Coherent control and enhancement of refractive index in an asymmetric double quantum well

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(Received 23 June 2000; revised manuscript received 11 August 2000)

We propose coherent control and enhancement of the refractive index with zero absorption in an *n*-type asymmetric double quantum well using intersubband transitions. These effects are caused by quantum coherence and interference whereby a strong infrared laser mixes upper conduction subbands of an intersubband transition with an auxiliary subband. In the GaAs/$Al_xGa_{1-x}As$ system considered here, an approximately 10% enhancement of the refractive index accompanied with zero absorption occurs at 10.5 μm, when the system is driven by a 6.2-μm laser field with an intensity of $\sim$1 MW/cm^2. The interplay between quantum coherence and electron population dynamics not only causes an enhancement of refractive index similar to that in atomic systems, but can also induce both low positive and negative group velocities.

The use of quantum interference to coherently control the dispersive part of the susceptibility of a medium is an appealing subject for fundamental and applied reasons. It has been previously shown that quantum interference can make an atomic medium transparent at a frequency where the refractive index is large.[1,2] This effect, enhancement of a resonant refractive index, usually occurs when the absorption associated with a transition becomes zero[2] or insignificant[3] while its refractive index remains large. Quantum interference can also be used to produce an atomic medium with very large dispersion and no absorption. These phenomena can have useful applications including effective control of optical nonlinearities,[4] drastic reduction of the speed of light,[5] and sum frequency generation.[6]

Counterintuitive effects involving quantum coherence and interference have also attracted tremendous attention in bulk and quantum well (QW) semiconductor systems. For example, it has been shown that they can lead to coherently controlled photocurrent generation,[7] electron intersubband transitions,[8] and laser induced transparency.[9] While these effects are mainly related to the manipulation of the absorptive properties of solids through coherent means, the issue of coherent control of the dispersive part of the susceptibility has remained largely unexplored. However, such control can also have important applications in optical and photonic devices such as all-optical switches.

Here we propose coherent control and enhancement of the refractive index associated with an intersubband transition in an *n*-type double QW. We show that the interplay between the quantum coherence generated by two lasers, one strong and one weak, and the electron dynamics via tunneling of electrons allows one to have a regime of refractive index enhancement which is unique in QW's. This includes increasing the refractive index contribution from an intersubband transition with an accompanying decrease in absorption. When the former reaches its maximum value the latter becomes zero. We also show that the coherent process can induce low positive and negative group velocities. These results may lead to various applications including pulse compression, and coherent control of optical length without absorption. In addition, since QW structures can be grown with submicron structural features or can be etched to make various structural forms, one may be able to tailor this effect for photonic devices.

To illustrate these phenomena, we consider a double *n*-type QW structure such as that shown in Fig. 1. This structure consists of two GaAs layers with 6.4- and 3.0-nm thickness separated by a 1-nm $Ga_{0.6}Al_{0.4}As$ layer. The left barrier is $Ga_{0.68}Al_{0.32}As$ while the right one is $Ga_{0.6}Al_{0.4}As$. In this structure $|a\rangle$ and $|1\rangle$ are well localized in the QW layers while the third subband ($|2\rangle$) is, respectively, $\sim$10 and $\sim$76 meV below the left and right barrier edges. These conditions lead to a large dipole moment for the transition between $|2\rangle$ and $|1\rangle$ ($\langle z\rangle_{21}=2.7\,\text{nm}$) and a reasonable value for that between $|2\rangle$ and $|a\rangle$ ($\langle z\rangle_{2a}=1.2\,\text{nm}$). The transition energy between $|1\rangle$ and $|a\rangle$ is $\sim$84 meV and that between $|2\rangle$ and $|1\rangle$ is $\sim$114 meV. The QW is taken to be *n* doped with a carrier density $8\times10^{11}\,\text{cm}^{-2}$.

To study coherent control and enhancement of refractive index in such a system we adopt a configuration similar to the so-called Λ configuration in atomic systems.[2] Here a driving field with frequency ω_c establishes quantum coherence between the upper transition level, $|2\rangle$, and the auxiliary state, $|a\rangle$. The $|a\rangle$ state is incoherently pumped by tunneling

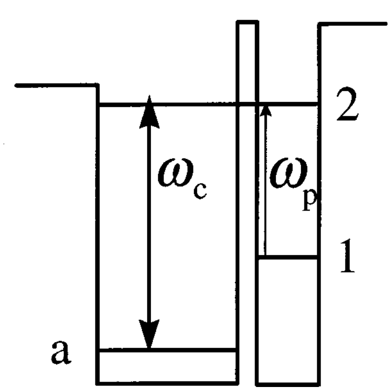

FIG. 1. Schematic diagram of the doubled quantum well structure. The two-sided arrow represents the strong coupling laser with frequency ω_c and the one-sided arrow to that of the probe field with frequency ω_p.

of electrons from the lower transition level, $|1\rangle$. A weak field with frequency ω_p probes the transitions between $|1\rangle$ and $|2\rangle$.

Since the QW is n doped the intersubband transitions induced by the optical fields are influenced by Coulomb interaction between electrons. Effects such as depolarization and excitonic shifts and exchange energy can play a role in infrared coupling of the QW. In addition, electron-electron scattering processes can have incoherent contributions to the intersubband transitions, including polarization scattering. Finally the coupling field can renormalize the polarization dephasing rates associated with the intersubband transitions.[10] However, since the carrier densities here are high, screening is very significant. Therefore the contribution of polarization scattering to the dephasing rates is small and we can ignore the nondiagonal dephasing rates.[11] In addition, the energy renormalization caused by the exchange terms and the exciton shifts counter each other, resulting in an insignificant net effect.[12,13] Based on these considerations the equations of motion of density matrix at a given $\mathbf{k}$ for the infrared-coupled QW are found to be

$$\frac{\partial\rho_{aa}^{\mathbf{k}}}{\partial t}=-i\Omega_c(\rho_{2a}^{\mathbf{k}}-\rho_{a2}^{\mathbf{k}})+\left.\frac{\partial\rho_{aa}^{\mathbf{k}}}{\partial t}\right|_{e-e}^{\mathrm{incoh}}+\left.\frac{\partial\rho_{aa}^{\mathbf{k}}}{\partial t}\right|_{e-p}^{\mathrm{diff}}+\Gamma_{2a}\rho_{22}^{\mathbf{k}}+r_t\rho_{11}^{\mathbf{k}}, \tag{1}$$

$$\frac{\partial\rho_{11}^{\mathbf{k}}}{\partial t}=-i\Omega_p(\rho_{12}^{\mathbf{k}}-\rho_{21}^{\mathbf{k}})-r_t\rho_{11}^{\mathbf{k}}+\Gamma_{21}\rho_{22}^{\mathbf{k}}, \tag{2}$$

$$\frac{\partial\rho_{22}^{\mathbf{k}}}{\partial t}=i\Omega_c(\rho_{2a}^{\mathbf{k}}-\rho_{a2}^{\mathbf{k}})-i\Omega_p(\rho_{21}^{\mathbf{k}}-\rho_{12}^{\mathbf{k}})-(\Gamma_{21}+\Gamma_{2a})\rho_{22}^{\mathbf{k}}, \tag{3}$$

$$\frac{\partial\rho_{a2}^{\mathbf{k}}}{\partial t}=i[\Delta_c^k+i\gamma_{a2}^{e-e}(k)+i\gamma_{a2}^{p}]\rho_{a2}^{\mathbf{k}}+i\Omega_p\rho_{a1}^{\mathbf{k}}+i\Omega_c(\rho_{aa}^{\mathbf{k}}-\rho_{22}^{\mathbf{k}}), \tag{4}$$

$$\frac{d\rho_{1a}^{\mathbf{k}}}{dt}=-i[\delta^k-i\gamma_{1a}^{e-e}(k)-i\gamma_{1a}^{p}]\rho_{1a}^{\mathbf{k}}-i\Omega_p\rho_{2a}^{\mathbf{k}}+i\Omega_c\rho_{12}^{\mathbf{k}}, \tag{5}$$

$$\frac{\partial\rho_{12}^{\mathbf{k}}}{\partial t}=i[\Delta_p^k-i\gamma_{12}^{p}]\rho_{12}^{\mathbf{k}}-i\Omega_p(\rho_{22}^{\mathbf{k}}-\rho_{11}^{\mathbf{k}})-i\Omega_c\rho_{a1}^{\mathbf{k}}. \tag{6}$$

Here $\rho_{ij}^{\mathbf{k}}$ (i and $j=a$, 1 and 2) are the density matrix elements, $\Omega_c=-\mu_{a2}E_c/\hbar$ and $\Omega_{12}=-\mu_{12}E_p/\hbar$ are the Rabi frequencies of the fields in which E_c and E_p are the amplitudes of the IR fields with frequencies ω_c and ω_p and μ_{a2} and μ_{12} are the dipole moments associated with the a-2 and 1-2 transitions, respectively; r_t is the rate of incoherent pumping of $|a\rangle$ caused by tunneling of electrons from $|1\rangle$. In Eqs. (4)–(6), $\Delta_c^k=\omega_2^k-\omega_a^k-\omega_c$ and $\Delta_p^k=\omega_2^k-\omega_1^k-\omega_p$ are the one-photon detunings of the fields and $\delta^k=\omega_1^k-\omega_a^k+\omega_p-\omega_c$ is the two-photon detuning while ω_i^k is the frequency associated with the energy of $|i,\mathbf{k}\rangle$. The transition energies are renormalized by the depolarization shifts. However, since the electron density in the conduction band of the QW is not changed by the fields, these shifts are nearly constant parameters. Therefore in practice these shifts do not change the results of this paper if the field detunings are adjusted properly.

In Eq. (1) $\partial\rho_{aa}^{\mathbf{k}}/\partial t|_{e-p}^{\mathrm{diff}}$ refers to the electron-photon scattering contribution to the damping of $|a,\mathbf{k}\rangle$. Since the Fermi level in this subband is less than 36 meV, this contribution mostly comes from scattering of electrons with acoustic phonons. Such a process transfers a small amount of energy and therefore is diffusive.[14] Γ_{ij}'s are the scattering rates of electrons from the ith to the jth subbands. Since the energy spacing between these subbands are larger than those of LO phonons, these rates are caused by electron-LO-phonon scattering. For the structure shown in Fig. 1, since the a-2 transition energy is ~84 meV larger than that of the 1-2 transition, the intersubband transitions associated with the former require LO phonons with larger in-plane wave vectors. Therefore, considering the typical values of Γ_{ij}'s in asymmetric quantum wells[15–16] and the fact that they are proportional to the inverse of squared phonon wave vectors, we take $1/\Gamma_{21}=1$ ps and $1/\Gamma_{2a}=1.5$ ps. In Eqs. (4)–(6), γ_{ij}^{p} refers to the dephasing rates of the nondiagonal elements of the density matrix due to the electron-phonon scattering processes. We have $\gamma_{a2}^{p}=\frac{1}{2}(\Gamma_a^p+\Gamma_{2a}+\Gamma_{21})$, $\gamma_{12}^{p}=\frac{1}{2}(\Gamma_{21}+\Gamma_{2a}+r_t)$, and $\gamma_{1a}^{p}=\frac{1}{2}(\Gamma_a^p+r_t)$. Here Γ_a^p is the damping rate caused by the acoustic phonons in $|a\rangle$.

In Eq. (1) $\partial\rho_{aa}^{\mathbf{k}}/\partial t|_{e-e}^{\mathrm{incoh}}$ refers to the contribution of the electron-electron scattering processes in the damping of $|a,\mathbf{k}\rangle$. This term is governed by the Boltzmann equation. In our system, however, since we seek a steady-state solution of Eqs. (1)–(6), we have[10]

$$\left.\frac{\partial\rho_{aa}^{\mathbf{k}}}{\partial t}\right|_{e-e}^{\mathrm{incoh}}+\left.\frac{\partial\rho_{aa}^{\mathbf{k}}}{\partial t}\right|_{e-p}^{\mathrm{diff}}=0. \tag{7}$$

The electron-electron scattering process strongly influences the polarization dephasing rates of the system. In Eqs. (4)–(6), these contributions are introduced by $\gamma_{ij}^{e-e}(k)$. Since electrons are mostly in $|a,\mathbf{k}\rangle$, these rates are introduced only for those transitions which involve this subband. As discussed in Ref. 10 in detail, these rates could change as the field intensity changes. Therefore they could renormalize the interaction of the coupling field with the QW. Here, however, since the electron distribution in $|a\rangle$ tends to be a Fermi-like distribution and the effective masses of electrons are nearly the same for all three subbands around $k=0$, we introduce $\gamma_{ij}^{e-e}(k)$ in a phenomenological way. For the carrier density considered here, based on the results of Ref. 10 we consider $\gamma_{a2}^{e-e}(k)=\gamma_{1a}^{e-e}(k)=2\ \mathrm{ps}^{-1}$.

We are interested in the response of the system for the 2-1 transition. The absorption coefficient and refractive index associated with this transition can be found from the following:

$$\alpha(\omega_p)=\frac{4\pi\omega}{cn_b}\mathrm{Im}[\chi_{21}(\omega_p)], \tag{8}$$

$$n-n_b=\frac{2\pi}{n_b}\mathrm{Re}[\chi_{21}(\omega_p)], \tag{9}$$

where n_b is the background refractive index. In GaAs $n_b\sim3.26$ at 114 meV. $\chi_{21}(\omega_p)$ is the susceptibility of the system for the particular transition, given by

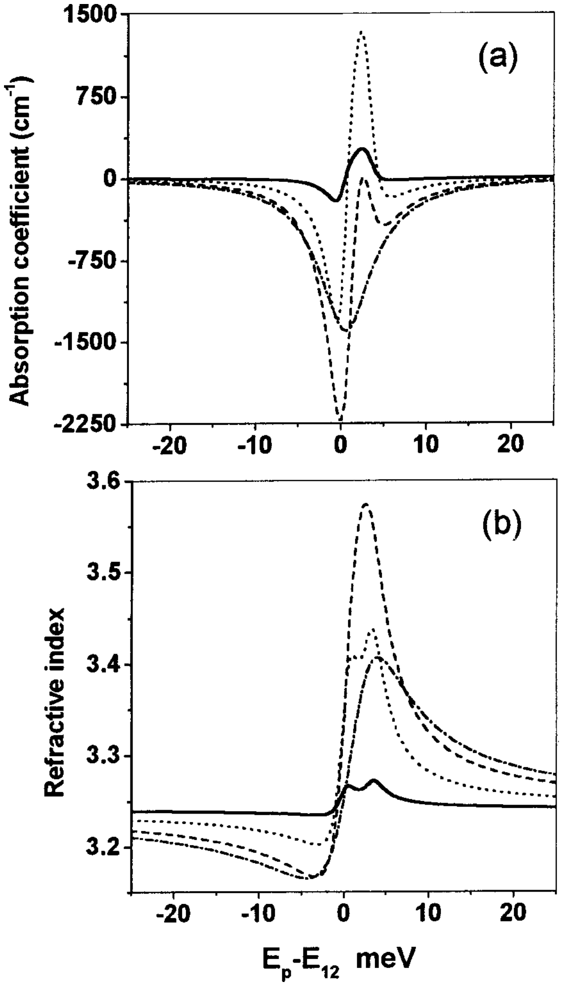

FIG. 2. The absorption coefficient (a) and refractive index (b) associated with the 1-2 transition. The solid, dotted, dashed, dotted-dashed lines refer, respectively, to 0.025, 0.22, 1, and 3.6 MW/cm^2 coupling field intensities. E_p refers to the probe photon energy and E_{12} to that of the 1-2 transition. (a) and (b) are obtained using Eqs. (8) and (9), respectively.

$$\chi_{21}(\omega)=\frac{1}{\pi\epsilon L_{\mathrm{eff}}}\int_0^{k_{\max}}kdk\,\frac{|\mu_{21}|^2(\rho_{11}^k-\rho_{22}^k)}{\hbar[\Delta_p^k-i\gamma_{12}^p]},\qquad(10)$$

where L_{eff} is the effective length over which the interaction occurs, taken to be the width of the narrower well, and ϵ is the dielectric constant of GaAs at the 114 meV.

To study the coherent control and enhancement of refractive index in our system we take $r_t=2\ \mathrm{ps}^{-1}$.[15,16] This value is chosen because the 1-a transition energy is small (~84 meV) and the QW is heavily doped. This situation allows fast tunneling between $|1\rangle$ and $|a\rangle$ via LO phonons of very small wave vector. In the following we also take the intensity of the probe field as ~1 kW/cm^2 or $\Omega_p=0.1\ \mathrm{ps}^{-1}$. Therefore the field near resonance with the 1-2 transition does not cause significant coupling effect. As shown in Fig. 2 for various values of the coupling intensities and with $\Delta_c^k=-3.3\,\mathrm{meV}$ the response to the probe field is interesting. When $\Omega_c=0.5\,\mathrm{ps}^{-1}$ (~$25\,\mathrm{kW/cm}^2$) the absorptive and refractive parts associated with the 1-2 transition are relatively small (solid lines) with a gain-absorption feature (a) and an increase in the index (b). These features can be related to the small number of electrons that populate $|2\rangle$ and to the coherent coupling of $|a\rangle$ and $|2\rangle$. When the intensity of the coupling field is increased to 220 kW/cm^2 (corresponding to $\Omega_c=1.5\,\mathrm{ps}^{-1}$, dotted lines), while the overall features of the absorption coefficient and refractive index remain the same, the population increase in $|1\rangle$ leads to an increase in the amplitudes of the probe response. So far the results can be characterized by two features: First, both absorption gain and refractive index increase as Ω_c increases, and second, for each Ω_c the absorption is zero at two frequencies while at those frequencies the change in the refractive index is nonzero. The latter is similar to the enhancement of refractive index studied in atomic systems.[2]

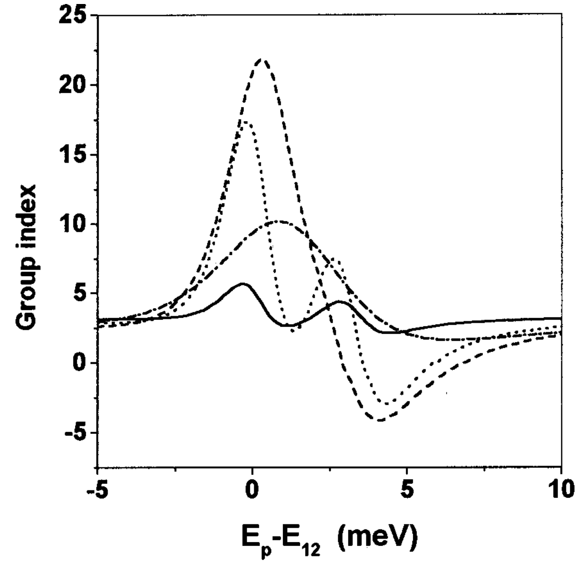

FIG. 3. Group indices associated with the refractive index spectra shown in Fig. 2(b). All other specifications are the same as those in Fig. 2(b).

The QW's response becomes more exotic and distinct when the coupling field intensity increases further. As Fig. 2 shows (dashed lines), when $\Omega_c=3.1\,\mathrm{ps}^{-1}$ (corresponding to a beam intensity of ~1 MW/cm^2) the refractive index reaches the maximum enhancement at an energy ~3.3 meV above the resonance (10.5 μm). At this energy the absorption is zero, elsewhere the system experiences gain. For higher field intensities both the refractive index and the amplitude of the gain spectrum decrease (dotted-dashed lines). This occurs as the latter becomes featureless and broad.

The corresponding development of the group index $[N=c/(d\omega_p/dk)]$ is also interesting. As Fig. 3 shows, before the maximum enhancement of the refractive index is reached, the system dispersion produces a fluctuating feature in the group index spectrum. This feature becomes stronger as the coupling field intensity increases from 25 (solid line) to 220 kW/cm^2 (dotted line). This also makes the group index negative. For a coupling field intensity of 1 MW/cm^2, where the maximum enhancement occurs, the group index attains its maximum, ~22, and minimum value, ~−4 (dashed line).

The overall mechanism behind the results seen in Figs. 2 and 3 can be understood when the role of the incoherent pumping rate r_t is considered. If $r_t=0$, in the presence of the coupling field nearly all population remains in the narrower well. Therefore the system has a Λ configuration similar to that studied for laser-induced transparency.[1] Here the optical coupling of $|a\rangle$ with $|2\rangle$ leads to quantum interference when the latter is detected from $|1\rangle$. This leads to a transparency hole in the absorption spectrum of the 1-2 transition. When r_t is not zero the auxiliary subband ($|a\rangle$) is populated. This alters the coherent nature of the system, changing the response of the system from being one of pure absorption with a transparency hole into a spectrum involving gain and absorption. The absorption and refraction spectra presented in Fig. 2 (solid and dotted lines) are indications of this case.

Before the maximum enhancement of the refractive index is reached, the partial gain-absorption feature dominates the system's response. Here, although enhancement of refractive index with zero absorption occurs at two frequencies, for the frequency at which the refractive index is peaked the absorption is considerable (dotted lines in Fig. 2). For the case of a coupling field intensity of $\sim$1 MW/cm^2, however, the balance between quantum coherence and the electron population effects in $|1\rangle$ and $|2\rangle$ led to the maximum enhancement of the refractive index. This was accompanied by a reduction to zero of the absorption at the peak frequency to zero. At higher field intensities, due to the high excitation of electrons, this balance changed and the refractive and absorptive responses of the system became similar to those of an incoherently inverted system.

In conclusion, we have proposed a method by which the interplay between quantum coherence and electron dynamics in an *n*-type quantum well could lead to the enhancement of a refractive index accompanied by zero absorption in the intersubband transitions. Our results show that in a GaAs/Al$_x$Ga$_{1-x}$As asymmetric double quantum well one could increase the refractive index by about 10%. We show that this distinct effect is caused by a coherently correlated increase of the refractive index and decrease of the absorption. Before reaching the limit of maximum refractive index and zero absorption, however, the system presents enhancement of the refractive index similar to that in atoms.

We are very grateful for the financial support of the Natural Sciences and Engineering Research Council of Canada.

[1] M. D. Lukin, S. F. Yelin, A. S. Zibrov, and M. O. Scully, Laser Phys. **9**, 759 (1999).
[2] U. Rathe, M. Fleischhauer, Shi-Yao Zhu, T. W. Hänsch, and M. O. Scully, Phys. Rev. A **47**, 4994 (1993).
[3] M. Löffler, D. E. Nikonov, O. A. Kocharovskaya, and M. O. Scully, Phys. Rev. A **56**, 5014 (1997).
[4] S. E. Harris, Opt. Lett. **19**, 2018 (1994).
[5] L. V. Hau, S. E. Harris, Z. Dutton, and C. H. Behroozi, Nature (London) **397**, 594 (1999).
[6] G. Z. Zhang, K. Hakuta, and B. P. Stoicheff, Phys. Rev. Lett. **71**, 3099 (1993).
[7] R. Atanasov, A. Haché, J. L. P. Hughes, H. M. van Driel, and J. E. Sipe, Phys. Rev. Lett. **76**, 1703 (1996).
[8] W. Pötz, Physica E (Amsterdam) **7**, 159 (2000).
[9] D. E. Nikonov, A. Imamoğlu, and M. O. Scully, Phys. Rev. B **59**, 12 212 (1999).
[10] S. M. Sadeghi, S. R. Leffler, J. Meyer, and E. Mueller, J. Phys.: Condens. Matter **10**, 2489 (1998).
[11] F. Rossi, S. Haas, and T. Kuhn, Phys. Rev. Lett. **72**, 152 (1994).
[12] S. L. Chuang, M. S. C. Luo, S. Schmitt-Rink, and A. Pinczuk, Phys. Rev. B **46**, 1897 (1992).
[13] W. L. Bloss, J. Appl. Phys. **66**, 3639 (1989).
[14] S. E. Esipov and Y. B. Levinson, Adv. Phys. **36**, 331 (1987).
[15] O. Gauthier-Lafaye, S. Sauvage, P. Boucaud, F. H. Julien, F. Glotin, R. Prazeres, J.-M. Ortega, V. Thierry-Mieg, and R. Planel, J. Appl. Phys. **83**, 2920 (1998).
[16] O. Gauthier-Lafaye, S. Sauvage, P. Boucaud, F. H. Julien, R. Prazeres, F. Glotin, J.-M. Ortega, V. Thierry-Mieg, R. Planel, J.-P Leburton, and V. Berger, Appl. Phys. Lett. **70**, 3197 (1997).