Vertex corrections in nearly ferroelectric superconductors

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We investigate the effect of an incipient ferrolectric transition on vertex corrections to the superconducting pairing interaction. The vertex corrections for small momentum transfers are largely independent of the type of boson responsible for the superconducting transition. The electron-phonon interaction is found to be enhanced by a nearly ferroelectric medium. We discuss application to the cuprate superconductors.

I. INTRODUCTION

High-temperature superconductivity is commonly believed to be associated with the exchange of antiferromagnetic spin fluctuations. A unified description of antiferromagnetism and superconductivity is given by an SO(5) group scenario.[1] However, it had been previously shown by Birman and Solomon[2] that a similar group-theoretical formulation is possible with superconductivity associated with charge-density waves, ferromagnetism, and ferroelectricity, as well as the ''common'' antiferromagnetic scenario. We consider here an effect associated with the nearness of a ferroelectric transition. A phenomenological pairing interaction has been proposed[3,4] for such a nearly ferroelectric system, and one purpose of this paper is to make a first step toward giving this pairing interaction a firmer microscopic basis.

Quite generally, the pair interaction can be viewed as the exchange of a boson between the members of a Cooper pair. Of primary interest here is the electron-boson vertex. We reevaluate the Bardeen calculation[5] of the correction to the electron-phonon vertex due to electron-electron interactions, for the case of an electron gas embedded in a dielectric medium with a large dynamic dielectric constant ε reflecting the nearness to a ferroelectric transition. We calculate the lowest order vertex correction that includes phenomenologically the important ionic dielectric effects. This turns out to be a large correction. Although we assume in the following that the boson is a phonon, it is important to note that the present result is not limited to this case and indeed would also yield a large vertex correction in the popular scenario in which the boson is an antiferromagnetic spin fluctuation.

Many perovskites are ferroelectrics. Perovskites like $SrTiO_3$ and the high-T_c cuprates are nearly ferroelectric; their ionic dielectric constant is abnormally high, although they do not undergo a ferroelectric transition. The dielectric constant of $La_{2-x}Sr_xCuO_4$ (LSCO) and $YBa_2Cu_3O_{7-\delta}$ (YBCO) was measured recently quite accurately as function of frequency and is of the order of 100 at frequencies of order 10 meV.[6,7] In ''conventional'' calculations of the electron-phonon interaction, this dielectric constant is not taken into account. The reason is that, although the electron-phonon interaction itself is a nonadiabatic contribution, it is *calculated* within the framework of the Born-Oppenheimer approximation (BOA); i.e., it is assumed that during the scattering of the electron, while it imparts momentum to the lattice, the lattice does not move. In contrast, the ionic dielectric constant describes motion of the lattice. Therefore, in order to take it into account, we must consider effects outside the BOA. This presents a *fundamental* difficulty, and not just ''technical'' considerations.

The phonon-mediated interaction considered by Bardeen and Pines in the 1950s involves an effect outside the BOA. The scattering of electron 1 causes an atomic motion, and this motion interacts with electron 2. The Bardeen-Pines interaction is given by : $V=g^2[2\Omega/(\omega^2-\Omega^2)]$, where Ω is the phonon frequency, g is the Frohlich constant given by $g=I/\sqrt{M\Omega}$, M is the ionic mass, I is the matrix element given by $I(q)=\langle k|\nabla V|k+q\rangle$, and V is the electron-ion potential. $I(q)$ is thus calculated *within* the BOA. This is an apparent paradox; this procedure is justified by Migdal's theorem, which states that the corrections to $I(q)$, arising from the fact that we consider an effect outside the BOA, are of order $\Omega/E_F \simeq \sqrt{m/M} \simeq 10^{-2}$, where m is the electronic band mass.

Pietronero *et al.*[8] suggest that in the high-T_c cuprates Ω/E_F is not so small, and, as a result, the deviations from Migdal's theorem are significant. Here, we also question the validity of the BOA, but for a different reason—namely, we suggest that the very large ionic dielectric constant has a very large effect on the electron-phonon matrix element $I(q)$. It renormalizes it, and thus makes it frequency dependent; at very low frequencies $I(q)$ is increased by a significant amount above the BOA value.

II. CONVENTIONAL APPROACH

The ''conventional'' way to calculate the electron-phonon matrix element $I(q)$ is based, first, on Bloch's calculation of 1928,[9] in which he assumed $I(q)=\langle k|\nabla V|k+q\rangle$, where V is the electron-ion potential Ze^2/r . Thus $I(q)=4\pi Ze^2/q$. $I(q)$ is seen to diverge at small q values. We denote this as the ''Bloch vertex'' Γ_{Bloch} [Fig. 1(a)]. This estimate neglects the electron-electron interaction that screens the electron-ion potential. This screening was first considered by Bardeen in 1937.[5] The electron-electron interaction propagator is given by $D_{ee}(q)=4\pi e^2/q^2$. Bardeen took it into account by considering the potential

$$V_{Bardeen}(q)=\frac{4\pi Ze^2/q^2}{1+D_{ee}(q)N(E_F)}=\frac{4\pi Ze^2}{q^2+q_D^2}, \qquad (1)$$

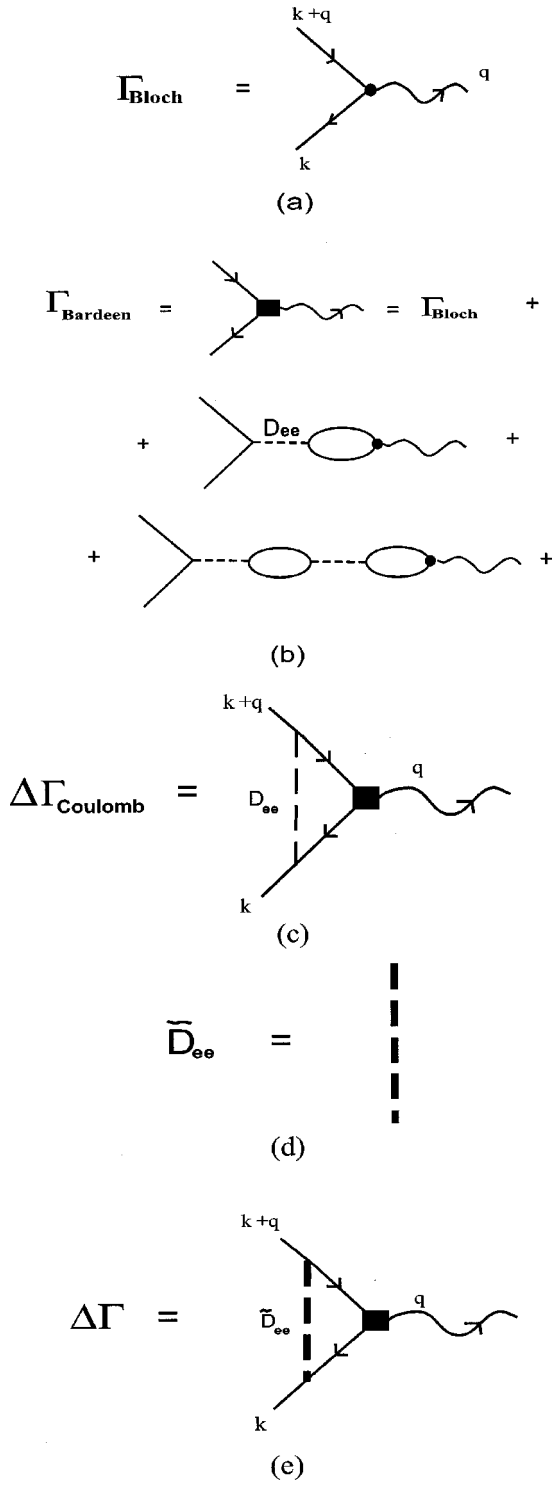

FIG. 1. The various electron-boson vertices discussed in the text: (a) the bare (Bloch) vertex; (b) the Bardeen vertex which includes electronic screening, $D_{ee}(q)=4\pi e^2/q^2$; (c) the lowest order Coulomb vertex correction; and (d) the electron-electron interaction including ionic dielectric effects [Eq. (2)].

where $q_D^2=4\pi e^2 N(E_F)$. This potential no longer diverges as $q\to 0$. The electron-phonon matrix element is given by $I(q)=4\pi Ze^2|q|/(q^2+q_D^2)$. Since $q_D>k_F$, the dependence of $I(q)$ on q is weak and the electron-phonon scattering is nearly isotropic.[10] The Cooper potential reflects this isotropic nature and leads to superconducting pairing with *s*-wave symmetry. What Bardeen did to deal with this (without saying it explicitly) was to sum up random-phase approximation bubble diagrams of the form of Fig. 1(b) to get the correct screened vertex. We call this the ''Bardeen vertex'' $\Gamma_{Bardeen}$.

The simplest vertex correction arising from the electron-electron interaction, $\Delta\Gamma_{Coulomb}$, is shown in Fig. 1(c). We calculate this contribution in the next section for comparison with the vertex correction including ionic effects. In Fig. 1(c) we could of course replace D_{ee} with a screened interaction of the form of Fig. 1(b). Note that Migdal's theorem does *not* apply here since D_{ee} does not possess a low-energy cutoff.

III. VERTEX CORRECTION INCLUDING THE IONIC DIELECTRIC FUNCTION

In a medium with a dielectric constant ε, the electron-electron interaction is given by $4\pi e^2/q^2\varepsilon$. Here ε is the dielectric constant of the medium *external* to the electron gas.[11] Thus we use a dressed electron-electron propagator:

$$\tilde{D}_{ee}(q,\omega)=\frac{4\pi e^2}{(q^2+q_D^2)\varepsilon(q,\omega)}. \tag{2}$$

Diagramatically, we denote $\tilde{D}_{ee}$ by a heavy broken line [Fig. 1(d)] that denotes the inclusion of the effect of the highly polarizable ions on the electron-electron interaction. The Thomas Fermi screening vector q_D is included to facilitate the numerical computations and to account for electronic screening. Thus we consider the vertex correction $\Delta\Gamma$ shown in Fig. 1(e). For simplicity in the present calculation we take $\Gamma_{Bardeen}$ to be a constant. We can thus write $\Gamma=\Gamma_{Bardeen}[1+(\Delta\Gamma/\Gamma_{Bardeen})]$ and we calculate $\Delta\Gamma/\Gamma_{Bardeen}$ and $\Delta\Gamma_{Coulomb}/\Gamma_{Bardeen}$, which are to be compared with the number 1.

We have to calculate the vertex correction as a function of q, paying particular attention to *small* q values. This is plausible for several reasons. First, the ionic dielectric corrections could also be included in the Bardeen screening by replacing D_{ee} with $\tilde{D}_{ee}$ and setting $q_D=0$ to avoid double counting of screening diagrams. Then if $\tilde{D}_{ee}$ were zero (as is the case for an infinite ε), Γ would be the Bloch vertex that diverges as $q\to 0$. Thus $\Delta\Gamma/\Gamma_{Bardeen}$ would diverge as $q\to 0$.

The necessity to consider small q values was also pointed out by Pietronero *et al.*,[8] for a different reason. They show that for their (phonon) vertex correction, the behavior when $q\to 0$ first, and $\omega\to 0$ afterward, is entirely different from the behavior when $\omega\to 0$ first, and $q\to 0$ afterward, which was previously calculated by Grabowski and Sham.[12]

The relevance of small q values for the cuprates has also been pointed out previously by Santi *et al.*,[13] Perali and Varelogiannis,[14] Abrikosov,[15] Zeyher and Kulic,[16] Bulut and Scalapino,[17] Tsuei and Kirtley,[18] Leggett,[19] and others. The observation of stripes[20] suggests the relevance of q values close to $q_{stripe}\simeq 0.24$ Å^{-1} (Ref. 21) and we calculate here the vertex correction for q values in this range.[22]

The dielectric constant has a sharp dispersion at a frequency of about 19 meV in YBCO.[6] This frequency is the frequency of phonons involving the displacements of barium atoms in the *c* direction. In LSCO, the dispersion frequency

is about 27 meV, the frequency of phonons involving motion of strontium in the c direction.[7] A theoretical expression for the ionic dielectric constant is given by the Lyddane-Sachs-Teller (LST) theory,[23] namely,

$$\varepsilon(\omega)=\frac{\omega^2-\omega_L^2}{\omega^2-\omega_T^2}\varepsilon_\infty, \tag{3}$$

where ω_L and ω_T are the frequencies of the longitudinal and transverse phonon modes. Although this expression is derived for $q=0$, we assume that it is valid for the small values of q of interest here.

The use of such a simplistic expression, however, raises several further questions.

(a) There are several phonon modes, and there is no *a priori* reason why $\varepsilon(\omega)$ should be dominated by one transverse and one specific longitudinal mode.

(b) We ignore the dispersion of the phonon modes for the relatively small values of q that we consider here (about 1/4–1/3 of the way to the Brillouin zone). This is justified here since there is no softening of the modes, as manifested by the absence of a strong temperature dependence of their frequency.[22]

(c) This expression applies to insulators. The cuprates possess a metallic Fermi surface.

(d) The expression is derived for an isotropic three-dimensional (3D) system.

We feel reasonably confident using Eq. (3) simply because experiment shows that it works quite well. For example, from optical infrared measurements, it is found that the simple LST expression applies extremely well to the c-axis component of ε for LaSrCuO (with $\omega_L=63$ meV),[7] and reasonably well for YBCO (Ref. 6) (there are two modes that serve as ω_L, at 40 meV and 70 meV; we can choose some average in between these two frequencies; about 50 meV gives good agreement, with $\varepsilon_\infty\simeq 4$).

Since the ''metallic'' layer is thin, the width being about $2a_c$, where a_c is the Bohr radius of the oxygen $2p\sigma$ orbitals in the c direction, $a_c\simeq 0.4$ Å, and the average distance between electrons is approximately the lattice constant ($\simeq 4$ Å), the c-axis component of ε screens out the electron-electron interaction effectively, and we can use it (as determined experimentally) in the expression for $\tilde{D}_{ee}$.

The contribution $\Delta\Gamma$, Fig. 1(e), is given at zero temperature by

$$\Delta\Gamma(\underline{k},\underline{q})=\Gamma_{Bardeen}\sum_{\mathbf{p}}\int\frac{dp_0}{2\pi}\tilde{D}_{ee}(\underline{p}-\underline{k})G_0(\underline{p})G_0(\underline{p}+\underline{q}), \tag{4}$$

where $\underline{k}=(\mathbf{k},k_0)$ and we assume a two-dimensional system. Substituting Eq. (3) for $\varepsilon(\omega)$ with $\varepsilon_\infty=1$ in Eq. (2) we have

$$\tilde{D}_{ee}(\underline{k})=\frac{4\pi e^2}{(\mathbf{k}^2+q_D^2)}\frac{k_0^2-\omega_T^2}{k_0^2-\omega_L^2+\imath\delta}. \tag{5}$$

This form is quite plausible but, as mentioned above, has not yet been derived from a truly microscopic theory. G_0 is the usual zero order electron propagator and we assume for now a free particle spectrum. The p_0 integration can be carried out as a contour integration picking up contributions from the poles of the G_0's and $\tilde{D}_{ee}$.

Since Γ is normally part of a larger diagram, the pair interaction, for example, its external momenta and energies are integrated over. Although for such applications we require the full complex $\Delta\Gamma$ as a function of all of its arguments, for present purposes it should suffice for a rough estimate of $\Delta\Gamma$ as a function of $q\equiv|\mathbf{q}|$ to take typical values for the other variables. We also neglect for now the imaginary part of $\Delta\Gamma$. We assume $0<q_0<\omega_L$ and, because the external electron lines usually represent members of a Cooper pair, we take $|\mathbf{k}|=k_F$ and $k_0=0$, measuring energies relative to E_F. Thus we have

$$\Delta\Gamma/\Gamma_{Bardeen}=\frac{1}{2\omega_L}\int\int\frac{dp_x dp_y}{(2\pi)^2}\frac{4\pi e^2}{(\mathbf{p}-\mathbf{k})^2+q_D^2}\times iI(\mathbf{p},q,\mathbf{p}\cdot\mathbf{q};\omega_T,\omega_L), \tag{6}$$

where $\mathbf{k}=k_F\hat{\mathbf{k}}$ and we have taken unit volume and set $\hbar=1$. I consists of six terms resulting from the p_0 integrations, which are displayed in the Appendix. For the p integrations we can set the p_x axis along the direction of $\mathbf{q}$ and then the direction of $\mathbf{k}$ with respect to $\mathbf{q}$ is given by $\hat{k}_x=\cos(\theta_k)$. For the present computation we have taken as typical values $q_D=0.1k_F$, $\hat{k}_x=0.8$, $\omega_L=0.16E_F$, and $\omega_T=0.04E_F$.

For diagram 1(c) the complications due to the ionic corrections are absent, and only two terms result from the p_0 integrations. Then $\Delta\Gamma_{Coulomb}/\Gamma_{Bardeen}$ is given by Eq. (6) without the factor $1/2\omega_L$ and with I replaced by $I_{Coulomb}$ where

$$iI_{Coulomb}=\frac{f(\varepsilon_p)[1-f(\varepsilon_{p+q})]}{\varepsilon_{p+q}-\varepsilon_p-q_0-i\delta}-\frac{f(\varepsilon_{p+q})[1-f(\varepsilon_p)]}{\varepsilon_{p+q}-\varepsilon_p-q_0+i\delta}, \tag{7}$$

and $f(\varepsilon_p)$ is the Fermi function at $T=0$. The principal value integrations over p_x and p_y must be done numerically.

In Fig. 2 we show $\Delta\Gamma/\Gamma_{Bardeen}$ and $\Delta\Gamma_{Coulomb}/\Gamma_{Bardeen}$ as functions of q/k_F for $q_0=0.08E_F$ (solid curves) and $q_0=0.02E_F$ (dashed curve). There are two surprising results here. First, in two dimensions the vertex correction $\Delta\Gamma_{Coulomb}/\Gamma_{Bardeen}$ is already very large and, second, the effect of the ionic screening is drastic: One would naively expect the inclusion of the large ionic dielectric function to greatly reduce the correction. Except very near the sign change just below $q=0.3k_F$, this is *not* the case. A similar sign change was also found by Pietronero *et al.*[8]

The structure in $\Delta\Gamma_{Coulomb}/\Gamma_{Bardeen}$ deserves a brief comment. It arises from the complicated structure of the integrands and does not seem to have any obvious physical relevance. It is due to the first term in Eq. (7), which is singular at $\mathbf{p}=\mathbf{p}_s(q,q_0)$ when $q_0=\varepsilon_{p_s+q}-\varepsilon_{p_s}$, i.e., when the external frequency can excite a particle-hole pair. The integrand of the principal value integration in Eq. (6) then contains the singularity at $\mathbf{p}=\mathbf{p}_s$ and a fixed (for $\mathbf{k}$ fixed) peak arising from the Coulomb factor. As q changes (for fixed $\mathbf{k}$ and q_0), the singularity moves through the Coulomb peak causing alternately positive and negative contributions, producing the structure. This structure does not occur in $\Delta\Gamma/\Gamma_{Bardeen}$ because the integrand on the negative side of

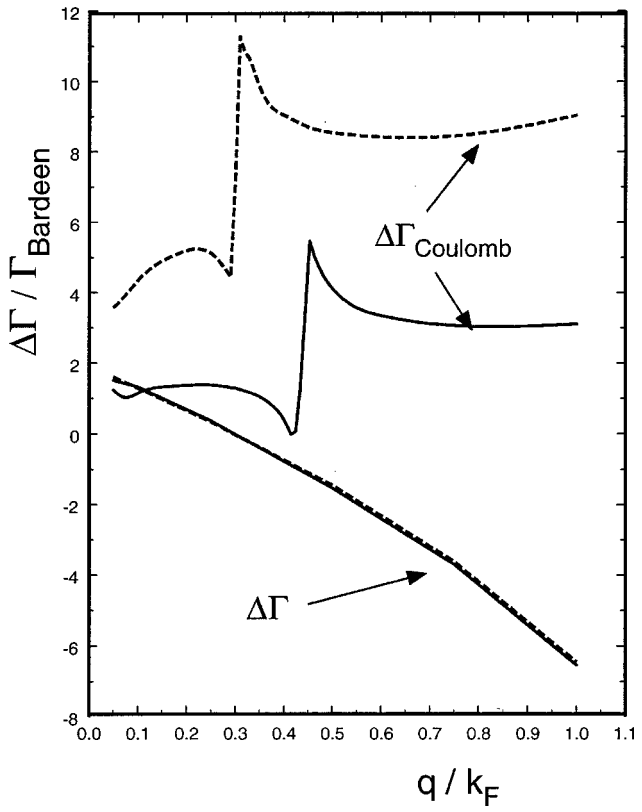

FIG. 2. The vertex corrections $\Delta\Gamma_{Coulomb}/\Gamma_{Bardeen}$ and $\Delta\Gamma/\Gamma_{Bardeen}$ as functions of the external momentum q for external frequency $q_0=0.08E_F$ (solid curves) and $q_0=0.02E_F$ (dashed curves).

the singularity becomes positive a short distance from the singularity and practically no negative contribution occurs.

IV. THE LARGE VALUE OF $\Delta\Gamma/\Gamma_{Bardeen}$

The extremely large value of $\Delta\Gamma/\Gamma_{Bardeen}$ that we find here seems at first sight to be unphysical, since (to the best of our knowledge) it was not considered before. We suggest that it *is* physical. We are assuming here that our lowest order correction is not too strongly reduced by higher order vertex corrections, see discussion below. We believe that the reason why such an effect was not considered previously is that, in a homogeneous system, the dielectric constant ε also enters into the expression for the bare vertex, which becomes $Ze^2|q|/q^2\varepsilon$, and not only into the expression for $\tilde{D}_{ee}$. Our ansatz considers an extremely inhomogeneous system.[21,3,4]

While we consider effects due to phonons, we consider three entirely distinct phonon modes, namely, the mode $\Omega=q_0$, which represents phonons associated with the momentum q of the bare vertex Γ_{Bloch}, Fig. 1(a); and the modes ω_T and ω_L, which dominate the ionic dielectric constant. Since the modes are distinct, they involve motion of different atoms, in different regions of the unit cell. As mentioned previously, the mode Ω could also be a spin fluctuation (or another type of boson). For now we assume it is a phonon involving motion of the planar oxygen; such motion could be longitudinal (along the Cu-O bond) or transverse, in the *a-b* plane or in the *c* direction. The frequency of the transverse motion is around 40 meV. This mode is seen as perhaps a McMillan-Rowell structure in tunneling experiments in $Bi_2Sr_2CaCu_2O_{8+\delta}$.[24] The mode ω_T involves the motion of the Ba (or Sr) atoms in the *c*-direction. The mode ω_L involves motion of the apex oxygen, as well as motion of the planar oxygens. These modes are in the range 15–80 meV; ω_T is in the range 15–30 meV, and ω_L is in the range 40–80 meV. This distinction between the modes is important not only to avoid double counting, but also because it causes the shielding of the electron-electron interaction D_{ee} by the ionic dielectric constant to be *entirely different* from the shielding of the electron-Ω phonon interaction (by the ionic dielectric constant, associated with the modes ω_T,ω_L). We estimated this last shielding and found that it is small.[3] Therefore we do not introduce it into the present calculation.

While the experimental determination of $\varepsilon(\omega)$ by the IR measurements[6,7] is definitive, it is instructive to consider possible microscopic causes for the anomalously large value of ε. We believe that the large ε may be related to a new degeneracy between the Zhang-Rice singlet and the anti-Jahn-Teller triplet of the CuO_5 complex. Kamimura and Sano[25] calculated the splitting between the singlet and triplet as a function of the occupation of the $3d$ shell of the *chain* copper, and found that for 0.55 holes there, these states are degenerate. This calculation is carried out using a quantum-chemistry algorithm. We can characterize this splitting by an effective Hubbard U, and write $U_{eff}=U_{bare}/\varepsilon\cdot U_{bare}$ is the Hubbard U in a ''normal'' complex, i.e., several electron volts. Thus the splitting of about one-tenth of an eV calculated by Kamimura and Sano indicates a value of ε of about 50. Also, this calculation shows clearly the *local* nature of this large dielectric constant.

Anisimov *et al.*[26] found that the singlet and triplet are close, by a rigorous local denisity approximation calculation. They also considered relaxation of the complex, i.e., different Cu-O distances in the singlet and triplet states. They found a rather large relaxation, substantiating the ionic nature of the large dielectric constant. Polinger *et al.*[27] found a near degeneracy of the singlet and triplet states, near the Sr in LSCO by extended x-ray-absorption fine-structure measurements. Gatt[28] carried out a quantum-chemistry calculation, and showed the important role of the *c*-axis motion of the alkaline-earth atom. All these calculations show that the ε's that describe the shielding of the electron-electron interaction D_{ee} and the electron-planar oxygen interaction Γ_{Bloch} are of an entirely different nature, and only the former one is greatly enhanced. Therefore the very large difference in their values is not ''unphysical.''

Since the diagram, Fig. 1(e), yields such a large result one cannot rule out that further vertex corrections may also be important. In fact, the natural class of diagrams to include is the sum of ''ladder'' diagrams. Such diagrams represent nothing more than the spin susceptibility, or spin fluctuation propagator, and describe the (antiferromagnetic) spin correlations thought to be particularly important in the cuprate superconductors. Previous calculations,[29,30] which considered the effect of these contributions on the phonon frequencies and on the screening of the electron-phonon vertex in nearly antiferromagnetic systems, could be modified to include the ionic screening considered here. Thus the nearness of both a ferroelectric as well as an antiferromagnetic transition could perhaps be accounted for.

Kim[30] first considered effects of the proximity to a ferromagnetic or an antiferromagnetic transition, and showed that this can enhance the electron-phonon coupling constant. The enhancement arises both from softening of the phonon, and from an increase of the matrix element $\langle k|\nabla V|k+q\rangle$. The first effect does not concern us here (it does not lead to an increase in T_c in the strong-coupling case). The second effect bears some similarity to the effect that we consider here, except that we consider primarily the nearness to a ferroelectric transition.

Kim considers the role of a sum rule that restricts the magnitude of his effect. This sum rule was also considered by McMillan[31] and originally proved by Heine, Nozieres, and Wilkins.[32] It states that $V(0) = Z/N(E_F)$. This rule applies to a *homogeneous* system. In an inhomogeneous system, it breaks down. Its origin is that when $\tilde{D}_{ee}$ is reduced from D_{ee} by ε, the bare vertex Γ is also reduced by ε [Fig. 1(b)] and these two effects cancel each other when $q=0$. [When $q>0$, a large ε actually diminishes $V(q)$.]

In an inhomogeneous system, the reduction of D_{ee} and the reduction of Γ are given by two different dielectric constants. We showed[21,3,4] that the ε for Γ is close to one, while the ε for D_{ee} is large. This follows from the *local* nature of ε.[33,25,4]

We note in passing that the relevant values of q are on the order of 0.25 Å^{-1}.[21] This is small enough that the use of the ε derived for $q=0$ is justified. On the other hand, the screening length q_D^{-1} is about 4 Å, which is small enough so the dielectric constant is local.

V. BARDEEN'S VERTEX FOR AN IONICALLY SCREENED MEDIUM

Bardeen calculated the sum over bubble diagrams [Fig. 1(b)]. The correction due to the ''bubble'' vertex is very large, and negative; i.e., for $\omega=0$, the sum over the bubble diagrams reduces the vertex from the Bloch value $4\pi Ze^2/q^2$ by a factor of $1/[1+D_{ee}N(E_F)]\approx 1/[1+(q_D/q)^2]$, as in Eq. (1). For $q\approx k_F$, this factor is typically about 1/3, and for small q values, it is even smaller. This reduction accounts for the resistivity of monovalent metals being an order smaller than the value calculated assuming the Bloch value of the vertex. Now, to include the ionic screening, we should replace D_{ee} in Fig. 1(b) by $\tilde{D}_{ee}$. Here we must set $q_D=0$ in Eq. (2) to avoid double counting the electronic screening. Since $\tilde{D}_{ee}$ is so much smaller (at low frequencies) than D_{ee}, the vertex correction is much smaller, and when $\tilde{D}_{ee}$ can be neglected, the vertex becomes the Bloch vertex; i.e., it is considerably *larger*. Thus a very large ionic dielectric constant restores the value of the vertex to the large original Bloch value at low frequencies ($\omega<\omega_T\approx 20$ meV). To illustrate this effect we consider $\Gamma_{Bardeen}$ replaced with $\tilde{\Gamma}_{Bardeen}$, where

$$\tilde{\Gamma}_{Bardeen}=\frac{\Gamma_{Bloch}}{\left[1+\dfrac{D_{ee}N(E_F)}{\varepsilon(0)}\right]}\approx\frac{\Gamma_{Bloch}}{\left[1+\dfrac{q_D^2}{q^2\varepsilon(0)}\right]}. \qquad (8)$$

In Fig. 3 we show $\tilde{\Gamma}_{Bardeen}/\Gamma_{Bloch}$ and $\Gamma_{Bardeen}/\Gamma_{Bloch}$ vs q/k_F for $\varepsilon(0)=40$ and $q_D=k_F/2$. It is clearly seen that, with increasing q, $\tilde{\Gamma}_{Bardeen}$ approaches Γ_{Bloch} much faster than $\Gamma_{Bardeen}$ does. At the q values of interest here, the corrected vertex is quite close to the original Bloch vertex, e.g., for $q=0.2k_F$ we have $\tilde{\Gamma}_{Bardeen}\approx 0.9\Gamma_{Bloch}$.

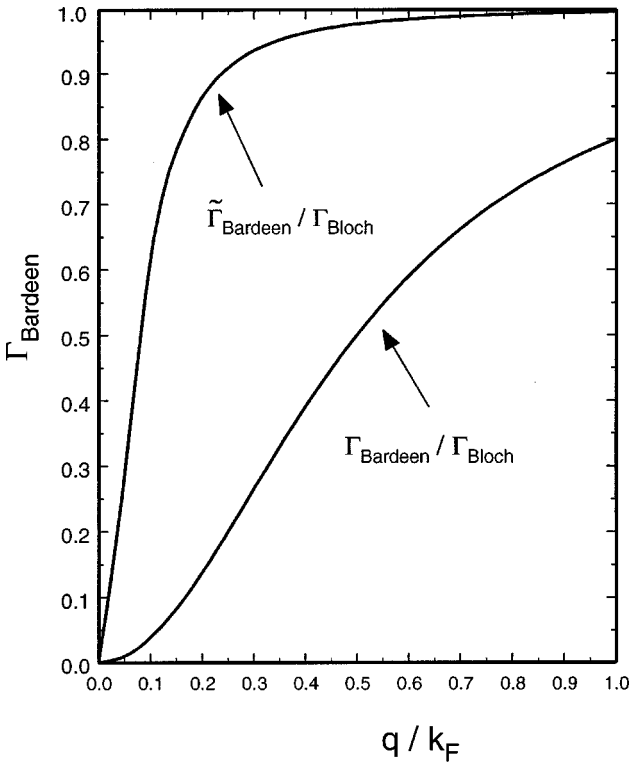

FIG. 3. The electronically screened vertex with ($\tilde{\Gamma}_{Bardeen}$) and without ($\Gamma_{Bardeen}$) ionic screening.

VI. CONCLUSIONS

We have shown that the lowest order vertex corrections including ionic dielectric effects are large. In particular, we show that the electron-phonon vertex for small q and small ω is greatly enhanced over the ''standard'' Bardeen value. Indeed, surprisingly, the Coulomb vertex correction, $\Delta\Gamma_{Coulomb}/\Gamma_{Bardeen}$, is already large. This may be related to the 2D nature of the system. We are, however, not aware of any explicit calculations of this quantity either in two or in three dimensions. It has apparently previously been considered phenomenologically to be included in the bare vertex. We have also shown that the inclusion of ionic effects also greatly affects the electronic screening. These corrections all tend to significantly increase the maximum superconducting T_c value. This may require us to reconsider the possibility of the high T_c as being due, at least partially, to a phonon-mediated interaction. At the current level of approximation, however, one cannot make a definite statement in this respect.

We point out that we have employed a ''hybrid'' formalism in which a 3D ε is combined with $2d$ band structure and 2D integrations. This is not done to make the calculations easier but to have a consistent picture in accordance with experiment. We do not believe that a pure 2D or a pure 3D scenario is physically correct in the high-T_c superconductors.

There are still a number of things to be done before a truly quantitative theory is obtained. We mentioned briefly in Sec. IV that the large contribution of the lowest order vertex correction indicates that further vertex corrections may be important, in particular the particle-hole ladder diagrams that contribute to the spin susceptibility. At the least, one must consistently include such diagrams in the bubbles of the electronic screening diagrams as well as directly as vertex corrections. If the higher order vertex corrections can be summed to infinite order the result could turn out to be smaller than the large lowest order contribution. In any case, this difficult task must be carried out before quantitative comparison with experiment can be attempted. In addition, a better theory must go beyond the present level of approximation in other respects, for example, the momentum dependence of $\Gamma_{Bardeen}$ should also be included. More fundamentally, there is at present no microscopic, field theoretic derivation of the Lyddane-Sachs-Teller expression for ε in Eq. (5), and in general it does not hold for a polyatomic unit cell. However, experimentally it applies extremely well for LSCO (Ref. 7) and reasonably well for YBCO.[6]

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APPENDIX

We display in detail the integrand of Eq. (6):

$$
\begin{aligned}
iI = & \frac{(\omega_L^2-\omega_T^2) f(\varepsilon_p) f(\varepsilon_{p+q})}{(\omega_L-\varepsilon_p)(\omega_L-\varepsilon_{p+q}+q_0)} \\
& + \frac{(\omega_L^2-\omega_T^2)[1-f(\varepsilon_p)][1-f(\varepsilon_{p+q})]}{(\omega_L+\varepsilon_p)(\omega_L+\varepsilon_{p+q}-q_0)} \\
& + \frac{(\varepsilon_p^2-\omega_T^2) f(\varepsilon_p)[1-f(\varepsilon_{p+q})]}{(\varepsilon_p-\omega_L)(\varepsilon_{p+q}-\varepsilon_p-q_0-i\delta)} \\
& + \frac{(\varepsilon_p^2-\omega_T^2)[1-f(\varepsilon_p)] f(\varepsilon_{p+q})}{(\varepsilon_p+\omega_L)(\varepsilon_{p+q}-\varepsilon_p-q_0)} \\
& - \frac{[(\varepsilon_{p+q}-q_0)^2-\omega_T^2][1-f(\varepsilon_p)] f(\varepsilon_{p+q})}{(\varepsilon_{p+q}-q_0-\omega_L)(\varepsilon_{p+q}-\varepsilon_p-q_0)} \\
& - \frac{[(\varepsilon_{p+q}-q_0)^2-\omega_T^2] f(\varepsilon_p)[1-f(\varepsilon_{p+q})]}{(\varepsilon_{p+q}-q_0+\omega_L)(\varepsilon_{p+q}-\varepsilon_p-q_0-i\delta)}.
\end{aligned}
$$

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