_0^2(\bar{y}'-Gl_0^2)]] \\ &+ \tilde{V}_{s0}^2[\Psi_0^2(\bar{y}+Gl_0^2)+\Psi_0^2(\bar{y}-Gl_0^2)] \\ &\times \int_{-\infty}^{\infty} d\bar{y}'\Psi_0^2(\bar{y}')\Bigg\} \\ &\times \int_{-\infty}^{\infty} d\bar{y}'' K_0(|q_x||\bar{y}'-\bar{y}''|)\rho_0(\omega,q_x,\bar{y}''),\end{aligned} \tag{12}$$

where $\bar{y} = y - y_{r0}$, and $\tilde{\omega} = \omega - q_x v_{g0} + i/\tau$. In order to simplify the notation we take $\rho_i(\omega,q_x,y) \equiv \rho_i(\omega,q_x,\bar{y})$, $i=0,\pm 1,\ldots$.

Similarly, omitting minor terms in Eq. (6) we obtain, for $m=1$,

$$\begin{aligned}\rho_1(\omega,q_x,\bar{y}) = \frac{e^2}{\pi\hbar\epsilon}\tilde{V}_{s0}\frac{q_x}{\tilde{\omega}}\Psi_0(\bar{y}) &\times [\Psi_0(\bar{y}+Gl_0^2) - \Psi_0(\bar{y}-Gl_0^2)] \\ &\times \int_{-\infty}^{\infty} d\tilde{y}\,\Psi_0^2(\tilde{y}) \\ &\times \int_{-\infty}^{\infty} d\bar{y}' K_0(|q_x||\tilde{y}-\bar{y}'|)\rho_0(\omega,q_x,\bar{y}').\end{aligned} \tag{13}$$

From Eq. (6) we find, for $m=-1$, $\rho_{-1}(\omega,q_x,y) \equiv \rho_1(\omega,q_x,y)$.

The general solution of the linear homogeneous integral equation, Eq. (12), can be sought in the form

$$\rho_0(\omega,q_x,y) = \rho_0^{(0)}(\omega,q_x)\Psi_0^2(\bar{y}) + \rho_0^{(1)}(\omega,q_x) \times [\Psi_0^2(\bar{y}+Gl_0^2)+\Psi_0^2(\bar{y}-Gl_0^2)]. \tag{14}$$

Substituting Eq. (14) into Eq. (12) and equating the coefficients of $\Psi_0^2(\bar{y})$ and $[\Psi_0^2(\bar{y}+Gl_0^2)+\Psi_0^2(\bar{y}-Gl_0^2)]$ on both sides of Eq. (12), we obtain two linear homogeneous equations for $\rho_0^{(i)}(\omega,q_x)$, $i=0,1$:

$$\rho_0^{(0)} = \frac{e^2}{\pi\hbar\epsilon}\frac{q_x}{\tilde{\omega}}([a_{00}(q_x) + 2\tilde{V}_{s0}^2 a_{00}^{00}(q_x, Gl_0^2)]\rho_0^{(0)} + 2\{a_{00}^{00}(q_x, Gl_0^2) + \tilde{V}_{s0}^2[a_{00}(q_x) + a_{00}^{00}(q_x, 2Gl_0^2)]\}\rho_0^{(1)}) \quad (15)$$

and

$$\rho_0^{(1)} = \frac{e^2}{\pi\hbar\epsilon}\tilde{V}_{s0}^2\frac{q_x}{\tilde{\omega}}\{a_{00}(q_x)\rho_0^{(0)} + 2a_{00}^{00}(q_x, Gl_0^2)\rho_0^{(1)}\}, \quad (16)$$

where the coefficients a_{nn}^{mm} are given by[5]

$$a_{nn}^{mm}(q_x, \Delta y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dx\, dx'\, \Psi_n^2(x)\Psi_m^2(x') \times K_0(|q_x||x - x' + \Delta y|). \quad (17)$$

Here $a_{nn}^{mm}(q_x, \Delta y) = a_{mm}^{nn}(q_x, \Delta y)$, $a_{nn}^{mm}(q_x, \Delta y) = a_{nn}^{mm}(q_x, -\Delta y)$, and $a_{00}^{00}(q_x, 0) = a_{00}(q_x)$. We will assume $2\pi q_x l_0^2/a \ll 1$. Notice that $a_{00}(q_x) \approx \ln(1/q_x l_0) + 3/4$ and, for $\Delta y/l_0 \gg 1$, $a_{00}^{00}(q_x, \Delta y) \approx \ln(2/q_x l_0) - \gamma - \ln(\Delta y/l_0) \approx \ln(1/q_x \Delta y) + 0.1$, where γ is the Euler constant.

A. Dispersion relations

The dimensionless frequencies $\omega' = \tilde{\omega}/(e^2 q_x/\pi\hbar\epsilon)$ of the branches resulting from the determinantal solution of the two linear homogeneous equations for $\rho_0^{(i)}(\omega, q_x)$, Eqs. (15) and (16), are given by

$$\omega'_{\pm} = \frac{1}{2}a_{00}(q_x) + 2\tilde{V}_{s0}^2 a_{00}^{00}(q_x, Gl_0^2) \pm \frac{1}{2}a_{00}(q_x) \times (1 + 8\tilde{V}_{s0}^2 a_{00}^{-1}(q_x)\{a_{00}^{00}(q_x, Gl_0^2) + \tilde{V}_{s0}^2[a_{00}(q_x) + a_{00}^{00}(q_x, 2Gl_0^2)]\})^{1/2}. \quad (18)$$

From Eq. (18) it follows that the effect of the modulation potential on the fundamental EMP is quite strong. Apart from the renormalization of the fundamental EMP of $n=0$ LL with dispersion

$$\omega'_+ \approx a_{00}(q_x) + 4\tilde{V}_{s0}^2 a_{00}^{00}(q_x, Gl_0^2) \quad (19)$$

it leads to the existence of a *novel* fundamental EMP of $n=0$ LL with dispersion

$$\omega'_- \approx 2\tilde{V}_{s0}^4 a_{00}^{-1}(q_x)\{2[a_{00}^{00}(q_x, Gl_0^2)]^2 - a_{00}(q_x) \times [a_{00}(q_x) + a_{00}^{00}(q_x, 2Gl_0^2)]\}. \quad (20)$$

Substituting the coefficients into Eq. (20), we obtain the dispersion relation (DR) of the novel fundamental EMP

$$\omega_- \approx \left[v_{g0} - \frac{12}{\epsilon}\tilde{V}_{s0}^4\sigma_{yx}^0 \ln(Gl_0)\right]q_x - i/\tau, \quad (21)$$

where $\sigma_{yx}^0 = \nu e^2/2\pi\hbar$ and $\nu = 1(2)$. Note that for $\tilde{V}_{s0} \leqslant 10^{-1}$ and $v_{g0} \geqslant 10^6$ cm/s, which is a typical value in GaAs-based heterostructures, the correction in the phase velocity of this novel mode should be quite small. However, in general, e.g., for a slightly larger $\tilde{V}_{s0}$ and a slightly smaller v_{g0}, this contribution should be taken into account and can lead to a substantial decrease of the phase velocity of the novel fundamental EMP from its maximum possible value v_{g0}. We point out that this contribution to $\mathrm{Re}\,\omega_-$ stems from the electron-electron interaction and the strength of the periodic modulation as well. Equation (21) is valid for

$$\tilde{V}_{s0}^2 \gg \frac{1}{3\ln(Gl_0)}\frac{\epsilon v_{g0}}{2\sigma_{yx}^0}\left(\frac{2\pi}{k_e^{(0)}a}\right)^2. \quad (22)$$

Notice that the RHS of the inequality (22) is typically very small. Under this condition, the second-order correction in the group velocity $v_{g0}(k_{r0}) = v_{g0}[1 - 2\tilde{V}_{s0}^2(2\pi/k_e^{(0)}a)^2]$ can be neglected. As discussed above this is equivalent to neglecting $E_0^{(2)}(k_{x\alpha})$.

From Eq. (19) the DR for the renormalized fundamental EMP can be written as

$$\omega_+ \approx q_x v_{g0} + \frac{2}{\epsilon}\sigma_{yx}^0 q_x\{\ln(1/q_x l_0) + \tfrac{3}{4} + 4\tilde{V}_{s0}^2 \times \ln[1/(q_x G l_0^2)]\} - i/\tau. \quad (23)$$

The term proportional to $\tilde{V}_{s0}^2$ shows a strong renormalization of the fundamental EMP that depends on the strength and the period of the modulation for given value of q_x.

For a GaAs-based 2DEG and negligible dissipation, the dispersion laws for the renormalized, by the superlattice potential and intra-LL Coulomb coupling, fundamental EMP, and for the novel fundamental EMP, caused by the periodic modulation $V_s(x)$, are shown in Fig. 1 by the top and bottom solid curves, respectively. The DR's corresponding to the ω_- and ω_+ modes here are obtained using Eqs. (21) and (23), respectively. For the assumed parameters these equations very well approximate the exact DR's given by Eq.

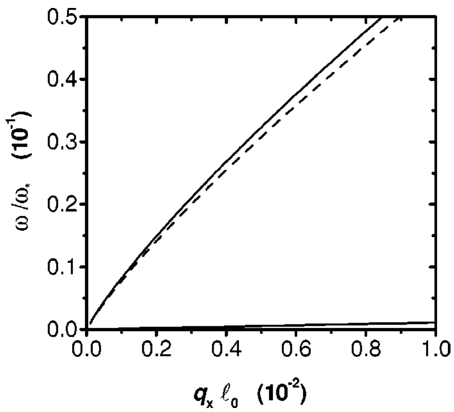

FIG. 1. Dispersion relations for $\omega_\pm$ modes, in units of $\omega_* = e^2/\pi\hbar\epsilon l_0$, for $\nu=1$ and $B=9$ T. The upper solid curve is the renormalized fundamental EMP of the $n=0$ LL and the lower solid one is the novel fundamental EMP, due to the effect of the modulation potential. The dashed curve is the fundamental EMP in the absence of the modulation. The parameters of a GaAs-based sample are given in the text and we took $\Delta_{F0} = \hbar\omega_c/2$, $a \approx 18.5$ nm, and $\tilde{V}_{s0} = \exp(-2)$.

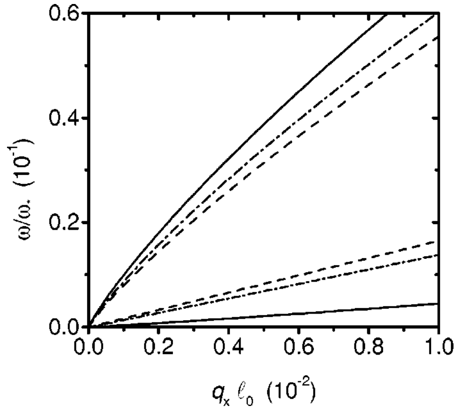

FIG. 2. Dispersion relation of $\omega_\pm$ modes for values of the modulation strength $\tilde{V}_{s0}=2.18$ meV (solid curve), 1.09 meV (dot-dashed), and 0.73 meV (dashed) and $\nu=1$ and $B=9$ T. Top (bottom) curves represent the dispersion laws for ω_+ and ω_- modes, respectively. The values of ω_- are multiplied by 30. The parameters are the same as in Fig. 1, except $\Delta_{F0}=\hbar\omega_c/8$, $a\approx 20.6$ nm. It follows that $v_{g0}\approx 3.25\times 10^5$ cm/s and $\exp[-(Gl_0/2)^2]\approx 0.2$.

(18). For the sake of comparison, the dashed curve in Fig. 1 shows the fundamental EMP of $n=0$ LL in the absence of the superlattice potential.

As we have discussed, the renormalization effect involves essentially intra-LL Coulomb coupling. In Fig. 1 the parameters are $m^*\approx 6.1\times 10^{-29}$ g, $\epsilon\approx 12.5$, and $\Omega\approx 7.8\times 10^{11}$ s^{-1}.[17] Assuming $\nu=1$ and $B=9$ T, these parameters lead to $\omega_c/\Omega\approx 30$. Here $\omega_*=e^2/\pi\hbar\epsilon l_0\approx 0.3\omega_c\approx 7\times 10^{12}$ s^{-1} is a characteristic frequency. We have also assumed $\Delta_{F0}=\hbar\omega_c/2$, $\tilde{V}_{s0}=\exp(-2)\ll 1$, and $a=\pi l_0/\sqrt{2}$. This gives $v_{g0}\approx\Omega l_0\approx 6.5\times 10^5$ cm/s, $a\approx 18.5$ nm, $V_s=2.9$ meV. Observe that for these parameters, the second term in the RHS of Eq. (21) is more than 50 times smaller than the first term, v_{g0}. Hence, the curve $\omega=q_x v_{g0}$ will practically coincide with the solid curve at the bottom of Fig. 1.

The dispersion laws corresponding to the ω_+ and ω_- modes, given by Eq. (18), are depicted in Fig. 2 by the top and bottom curves, respectively. The solid, dot-dashed, and dashed curves correspond to $\tilde{V}_{s0}=0.3$, 0.2, and 0.1, respectively. The data of the bottom curves were multiplied by the factor 30. The parameters are the same as in Fig. 1 except $\Delta_{F0}=\hbar\omega_c/8$, $a\approx 20.6$ nm. This leads to $v_{g0}\approx 3.25\times 10^5$ cm/s and to modulation strengths $V_s\approx 2.18$ meV, 1.09 meV, and 0.73 meV for $\tilde{V}_{s0}=0.3$, 0.2, and 0.1, respectively. Notice that in this case $\exp[-(Gl_0/2)^2]\approx 0.2$. It is seen that by varying the amplitude V_s of the periodic potential, strong modifications in the DR of the fundamental modes can occur. We observe that the phase velocity of the novel EMP decreases from its maximum value v_{g0} by increasing $\tilde{V}_{s0}$. It can be shown that the DR's given by Eqs. (21) and (23) still represent well all curves in Fig. 2.

B. Spatial structure

If we substitute Eq. (19) into Eq. (16), we obtain $\rho_0^{(1)}(\omega_+,q_x)/\rho_0^{(0)}(\omega_+,q_x)\approx\tilde{V}_{s0}^2$. As a consequence, only a small distortion of the edge charge occurs at $\overline{y}=\pm Gl_0^2$, in comparison with the usual one at $\overline{y}=0$. Furthermore, substituting Eq. (20) in Eq. (16), we obtain $\rho_0^{(0)}(\omega_-,q_x)/\rho_0^{(1)}(\omega_-,q_x)\approx -2[\ln(1/q_x l_0)-\ln(Gl_0)]/[\ln(1/q_x l_0)+3/4]>-2$. This means that the amplitude of the edge charges localized at $\overline{y}=\pm Gl_0^2$ has absolute value approximately twice smaller in comparison with that of the charge distortion localized at $\overline{y}=0$; in addition, it has the opposite sign. Furthermore, the ratio of amplitudes of the novel fundamental EMP is independent on V_{s0} for the assumed conditions, while in the case of the renormalized fundamental EMP such ratio tends to zero. The same results hold for $\nu=2$. Thus the novel mode has a spatial structure quite different both from the spatial structure of the fundamental EMP (i.e., in the absence of modulation) and from the renormalized mode.

We proceed now to evaluate the charge density $\rho_1(\omega,q_x,y)$ induced by $\rho_0(\omega,q_x,y)$ for the two new branches: the renormalized fundamental EMP and the novel fundamental EMP. For both fundamental EMP's we obtain, from Eq. (13),

$$\rho_1(\omega,q_x,y)=\rho_1(\omega,q_x)\Psi_0(\overline{y})[\Psi_0(\overline{y}+Gl_0^2)-\Psi_0(\overline{y}-Gl_0^2)], \tag{24}$$

where, using Eq. (16), we find

$$\rho_1(\omega,q_x)=\rho_0^{(1)}(\omega,q_x)/\tilde{V}_{s0}. \tag{25}$$

Then for the renormalized fundamental EMP the relative amplitude

$$\xi_+\equiv\rho_1(\omega_+,q_x,y)/\rho_0(\omega_+,q_x,y)\approx\tilde{V}_{s0}\exp[-(Gl_0/2)^2],$$

where the small factor $\exp[-(Gl_0/2)^2]\lesssim\tilde{V}_{s0}$ comes from the exponentially small overlapping of the wave functions in the products of Eq. (24). Similarly, for the novel fundamental EMP, ω_-, we obtain that

$$\frac{\rho_1(\omega_-,q_x)}{\rho_0^{(0)}(\omega_-,q_x)}\approx-\frac{a_{00}(q_x)}{2\tilde{V}_{s0}a_{00}^{00}(q_x,Gl_0^2)}\to-\frac{1}{2\tilde{V}_{s0}}, \tag{26}$$

where the limit holds for $q_x\to 0$. Now the relative amplitude is $\xi_-\equiv\rho_1(\omega_-,q_x,y)/\rho_0(\omega_-,q_x,y)=\exp[-(Gl_0/2)^2]/2\tilde{V}_{s0}$. Hence, ξ_- lies in the interval $[0.1,1]$, i.e., the amplitude of oscillations of the charge distortion $\propto\rho_{\pm 1}(\omega,q_x,y)$ can be of the same order of magnitude as that $\propto\rho_0(\omega,q_x,y)$. A further treatment of Eq. (6) for $m=2$ (and $\overline{n}=0$) shows that the charge distortions $\rho_2(\omega,q_x,y)=\rho_{-2}(\omega,q_x,y)$ as compared with $\rho_0(\omega,q_x,y)$ for these two new branches have an additional small factor $\propto\tilde{V}_{s0}\exp[-3(Gl_0/2)^2]\lesssim\tilde{V}_{s0}^4$ with respect to the relative strength $\xi_\pm$ of $\rho_{\pm 1}(\omega,q_x,y)$. Therefore, terms with $|l|\geqslant 2$ in Eq. (3) can be neglected both for the renormalized fundamental EMP and the novel fundamental EMP. As a result, from Eq. (3), we obtain straightforwardly the dimensionless form factors, $\rho_\pm(x,y)\equiv\rho_\pm=\sqrt{\pi}l_0\exp(-iq_x x)\rho(\omega_\pm,x,y)/\rho_0^{(0)}(\omega_\pm,q_x)$, as

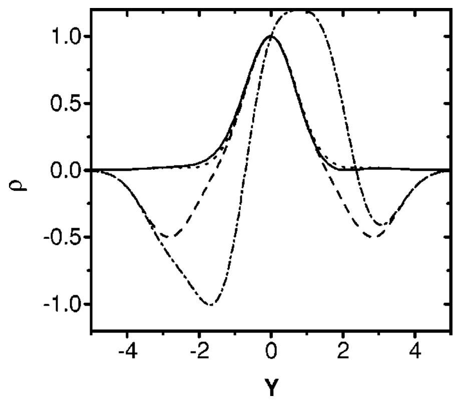

FIG. 3. Form factor for the fundamental EMP as a function of $Y=\overline{y}/l_0$, where $\overline{y}=y-y_{r0}$, and y_{r0} is the edge of $n=0$ LL. The renormalized mode is indicated respectively by solid and dotted curves for $x_1^{(m)}=ma$ and $x_0^{(m)}=a(m+1/2)/2$, with $m=0,\pm 1, \pm 2, \ldots$, and the novel EMP by dot-dashed and dashed curve for $x_1^{(m)}$ and $x_0^{(m)}$, respectively. The parameters used are the same as in Fig. 1.

$$\rho_+(x,y)=\sqrt{\pi}l_0\{\Psi_0^2(\overline{y})+\tilde{V}_{s0}^2[\Psi_0^2(\overline{y}+Gl_0^2)+\Psi_0^2(\overline{y}-Gl_0^2)]+2\tilde{V}_{s0}\cos(Gx)\Psi_0(\overline{y})\times[\Psi_0(\overline{y}+Gl_0^2)-\Psi_0(\overline{y}-Gl_0^2)]\} \quad (27)$$

for the renormalized fundamental mode, and

$$\rho_-(x,y)=\sqrt{\pi}l_0\left\{\Psi_0^2(\overline{y})-\frac{1}{2}[\Psi_0^2(\overline{y}+Gl_0^2)+\Psi_0^2(\overline{y}-Gl_0^2)]-\frac{1}{\tilde{V}_{s0}}\cos(Gx)\Psi_0(\overline{y})[\Psi_0(\overline{y}+Gl_0^2)-\Psi_0(\overline{y}-Gl_0^2)]\right\} \quad (28)$$

for the novel fundamental mode. In order to exhibit explicitly the x dependence of the form factors, for the same parameters as used in Fig. 1, we show them in Fig. 3 for $x_0^{(m)}=a(m+1/2)/2$, $m=0,\pm 1,\pm 2,\ldots$, with $\cos(2\pi x_0^{(m)}/a)=0$, and for $x_1^{(m)}=ma$, with $\cos(2\pi x_1^{(m)}/a)=1$. The solid and dotted curves show $\rho_+(x,y)$ as a function of $Y=\overline{y}/l_0$ for $x_1^{(m)}=ma$ and $x_0^{(m)}=a(m+1/2)/2$, respectively. We see that the dotted curve is exactly symmetrical with respect to the $Y=0$ axis. The deviations of the solid curve from this form come from contributions to the form factor that are commensurate with the unidirectional modulation. Also in Fig. 3, $\rho_-(x,y)$ is shown by dot-dashed and dashed curves for $x_1^{(m)}$ and $x_0^{(m)}$, respectively. Notice that the dashed curve is symmetric while the dot-dashed curve is clearly asymmetric. In Fig. 4, we present results for the charge densities of the renormalized and novel fundamental EMP's for the same parameters that are used to obtain the solid curves in Fig. 2. That is, in Fig. 4 we use $\tilde{V}_{s0}=0.3$ and $Gl_0\approx 2.53$ in Eqs. (27) and (28).

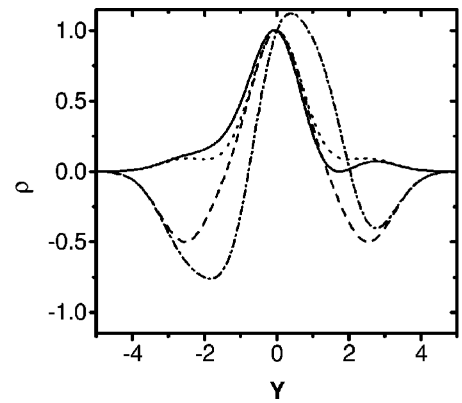

FIG. 4. The same as in Fig. 3, but with the parameters used for plotting the solid curves in Fig. 2.

IV. CONCLUSIONS

We have presented a fully *microscopic* model for EMP's in the RPA framework valid for integer $\nu=1$ and 2 in the case of an applied 1D weak modulation $V_s(x)=V_s\cos(2\pi x/a)$, and confining potentials that are smooth on the l_0 scale but still sufficiently steep at the edges that LL flattening[14] can be neglected.[15] The model also takes into account nonlocal responses and incorporates only very weak dissipation. The main results of the present work are as follows.

(i) The strength of the periodic modulation, if not too small, reshapes noticeably the spatial structure of the usual fundamental EMP of $n=0$ LL,[5] normal and parallel to the edge, and substantially modifies the dispersion relation leading to a renormalized fundamental EMP of $n=0$ LL. For instance, in Fig. 1, we have seen that the group velocity of the renormalized fundamental EMP is more than 4% greater than that of the fundamental EMP without modulation for $q_x l_0=0.8\times 10^{-2}$. Therefore, in time-resolved experiments, the periodic potential will imply a modulation of the propagation time of the signal due the renormalized fundamental EMP. As we have seen, this renormalization depends on the strength and period of the modulation potential.

(ii) The strength of the periodic modulation, even quite small, leads to the appearance of the *novel* fundamental EMP with acoustical dispersion relation and phase velocity typically equal, in a GaAs-based sample, to the group velocity of the edge states, v_{g0}, independent of V_s and a, if $\tilde{V}_{s0}=(V_s a/4\pi\hbar v_{g0})\exp[-(\pi l_0/a)^2]\leqslant 10^{-1}$ and $v_{g0}\geqslant 10^6$ cm/s. That is, this holds for a sufficiently weak periodic modulation. However, already for $\tilde{V}_{s0}\approx 0.2$ and $v_{g0}\lesssim 10^6$ cm/s, the phase velocity of the novel fundamental EMP can be substantially smaller than v_{g0}, as one can see in Fig. 2, due to the combined effect of a short-period lateral superlattice and the electron-electron interaction. In addition, its spatial structure is strongly dependent on both $\tilde{V}_{s0}$ and a. The spatial structure, with respect to the edge of the $n=0$ LL, becomes substantially asymmetric for some regions of x, as one can see by the dot-dashed curve in Figs. 3 and 4. We have also obtained that in the latter case the contributions to the spatial structure of the novel fundamental EMP that are commensurate with the periodic modulation can be of the same order of magnitude as those that are independent of x.

(iii) The measurement of the velocity of the novel EMP, due to its independence of the modulation parameters, in a wide range of them, can be a useful tool for obtaining directly the group velocity of edge states. Furthermore, a qualitative analysis, using results of previous studies[4] as well the above findings, shows that the dominant contribution to the damping rate of the novel EMP is absent. Then we may speculate that the damping rate of the novel mode could be rather small.

The simple analytical form of the lateral confining potential V_y used here is a rather good approximation of that calculated numerically in the Hartree approximation,[15] when the bare confining potential is sufficiently steep such that the LL do not have a flat region at the Fermi level. Furthermore, our main results still hold for confining potentials that are smooth on the scale of l_0 in the edge region, i.e., when the typical group velocity in this region satisfies the relation $0 < v_{g0} \ll \omega_c l_0$. Moreover, the confining potential here should be sufficiently smooth in order to hold the condition $v_{g0} \ll \omega_c / G$, where $G = 2\pi/a$. Notice that the consideration of the smoothness of the confining potential essentially simplifies the calculation of the eigenfunctions and eigenvalues for $\hat{h}^0$ and $\hat{H}_0$ and the analysis of the integral equation (5). Notice that the condition $v_{g0} \ll \omega_c l_0$ can be achieved in the Hartree approximation but not in the Hartree-Fock approximation because the exchange term leads to a logarithmically divergent v_{g0}.[18,19] However, when electron correlations are taken into account, a smooth spatial behavior of LL results near the edges and v_{g0} is small.[18]

Finally we discuss the physical origin of the new fundamental mode. We emphasize that this novel mode is obtained only when the relation $V_s \gtrsim 2\hbar v_{g0} G$ is assumed, which does not violate the condition of weak modulation, $V_s \ll 2\hbar\omega_c$. This mode arises from a rather strong quantum-mechanical coupling of the charge distortions at the edge of LL y_{r0} with those due to the periodic potential, at $y_{r0} \pm G l_0^2$. Furthermore, these three charge distortions are strongly coupled by the Coulomb interaction. We speculate that the frequency of the novel fundamental EMP is close to some characteristic frequency for the system of these charge distortions. At this ''resonance,'' the charge distortions for the novel fundamental EMP have comparable intensities (see Sec. III B). On the other hand, the ''resonance'' condition does not hold for the renormalized fundamental EMP and the charge distortion at y_{r0} is always much larger than those at $y_{r0} \pm G l_0^2$, i.e., in this case the effect of the periodic potential on the mode characteristics is rather small. This picture explains why the new fundamental EMP cannot be obtained in the limiting case $V_s \to 0$, but it is possible to obtain the well-known fundamental EMP (Ref. 5) from the renormalized fundamental EMP found here. We point out that the dipole and other multipole modes for $\nu = 1$, obtained in Refs. 4 and 5, should not be confused with the novel fundamental EMP. In particular, for the latter mode the normalized total charge density transverse to the edge $|\int \rho dy|$ is finite and rather large. For parameters taken in Fig. 1 and for $0.5 \times 10^{-2} \geqslant q_x l_0 \geqslant 0.5 \times 10^{-4}$, we obtain $0.2 > |\int \rho dy| \gtrsim 0.1$. As $|\int \rho dy| \approx 1$ for the renormalized fundamental EMP, we have exactly $|\int \rho dy| = 0$ for dipole and other multipole EMPs.[4,5]

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