T_c for non-s-wave pairing superconductors correlated with coherence length and effective mass

G. G. N. Angilella,[1] N. H. March,[2,3] and R. Pucci[1]
[1]*Dipartimento di Fisica e Astronomia, Università degli Studi di Catania, and Istituto Nazionale per la Fisica della Materia, Sezione di Catania, Corso Italia, 57, I-95129 Catania, Italy*
[2]*Oxford University, Oxford, England*
[3]*Department of Physics, University of Antwerp (RUCA), Antwerp, Belgium*

(Received 7 July 2000)

For unconventional heavy fermion superconductors, typified by UBe_{13}, the superconducting transition temperatures T_c are shown to correlate with a characteristic energy $\hbar^2/(m^*\xi^2)$, m^* being the effective mass, and ξ the coherence length. For four of the six materials for which T_c, m^*, and ξ are available, $k_B T_c \sim 20\hbar^2/(m^*\xi^2)$. One heavy fermion material, $\mathrm{UPd}_2\mathrm{Al}_3$, reveals a tendency for the above linear behavior to saturate at substantially larger $\hbar^2/(m^*\xi^2)$ than for UBe_{13}. The sixth material considered, $\mathrm{URu}_2\mathrm{Si}_2$, falls between UBe_{13} and $\mathrm{UPd}_2\mathrm{Al}_3$. To embrace d-wave pairing in cuprates, a log-log plot reveals that $k_B T_c \sim \hbar^2/(m^*\xi^2)$, but more materials for which m^* and ξ are measured will be required to substantiate the correlation in these high-T_c substances.

Although heavy fermion superconductors were discovered more than a decade and a half ago,[1,2] interest in their physical properties remains considerable. Thus, in a very recent study,[3] the differential conductivity of a UBe_{13}-Au junction has been measured in both superconducting and normal states, yielding in particular an energy gap Δ in this unconventional superconductor very different quantitatively from a BCS relation which at $T=0$ reads $2\Delta/k_B T_c = 3.5$. This figure is ''enhanced'' to around 7 from this experiment on UBe_{13}. We shall return briefly, at the end of this report, to this matter of the energy gap in this non-s-wave heavy fermion superconductor. However, the main focus of the present investigation is to address the question of whether there is a ''natural'' energy scale on which to measure $k_B T_c$ in non-s-wave superconductors. Then, leaving aside $l(l+1)$ in the eigenvalues of $L^2/\hbar^2$, with L the orbital angular momentum of a superconducting Cooper pair, a characteristic energy ϵ_c would appear to be

$$\epsilon_c = \frac{\hbar^2}{m^* l_c^2}, \tag{1}$$

where m^* is the effective mass, while l_c is a characteristic length that remains to be chosen. That m^* should enter inversely in determining the scale of $k_B T_c$ was clearly recognized in the study of Uemura *et al.*,[4] who did not, however, address the question of the length l_c below T_c. In the superconducting state of the heavy fermion materials which we first focus on below, it seemed to us that the natural physical choice was to take for l_c in Eq. (1) the coherence length ξ. We have then found in the available literature simultaneously data on T_c, m^*, and ξ for the six heavy fermion systems listed in Table I (see Ref. 5). These are the data we have therefore used to construct Fig. 1, in which $k_B T_c$ has been plotted against the ''independent variable'' $\hbar^2/(m^*\xi^2)$ from Eq. (1) with $l_c = \xi$. That there is a marked correlation between these two energies (both measured in meV in Fig. 1) is clear. The dashed curve, though mainly plotted as a guide to the eye, is represented over the range shown by the power series in ϵ_c

$$k_B T_c = b\epsilon_c(1 + c_1\epsilon_c + c_2\epsilon_c^2) + \mathcal{O}(\epsilon_c^3), \tag{2}$$

where $b \simeq 22$. For the three materials shown in Fig. 1 with the lowest T_c values, $k_B T_c/[\hbar^2/(m^*\xi^2)] \simeq 20$, as follows from the first term in the fitting series Eq. (2).

Below we shall briefly compare and contrast this linear behavior at low T_c with that for the d-wave pairing in the cuprates. However, the further point to be stressed is that the material $\mathrm{UPd}_2\mathrm{Al}_3$, though having somewhat different coherence lengths in different crystal directions, shows a clear tendency of the (assumed) relation

$$k_B T_c = f_{\mathrm{HF}}\left(\frac{\hbar^2}{m^*\xi^2}\right) \tag{3}$$

to go from the linear form $\sim 20\hbar^2/(m^*\xi^2)$ at small argument to $f_{\mathrm{HF}} \to$ const in these heavy fermion (HF) materials as the independent variable ϵ_c is increased by a factor of about 5 from UBe_{13} to $\mathrm{UPd}_2\mathrm{Al}_3$.

TABLE I. Selected physical properties for six heavy fermion materials [from Heffner and Norman (Ref. 5)].

	UPt_3	UBe_{13}	$\mathrm{UNi}_2\mathrm{Al}_3$	$\mathrm{UPd}_2\mathrm{Al}_3$	$\mathrm{URu}_2\mathrm{Si}_2$	$\mathrm{CeCu}_2\mathrm{Si}_2$
T_c [K]	0.55	0.9	1.0	2.0	1.2	0.7
ξ [Å]	$100(\parallel ab)$ $120(\parallel c)$	100	240	85	$100(\parallel ab)$ $150(\parallel c)$	90
m^*/m_e	180	260	48	66	140	380

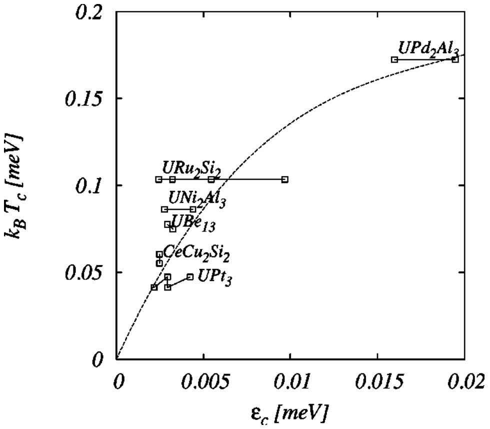

FIG. 1. Thermal energy $k_B T_c$ corresponding to superconducting transition T_c versus characteristic energy $\epsilon_c = \hbar^2/(m^* \xi^2)$, with m^* the effective mass and ξ the coherence length (see also Table I). Six heavy fermion materials are considered. The fitted (dashed) curve is represented in Eq. (2) and should be regarded mainly as a guide to the eye.

In light of the above findings for the unconventional heavy fermion superconductors considered in Table I and Fig. 1, it seemed of obvious interest to compare and contrast their behavior with corresponding results for the high-T_c cuprates, known also to have non-s-wave pairing. But then the difficulty comes up that for only very few cuprates are data simultaneously available on the same materials for T_c, m^*, and ξ, entering the correlation proposed in this report.

Nevertheless, in spite of the sparseness of the data, we felt it of obvious interest to show in Fig. 2 a log-log plot in which $k_B T_c$ is again displayed versus $\hbar^2/(m^* \xi^2)$. The plot is, to our mind, sufficiently encouraging to warrant further work in measuring both m^* and ξ in other high-T_c cuprates. The striking difference from the heavy fermion cases is that now, accepting the wide spread of data, $k_B T_c \sim \hbar^2/(m^* \xi^2)$, which is, roughly speaking, one order of magnitude different from the linear limit of Eq. (2) for the heavy fermion materials.

To return briefly to the new measurements of Wälti *et al.*[3] on the energy gap $\Delta(T)$ in UBe$_{13}$, we have plotted their data in somewhat unorthodox form in Fig. 3, with T/T_c on the ordinate, and $\Delta/\Delta(0)$ on the abscissa. Furthermore we have, admittedly with some small degree of arbitrariness, extrapolated the measured data to pass through twice the BCS value. What we wish to emphasize, in the present context of non-s-wave pairing superconductors, is that the renormalized BCS curve near $T=0$ can be referred to as a ‘‘gapped’’ phase, whereas the experimental curve shows excitations (gapless as well as gapped) characteristic of non-s-wave pairing. We expect, near $T=T_c$, that the difference between the BCS and the UBe$_{13}$ curves will reflect in general terms the specific heat low-temperature behavior in the normal state of UBe$_{13}$, namely,

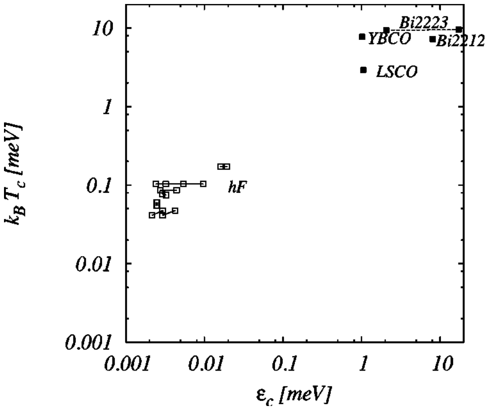

FIG. 2. log-log plot of Fig. 1, but now with high-T_c cuprates in the top right-hand corner [data taken from Poole *et al.* (Ref. 7)], in addition to heavy fermion data of Fig. 1 (here collectively marked by HF). For the cuprates, use has been made of the coherence length ξ_{ab}.

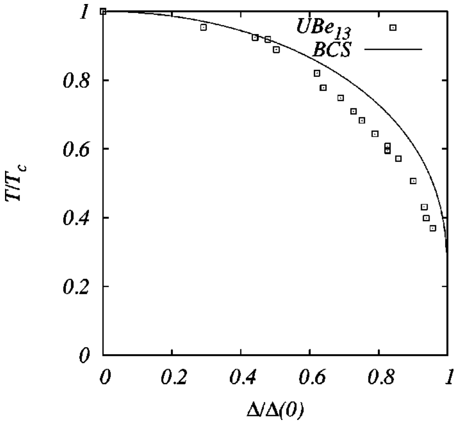

FIG. 3. T/T_c (on ordinate) vs $\Delta(T)/\Delta(0)$. Points are experimental data for UBe$_{13}$ [from Wälti *et al.* (Ref. 3)], with $\Delta(0)$ extrapolated to $\sim 7 k_B T_c$, while the solid line is BCS curve.

$$C_V = \gamma T + B T^3, \tag{4}$$

but it would take us well beyond the scope of the present study to attempt further, quantitative analysis on this issue.

In summary, motivated by recent continuing interest[3] in heavy fermion materials such as UBe$_{13}$ and related compounds,[6] we have reopened the question as to whether there is a ‘‘natural’’ energy scale on which to measure $k_B T_c$. For the heavy fermion cases with the lowest transition temperatures, we have presented evidence that $k_B T_c \sim 20\hbar^2/(m^* \xi^2)$ in these non-s-wave pairing superconductors. However, over a wider range of $\hbar^2/(m^* \xi^2)$ for these materials, the form Eq. (3) has been proposed, where $f_{\mathrm{HF}}(x) \rightarrow$ const for values of x substantially larger than the value for, say, UBe$_{13}$—a situation which occurs in fact for UPd$_2$Al$_3$. Figure 2 shows the non-s-wave cuprates on the same diagram as the heavy fermion superconductors, and now, roughly speaking, we are in a regime where $k_B T_c$ and $\hbar^2/(m^* \xi^2)$ are the same to better than order of magnitude.

One of us (N.H.M.) made his contribution to the present Report during a visit to the Physics Department, University of Catania, in the year 2000. Thanks are due to the Department for the stimulating environment and for much hospitality. N.H.M. also wishes to thank Professor V. E. Van Doren for his continuous interest and support. G.G.N.A. acknowledges support from the EU through the FSE program.

[1] D.L. Cox and A. Zawadowski, Adv. Phys. **47**, 599 (1998).

[2] J.A. Sauls, Adv. Phys. **43**, 113 (1994).

[3] Ch. Wälti, H.R. Ott, Z. Fisk, and J.L. Smith, Phys. Rev. Lett. **84**, 5616 (2000).

[4] Y.J. Uemura, L.P. Le, G.M. Luke, B.J. Sternlieb, W.D. Wu, J.H. Brewer, T.M. Riseman, C.L. Seaman, M.B. Maple, M. Ishikawa, D.G. Hinks, J.D. Jorgensen, G. Saito, and H. Yamochi, Phys. Rev. Lett. **66**, 2665 (1991).

[5] R.H. Heffner and M.R. Norman, Comments Condens. Matter Phys. **17**, 361 (1996).See also Zb. Koziol, Ph.D. thesis, University of Amsterdam, The Netherlands, 1994.

[6] V. Martisovits, G. Zaránd, and D.L. Cox, Phys. Rev. Lett. **84**, 5872 (2000).

[7] Ch. P. Poole, H.A. Farach, and R.J. Creswick, *Superconductivity* (Academic Press, San Diego, 1995).