Interaction-induced localization-delocalization transition in the double-layer quantum Hall system

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We report on numerical studies of the energy spectrum and the localization properties in the double-layer quantum Hall system at $\nu=1$. The Coulomb interaction is treated by the Hartree-Fock approximation, and the localization properties in the presence of disorder are studied by evaluating participation ratios for the Hartree-Fock eigenfunctions. We show that the extended states seem to exist only near each center of the two subbands split by the exchange-enhanced energy gap. It is also shown that the self-consistent orbitals whose energies are close to the Fermi energy appear to become extended together with the reduction in the energy gap as the layer separation increases. The collapse of the energy gap expected from our results is consistent with the incompressible-compressible transition observed in recent experiments, and the change of the localization properties near the Fermi energy can explain the disappearance of the quantum Hall effect for large layer separations very well.

I. INTRODUCTION

When the integer quantum Hall effect (QHE) is studied theoretically, it is usually assumed without justification that the Coulomb interaction between electrons can be safely ignored. In a strong magnetic field, the eigenfunctions of the single-particle Hamiltonian for two-dimensional noninteracting electron systems are localized by a disorder potential at almost all energies except for a discrete set of critical energies $\{\varepsilon_{cN}\}$ near the center of each disorder-broadened Landau level. Theoretical studies suggest that at $T=0$, the Hall conductivity jumps by e^2/h each time the Fermi energy ε_F crosses one of the critical energies, and that the longitudinal conductivity is zero if $\varepsilon_F \neq \varepsilon_{cN}$. These suggestions are supported by many experimental studies.[1]

However, such localization properties are not always guaranteed when two different Landau levels are nearly degenerate. In fact, the numerical studies of the double-layer quantum Hall (QH) system in the absence of electron-electron interactions[2,3] could not obtain the reasonable localization properties in cases of nearly degenerate Landau levels (i.e., in the weak interlayer-tunneling case). This is a typical case where the Coulomb interaction should be considered even for understanding the integer QHE qualitatively. Thus we consider this system in the presence of interactions in this paper. We show that the exchange-enhanced energy gap[4] appears in this interacting system and that the localization properties consistent with the observation of the QHE can be obtained for small layer separations even in the weak interlayer-tunneling case.

In double-layer QH systems,[5] the interlayer tunneling of electrons brings about the mixing of the Landau levels in the two layers, and the Landau levels split into symmetric and antisymmetric combinations about the center of the double-layer structure. The energy gap, Δ_{SAS}, between them is proportional to the tunneling amplitude, and it needs to be small for the nearby degeneracy of the Landau levels.

Such samples are realized experimentally and the transport properties have been investigated.[6,7] In these experiments using high-mobility samples, it has been reported that the Coulomb interaction plays an important role on the ground-state properties and low-lying excitations. For $\nu=1$, the phase diagram against the layer separation d, which controls the strength of the interlayer interactions, and Δ_{SAS} is obtained experimentally.[7] The phase diagram shows that the QH state disappears for $d>d_c$ and that the critical separation d_c increases as Δ_{SAS} increases.

In a more recent experiment using high-mobility samples with weak interlayer tunneling, the zero in the longitudinal resistivity is replaced for $d>d_c$ by a broad minimum similar to that observed in the single-layer QH system at $\nu=1/2$.[8] This suggests a transition from an incompressible QH state with strong interlayer correlations to compressible state consisting of two (weakly correlated) layers, where the metallic states of composite fermions are formed.

Theoretically, the pseudospin formalism is often introduced to describe the layer degrees of freedom in double-layer systems. This is done by assigning the upper/lower layers to the pseudospin $\uparrow/\downarrow$. At $\nu=1/m$ (m an odd integer), the pseudospin ferromagnetism results from the interlayer tunneling and exchange interactions between electrons.[5] The phase boundary between the QHE and the non-QHE phase was determined theoretically by assuming that the QHE phase is destroyed together with the collapse of the pseudospin ferromagnetism.[9,10] The pseudospin-ferromagnetic ground state is shown to evolve continuously from tunneling dominated to correlation dominated, as Δ_{SAS} decreases.[11]

The Hartree-Fock calculations have also been done to study the QH systems, and this approximation is expected to describe the electronic properties well especially for the integer filling factors. In fact, this approximation has been used for the study of the double-layer $\nu=1$ QH system in the absence of random disorder potential.[12,13]

In this paper, we investigate the energy spectrum and the localization properties in disordered double-layer QH systems at $\nu=1$. The Coulomb interaction is treated by the Hartree-Fock approximation, and the localization properties are studied by evaluating participation ratios for the Hartree-Fock eigenfunctions. This method has been used in the stud-

ies on the interaction effects in the single-layer QH system.[14,15] We show that the localization properties change qualitatively because of the interaction effects.[16]

Our paper is organized as follows. In Sec. II, we explain the model and calculation methods which we use in this study. In Sec. III, we first discuss the results of the numerical calculations in the absence of Coulomb interactions. Through this discussion about the previous results[2,3] and ours, it is shown that the Coulomb interaction should be considered to understand the $\nu=1$ QHE in double-layer system with weak interlayer tunneling. After this discussion, we show our numerical results in the presence of Coulomb interactions. These are our main results in this paper, and it is shown that the localization properties change together with the reduction in the exchange-enhanced energy gap as the layer separation increases. Finally in Sec. IV, we briefly summarize our findings.

II. MODEL AND METHOD

We consider a double-layer system of spin-polarized electrons in a strong magnetic field perpendicular to the layers. In double-layer systems, there exists the interlayer tunneling of electrons. The single-particle wave functions then split into symmetric and antisymmetric ones about the center of the double-layer structure, and the energy gap between them, $\Delta_{\rm SAS}$, enters as an energy scale. The thickness of the wave function in each layer is neglected for simplicity.

The two-dimensional coordinates in the two parallel planes are denoted by $\mathbf{r}=(x,y)$, and the layer degrees of freedom are described by the pseudospin $\sigma=\uparrow,\downarrow$. The Coulomb interaction between electrons is then dependent on pseudospin σ for a finite layer separation d. Its Fourier transform $V_{\sigma\sigma'}(q)$ is $2\pi e^2/\epsilon q$ if $\sigma=\sigma'$ (i.e., for the intralayer interaction) and $(2\pi e^2/\epsilon q)e^{-qd}$ if $\sigma\neq\sigma'$ (i.e., for the interlayer one), where ϵ is the dielectric constant of the host material. The Coulomb interaction is treated self-consistently within the Hartree-Fock (HF) approximation.

In the strong-magnetic-field limit, it is enough to consider only the lowest Landau level because one can neglect the Landau level mixing by interactions or disorders. The real spin degrees of freedom are also ignored by assuming the spin polarization due to the Zeeman energy. Our attention is restricted to this strong-field limit. We apply the periodic boundary condition to the single-particle wave functions inside the two parallel rectangles of dimensions L_x, L_y, and use the Landau gauge $\mathbf{A}(\mathbf{r})=(0,Bx,0)$. One can then use the following set of basis functions for the lowest Landau level:

$$\phi_j(\mathbf{r})=\left(\frac{1}{L_y\sqrt{\pi}l}\right)^{1/2}\times\sum_{k=-\infty}^{\infty}\exp\left[i\frac{X_j+kL_x}{l^2}y-\frac{(X_j+kL_x-x)^2}{2l^2}\right], \quad (1)$$

where $l\equiv\sqrt{c\hbar/eB}$ is the magnetic length, and $X_j=2\pi l^2 j/L_y$ is the center coordinate of the jth Landau orbit.[17] The orbital degrees of freedom in each layer are described by this set of the Landau orbits, and the two layers can be distinguished by the pseudospin index σ. Thus the set $\{|j\sigma\rangle\}$ $(j=1,2,\ldots,N_L,\sigma=\uparrow,\downarrow)$ can be used as a basis set for representing the Hartree-Fock Hamiltonian, where $N_L=L_xL_y/2\pi l^2$ is the Landau level degeneracy in each layer.

In terms of the set of basis functions, $\{|j\sigma\rangle\}$, the matrix element of the Hartree-Fock Hamiltonian is given by

$$\langle j\sigma|H_{\rm HF}|j'\sigma'\rangle=-\frac{\Delta_{\rm SAS}}{2}\delta_{jj'}\delta_{\sigma,-\sigma'}+\delta_{\sigma\sigma'}\langle j\sigma|v_{\rm imp}|j'\sigma\rangle+\langle j\sigma|V_{\rm HF}|j'\sigma'\rangle, \quad (2)$$

where $\delta_{jj'}$ is a usual Kronecker delta; the first term in the right-hand side of Eq. (2) is due to the interlayer tunneling. The second and third terms in Eq. (2) result from the impurity scattering and the Coulomb interaction between electrons, respectively. The amplitudes of interlayer impurity scatterings are neglected because of their small values and for simplicity.

The Hartree-Fock single-particle equation is given by

$$H_{\rm HF}|\varphi_\alpha\rangle=\varepsilon_\alpha|\varphi_\alpha\rangle, \quad (3)$$

where ε_α and $|\varphi_\alpha\rangle$ are an eigenvalue and corresponding eigenstate of this equation, respectively. Because the third term, $\langle j\sigma|V_{\rm HF}|j'\sigma'\rangle$, in Eq. (2) is dependent on the set, $\{|\varphi_\alpha\rangle\}$, as seen in Eq. (6), this single-particle equation must be solved self-consistently. This is done by diagonalizing the $2N_L\times 2N_L$ matrix $\langle j\sigma|H_{\rm HF}|j'\sigma'\rangle$ in Eq. (2) numerically and solving Eq. (3) iteratively until self-consistency is achieved. Among the obtained eigenstates, $\{|\varphi_\alpha\rangle\}$ in Eq. (3), the N lowest-energy ones are occupied in N-electron systems, and $N=N_L$ in the case of $\nu=1$.

The matrix element of the impurity scattering, which is the second term in Eq. (2), is given by

$$\langle j\sigma|v_{\rm imp}|j'\sigma\rangle=\frac{1}{L_xL_y}\sum_{\mathbf{q}}v_\sigma(\mathbf{q})\delta\left(j-j',\frac{q_yL_y}{2\pi}\right)\times\exp\left[-\frac{q^2l^2}{4}+i\left(q_xX_j-\frac{q_xq_yl^2}{2}\right)\right], \quad (4)$$

where $v_\sigma(\mathbf{q})$ is the Fourier transform of the impurity potential $v_\sigma(\mathbf{r})$ in the layer σ, $\delta(j,j')$ is 1 if $j=j'$ (mod N_L) and 0 otherwise. The sum over the wave vector $\mathbf{q}$ is over $q_x=(2\pi/L_x)n_x$, $q_y=(2\pi/L_y)n_y$ (n_x and n_y are integers), because the periodic boundary condition is used.

Our model disorder consists of randomly located δ-function scatterers with a random strength uniformly distributed between $-V_0$ and V_0. The disorder potential is then given by

$$v_\sigma(\mathbf{r})=\sum_i V_i^\sigma\delta(\mathbf{r}-\mathbf{R}_i^\sigma), \quad (5)$$

where V_i^σ and $\mathbf{R}_i^\sigma$ are the strength and position of the ith impurity in the layer σ, respectively. There exist $N_{\rm imp}$ impurities in each layer, and we assume that the disorder potentials in the two layers are *uncorrelated*, i.e., there is no cor-

relation about $\{\mathbf{R}_i^\sigma\}$ and $\{V_i^\sigma\}$ between the two layers.[3] For this model disorder, the energy scale that characterizes the Landau subband width is given by $\Gamma=(V_0^2 N_{\rm imp}/l^2 L_x L_y)^{1/2}$.[18] We choose to work with $N_{\rm imp}/N_L=2\pi l^2 N_{\rm imp}/L_x L_y=5$, and keep V_0/l^2 constant in order to use Γ as the unit of energy.

The matrix element of the Coulomb interaction, $\langle j\sigma|V_{\rm HF}|j'\sigma'\rangle$, which is the third term in Eq. (2), is given by

$$\begin{aligned}\langle j\sigma|V_{\rm HF}|j'\sigma'\rangle=&\sum_\alpha\ \theta(\varepsilon_F-\varepsilon_\alpha)\\&\times\sum_{j_1,j_2}\Bigg[\delta_{\sigma\sigma'}\sum_{\sigma''}\langle jj_1|V_{\sigma\sigma''}|j'j_2\rangle\langle\varphi_\alpha|j_1\sigma''\rangle\\&\times\langle j_2\sigma''|\varphi_\alpha\rangle-\langle jj_1|V_{\sigma\sigma'}|j_2j'\rangle\langle\varphi_\alpha|j_1\sigma'\rangle\\&\times\langle j_2\sigma|\varphi_\alpha\rangle\Bigg]\\=&\sum_{\mathbf{q}\in{\rm BZ}}e^{iq_xX_j}\delta\left(j-j',\frac{q_yL_y}{2\pi}\right)\\&\times\Bigg[\delta_{\sigma\sigma'}\sum_{\sigma''}\Delta_{\sigma''\sigma''}(\mathbf{q})U_{\rm H}^{\sigma\sigma''}(\mathbf{q})\\&-\Delta_{\sigma'\sigma}(\mathbf{q})U_{\rm F}^{\sigma\sigma'}(\mathbf{q})\Bigg],\end{aligned}\tag{6}$$

$$\begin{aligned}U_{\rm H}^{\sigma\sigma'}(\mathbf{q})=&\frac{1}{2\pi l^2}\sum_{\mathbf{q}'\neq\mathbf{0}}\delta\left(\frac{q_xL_x}{2\pi},\frac{q_x'L_x}{2\pi}\right)\\&\times\delta\left(\frac{q_yL_y}{2\pi},\frac{q_y'L_y}{2\pi}\right)V_{\sigma\sigma'}(q')e^{-q'^2l^2/2},\end{aligned}\tag{7}$$

$$\begin{aligned}U_{\rm F}^{\sigma\sigma'}(\mathbf{q})=&\frac{1}{L_xL_y}\sum_{\mathbf{q}'\neq\mathbf{0}}V_{\sigma\sigma'}(q')\\&\times\exp\left[-\frac{q'^2l^2}{2}+i(q_x'q_y-q_y'q_x)l^2\right],\end{aligned}\tag{8}$$

$$\begin{aligned}\Delta_{\sigma\sigma'}(\mathbf{q})\equiv&\frac{1}{N_L}\exp\left(\frac{q^2l^2}{4}-i\frac{q_xq_yl^2}{2}\right)\langle\hat\rho_{\sigma\sigma'}(\mathbf{q})\rangle\\=&\frac{1}{N_L}\sum_{j,j'}e^{-iq_xX_{j'}}\delta\left(j'-j,\frac{q_yL_y}{2\pi}\right)\\&\times\sum_\alpha\ \theta(\varepsilon_F-\varepsilon_\alpha)\langle\varphi_\alpha|j\sigma\rangle\langle j'\sigma'|\varphi_\alpha\rangle,\end{aligned}\tag{9}$$

$$\begin{aligned}\langle j_1j_2|V_{\sigma\sigma'}|j_3j_4\rangle=&\frac{\delta(j_1+j_2,j_3+j_4)}{L_xL_y}\sum_{\mathbf{q}\neq\mathbf{0}}V_{\sigma\sigma'}(q)\\&\times\delta\left(j_1-j_3,\frac{q_yL_y}{2\pi}\right)\\&\times\exp\left[-\frac{q^2l^2}{2}+iq_x(X_{j_1}-X_{j_4})\right],\end{aligned}\tag{10}$$

$$\hat\rho_{\sigma\sigma'}(\mathbf{q})=\int_0^{L_x}dx\int_0^{L_y}dy\,e^{-i\mathbf{q}\cdot\mathbf{r}}\Psi_\sigma^\dagger(\mathbf{r})\Psi_{\sigma'}(\mathbf{r}),\tag{11}$$

where the sum over $\mathbf{q}$ in Eq. (6) is over ''the Brillouin zone'', $q_x=(2\pi/L_x)n_x$, $q_y=(2\pi/L_y)n_y$ $(n_x,n_y=1,2,\ldots,N_L)$, and the quantities $U_{\rm H}^{\sigma\sigma'}(\mathbf{q})$ and $U_{\rm F}^{\sigma\sigma'}(\mathbf{q})$ correspond to the Hartree and Fock potential, respectively.[19] The quantity $\Delta_{\sigma\sigma'}(\mathbf{q})$ is proportional to the expectation value of the density operator $\hat\rho_{\sigma\sigma'}(\mathbf{q})$. The two-body matrix element $\langle j_1j_2|V_{\sigma\sigma'}|j_3j_4\rangle$ in Eq. (10) is determined by the Fourier transform $V_{\sigma\sigma'}(q)$ of the Coulomb interaction, and a field operator $\Psi_\sigma(\mathbf{r})$ for pseudospin σ is considered within the subspace of the lowest Landau level. In Eq. (6) and Eq. (9), $\theta(x)$ and ε_F are the Heaviside step function and the Fermi energy, respectively, and only the N lowest-energy eigenstates in Eq. (3) contribute to the sum over α. About the quantities $U_{\rm F}^{\sigma\sigma'}(\mathbf{q})$, $U_{\rm H}^{\sigma\sigma'}(\mathbf{q})$, and $\Delta_{\sigma\sigma'}(\mathbf{q})$, it is enough to consider them only within ''the Brillouin zone'' because of their periodicity.

When the HF single-particle equation is solved, the quantity $\Delta_{\sigma\sigma'}(\mathbf{q})$ can be used to check the self-consistency of the calculated results. We have judged the convergence of the calculated ones by the following condition:

$$\delta\equiv\frac{1}{4N_L^2}\sum_{\sigma,\sigma'}\sum_{\mathbf{q}\in{\rm BZ}}|\Delta_{\sigma\sigma'}^{k+1}(\mathbf{q})-\Delta_{\sigma\sigma'}^{k}(\mathbf{q})|<10^{-6},\tag{12}$$

where k represents each iteration step. We note that our calculations have been done under the constraint that the average numbers of electrons are the same in the two layers, i.e., $\Delta_{\sigma\sigma}(\mathbf{0})=\nu/2=1/2$ for $\sigma=\uparrow,\downarrow$.[12]

In order to investigate the localization properties, we evaluate the participation ratios for the self-consistent Hartree-Fock eigenstates.[15] The participation ratio is given by

$$P_\alpha\equiv\left[L_xL_y\int_0^{L_x}dx\int_0^{L_y}dy|\varphi_\alpha(\mathbf{r})|^4\right]^{-1}\tag{13}$$

for a normalized eigenstate $\varphi_\alpha(\mathbf{r})=\langle\mathbf{r}|\varphi_\alpha\rangle$, and $P_\alpha\sim\xi_\alpha^2/L_xL_y$, where ξ_α is the localization length of an eigenstate, $\varphi_\alpha(\mathbf{r})$. As the eigenstate becomes more extended, the participation ratio becomes larger. That is, the participation ratio shows how extended the eigenstate is.

For our numerical calculations, $\Delta_{\rm SAS}$ and $e^2/\epsilon l$ can be used as the energy scales for the interlayer tunneling and the Coulomb interaction, respectively. As the unit of energy, the strength Γ of the impurity potential is used, and we consider the following cases: $\Delta_{\rm SAS}/\Gamma=0.1$, $(e^2/\epsilon l)/\Gamma=20$, and $d/l=1.2,1.5$, and 1.8. For these values of the parameters, the amplitude of the interlayer tunneling is very small and the Coulomb interaction is much stronger than the disorder potential ($e^2/\epsilon l\sim100$ K for typical GaAs samples). Thus we can compare the calculated results with the experimental ones for high mobility samples with weak interlayer tunneling.

In the process of our self-consistent calculations, we first solve Eq. (3) in the absence of Coulomb interactions, i.e., for $(e^2/\epsilon l)/\Gamma=0$, where no self-consistency is required. By increasing the ratio $(e^2/\epsilon l)/\Gamma$ gradually to the required value

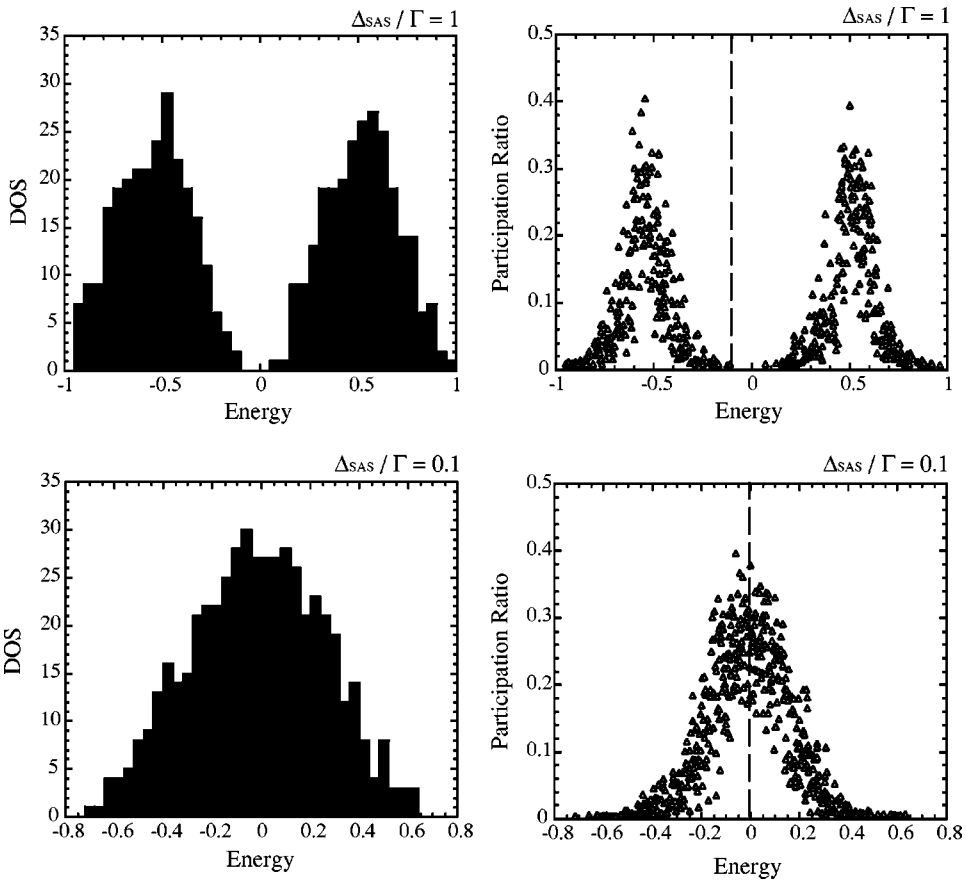

FIG. 1. The density of states (DOS) and participation ratio in the noninteracting double-layer QH system at $\nu=1$ are shown in both strong ($\Delta_{\mathrm{SAS}}/\Gamma=1$) and weak ($\Delta_{\mathrm{SAS}}/\Gamma=0.1$) interlayer-tunneling cases. In the figures of the DOS, the numbers of eigenstates within a finite width of energy, ΔE, are plotted. Each of the vertical broken lines in the figures of the participation ratio indicate the highest-energy eigenvalue among those of the occupied eigenstates. The Landau level degeneracy N_L is 256, and the energy E in the figures is given in units of Γ.

and using the latest results as input data in a new calculation step,[14] we have obtained the self-consistent solution of Eq. (3).

We also show the results in the absence of Coulomb interactions, because the comparison between them and those in the presence of interactions clarifies the importance of Coulomb interactions for the localization properties in the double-layer QH system at $\nu=1$. The calculations in the absence of interactions are done for $\Delta_{\mathrm{SAS}}/\Gamma=1$ and 0.1, i.e., in both strong- and weak-tunneling cases.

III. NUMERICAL RESULT AND DISCUSSION

A. In the absence of Coulomb interactions

We first discuss the double-layer QH system in the absence of Coulomb interactions. The localization properties in this noninteracting system were studied previously.[2,3] Although our numerical results in the noninteracting case are almost similar to previous ones,[3] we show them in Fig. 1 to make our discussion easy to understand. The importance of Coulomb interactions in double-layer QH systems can be understood by comparing these results with ones in the presence of interactions, which will be given in the next subsection.

We calculated the density of states (DOS) and participation ratio for several impurity configurations in the noninteracting double-layer QH system with the Landau level degeneracy $N_L=256$. Because the square ($L_x=L_y=L$) systems are now considered, the dimension L is given by $L=l\sqrt{2\pi N_L}\simeq 40l$ (l is the magnetic length). Figure 1 shows the result for one of these impurity configurations in the cases of $\Delta_{\mathrm{SAS}}/\Gamma=1$ and 0.1. In the noninteracting case, the parameter $\Delta_{\mathrm{SAS}}/\Gamma$ is needed to characterize the disordered double-layer QH system. The layer separation, which controls interlayer interactions, does not need to be specified in spite of its importance in the real experiments.

In the figures of the DOS in Fig. 1, the numbers of eigenstates within a finite width of energy ΔE are plotted. The width ΔE is several times as large as the average energy-level spacing. We note that the energy E in the figures is given in units of Γ all through this paper and that each of the vertical broken lines indicates the highest-energy eigenvalue among those of the occupied eigenstates.

For $\Delta_{\mathrm{SAS}}/\Gamma=1$, which corresponds to the strong interlayer-tunneling case, there exist two subbands that mainly consist of symmetric and antisymmetric combinations of isolated layer states, respectively. The localization properties within each subband are similar to those in the single-layer QH systems as pointed out previously.[2,3] That is, the eigenfunctions are extended only near the center of each disorder-broadened subband.

For $\Delta_{\mathrm{SAS}}/\Gamma=0.1$, however, the symmetric and antisymmetric subbands are not well developed in the DOS, and the localization properties are quite different from those for $\Delta_{\mathrm{SAS}}/\Gamma=1$. In fact, the peak in the participation ratios has a much broader width than that for $\Delta_{\mathrm{SAS}}/\Gamma=1$, and the participation ratios near the Fermi energy take much larger values than those for $\Delta_{\mathrm{SAS}}/\Gamma=1$.

In the previous study,[3] the finite-size scaling method was also used for the noninteracting system with weak interlayer tunneling. Then it was claimed that the extended states exist only at the two energies, which are split by somewhat larger

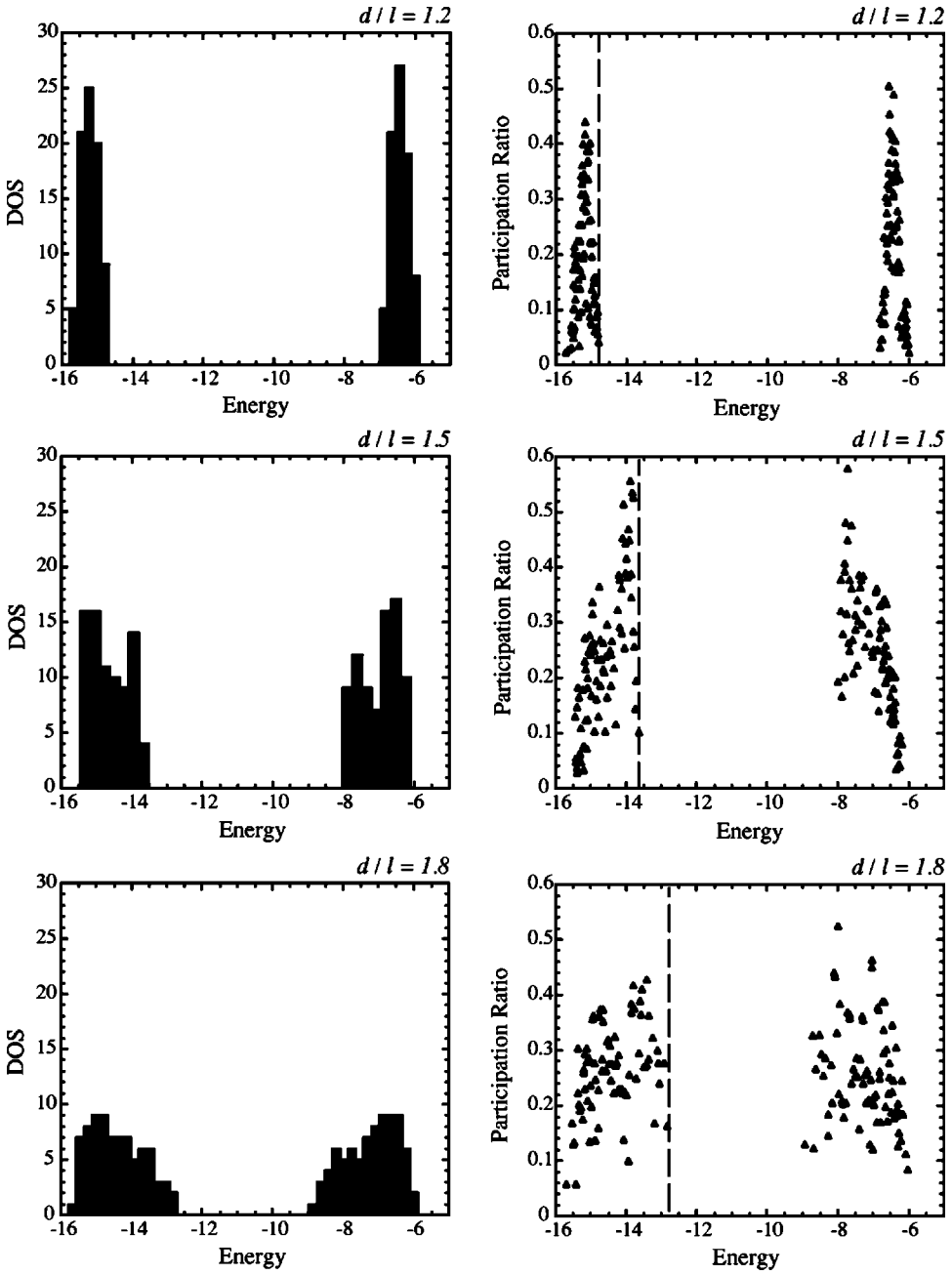

FIG. 2. The density of states (DOS) and participation ratio for the self-consistent eigenstates in the interacting double-layer QH system at $\nu=1$ are shown in the weak interlayer-tunneling case ($\Delta_{\mathrm{SAS}}/\Gamma=0.1$). As the layer separation increases, the energy gap decreases and the electronic eigenstates whose energies are close to the Fermi energy seem to become extended. The values used for the layer separation d/l are 1.2, 1.5, and 1.8, and the Landau level degeneracy N_L is 80.

than Δ_{SAS}, rather than across a band of finite width between the low- and high-energy mobility edges in the thermodynamic limit. In the energy interval between the two extended-state energies, however, their numerical values of the localization length in the weak-tunneling case are much larger than those in the strong-tunneling case. Moreover it is not clear from the system-size dependence of their results whether the extended states do not exist across a band of finite width in the thermodynamic limit either. Thus, their claim seems to be controversial as far as one judges from their numerical results. Therefore we consider the effects of the Coulomb interaction ignored in their study, and show that the extended states seem to exist only near each center of the two subbands split by the interaction.

B. In the presence of Coulomb interactions

In the presence of Coulomb interactions, it is necessary to specify the layer separation d/l, which controls interlayer interactions, as well as the interlayer tunneling amplitude $\Delta_{\mathrm{SAS}}/\Gamma$, in order to characterize the disordered double-layer QH system. In this subsection, we consider the double-layer QH system with $\Delta_{\mathrm{SAS}}/\Gamma=0.1$, i.e., the weak interlayer-tunneling case. The localization properties in this case remain unclear as seen in the previous subsection. About the layer separation, we consider the three cases of $d/l=1.2$, 1.5, and 1.8. By using these values, we can see that the localization properties change qualitatively as the layer separation increases.

We calculated the DOS and participation ratio for several impurity configurations in the double-layer QH system with the Landau level degeneracy $N_L=80$. The dimension L is then given by $L=l\sqrt{2\pi N_L}\simeq 22l$. Figure 2 shows the result for one of these impurity configurations in the cases of $\Delta_{\mathrm{SAS}}/\Gamma=0.1$, $d/l=1.2$, 1.5, and 1.8. In the figures of the DOS, the numbers of eigenstates within a finite width of energy ΔE are plotted, and the energy E in the figures is

given in units of Γ. Each of the vertical broken lines in the figures of the participation ratio indicates the highest-energy eigenvalue among those of the occupied eigenstates.

Let us first consider the case of $d/l=1.2$ in Fig. 2. We can see that the DOS has a large energy gap near the Fermi energy. The participation ratios take small values (nearly zero) at the edges of each subband, and take much larger values around the center of each subband. These localization properties are almost similar to those for $\Delta_{\mathrm{SAS}}/\Gamma=1$ in Fig. 1, and are quite different from those for $\Delta_{\mathrm{SAS}}/\Gamma=0.1$ in Fig. 1. This difference results from the electron correlation effects, and the energy gap in Fig. 2 is due to the exchange interactions between electrons. In fact, for small layer separations d/l, the ground state is a pseudospin-ferromagnetic one,[11] and the symmetric and antisymmetric combinations of isolated layer states are separated from each other by an exchange-enhanced energy gap. Then the localization properties within each subband are expected to be similar to those in the single-layer QH systems and be consistent with the observation of the integer QHE.

For larger layer separations ($d/l=1.5$,1.8), however, the localization properties become complicated. The participation ratios for the electronic eigenstates whose energies are close to the Fermi energy take larger values than those in the case of $d/l=1.2$, i.e., they are more extended than the ones for $d/l=1.2$. Moreover, the energy gap seen in the DOS seems to decrease as the layer separation increases. This decrease in the energy gap corresponds to the theoretical results from the viewpoint of the pseudospin ferromagnetism[9,10] that the pseudospin-ferromagnetic order are broken gradually with the increase of layer separation. If the energy gap continues to decrease and eventually collapses with the increase of layer separation, it is consistent with the incompressible-compressible transition reported in recent experiments.[8]

Although the size-scaling calculations are needed to make the localization properties clear, we claim the existence of the extended states only near the center of each subband in the case of $d/l=1.2$, from the following facts. One is that the dependence of such localization properties on the layer separation is consistent with the transition observed experimentally between the QHE and the non-QHE phase.[7,8] The other is that the localization properties in the case of $d/l=1.2$ are almost similar to those in the case of $\Delta_{\mathrm{SAS}}/\Gamma=1$ in Fig. 1, although the origins of the energy gap are different. We expect that our claim will be confirmed by the size-scaling calculations.

We also performed numerical calculations in the cases of $\Delta_{\mathrm{SAS}}/\Gamma=1$, $d/l=1.5$, 1.8, 2.0, and 2.3. Although these results are not shown graphically in this paper, we obtained results that are similar to those for $\Delta_{\mathrm{SAS}}/\Gamma=0.1$. The energy gap then survives for larger layer separations than that in the case of $\Delta_{\mathrm{SAS}}/\Gamma=0.1$. This corresponds to the experimental[7] and theoretical results[9,10] that the critical separation d_c increases as Δ_{SAS} increases.

Thus the exchange-enhanced energy gap under the pseudospin-ferromagnetic order appears between the two subbands, and the existence of the extended states only near the center of each subband seems to be realized for small layer separations by the electron correlation effects. Unfortunately our Hartree-Fock calculations in this paper are limited to those for small system sizes and relatively small layer separations, because it is difficult to obtain the numerical convergence for larger system sizes or larger layer separations. Therefore, for the confirmation of our claim about the localization properties in weak interlayer-tunneling cases, further numerical studies are needed for larger systems. The size-scaling calculations should especially be done to discuss the change of the localization properties quantitatively.

IV. SUMMARY

We investigated the disordered double-layer QH system at $\nu=1$ numerically, and then the Coulomb interaction was taken into consideration within the Hartree-Fock approximation. We examined the density of states and the participation ratios for the self-consistent eigenstates of the Hartree-Fock Hamiltonian. By considering the interaction effects in this disordered system, it was found that the extended states seem to exist only near each center of the two subbands split by the exchange-enhanced energy gap for small layer separations. It was shown that the eigenstates whose energies are close to the Fermi energy appear to become extended together with the decrease in the energy gap as the layer separation increases. The collapse of the energy gap expected from our results is also consistent with the incompressible-compressible transition reported in recent experiments. The change of the localization properties near the Fermi energy can especially explain the disappearance of the QHE phase for large layer separations very well.

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