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Metal-insulator transitions in the cyclotron resonance of periodic semiconductor nanostructures due to avoided band crossings

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A recently found metal-insulator transition in a model for cyclotron resonance in a two-dimensional periodic potential is investigated by means of spectral properties of the time evolution operator. The previously found dynamical signatures of the transition are explained in terms of avoided band crossings due to the change of the external electric field. The occurrence of a crosslike transport is predicted and numerically confirmed.

I. INTRODUCTION

Since the pioneering work of Bloch[1] and Landau[2] the investigation of the spectral properties and transport features of Bloch electrons in magnetic fields[3] has attracted much attention. While infinitely degenerate Landau levels are found for free electrons in a static magnetic field, a spatially periodic potential without a field yields a band structure in the spectrum. The general situation of a periodic potential *and* a magnetic field is still a matter of intensive research.[4–7] A particular simple realization for a two-dimensional (2D) periodic potential is, e.g., given by

$$V(\mathbf{r})=V_x\cos\left(\frac{2\pi x}{a}\right)+V_y\cos\left(\frac{2\pi y}{b}\right), \tag{1}$$

where a,b are the periods in the corresponding directions. In a one-band approximation this potential leads to the well-known Harper model,[8] where a metal-insulator transition (MIT) is observed upon changing the modulation amplitudes from $V_y>V_x$ to $V_x>V_y$.[9] At the critical point, i.e., $V_x=V_y$, the spectrum and the eigenfunctions are multifractals[10] and give rise to anomalous diffusion.[11–13] The one-band Harper model, however, has an integrable classical limit, whereas Bloch electrons in a magnetic field show chaotic dynamics in the classical limit. In quantum mechanics this is reflected in the coupling of Landau bands. This coupling leads to avoided band crossings, which induce an unusual transport phenomenon: For $V_x\neq V_y$ electrons show ballistic transport in the direction of strong potential modulation and localization in the direction of weaker modulation.[7]

Analogous phenomena can be found in driven systems with a chaotic classical limit even within the one-band approximation. A well-studied example is the kicked Harper model (KHM).[14–19] For small kicking strength its behavior is analogous to the Harper model,[15] while for increasing kicking strength avoided band crossings due to the classical nonintegrability induce a variety of MIT's.[18] Recently, it was demonstrated that a variety of kicked models, including the KHM, can be realized for electrons on a lattice in the presence of a magnetic field driven by a smooth electric field.[20] The effective kicking is a result of a resonant interaction between the electronic motion and the driving field. These models correspond to cyclotron resonance experiments in antidot arrays and in organic metals.[20,21] By investigating the dynamics, a localization-delocalization transition induced by the amplitude of the electric field was observed[21] and remained unexplained so far.

In this paper we investigate these dynamical properties in terms of spectral properties in a wide range of driving strengths. We show that the previously found localization-delocalization transition is a consequence of avoided band crossings due to the change of the driving field strength. Moreover, the spectral analysis predicts a crosslike ballistic transport, which is numerically confirmed. We start in Sec. II by briefly introducing the model and explain the numerical approach in Sec. III. In Sec. IV we present our main results, which give a clear picture for the MIT in terms of the spectrum. We finally summarize our findings in Sec. V.

II. MODEL

The one-particle Hamiltonian, describing the cyclotron resonance of an electron in the 2D periodic potential $V(\mathbf{r})$ of Eq. (1), can be written in the following form

$$\mathcal{H}=\frac{1}{2m^*}[\mathbf{p}-e\mathbf{A}(\mathbf{r},t)]^2+V(\mathbf{r}), \tag{2}$$

where $\mathbf{p}=(p_x,p_y)$ is the momentum of an electron with effective mass m^* and charge e moving in the xy plane perpendicular to the constant magnetic field B pointing in the z direction. We choose the Landau gauge, which yields a vector potential $\mathbf{A}(\mathbf{r},t)=((E_x/\nu)\sin\nu t, xB-(E_y/\nu)\cos\nu t)$ containing also the alternating electric field $\mathbf{E}=(-E_x\cos\nu t, E_y\sin\nu t)$ with frequency ν.

Applying a gauge transformation to the Hamiltonian (2), the time dependence can be shifted to the periodic potential $V(\mathbf{r})\rightarrow V(\mathbf{r},t)$:[21] If the fields B,E_x, and E_y satisfy $\omega^*E_y=\nu E_x$, then one can rewrite the Hamiltonian as $\mathcal{H}=\mathcal{H}_0(\mathbf{p},x)+V(\mathbf{r},t)$, where $\mathcal{H}_0=\mathcal{H}_0(\mathbf{p},x)$ describes the cyclotron motion with frequency $\omega^*=eB/m^*$. The perturbation $V(\mathbf{r},t)$ splits the infinitely degenerate Landau levels of $\mathcal{H}_0$ into a miniband structure. For strong electric fields and $\nu=1$ this Hamiltonian $\mathcal{H}$ reduces to a kicked system[20,21]

$$\tilde{\mathcal{H}} = L\cos p + K\sum_{n=-\infty}^{\infty}\{\cos(q-\kappa_0)\,\delta(t-2n+1) + \cos(q+\kappa_0)\,\delta(t-2n)\}, \tag{3}$$

where the canonical operators q and p corresponding to the spatial coordinates x,y satisfy $[p,q]=-i2\pi\hbar_{\text{eff}}$ and $\kappa_0 = [2\pi eE_y/(bm^*)-\pi/4] \bmod 2\pi$; L,K denote the rescaled amplitudes of the potential (1). The effective Planck constant $\hbar_{\text{eff}}=\hbar/(abeB)$ corresponds to the inverse number of magnetic flux quanta through a unit cell $a\times b$. The system described by Eq. (3) has a chaotic classical limit[20] and for $\kappa_0=0$ reduces to the KHM.

III. NUMERICAL APPROACH

The time evolution operator for one period corresponding to Eq. (3) reads

$$\mathcal{U} = \exp\left(-\frac{iL}{\hbar_{\text{eff}}}\cos p\right)\exp\left(-\frac{iK}{\hbar_{\text{eff}}}\cos(q-\kappa_0)\right) \times \exp\left(-\frac{iL}{\hbar_{\text{eff}}}\cos p\right)\exp\left(-\frac{iK}{\hbar_{\text{eff}}}\cos(q+\kappa_0)\right). \tag{4}$$

In order to obtain its spectrum, one has to approximate the irrational effective Planck constant by a rational approximant, $\hbar_{\text{eff}}=2\pi M/N$. Following standard techniques for kicked systems,[22] this leads to a $N\times N$ matrix for the evolution operator $\mathcal{U}$, which depends on two Bloch phases $\theta_q,\theta_p \in [0,2\pi]$. These Bloch phases arise from periodic boundary conditions in the q and p directions, respectively. Diagonalization of $\mathcal{U}$ for all pairs (θ_q,θ_p) gives its spectrum. Keeping θ_q fixed and varying θ_p yields a part of this spectrum, from now on referred to as the ''p spectrum.'' Eigenfunctions, which are localized in the p direction, will be associated with very narrow bands in the p spectrum, whereas extended states give rise to broad bands. Since for higher approximants of $\hbar_{\text{eff}}$ the width of the narrow bands decreases exponentially, we will call them levels. In an analogous way one can define the ''q spectrum,'' which reflects the properties of the eigenfunctions in the q direction. Figure 1 shows a q and p spectrum, where one observes broad bands and levels.

In order to investigate the dynamics, we will study the spreading of wave packets, initially δ localized in either the q or p direction by determining the corresponding variances $M_q(t)$ and $M_p(t)$. For spreading in the p direction we use periodic boundary conditions in the q direction after one unit cell, giving a discrete lattice in momentum space with spacing $\hbar_{\text{eff}}$. An analogous construction is used for spreading in the q direction. When considering transport in phase space in both directions, we use periodic boundary conditions after r unit cells in the q direction, which gives r different Bloch phases $\theta_q=2\pi j/r$ $(j=0,\ldots,r-1)$. The complete wave packet is then obtained by combining the wave packets for each Bloch phase and it is defined on a grid with spacing $\hbar_{\text{eff}}/r$ in momentum space.

IV. RESULTS

We will study the spectrum of Eq. (4) as a function of κ_0 and its consequences for dynamical properties. In particular, we will focus on the parameters used in Ref. 21 and show that they reflect the generic behavior of the model (3).

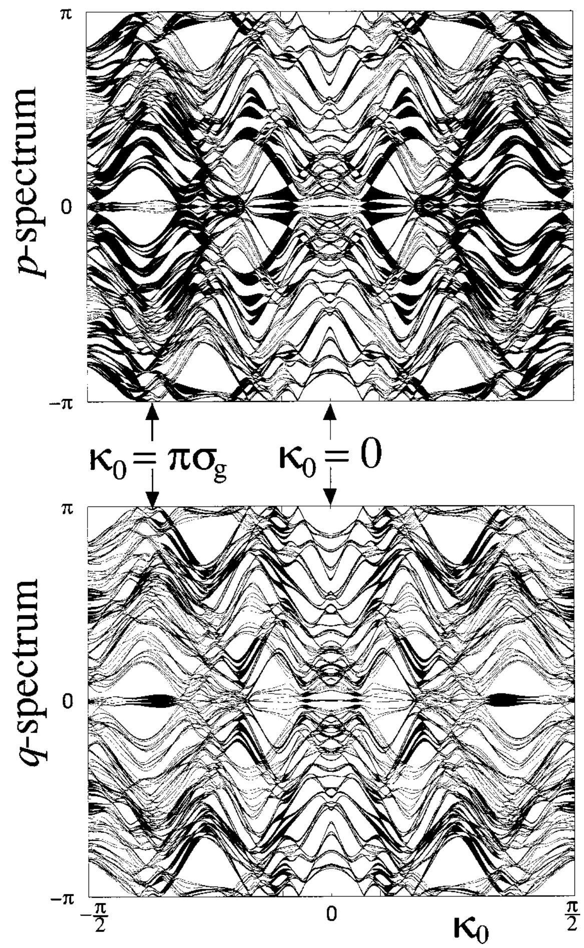

FIG. 1. The p spectrum (top) and q spectrum (bottom) of the evolution operator (4) for $K=L=5$ vs κ_0 (one period $[-\pi/2,\pi/2]$ is shown). The approximant for the irrational effective Planck constant is $\hbar_{\text{eff}}=2\pi\times 34/259$. Arrows denote the values $\kappa_0=0$ and $\kappa_0=\pi\sigma_g$ used in Fig. 2. For $\kappa_0=0$, Eq. (3) reduces to the KHM and the spectrum is multifractal, while for $\kappa_0\neq 0$ it consists of bands and levels. The spectra appear to be dual to each other, since broad bands in one of them are associated with levels in the other.

In Ref. 21 for $K=L=5$ and $\hbar_{\text{eff}}=2\pi/(7+\sigma_g)$, where $\sigma_g=(\sqrt{5}-1)/2$ denotes the golden mean, a transition from diffusive to ballistic dynamics was found, when κ_0 was varied from 0 to $\pi\sigma_g$ (cf. Fig. 1 of Ref. 21). In Fig. 1 we present the q and p spectra for one period of κ_0. Apparently, the duality of the q and p spectra is conserved for all κ_0; i.e., the q spectrum shows wide bands, where the p spectrum is level like and vice versa.

For $\kappa_0=0$ the variances increase almost linearly in both the q and p directions, corresponding to diffusive spreading in either direction [top of Fig. 2(a)]. This well-known behavior of the KHM is a direct consequence of the multifractal nature of spectrum and eigenfunctions.[14–16]

For $\kappa_0=\pi\sigma_g$ bands occur in the p spectrum explaining the ballistic transport in the p direction observed in Ref. 21. There are, however, also levels which are related to bands in the q spectrum, predicting a ballistic spreading in the q direction. This is confirmed in Fig. 2(b). All eigenstates are

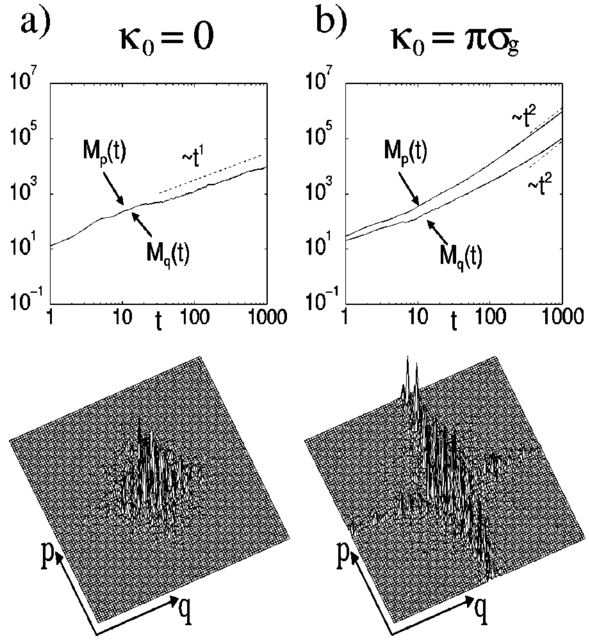

FIG. 2. (a) Variances $M_q(t)$, $M_p(t)$ and the Husimi plot of a 2D wave packet after 100 kicks (one point per unit cell and $r=89$ cells in each direction) for $K=L=5$, $\hbar_{\rm eff}=2\pi/(7+\sigma_g)$, and $\kappa_0=0$. (b) Same for $\kappa_0=\pi\sigma_g$. In the first case, the variances show a diffusivelike behavior while the wave packet spreads ballistically in both directions in the second case. This is also reflected in the Husimi plots, where one observes that the wave packet spreads isotropically in phase space in (a) and shows a crosslike spreading in (b).

extended in either of the directions and localized in the other one. A typical initial wave packet will excite both types of eigenfunctions. The part of the wave packet consisting of eigenfunctions extended in the q direction gives ballistic transport in the q direction. The other part, which is a superposition of eigenfunctions extended in the p direction, yields ballistic transport in the p direction. Therefore one finds a superposition of ballistic transport in the q and p directions, namely, a crosslike transport [bottom of Fig. 2(b)]. The crosslike form of the wave packet is clearly different from the isotropic form for $\kappa_0=0$ [Fig. 2(a)]. There are several other transport features one can observe from the Husimi plot in Fig. 2(b). The weight of the wave packet spreading ballistically along the p direction is much bigger than the corresponding weight for the q direction. This is a signature of the spectrum, namely, that there are more bands in the p spectrum than there are bands in the q spectrum at $\kappa_0=\pi\sigma_g$, as can be seen in Fig. 1. It should be noted that the velocities in the p and q directions [which can be inferred from the variances $M_p(t)$ and $M_q(t)$] are in general not related to the number of bands but are determined by the bandwidths.

Now we will turn away from the symmetry line $K=L$ and focus on the parameters $K=2$, $L=4$, and $\hbar_{\rm eff}=2\pi/(7+\sigma_g)$. In Fig. 3 we show the q and p spectra as a function of κ_0, which again appears to be dual. For the KHM ($\kappa_0=0$) the q spectrum shows broad bands, whereas the p spectrum is level like, indicating transport in the q direction, but localization in the p direction.[14] This is confirmed by the dynamics [Fig. 4(a)]. Variation of κ_0 induces avoided band crossings leading to the occurrence of bands in the p spectrum

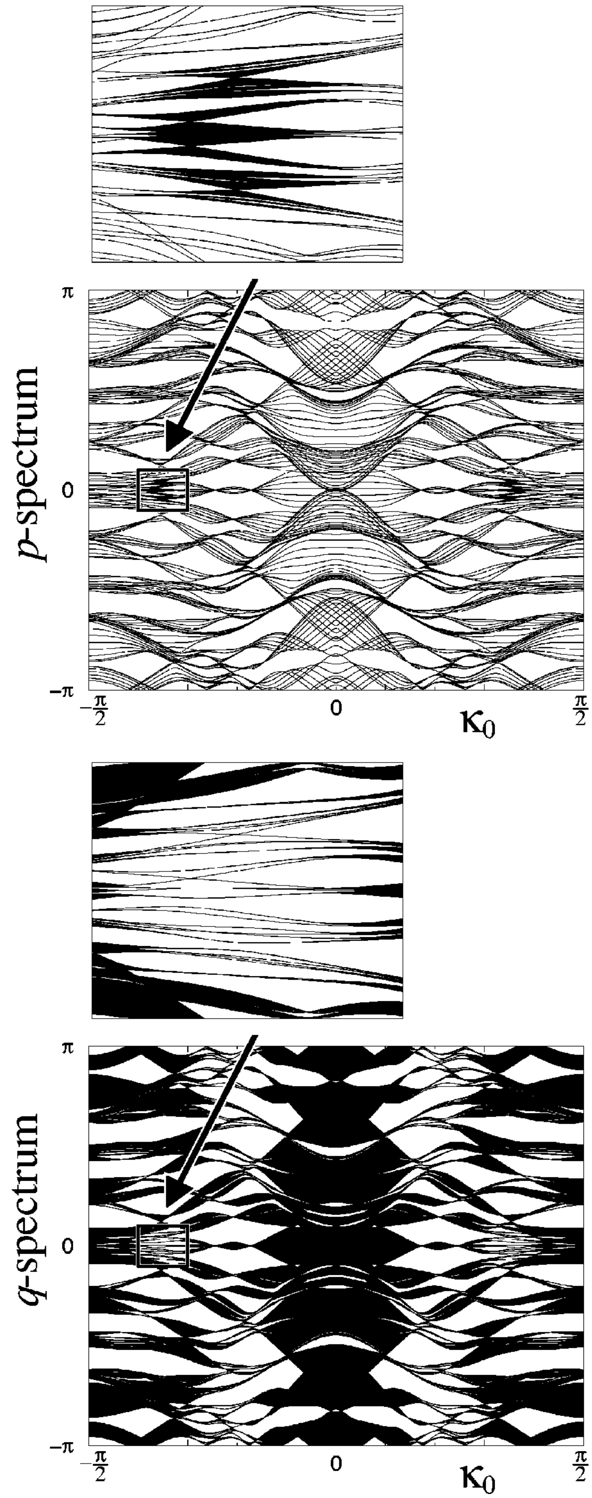

FIG. 3. Same as in Fig. 1 for $K=2$, $L=4$. For $\kappa_0=0$ the p spectrum consists of levels only and the q spectrum shows bands. Magnifications ($\hbar_{\rm eff}=2\pi 55/419$) show part of the spectra around $\kappa_0=\pi\sigma_g$, where one finds bands in the p spectrum and levels in the q spectrum. Duality again seems to be conserved.

(see the magnification in Fig. 3). This is associated with a localization-delocalization transition in the p direction as observed in Ref. 21. As there are still bands in the q spectrum one again finds the generic crosslike transport [Fig. 4(b)]. Here, the weight of the wave packet spreading along the q direction is bigger than the corresponding weight in the p

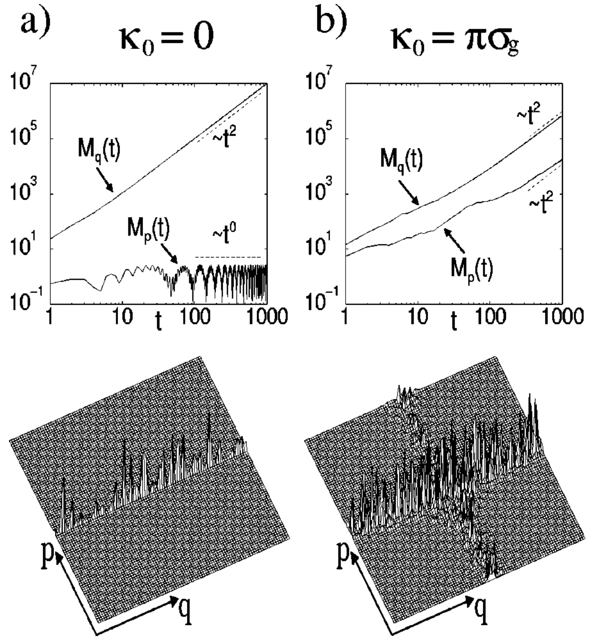

FIG. 4. Same as in Fig. 2 for $K=2$, $L=4$. (a) Ballistic spreading in the q direction and localization in the p direction are observed. (b) Ballistic spreading in both directions leads to a crosslike transport.

direction, as is implied by the much larger number of bands in the the q spectrum (Fig. 3). Another characteristic of the wave packet is the increased localization length in the p direction of the part of the wave packet spreading in the q direction compared to Fig. 4(a). This is due to the increased localization length of the eigenstates corresponding to the levels in the p spectrum. This change in the localization length can be explained by avoided band crossings.[18]

V. SUMMARY

In conclusion, we have shown here that the previously found MIT in cyclotron resonance in a 2D-periodic potential in the presence of a magnetic field can be fully understood by investigating the spectrum of the corresponding time evolution operator. We have presented numerical evidence that the duality of the spectra is conserved under changes of the driving strength. We were able to explain the previously observed dynamics,[21] which has shown a transition from diffusive to ballistic transport as well as a transition from localization to ballistic transport in the p direction. Furthermore, the spectra allowed us to predict the transport behavior in the q direction leading to a crosslike transport. These predictions were also confirmed numerically. We would like to mention that crosslike transport is generic for systems with dual spectra and a chaotic classical limit.

The observed transport phenomena in phase space of the effective Hamiltonian (3) correspond to analogous transport phenomena in the xy plane. The required conditions for realization of the observed MIT may well be achieved in experiments on cyclotron resonance[23] of a 2D electron gas embedded in lateral superlattices fabricated on GaAs heterostructures.[24]

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