Possible composite-fermion liquid as a crossover from Wigner crystal to bubble phase in higher Landau levels

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The ground-state cohesive energy per electron of the composite fermion (CF) Fermi sea, the Laughlin state, and the charge density wave (CDW) at higher Landau levels (LL's) are computed. It is shown that whereas for $n \geqslant 2$ LL, the CDW state is generally more energetically preferable than those of the CF liquid and the Laughlin liquid, the $\nu = 4 + 1/6$ CF liquid state unexpectedly has lower ground-state energy than that of the CDW state. We suggest this CF liquid between the Wigner crystal and the bubble phase may lead to the crossover from the normal integer quantum Hall liquid to the re-entrant integer quantum Hall state observed in the recent magnetotransport experiments.

While fractional quantum Hall effects (FQHE) are impressive for their odd-denominator fillings, the remarkable phenomena for even-denominator fillings have caused great interest in the last decade.[1] By using the concept of composite fermion (CF),[2,3] Halperin *et al.* suggested that the $\nu = 1/2$ system can be viewed as a spin-polarized Fermi liquid of CF.[4] The only incompressible state at even-denominator filling ($\nu = 5/2$) in the single-layer two-dimensional electron gas is now widely accepted as the pairing of CF in a vanishing effective magnetic field.[5–7] For higher Landau levels (LL's), recent magnetotransport experiments on high-mobility samples in GaAs/Al_xGa_{1-x}As heterostructures revealed new classes of correlated many-electron states.[8] The most prominent findings are the discoveries of the giant anisotropy in the resistivity near-half-filling of the topmost LL (Refs. 9 and 10) and the observation of re-entrant integral quantum Hall (RIQH) states in the flanks of these same levels,[11] as well as the substantial nonlinearity of the resistivity. It is considered that the highly anisotropic transport is related to the formation of the unidirectional charge density wave (UCDW) state, i.e., the stripe phase.[12,13] Specifically, the state of this system may be classified by its symmetries, which are highly analogous to those of liquid crystals.[14] The possible states include stripe crystals,[15] smectic, and nematic phases.[16–19] The RIQH effect was thought to be the depinning and sliding of the Wigner crystal (WC) and reformation of the bubble phase. But no further discussions were given on the details of this transition.

In the present work, we carry out systematic calculations of the cohesive energies of various competitive ground states in the lowest, as well as higher, LL's. We find that while in the lowest and second LL's, the CF and the Laughlin liquids prevail for $\nu_n < 1/7$ (ν_n is the filling factor at the nth LL), the charge density wave (CDW) generally has a lower ground energy than those of the corresponding CF or Laughlin liquid for LL indices $n \geqslant 2$. In particular, the stripe phase dominates around $\nu_n = 1/2$ ($n \geqslant 2$), which is in agreement with previous calculations.[20–22] However, we find an interesting exception occurs at $\nu_2 = 1/6$, where the CF liquid has a lower cohesive energy than that of the CDW phase. This result may lead to an important experimental phenomenon. Our calculation shows that the CDW has one electron in each bubble (the WC phase) for $\nu_2 < 1/6$, while it has two electrons for $\nu_2 > 1/6$ (the bubble phase). We relate this result to the recent observed phenomenon of the RIQH, i.e., the metallic CF liquid phase may appear as an intermediate state between the WC phase and the bubble phase. We suggest that the WC first melts into the CF liquid and then the system recrystalizes into the bubble phase as the filling at the topmost LL increases.

The fractional quantum Hall states at higher LL's were first suggested by MacDonald and co-worker[23] by raising the Laughlin wave function to higher LL's. The state wave function at the nth LL is defined as follows:

$$|\Psi_L^n\rangle = \prod_i \frac{(a_i^\dagger)^n}{\sqrt{n!}} |\Psi_L^0\rangle. \tag{1}$$

Here, $a_i^\dagger$ is the inter-LL ladder operator, promoting the ith electron to the next LL, and $|\Psi_L^0\rangle$ is the Laughlin state in the lowest LL. It should be noted that this state in the higher LL is in fact not the realistic electronic state because the lower LL's are empty. The effect of electrons in the lower-filled LL's was treated by Aleiner and Glazman[24] and Wang[25] by integrating out the electron degrees of freedom in the lower LL's, which leads to a renormalization of the Coulomb interaction between electrons at the topmost LL.

The correlation energy per electron can be calculated using the density-density correlation function in the higher LL:

$$h_n(\mathbf{r}) \equiv \frac{\langle \rho_n(\mathbf{r})\rho_n(0)\rangle - \langle \rho_n\rangle^2}{\langle \rho_n\rangle}, \tag{2}$$

where $\rho_n(\mathbf{r})$ is the projection of the density operator onto the nth LL. This can be most effectively done in the Fourier space because $h_n(q)$ is very simply related to $h_0(q)$ (the correlation for $n=0$):[26]

$$h_n(q) = h_0(q)\left[L_n\left(\frac{q^2 l_0^2}{2}\right)\right]^2, \tag{3}$$

where $L_n(q^2 l_0^2/2)$ is the Laguerre polynomial. The correlation energy can be written as

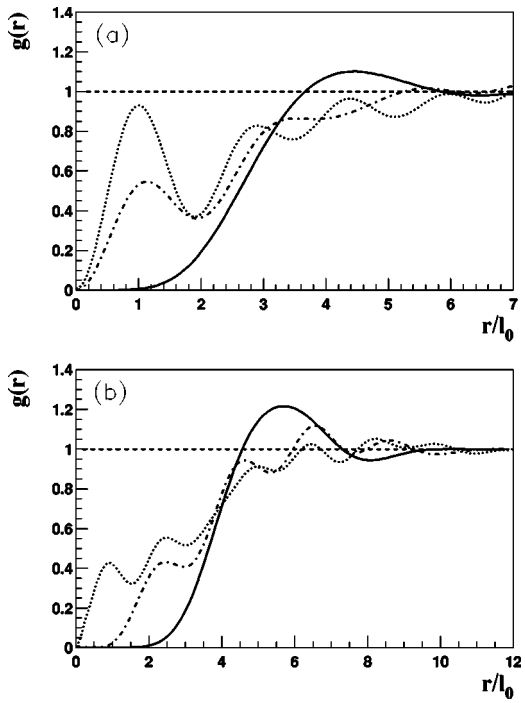

FIG. 1. The pair-distribution functions of Laughlin states at $n=0$ (the solid line), $n=1$ (the dash-dotted line), and $n=2$ (the dotted line). (a) $\nu_n=1/3$. (b) $\nu_n=1/5$.

$$E_{cor}=\frac{1}{2}\int \frac{d^2q}{(2\pi)^2}\frac{v(q)}{\varepsilon(q)}h_n(q). \tag{4}$$

The cohesive energy per electron is then, according to the definition

$$E_{coh}=E_{cor}-E^{UEL}, \tag{5}$$

where

$$E^{UEL}=-\frac{\nu_n}{2}\int dq\frac{e^2}{\kappa l_0}\frac{F_{nn}^2(q)}{\varepsilon(q)} \tag{6}$$

is the interaction energy per particle in the uncorrelated electron liquid formed at high temperature. Here, $F_{nn}(q)=L_n(q^2l_0^2/2)e^{-q^2l_0^2/4}$ and $\epsilon(q)$ is the dielectric function that accounts for the screenings of the Coulomb interaction between electrons at the topmost LL by electrons in the lower LL's. We make use of the analytical expressions of the pair-distribution functions for $\nu=1/3$ and 1/5 in the lowest LL obtained earlier by MacDonald and co-worker[23] We reproduce the pair-distribution functions of the Laughlin states for $\nu_n=1/3$ and 1/5 in the $n=0,1,2$ LL's (Fig. 1). It shows that the electrons have a larger probability to approach each other in higher LL's than in lower LL's.

The trial wave function for the CF liquid ground state at higher LL can also be obtained in the same way described above[27] if the Rezayi-Read wave functions for the CF at the lowest LL are considered.[28] We consider a system of N electrons in the external magnetic field. Through attaching $2p$ (p integer) flux quanta to each electron, one constructs the CF that experiences a vanishing effective magnetic field at even-denominator fillings. The interaction energy of electrons is transmuted into the kinetic energy of CF's. To compute the pair-distribution function of the CF liquid, we work on a spherical geometry in which the N electrons move on the two-dimensional surface of a sphere under the influence of a radial magnetic field B originating from a magnetic monopole of strength Q at the center, which corresponds to a total flux of $2Q\phi_0$, where $\phi_0=hc/e$ is the flux quantum. We consider a filled shell system with $N=n^2$, because the ground state is a single Slater determinant, which makes computations easier. Employing the projection scheme proposed by Jain and Kamilla,[29,3] we compute the pair-distribution function of CF's for $N=25$ electrons in the lowest LL by using the standard Metropolis Monte Carlo method. We carry out 5×10^6 steps and the results are shown in Fig. 2. Although the CF's hardly interact with each other in the Fermi sea state, the electrons carrying more flux avoid each other more steadily. The cohesive energies of CF liquids E_{coh}^{CF} and Laughlin states E_{coh}^{L} for $\nu_n=1/2p(p=1, 2, 3,$ and 4) are listed in Tables I–IV.

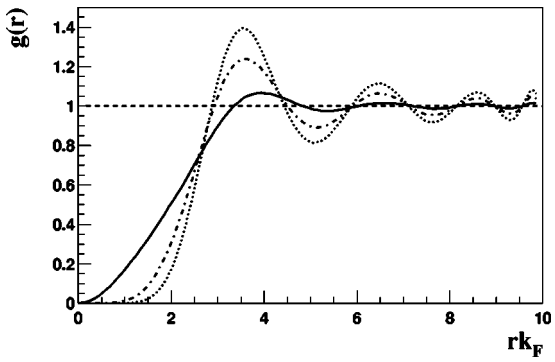

FIG. 2. The pair-distribution functions of CF liquid states in the lowest LL at $\nu=1/2$ (the solid line), $\nu=1/4$ (the dash-dotted line), and $\nu=1/6$ (the dotted).

Due to the screenings by the lower LL's, the repulsive interaction between any two electrons at the topmost LL as a function of the separation between the guiding centers of their orbits abruptly drops at the distance of two cyclotron radii. Such a ''boxlike'' component in the interaction potential makes the uniform distribution of the electron density at the topmost LL unstable. The translational symmetry is spontaneously broken, and crystalline domains with filling factor equal to one and zero are formed.[20] The cohesive energy of the CDW state in the Hartree-Fock approximation is expressed as follows:

TABLE I. The cohesive energy per electron for $n=0$ LL. The last column are energies for Laughlin states (odd-denominator fillings) or CF liquids (even-denominator fillings), respectively. The energy is in units of $e^2/\kappa l_0$.

ν_n	E_{coh}^{B}	E_{coh}^{UCDW}	E_{coh}^{L}/E_{coh}^{CF}
1/2	−0.1320	−0.1120	−0.1551
1/3	−0.1796	−0.1469	−0.2005
1/4	−0.1945	−0.1574	−0.2040
1/5	−0.1967	−0.1584	−0.2018
1/6	−0.1941	−0.1547	−0.1971
1/8	−0.1850	−0.1466	−0.1857

TABLE II. The same as Table I for $n=1$ LL.

ν_n	E^B_{coh}	E^{UCDW}_{coh}	E^L_{coh}/E^{CF}_{coh}
1/2	−0.0721	−0.0687	−0.0768
1/3	−0.0996	−0.0918	−0.1062
1/4	−0.1145	−0.1014	−0.1169
1/5	−0.1229	−0.1055	−0.1256
1/6	−0.1272	−0.1069	−0.1283
1/8	−0.1295	−0.1063	−0.1288

$$E_{coh}=\frac{n_L}{2\nu_n}\sum_{\mathbf{q}\neq 0} u_{HF}(q)|\Delta(\mathbf{q})|^2 \tag{7}$$

where $n_L=1/2\pi l_0^2$ is the density of one completely filled LL, and $\Delta(q)$ is the order parameter of the CDW.

For the bubble state, in the limit of a weak magnetic field, a simple quasiclassical picture can be given. In this case, electrons can be viewed as classical particles rotating in cyclotron orbits. It is shown that the optimum number of electrons in a bubble is $\tilde{M}\simeq 3n\nu_n$, which corresponds to the separation $\simeq 3R_c$ between nearest bubbles. However, to calculate the cohesive energy accurately, one cannot use the quasiclassical approach and the CDW should be defined more precisely. According to Fogler and Koulakov,[26] it can be shown that for the bubble states

$$\Delta(q)\approx 2\nu_n A\frac{J_1(ql_0\sqrt{2M})}{ql_0\sqrt{2M}}, \tag{8}$$

which is just the Fourier transform of a uniform disk with a radius $l_0\sqrt{2M}$. Here the asymptotic formula for the Laguerre polynomials for $q\ll\sqrt{M}/l_0$ is used. The cohesive energy of the bubble state can be calculated in the same way as it has been done for the Wigner crystal. The results are different for each M. Therefore, one has to find the most optimum $\tilde{M}$ corresponding to the lowest energy. The final results for the stripe phase E^{UCDW}_{coh} and the bubble phase E^B_{coh} are also listed in Tables I–IV for comparison.

The cohesive energies of the ground states for the Laughlin liquid, the CF liquid, and the CDW are plotted in Fig. 3. It can be seen that in the lowest LL, the Laughlin liquids and the CF liquids have lower ground energies than those of corresponding CDW states for $\nu<1/7$. In the second LL, an incompressible pairing state of CF is considered to be more energetically preferable at $\nu_1=1/2$. The energy gap disappears when an in-plane magnetic field is applied to the system.[30,31] It was long interpreted as indicating the existence of substantial spin reversal in the ground state.[30] The early tilt-field experiments, however, missed an important point. In addition to suppressing the $\nu=5/2$ FQHE state, the tilted field leaves the transport in the $n=1$ LL highly anisotropic. Recently, the present authors demonstrated that the pairing of CF's is destroyed by the in-plane field and finally transforms to the UCDW.[32] On the other hand, our calculations show that a CF liquid may exist at $\nu_1=1/4$ and 1/6, in agreement with Morf and d'Ambrumenil.[27]

TABLE III. The same as Table I for $n=2$ LL.

ν_n	E^B_{coh}	E^{UCDW}_{coh}	E^L_{coh}/E^{CF}_{coh}
1/2	−0.0448	−0.0456	−0.0387
1/3	−0.0635	−0.0598	−0.0603
1/4	−0.0713	−0.0659	−0.0681
1/5	−0.0762	−0.0690	−0.0764
1/6	−0.0798	−0.0707	−0.0802
1/8	−0.0834	−0.0719	−0.0829

TABLE IV. The same as Table I for $n=3$ LL.

ν_n	E^B_{coh}	E^{UCDW}_{coh}	E^L_{coh}/E^{CF}_{coh}
1/2	−0.0318	−0.0329	−0.0290
1/3	−0.0441	−0.0435	−0.0420
1/4	−0.0509	−0.0480	−0.0466
1/5	−0.0531	−0.0502	−0.0490
1/6	−0.0560	−0.0514	−0.0530
1/8	−0.0593	−0.0523	−0.0587

For the $n\geq 2$ LL, the CDW states are generally favorable. The Laughlin liquid and the CF liquid are ruled out in the region of $\nu_n<1/6$, and specifically, the stripe phase dominates around $\nu_n=1/2$. This result agrees with earlier works using the Hartree-Fock method and recent numerical studies carried out by Rezayi, Haldane, and Yang.[22] However, for $\nu_2=1/6$, the CF liquid state is unexpectedly lower in energy than the CDW state. There is one electron in each bubble for $\nu_2<1/6$ (the WC phase) and two electrons for $\nu_2>1/6$ (the bubble phase). This means that a possible metallic phase may exist in the crossover from the WC to the bubble phase. The RIQH effect was previously explained as depinning and sliding of the WC and crystalizing of the bubbles.[11] Unlike at the lowest LL, however, the lattice con-

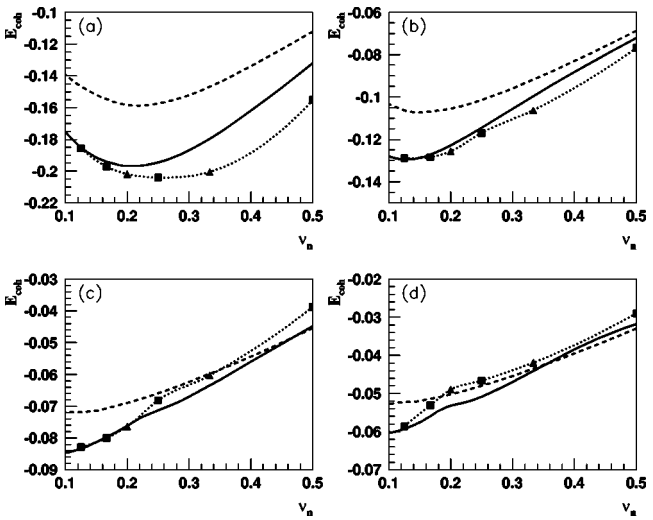

FIG. 3. The cohesive energy per electron versus ν_n at the nth LL's. The bubble state and the stripe state are represented by the solid line and dashed line, respectively. The triangle and square denote the Laughlin state and the CF liquid state, respectively. The dotted line is a guide to the eye. (a) $n=0$. (b) $n=1$. (c) $n=2$. (d) $n=3$.

stant at higher LL's does not change as one increases the LL filling, but remains of the order of $R_c \gg l_0$. This implies the quantum fluctuations are hard to depin the WC in the higher LL's.[26] In our picture, the WC *melts* into the CF liquid and subsequently, the bubbles containing $\tilde{M}=2$ electrons form and are pinned as the filling increases. In fact, the feature of an isotropic metallic phase around $\nu_2=1/6$ can be identified in Fig. 1 of Ref. 11. In addition, the $\nu_2=1/5$ state in our calculations is also slightly lower in energy than the corresponding CDW state while no FQHE was observed in this region. This may be explained as the ground-state energies of compressible states (CDW and CF liquid) being lowered by disorder, while the incompressible FQHE state is hardly affected. For $n \geqslant 3$, the RIQH effects are even weaker to be identified in the experiment.[11] Our calculation also shows that it is very delicate. The CF liquid is slightly higher in energy than the CDW at $\nu_3=1/6$ (Fig. 3). However, the disorder should lower the ground-state energy of the CF liquid a little bit more than that of the CDW because of its gapless ground state.

In this work, we have presented a systematic computation of the cohesive energies of the CF liquid, the Laughlin liquid, and the CDW in the lowest, as well as in higher, LL's. We find that the CF liquid and Laughlin liquid generally dominate at the lowest and second LL's while the CDW dominates at the higher LL's. The energy of the CF liquid at $\nu=4+1/6$, anomalously lower than that of the CDW, may lead to the remarkable phenomenon of RIQH. Since the difference of energies in this region is very slight and the treatment of the screenings from the lower LL's is based on the Hartree-Fock approximation in the large n limit, the results may not be completely decisive for $n=2,3$. Further numerical experiments are required to justify our conclusion.

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