Electron transmission through one-dimensional chains of randomly spaced identical potentials

Ashot Zh. Khachatrian*
Physics Department, State Engineering University of Armenia, 375049 Yerevan, Armenia

Gerd Roepke†
Fachbereich Physik, Universität Rostock, D-18051 Rostock, Germany

David H. Badalyan and David M. Sedrakian‡
Physics Department, Yerevan State University, 375046 Yerevan, Armenia
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An alternative method is developed for calculating the average kinetic characteristics of a one-dimensional chain consisting of N randomly spaced potentials. A finite-difference equation for the average resistance $\langle\rho_N\rangle$ is found. Its solution provides the $\langle\rho_N\rangle$ as a function of N, the energy of the infalling electron and the chain disorder parameter. It is shown that the relationship between the average resistance and chain length is a sum of three power-law functions. In the limiting case of infinitely long chains, the dependence $\langle\rho_N\rangle$ on N is exponential for arbitrary electron energies and values of the disorder parameter. In this case all the states of the single-electron spectrum are localized.

I. INTRODUCTION

The description of the elementary excitations and energy spectrum of a random medium is an important problem of condensed matter theory.[1–3] The interest in this topic is motivated by the necessity of finding the average kinetic characteristics and different correlation functions of quasiperiodic, nonregular, or random systems.[4–11]

It is well known that the electric conductivity at zero temperature vanishes in the case of a random field with a strong degree of disorder. This critical phenomenon is a consequence of the localization of all electron states. An exact mathematical method and universal approach to the description of the crossover from delocalized to localized states in arbitrary dimensions do not exist so far.[12] In particular, in the case of a one-dimensional (1D) system even a weak disorder leads to localization of all states of the single-electron spectrum.[13,14] Recent investigations[15,16] demonstrate that in 1D systems with correlated disorder delocalized states can appear.

Following the pioneering work by Anderson,[1] much effort has been devoted to the understanding of the transport properties of 1D disordered and quasiperiodic systems. Along with the practical interest, related to the technical advance in fabrication of structures with predetermined parameters,[17,18] the quantitative description of the properties of 1D systems is important for qualitative understanding of physical phenomena in two- and three-dimensional systems.[19–22] The exactly solvable 1D models can be used as a test for new approaches, which can also be applied to the higher $(d>1)$ dimensional problems.

A quantitative analysis of the localization of electron states in 1D systems has been carried out in Refs. 23–26 starting from the kinetic theory. Landauer[27] found the dimensionless resistance ρ for spinless electrons in a 1D disordered medium, by dividing the current by the difference of the chemical potentials between both sides of the sample. The localization radius ξ is then determined by the asymptotic behavior of the average resistance, which depends on the system size as[14] $(\hbar=e^2=1)$

$$\langle\rho\rangle=\tfrac{1}{2}\,(e^{L/\xi}-1), \quad L\to\infty \tag{1}$$

where $\langle\rho\rangle$ is the Landauer resistance averaged over the random field, and L is the system's length. Note that for a localized state the radius ξ depends on the energy of the electron and the disorder parameters of the system, while for a delocalized state ξ is infinite.

In this work we develop an alternative method for calculating average kinetic characteristics of a 1D chain consisting of identical, randomly spaced potentials. In the first step we solve the general scattering problem for a chain consisting of identical, periodically spaced potentials. In the second step we obtain analytical expressions for the average resistance of a disordered system. We show that in the case of a random field all the states of the single-electron spectrum are localized. Our approach is based on the method of recurrent relations for scattering amplitudes of arbitrary 1D chains.[28]

II. SCATTERING PROBLEM

Let us consider a model, for which the electron potential energy is a superposition of nonoverlaping atomic potentials $V_n(x-x_n)$ spaced at $x_1, x_2, \ldots, x_N$ of 1D chain points:

$$V(x)=\sum_{n=1}^{N} V_n(x-x_n), \tag{2}$$

where $x_n<x_{n+1}$ and N is the number of potentials. We assume that the asymptotic behavior of the wave function has the form

$$\psi(x)=e^{ikx}+R_N e^{-ikx}, \quad x\to-\infty \tag{3}$$

$$\psi(x)=T_N e^{ikx}, \quad x\to\infty \tag{4}$$

where R_N and T_N are electron reflection and transmission amplitudes, respectively, and $k^2=E$ ($2m_0=1$, m_0 is electron mass) is the energy of an incident electron.

Due to the conservation of the current, the reflection and transmission coefficients satisfy the well-known relation

$$|R_N|^2+|T_N|^2=1. \tag{5}$$

In the general form, the determination of scattering amplitudes R_N and T_N for potential (2) can be reduced to solving of the following second-order finite-difference equations:[28]

$$D_N=A_N D_{N-1}-B_N D_{N-2}, \tag{6}$$

$$\tilde{D}_N=A_N \tilde{D}_{N-1}-B_N \tilde{D}_{N-2}, \tag{7}$$

where

$$D_N=1/T_N, \quad \tilde{D}_N=R_N/T_N. \tag{8}$$

The coefficients A_N and B_N in Eqs. (6) and (8) are of the form

$$A_N=1/t_N+B_N/t^*_{N-1}, \quad B_N=r_N t_{N-1}/t_N r_{N-1}, \quad \text{for } N>1 \tag{9}$$

$$A_1=1/t_1, \quad B_1=0,$$

where t_n is the transmission amplitude (TA) and r_n is the reflection amplitude (RA) of the nth atomic potential. Note that in Eqs. (6) and (A8) N is a discrete variable, so that $D_{N-j}=1/T_{N-j}$ and $\tilde{D}_{N-j}=R_{N-j}/T_{N-j}$ ($j=1,2,\ldots,N-1$) correspond to the TA and RA for the system constructed from the first $N-j$ potentials of the chain, i.e., the potentials $\Sigma_{n=1}^{N-j}V_n(x-x_n)$.

The quantities D_N and $\tilde{D}_N$ satisfy the same equations (6) and (A8). To obtain the special solutions, the initial conditions must be specified. These conditions differ for D_N and $\tilde{D}_N$ and have the form

$$D_0=1,\ D_1=1/t_1,\ \ \tilde{D}_0=0,\ \tilde{D}_1=r_1/t_1, \tag{10}$$

where D_0 and $\tilde{D}_0$ correspond to the electron free motion and D_1 and $\tilde{D}_1$ to the scattering on the first atomic potential $V_1(x-x_1)$. For example, if the potential is a δ function [$V_1(x-x_1)=u_1\delta(x-x_1)$], then

$$D_1=1+\frac{iu_1}{2k},\ \ \tilde{D}_1=\frac{-iu_1}{2k}e^{i2kx_1}. \tag{11}$$

The solution (6) or (A8) can be represented in the form

$$D_N=d_1 d_2\cdots d_N. \tag{12}$$

Substituting d_n from Eq. (12) into Eq. (6) we find the following nonlinear algebraic system:

$$d_n=A_n-B_n/d_{n-1} \quad (n=2,3,\ldots,N), \tag{13}$$

which, in the general case, can be solved numerically.

III. THE ELECTRON SCATTERING ON THE PERIODIC STRUCTURES

Let us consider the particular case of the potential (2), when N identical nonoverlaping and arbitrary shaped potentials are spaced periodically, in which case the electron potential energy is

$$V(x)=\sum_{n=1}^{N} u(x-an), \tag{14}$$

where a is the chain period. Since the atomic potentials are identical, the RA's r_n and r'_n of the n and n' scattering centers differ by the phase factor[6]

$$r_{n'}=r_n\exp[i2ka(n'-n)], \tag{15}$$

while at the same time the TA are equal:

$$t_{n'}=t_n. \tag{16}$$

Using Eqs. (15), (16), and (A10), one can see that for a periodic chain the coefficients A_N and B_N are independent of N:

$$A_N=A=1/t+(t^*)^{-1}e^{i2ka}, \quad B_N=B=e^{i2ka}, \quad N>1 \tag{17}$$

where t is the transmission amplitude of an individual potential of the periodic system. Taking into account Eq. (17), Eq. (6) can be transformed to the form

$$D_N=AD_{N-1}-BD_{N-2}, \tag{18}$$

subject to initial conditions

$$D_0=1, \quad D_1=1/t. \tag{19}$$

The solution of Eq. (18) is

$$D_N=C_1\alpha_1^N+C_2\alpha_2^N, \tag{20}$$

where the constants C_1 and C_2 have to be determined from the initial conditions (19). The quantities α_1 and α_2 are the roots of the following characteristic equation:

$$\alpha^2-A\alpha+B=0 \tag{21}$$

and can be represented as

$$\alpha_{1,2}=e^{i(ka\pm\beta)}, \tag{22}$$

where

$$\cos\beta=\mathrm{Re}(e^{-ika}t^{-1}). \tag{23}$$

It is important to note that expression (23) determines the electron energy spectrum for a periodical system. It relates the electron quasimomentum with the TA of the atomic potential of the system.[29]

Furthermore using Eqs. (19), (20), and (22), we find

$$C_{1,2}=\frac{1}{2}\pm\frac{\mathrm{Im}(e^{-ika}t^{-1})}{2\sin\beta}. \tag{24}$$

Substituting Eqs. (22) and (24) into Eq. (20) we finally obtain

$$D_N = e^{iNka}\left(\cos N\beta + \mathrm{Im}(e^{-ika}t^{-1})\frac{\sin N\beta}{\sin\beta}\right). \quad (25)$$

The result (25) determines the TA T_N [Eq. (8)] for an arbitrary periodical potential (14).

To determine the quantity $\tilde{D}_N$, Eq. (8) has to be solved with the initial conditions

$$\tilde{D}_0 = 0, \quad \tilde{D}_1 = r/t. \quad (26)$$

Determining C_1 and C_2 from Eqs. (26) and (20) we obtain for $\tilde{D}_N$ the following expression:

$$\tilde{D}_N = \frac{r}{t}\frac{\sin N\beta}{\sin\beta}e^{i(N-1)ka}. \quad (27)$$

Expressions (27) and (25) determine the RA for a periodic system ($R_N = \tilde{D}_N/D_N$). Note that the functions $\tilde{D}_N$ and D_N are directly related to the dimensionless Landauer resistance, which can be expressed as the ratio of reflected and transmitted intensities:[14,27]

$$\rho = |D_N|^2 - 1 = |\tilde{D}_N|^2. \quad (28)$$

The differences between the Landauer conductance and the Kubo formula were analyzed in many papers.[30,31]

Using Eqs. (25), (27), and (28), we obtain for the Landauer resistance of the periodic chain

$$\rho_N = \left|\frac{r}{t}\right|^2 \frac{\sin^2 N\beta}{\sin^2\beta}. \quad (29)$$

It is evident from Eqs. (29) and (23) that, when $|\cos\beta| \leqslant 1$, the dependence of ρ_N on N is a periodic function. If $|\cos\beta| > 1$, then $\beta = i\gamma$ (γ is real), and ρ_N can be written in the form

$$\rho_N = \left|\frac{r}{t}\right|^2 \frac{\sinh^2 N\beta}{\sinh^2\beta}. \quad (30)$$

Relation (30) means that for states outside of a conductivity band the resistance exponentially tends to infinity, when $N \to \infty$. Expressions (25) and (29) generalize previous results obtained for layered homogeneous media and for chains composed of periodic δ-like potentials.[32]

IV. THE RESISTANCE OF THE CHAIN WITH STRUCTURAL DISORDER

Let us consider a 1D model consisting of identical, randomly spaced potentials. We will assume that for an arbitrary realization of random field the distances between potentials are larger compared with the radius of the action of an individual potential. This form of disorder is usually called a structural disorder.[2,33]

As follows from Eqs. (6), (8), and (28) the resistance of a chain of N identical potentials depends on the distances between the nearest scatterers only, i.e., ρ_N can be considered as a function of the quantities $|x_{l+1} - x_l|$ ($l = 1,2,\ldots,N-1$). Here we assume $|x_{l+1} - x_l|$ to be random and independent, so that

$$|x_{l+1} - x_l| = a + \Delta x_l, \quad (31)$$

where a is the average distance between the scatterers, and the average length of the system L is equal to $(N-1)a$. We take the average value of the random quantities Δx_l to be equal to zero:

$$\int f(\Delta x_l)\Delta x_l d\Delta x_l = 0, \quad \int f(\Delta x_l) d\Delta x_l = 1, \quad (32)$$

where $f(\Delta x_l)$ are the distribution functions for Δx_l, which are supposed to be the same for all Δx_l ($l = 1,2,\ldots,N-1$). The region of integration of the function $f(\Delta x_l)$ has to be chosen in such a manner that the potentials of the nearest scatterers, for an arbitrary realization of random field, do not overlap.

Using the random field averaging procedure, the resistance can be written as

$$\langle\rho_N\rangle = \int \cdots \int |\tilde{D}_N|^2 f(\Delta x_1)\cdots f(\Delta x_{N-1}) \times d\Delta x_1 \cdots d\Delta x_{N-1}. \quad (33)$$

As is clear from Eq. (33), the problem of the determination of the average resistance is reduced to the averaging of the random quantity $|\tilde{D}_N|^2$. A direct derivation of an explicit expression for $\tilde{D}_N$ does not appear to be possible.[34] Therefore, in order to evaluate the integral (33), we attempt to obtain the equation for the unknown quantity $|\tilde{D}_N|^2$, whose solution gives the dependence of the average resistance on the model parameters. This equation is derived in the Appendix and has the following form,

$$\langle\rho_N\rangle = (n^2 l + m)\langle\rho_{N-1}\rangle - n^2(l-m)\langle\rho_{N-2}\rangle + n^4\langle\rho_{N-3}\rangle + (1-n^4)\langle\rho_1\rangle, \quad (34)$$

where

$$m = (1-n^2)(2\langle\rho_1\rangle + 1), \quad l = 4\cos^2\beta - 1. \quad (35)$$

The initial conditions for Eq. (34) are

$$\langle\rho_0\rangle = 0, \quad \langle\rho_1\rangle = \left|\frac{r}{t}\right|^2, \quad \langle\rho_2\rangle = (n^2 l + m + 1)\langle\rho_1\rangle. \quad (36)$$

Conditions (36) follow directly from Eq. (33). In Eqs. (34)–(36)

$$n^2 = \langle e^{i2k\Delta x_l}\rangle = \int f(\Delta x_l) e^{i2k\Delta x_l} d\Delta x_l. \quad (37)$$

For example, when $f(\Delta x_l)$ is a Gaussian distribution function,

$$f(\Delta x) = \frac{1}{\sqrt{2\pi\langle(\Delta x)^2\rangle}} e^{-\Delta x^2/2\langle(\Delta x)^2\rangle}, \quad (38)$$

then n^2 depends on $\langle(\Delta x)^2\rangle$ and has the form

$$n^2 = e^{-2k^2\langle(\Delta x)^2\rangle}, \quad (39)$$

where $\langle(\Delta x)^2\rangle$ is the disorder parameter of the system.

Equation (34) is an inhomogeneous finite-difference equation with constant coefficients. Its solution can be written as

$$\langle \rho_N \rangle = \sum_{j=1}^{3} C_j y_j^N + C_0, \tag{40}$$

where C_0 and C_j $(j=1,2,3)$ are constants, y_j $(j=1,2,3)$ are the roots of the characteristic equation

$$y_j{}^3 - (n^2 l + m) y_j{}^2 + n^2 (l-m) y_j - n^4 = 0, \tag{41}$$

which are calculated using the standard formulas for the solution of a cubic equation:

$$y_1 = a + b + (n^2 l + m)/3,$$

$$y_{2,3} = -\frac{a+b}{2} \pm i \left(\frac{a-b}{2} \right) \sqrt{3} + (n^2 l + m)/3, \tag{42}$$

where

$$a = \sqrt[3]{-\frac{q}{2} + \sqrt{Q}}, \quad b = \sqrt[3]{-\frac{q}{2} - \sqrt{Q}},$$

$$q = -2 \left(\frac{n^2 l + m}{3} \right)^3 + \frac{n^2 (l-m)(n^2 l + m)}{3} - n^4,$$

$$Q = \left(\frac{q}{2} \right)^2 + \left(\frac{p}{3} \right)^3, \quad p = -\frac{(n^2 l + m)^2}{3} + n^2 (l-m). \tag{43}$$

As follows from Eqs. (42) and (43), for $Q>0$ Eq. (41) has one real- and two complex-valued roots. When $Q \leqslant 0$, all three roots are real.

Let us now find the constants C_0 and C_j. Constant C_0 is a particular solution of Eq. (34) and it is easy to see that

$$C_0 = -1/2. \tag{44}$$

To determine C_j $(j=1,2,3)$ we first derive, using Eqs. (36), (41), and (44), the following linear system of equations:

$$\sum_{j=1}^{3} C_j = \langle \rho_0 \rangle + 1/2,$$

$$\sum_{j=1}^{3} C_j y_j = \langle \rho_1 \rangle + 1/2, \tag{45}$$

$$\sum_{j=1}^{3} C_j y_j{}^2 = \langle \rho_2 \rangle + 1/2.$$

The solution of system (45) is

$$C_1 = \frac{1}{2} \frac{(2\langle \rho_2 \rangle + 1) - (2\langle \rho_1 \rangle + 1)(y_2 + y_3) + y_2 y_3}{(y_2 - y_1)(y_3 - y_1)}. \tag{46}$$

The constants C_2 and C_3 can be obtained from Eq. (46) via a cyclic permutation of y_1, y_2, and y_3.

Formulas (40)–(46) solve the problem of determination of the average resistance for a general specified random field. As we have shown, the dependence of $\langle \rho_N \rangle$ on the number of scatterers is a sum of three power law functions. Finally, note that taking the disorder parameter in the solution (40) as $n^2 = 1$ (ordered chain), one recovers the form given by Eq. (30).

V. THE LOCALIZATION RADIUS OF SINGLE-ELECTRON STATES

In this section we prove that in the considered 1D random model all single-electron states are localized, i.e., the localization radius

$$\xi = \lim_{N \to \infty} \frac{Na}{\ln \langle \rho_N \rangle} \tag{47}$$

is a finite function, which is independent of N for arbitrary values of parameters r, t, and k. To do this, we investigate the properties of the roots of the characteristic equation (41). According to Viete's theorem, we have the following relationship for the roots of Eq. (41):

$$y_1 y_2 y_3 = n^4. \tag{48}$$

Note that $n^2 > 0$ by definition. As follows from Eq. (48) all three roots are real $(Q \leqslant 0)$, when they are positive. If one of the roots is positive while the remaining ones are negative, the characteristic equation has only one positive real root.

Let us now demonstrate, that among the roots of Eq. (41) there is one root which is greater than or equal to unity. Let us consider the function

$$F(y) = y_j{}^3 - (n^2 l + m) y_j{}^2 + n^2 (l-m) y_j - n^4. \tag{49}$$

The zeros of the function $F(x)$ determine the roots of the characteristic equation.

It is easy to see that for all values of l, m, and n^2 the function $F(x)$ is negative at the point $y=1$. If $x \to \infty$ the function $F(x) \to \infty$ and when $x \to -\infty$, then $F(x) \to -\infty$. From these properties, it follows, in particular, that the equation $F(x)=0$ contains a root that is larger than unity and larger than the absolute value of the other roots. Therefore the asymptotic behavior of $\langle \rho_N \rangle$ for $N \to \infty$ has the form

$$\langle \rho_N \rangle = C_1 z_1^N - 1/2. \tag{50}$$

Here z_1 is the root of the characteristic equation (41), which is greater than or equal to unity $(z_1 \geqslant 1)$ and $z_1 \geqslant |z_2|, |z_3|$, where z_2 and z_3 are the two remaining roots. Substituting Eq. (50) into Eq. (48) we obtain the following expressions for the radius of localization:

$$\xi = \frac{a}{\ln z_1}. \tag{51}$$

As follows from Eq. (42), ξ has a complicated transcendental dependence on the disorder parameter of the system, the electron energy, and the scattering amplitudes.

In Eq. (51), the case $z_1 = 1$ corresponds to delocalized states $(\xi \to \infty)$. As can be seen from Eqs. (35) and (41) this condition is satisfied in the case of periodic chain $(n^2 = 1)$ for the conductivity band states. For a random chain, the states with energies equal to the resonance energies of a single potential of the chain $(\rho_1 = 0)$ are also delocalized.

To illustrate the obtained results, we consider a structural disordered chain consisting of identical rectangular barriers when the deviation from a periodic chain is described by means of the Gaussian distribution function (38). Using the expression for TA of a rectangular potential,

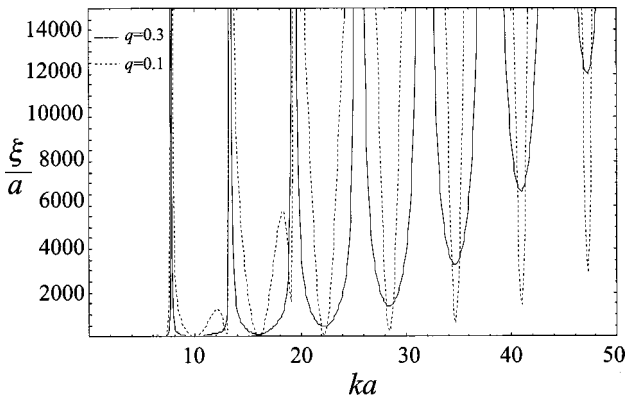

FIG. 1. The dependence of ξ/a for a chain of randomly spaced identical rectangular potentials as a function of ka for $Va^2=20$, $d/a=0.5$ and different values of $q=\sqrt{\langle(\Delta x)^2\rangle/a^2}$ [if $a=5$ Å, then $(\hbar^2/2m)V\approx 2.5$ eV] .

$$t^{-1}=\exp(ikd)\left\{\cos\ \chi d-i\frac{\chi^2+k^2}{2k\chi}\sin\chi d\right\}, \tag{52}$$

where $\chi=\sqrt{E-V}$, V is the magnitude of the potential, and d is its width, we find from Eq. (23)

$$\cos\beta=\cos(a-d)k\cos\chi d-\frac{\chi^2+k^2}{2k\chi}\sin(a-d)k\sin\chi d, \tag{53}$$

which is the well-known formula for the energy spectrum of periodic Kronig-Penney chain, where a is the system period.

From Eqs. (30) and (52), for the Landauer resistance of a single rectangular potential we obtain

$$\rho_1=\frac{(\chi^2-k^2)^2}{4\chi^2k^2}\sin^2\chi d. \tag{54}$$

Using Eqs. (52)–(54), (41), and (51) we carried out numerical calculations of the dependence ξ/a on ka for different values of dimensionless disorder parameter $q=\sqrt{\langle(\Delta x)^2\rangle/a^2}$(see Fig. 1). As can be seen, the localization radius is a multivalued function of the electron energy. For states with energies equal to the resonance energies of a single barrier ($\chi d=\pi N, N=1,2,\ldots$), the radius of localization ξ tends to infinity. For the states of the band gap, the increase of the disorder leads to an increase of the localization radius. In the conductivity band ξ/a attains its maximum at the end of the band.

It is interesting to consider for the general model the case of weak disorder ($1-n^2=s\ll 1$) when the energy of an electron corresponds to the conductivity band of the periodic system $|\mathrm{Re}(e^{-ika}t^{-1})|<1$. In this limiting case the root z_1 determining the localization radius is

$$z_1=1+\Delta z, \tag{55}$$

where $0<\Delta z\ll 1$.

Inserting Eq. (55) in Eq. (41) and keeping only terms linear in Δz, we arrive at the following expression for Δz:

$$\Delta z=\frac{4\langle\rho_1\rangle(1-n^2)}{\sin^2\beta}. \tag{56}$$

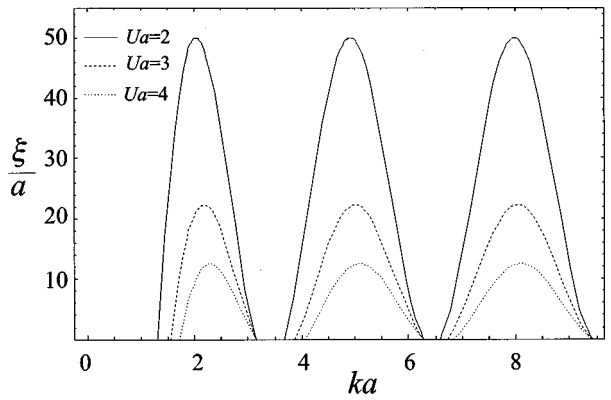

FIG. 2. The dependence of ξ/a on ka for a weakly disordered chain of δ potentials for $q=\sqrt{\langle(\Delta x)^2\rangle/a^2}=0.01$ and different values of Ua [if $a=5$ Å, then $(\hbar^2/2m)U$ is of the order of 2 eV Å].

Inserting Eq. (56) into Eq. (55), from Eq. (51) we obtain

$$a\xi^{-1}=\langle\rho_1\rangle(1-n^2)/\sin^2\beta. \tag{57}$$

According to Eq. (57), the radius of localization is infinity only for the case where $n^2=1$ (periodic potential).

In Fig. 2 we present a numerical evaluation of Eq. (57) for a random chain composed of δ potentials. In this case, the energy spectrum is given by the well-known formula $\cos\beta=\cos ka+(U/k)\sin ka$ with $\rho_1=U^2/4k^2$ (U is the strength of the δ potential). As a small dimensionless disorder parameter we adopted the ratio $q=\sqrt{\langle(\Delta x)^2\rangle/a^2}$. According to Fig. 2, for fixed q the increase of U leads to a decrease of the localization radius of the conductivity band states. As we see in Fig. 2, the localization radius increases when the value of ka moves away from the ends of conductivity band and takes its maximum value at the center of the band.

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APPENDIX

Here we present the derivation of the equation for the average resistance of a chain consisting of identical potentials of an arbitrary shape with structural disorder:

$$\langle\rho_N\rangle=\langle|D_N|^2\rangle-1. \tag{A1}$$

To do this, we first derive certain recurrent relations for quantity $|D_N|^2$, which can be averaged directly. For the present case the equation for D_N is

$$D_N=A_ND_{N-1}-B_ND_{N-2}, \tag{A2}$$

where

$$A_N=1/t+(t^*)^{-1}e^{i2k|x_N-x_{N-1}|},\quad B_N=e^{i2k|x_N-x_{N-1}|},\quad N>1 \tag{A3}$$

and $|x_N-x_{N-1}|=a+\Delta x_{N-1}$, where Δx_{N-1} is a random variable.

Using Eq. (A2) we can write

$$|D_N|^2=|A_N|^2|D_{N-1}|^2+|D_{N-2}|^2-A_NB_N^*D_{N-1}D_{N-2}^*-A_N^*B_ND_{N-1}^*D_{N-2}. \quad (A4)$$

The last two terms of Eq. (A4) can be rewritten in terms of the quantities D_{N-3} and D_{N-4} using Eq. (A2):

$$\begin{aligned}|D_N|^2=&|A_N|^2|D_{N-1}|^2+(1-A_NA_{N-1}B_N^*\\&-A_N^*B_NA_{N-1}^*)|D_{N-2}|^2+(A_NB_N^*A_{N-2}^*B_{N-1}\\&+A_N^*B_NA_{N-2}^*B_{N-1}^*)|D_{N-3}|^2\\&-A_N^*B_{N-1}^*B_NB_{N-2}D_{N-3}^*D_{N-4}\\&-A_NB_{N-1}B_N^*B_{N-2}^*D_{N-3}D_{N-4}^*. \end{aligned}\quad (A5)$$

Replacing in Eq. (A4) N by $N-2$ we find:

$$|D_{N-2}|^2=|A_{N-2}|^2|D_{N-3}|^2+|D_{N-4}|^2-A_{N-2}B_{N-2}^*D_{N-3}D_{N-4}^*-A_{N-2}^*B_{N-2}D_{N-3}^*D_{N-4}. \quad (A6)$$

In Eqs. (5) and (6) the corresponding coefficients are averaged separately. For example,

$$\langle|A_{N-2}|^2|D_{N-3}|^2\rangle=\langle|A_{N-2}|^2\rangle\langle|D_{N-3}|^2\rangle, \quad (A7)$$

and so on.

Calculating the average values of coefficients and excluding the terms $\langle D_{N-3}^*D_{N-4}\rangle$ and $\langle D_{N-3}D_{N-4}^*\rangle$ we obtain from Eqs. (5) and (6)

$$\begin{aligned}&\langle|D_N|^2\rangle-(n^2l+m+1)\langle|D_{N-1}|^2\rangle+[2n^2l+(1-n^2)m]\\&\times\langle|D_{N-2}|^2\rangle-[n^2(l-m)+n^4]\langle|D_{N-3}|^2\rangle\\&+n^4\langle|D_{N-4}|^2\rangle=0, \end{aligned}\quad (A8)$$

where l and m are given in Eq. (35). The finite-difference equation (A8) has the particular solution $\langle|D_N|^2=C\rangle$, where C is an arbitrary constant, which can be immediately verified by a direct substitution. Therefore the homogeneous equation (A8) can be reduced to an inhomogeneous third-order finite-difference equation.

Indeed, let us consider the equation

$$\langle|D_N|^2\rangle+P_1\langle|D_{N-1}|^2\rangle+P_2\langle|D_{N-2}|^2\rangle+P_3\langle|D_{N-3}|^2\rangle+P_4=0, \quad (A9)$$

where the coefficients P_J are unknown. Replacing N by $N-1$ in Eq. (A9) and subtracting the resulting equation Eq. (A9), we find

$$\langle|D_N|^2\rangle+(P_1-1)\langle|D_{N-1}|^2\rangle+(P_2-P_1)\langle|D_{N-2}|^2\rangle+(P_3-P_2)\langle|D_{N-3}|^2\rangle+P_3\langle|D_{N-4}|^2\rangle=0. \quad (A10)$$

This equation takes the same form as (A8), when

$$P_1=-(n^2l+m),\quad P_2=(n^2)(l-m),\quad P_3=-n^4. \quad (A11)$$

Using Eqs. (A1) and (A11), we obtain from Eq. (A9) the equation (34) for the unknown function $\langle\rho_N\rangle$.

*Email address: akhachat@www.physdep.r.am
†Email address: gerd@darss.mpg.uni-rostock.de
‡Email address: dsedrak@www.physdep.r.am

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