Interaction between a vortex and a columnar defect in the London approximation

H. Nordborg* and V. M. Vinokur
Materials Science Division, Argonne National Laboratory, Argonne, Illinois 60469
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We calculate the interaction between a vortex and an insulating cylindrical cavity in a type-II superconductor using the London approximation, thus extending the result in an earlier work by Mkrtchyan and Shmidt (Zh. Éksp. Teor. Fiz. **61**, 367 (1971) [Sov. Phys. JETP **34**, 195 (1972)]) to an arbitrarily large cavity radius. In the limit of an infinitely large radius our result reduces to the well-known Bean-Livingston formula for the interaction between a vortex and an insulating wall.

I. INTRODUCTION

The interaction between a vortex and an insulating cylindrical cavity in a type-II superconductor was calculated in 1972 by Mkrtchyan and Shmidt[1] and was long regarded as a somewhat academic exercise in the phenomenology of the vortex state. Their result has gained importance in recent years in connection with the extensive study of high-T_c superconductors containing artificially manufactured columnar defects due to irradiation with heavy ions (see, for example Ref. 2 for a recent review). Modeling an amorphous track left by a heavy ion as an insulating cylindrical cavity, the Mkrtchyan-Shmidt maximal pinning force gives a reasonable estimate for a low-field–temperature columnar defect critical current. Making use of the full interaction between a vortex and a cavity one derives a temperature dependence of the critical current,[3,4] which agrees excellently with experiments. The work by Mkrtchyan and Shmidt, which henceforth will be referred to as Ref. 1, has thus become the basic reference to researchers working in the field of high-temperature superconductivity (HTS).

Interestingly, Ref. 1, which was completed long before the discovery of HTS, seems to be designed specifically for a heavy ion tracks as it only considers a cylinder of radius $b \ll \lambda$, where λ is the London penetration depth. Recent developments in artificially engineered pinning structures (see, for example Ref. 5) raise the question of the interaction between a vortex and a large cavity, with the radius b comparable to or even exceeding λ. The results of Ref. 1 could have been easily extended to arbitrary b if not for an unfortunate oversight in the calculations, which invalidates the result at large values of b. One can easily see, in particular, that taking the limit of an infinitely large cavity in Ref. 1 does not reproduce the Bean-Livingston barrier for a vortex interacting with a flat interface. Motivated by the quest for a general result valid in the whole range of parameters we present more general and somewhat more transparent derivation for the interaction of a vortex with a cylindrical cavity. We also demonstrate that the result of Ref. 1 for small cavities can very easily and instructively be obtained using the method of image vortices similar to Ref. 6, as has already been pointed out by Buzdin and Feinberg.[7,8] A very similar problem has been discussed by Wei *et al.*, who considered the interaction between a vortex and a cylindrical *superconducting* column with a different penetration depth.[9] There is no a simple way, however, to generalize their result into the result presented here, since the boundary conditions at the edge of the cavity are completely different.

We consider a superconductor with an infinitely long cylindrical cavity, i.e., a columnar defect, of radius b. The induction is taken to be parallel with the column, which is directed along the z axis. We thus have a two-dimensional problem and only have to solve the London equation for the z component $B(x,y)$ of the induction. We measure all lengths in units of the London penetration depth λ and measure the induction in units of $\sqrt{2}H_c = \Phi_0/2\pi\lambda\xi$, where H_c is the thermodynamic critical field, Φ_0 is the flux quantum, and ξ is the superconducting coherence length. The London equation for the magnetic induction from a vortex is then given by

$$B - \Delta B = \frac{2\pi}{\kappa}\delta^{(2)}(\mathbf{r}-\mathbf{r}_0), \tag{1}$$

where $\kappa = \lambda/\xi$ is the Ginzburg parameter and $\mathbf{r}_0$ is the position of the vortex. The boundary condition for the cavity is

$$\mathbf{J}_s \cdot \mathbf{n} = 0, \tag{2}$$

i.e., the supercurrent is tangential to the boundary. Since the current is given by the curl of the magnetic induction, it follows that $B(\mathbf{r})$ is constant on the boundary of the cavity. From the fact that B is a harmonic function inside the cavity, it then follows that the induction is constant, $B(\mathbf{r}) = B_0$, inside the entire cavity.

II. SOLVING THE LONDON EQUATION

We take the center of the column to be the origin of our coordinate system and use polar coordinates (r,ϕ). It is convenient to split the induction in two parts

$$B(r,\phi) = B_1(r) + B_2(r,\phi), \tag{3}$$

where B_1 is the radially symmetric field due to the columnar defect and B_2 is the contribution from the vortex. The field B_1 satisfies the homogeneous London equation

$$\frac{\partial^2 B_1}{\partial r^2} + \frac{1}{r}\frac{\partial B_1}{\partial r} - B_1 = 0, \tag{4}$$

with the boundary condition $B_1(b)=B_0$. The solution is easily found to be

$$B_1(r)=B_0\frac{K_0(r)}{K_0(b)}, \quad r\geqslant b. \tag{5}$$

The field B_2 is the solution to the inhomogeneous equation

$$\frac{\partial^2 B_2}{\partial r^2}+\frac{1}{r}\frac{\partial B_2}{\partial r}+\frac{1}{r^2}\frac{\partial^2 B_2}{\partial \phi^2}-B_2=-\frac{2\pi}{\kappa r}\delta(\phi)\delta(r-r_0), \tag{6}$$

with the boundary condition $B_2(b,\phi)=0$. This equation is solved using separation of variables

$$B_2(r,\phi)=\sum_n \tilde{B}_n(r)e^{in\phi}, \tag{7}$$

and the solution has the form

$$\tilde{B}_n(r)=\alpha_n I_n(r)+\beta_n K_n(r), \quad b\leqslant r\leqslant r_0, \tag{8a}$$

$$\tilde{B}_n(r)=\gamma_n K_n(r), \quad r_0\leqslant r. \tag{8b}$$

In order to find the constants α_n, β_n, and γ_n we use the fact that $\tilde{B}_n(b)=0$, $\tilde{B}_n(r)$ should be continuous at $r=r_0$, and the discontinuity in the derivate of $\tilde{B}_n(r)$ due to the δ function is given by

$$\left.\frac{d\tilde{B}_n}{dr}\right|_{r_0+}-\left.\frac{d\tilde{B}_n}{dr}\right|_{r_0-}=-\frac{1}{\kappa r_0}. \tag{9}$$

Solving for the constants, we find the solution

$$\tilde{B}_n(r)=\frac{1}{\kappa}K_n(r_0)\left\{I_n(r)-\frac{I_n(b)K_n(r)}{K_n(b)}\right\}, \quad b\leqslant r\leqslant r_0, \tag{10a}$$

$$\tilde{B}_n(r)=\frac{1}{\kappa}K_n(r)\left\{I_n(r_0)-\frac{I_n(b)K_n(r_0)}{K_n(b)}\right\}, \quad r_0\leqslant r. \tag{10b}$$

In order to better understand the solution for the field B_2, and to obtain a form which can be implemented numerically, we compare it to the solution for a vortex at $\mathbf{r}_0$ without the cavity. We call this solution $B^v(r,\phi)$ and find

$$B^v(r,\phi)=\sum_n \tilde{B}_n^v(r)e^{in\phi} \tag{11}$$

with

$$\tilde{B}_n^v(r)=\frac{1}{\kappa}K_n(r_0)I_n(r), \quad 0\leqslant r\leqslant r_0, \tag{12a}$$

$$\tilde{B}_n^v(r)=\frac{1}{\kappa}I_n(r_0)K_n(r), \quad r_0\leqslant r. \tag{12b}$$

On the other hand, we know that the field from the vortex at $\mathbf{r}_0$ is given by

$$B^v(\mathbf{r})=\frac{1}{\kappa}K_0(|\mathbf{r}-\mathbf{r}_0|). \tag{13}$$

A comparison of Eqs. (10) and (12) shows that we can write

$$B_2(\mathbf{r})=\frac{1}{\kappa}K_0(|\mathbf{r}-\mathbf{r}_0|)+B_2^c(r,\phi), \tag{14}$$

where B_2^c, which represents the modification of the vortex field due to the column, is given by

$$B_2^c(r,\phi)\equiv B_2^c(r,\phi;r_0)=\sum_n \tilde{B}_n^c(r)e^{in\phi}, \tag{15}$$

where

$$\tilde{B}_n^c(r)=-\frac{1}{\kappa}\frac{I_n(b)K_n(r_0)}{K_n(b)}K_n(r), \quad r_0>b. \tag{16}$$

The advantage of writing the field $B_2(\mathbf{r})$ in the form of Eq. (14) is that the divergence at $\mathbf{r}=\mathbf{r}_0$ is taken care of explicitly and that the term $B_2^c(r,\phi;r_0)$ is well defined for all $r>b$. It follows directly from Eq. (16), or from the fact that $B_2(b,0)=0$, that for $\mathbf{r}_0=(b,0)$ we have

$$B_2^c(r,\phi;b)=-\frac{1}{\kappa}K_0[|\mathbf{r}-(b,0)|]. \tag{17}$$

This property will be useful when computing the energy.

In order to find the induction inside the cavity we use the fact that the order parameter has to be single valued. This leads to the usual condition for flux quantization (see, e.g., Ref. 10),

$$\nabla\times\mathbf{B}=\kappa^{-1}\nabla\theta-\mathbf{A}, \tag{18}$$

where θ is the phase of the order parameter and $\mathbf{A}$ is the vector potential. Integrating this along the perimeter of the cavity we obtain

$$b\int d\phi(\nabla\times\mathbf{B})_\phi=\frac{2\pi q}{\kappa}-\pi b^2 B_0, \tag{19}$$

where q is the number of flux quanta in the cavity. The integral over ϕ can easily be computed using

$$(\nabla\times\mathbf{B})_\phi=-\frac{\partial B}{\partial r}. \tag{20}$$

and the fact that only the terms with $n=0$ survive. From this we obtain

$$B_0=\frac{1}{\kappa b}\frac{K_0(r_0)+qK_0(b)}{K_1(b)+\frac{1}{2}bK_0(b)}, \tag{21}$$

which is identical to Eq. (8) in Ref. 1.

III. COMPUTING THE INTERACTION ENERGY

The next task is to compute the energy of the system, which in the London approximation is given by

$$\mathcal{F}=\int d^2r\{\mathbf{B}^2+(\nabla\times\mathbf{B})^2\}, \tag{22}$$

where the energy is measured in units of $B_c^2\lambda^2/4\pi$, lengths in units of λ, and the induction in units of $\sqrt{2}B_c$. Here, and in the following, we always refer to the energy per unit length of the vortex. The integral (22) can be evaluated directly over the area inside the cavity and yields

$$\mathcal{F}_{cavity}=\pi b^2 B_0^2. \tag{23}$$

In order to evaluate the integral for the outside region, we rewrite it according to

$$\mathcal{F}=\int d^2r\,\mathbf{B}\cdot\{\mathbf{B}+\nabla\times(\nabla\times\mathbf{B})\}+\oint d\mathbf{S}\cdot\mathbf{B}\times(\nabla\times\mathbf{B}). \tag{24}$$

The contribution from the volume integral in Eq. (24) follows directly from the London equation, and we find

$$\int d^2r\,\mathbf{B}\cdot\{\mathbf{B}+\nabla\times(\nabla\times\mathbf{B})\}=\frac{2\pi}{\kappa}B(r_0,0). \tag{25}$$

The surface integral can be rewritten as

$$\oint d\mathbf{S}\cdot\mathbf{B}\times(\nabla\times\mathbf{B})=-\frac{b}{2}\frac{\partial}{\partial r}\int d\phi B^2(r,\phi). \tag{26}$$

Performing first the integration over ϕ we find

$$\int d\phi B^2(r,\phi)=2\pi\left\{[B_1(r)+\tilde{B}_0(r)]^2+\sum_{n\neq 0}\tilde{B}_n(r)^2\right\}. \tag{27}$$

Using the fact that

$$\tilde{B}_n(b)=0,\quad \left.\frac{\partial\tilde{B}_n(r)}{\partial r}\right|_{r=b}=\frac{K_n(r_0)}{\kappa b K_0(b)}, \tag{28}$$

we arrive at

$$\oint d\mathbf{S}\cdot\mathbf{B}\times(\nabla\times\mathbf{B})=\frac{2\pi B_0}{\kappa K_0(b)}[\kappa b B_0 K_1(b)-K_0(r_0)]. \tag{29}$$

Adding all the pieces together we obtain the final result

$$\mathcal{F}=\pi b^2 B_0^2+\frac{2\pi B_0}{\kappa K_0(b)}[\kappa b B_0 K_1(b)-K_0(r_0)]+\frac{2\pi}{\kappa}B(r_0,0). \tag{30}$$

The latter formula can be simplified further by inserting the expressions for B_0 and $B(\mathbf{r})$. After some straightforward algebra we arrive at

$$\mathcal{F}=\frac{2\pi}{\kappa^2 b K_0(b)}\frac{[K_0(r_0)+qK_0(b)]^2}{K_1(b)+\frac{1}{2}bK_0(b)}+\frac{2\pi}{\kappa^2}K_0(\kappa^{-1})+\frac{2\pi}{\kappa}B_2^c(r_0,0;r_0), \tag{31}$$

where we have cut off the divergence of $B_2(r_0,0)$ at a distance κ^{-1}. The first term in this expression differs significantly from the result given in Eq. (10) in Ref. 1 if the condition $b\ll 1$ is not fulfilled. Taking the limit of large b, with $q\propto b^2$, one sees that the induction and the energy take the form

$$B_0=\frac{2q}{\kappa b^2},\quad \mathcal{F}=\pi b^2 B_0^2, \tag{32}$$

as should be expected. It is easy to derive the result for a cavity containing q flux quanta, without an external vortex. In this case we obviously find a rotationally symmetric solution with the induction given by

$$B(r)=B_0\frac{K_0(r)}{K_0(b)},\quad B_0=\frac{1}{\kappa b}\frac{qK_0(b)}{K_1(b)+\frac{1}{2}bK_0(b)} \tag{33}$$

and the energy

$$\mathcal{F}=\frac{2\pi q^2}{\kappa^2 b}\frac{K_0(b)}{K_1(b)+\frac{1}{2}bK_0(b)} \tag{34}$$

It follows from Eq. (31) and from the observation

$$B_2^c(b,0;b)=-\frac{1}{\kappa}K_0(\kappa^{-1}), \tag{35}$$

that the energy for a vortex at a position $\mathbf{r}=\mathbf{r}_0$ and a cavity containing q flux quanta smoothly changes into the result for a cavity with $q+1$ flux quanta as the vortex enters the cavity, i.e., $r_0\to b$.

The potential for a vortex interacting with a column containing q flux quanta is obtained directly from Eq. (31) by dropping all terms which do not involve r_0. We find

$$U_{VC}(r)=\frac{2\pi}{\kappa^2 b K_0(b)}\frac{K_0(r)^2+2qK_0(b)K_0(r)}{K_1(b)+\frac{1}{2}bK_0(b)}-\frac{2\pi}{\kappa^2}\sum_n\frac{I_n(b)}{K_n(b)}K_n^2(r)\quad r>b, \tag{36}$$

where r is the distance from the vortex to the center of the column. The interaction potential (36) is our main result and gives the interaction of a vortex with an insulating column of radius b containing q flux quanta, where both b and q can be arbitrarily large. The potential $U_{VC}(R)$ is plotted in Fig. 1 for a number of different values of b. It vanishes exponentially fast at large distances, i.e., for $r\ll 1$.

IV. SOLUTION USING IMAGE VORTICES

If $b,r_0\ll 1$, the problem can be solved easily using image vortices, transcribing a problem from the textbook by Landau and Lifshits[6] for a charged line in an infinite dielectric near a cylindrical cavity with different dielectric constant, as has first been pointed out by Buzdin and Feinberg.[7,8] Assuming the magnetic induction from a vortex to decay logarithmically, we can satisfy the boundary conditions for the cavity by placing $q+1$ positive vortices in the center and a negative vortex at the distance $l=b^2/r_0$ from the center. The magnetic induction is then given by

$$B(\mathbf{r})=-\frac{1}{\kappa}[(q+1)\ln(|\mathbf{r}|)+\ln(|\mathbf{r}-\mathbf{r}_0|)-\ln(|\mathbf{r}-\mathbf{l}|)], \tag{37}$$

where $\mathbf{l}=(b^2/r_0,0)$. The energy of the system can then be computed by integration as above and becomes particularly

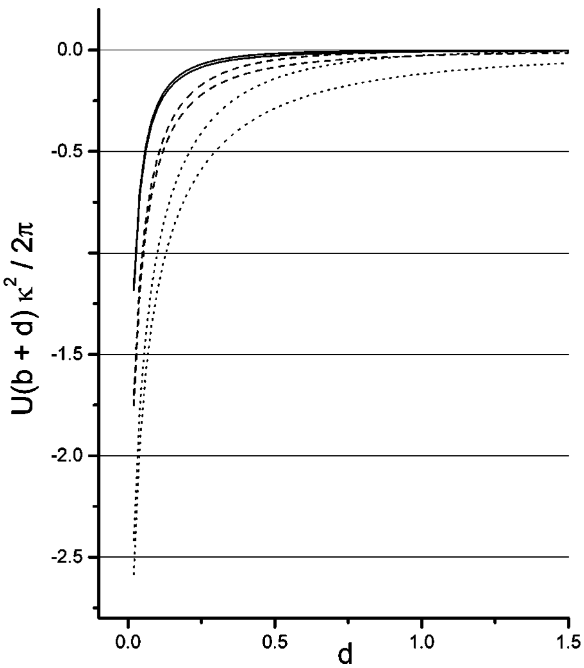

FIG. 1. The vortex-column potential for a number of different values of b: $b=0.1$ (solid lines), $b=0.2$ (dashed lines), and $b=0.5$ (dotted lines). The upper curve corresponds to the exact solution $U_{VC}(r)$ and the lower curve to the potential $U_{IV}(r)$ obtained using the method of image vortices in all three cases.

simple in the case of $q=0$, since the surface integral vanishes. We then find the energy

$$\mathcal{F}=\frac{2\pi}{\kappa^2}\left\{\ln\left(1-\frac{b^2}{r_0^2}\right)-\ln(\kappa^{-1})\right\}. \tag{38}$$

The interaction is again given by the r_0 dependent part of Eq. (38) and we finally obtain[4]

$$U_{IV}(r)=\frac{2\pi}{\kappa^2}\ln\left(1-\frac{b^2}{r^2}\right). \tag{39}$$

The interaction potential (39) is also shown in Fig. 1 and compares favorably with the exact result for $b\lesssim 0.2$. In the case of larger columns, the image vortex solution produces a too long-ranged interaction. Note in particular that the $U_{IV}(r)$ approaches zero as $(b/r)^2$, whereas the exact potential $U_{VC}(r)$ decays exponentially for large distances.

V. BEAN-LIVINGSTON BARRIER

The main problem with the result in Ref. 1 is that it fails to describe the limit of a large column. Sending the column radius to infinity, one expects to obtain the interaction between a vortex and a flat insulating wall, i.e., the Bean-Livingston barrier. We show below that this is indeed the case for the interaction in Eq. (36). For the sake of completeness, however, we first derive the Bean-Livingston barrier using methods similar to those above. Consider an interface to an insulator, which we take to coincide with the y axis and a vortex at a distance d from the wall at position $\mathbf{r}=(d,0)$. The symmetry of the problem makes it possible to solve the London equation exactly using image vortices and we find the induction

$$B(x,y)=He^{-x}+\frac{1}{\kappa}\{K_0(\sqrt{(x-d)^2+y^2})-K_0(\sqrt{(x+d)^2+y^2})\}, \tag{40}$$

for $x>0$. The energy can again be calculated using partial integration. The bulk term gives the contribution

$$\mathcal{F}_{bulk}=\frac{2\pi}{\kappa}B(d,0)=\frac{2\pi}{\kappa}He^{-d}+\frac{2\pi}{\kappa^2}K_0(\kappa^{-1})-\frac{2\pi}{\kappa^2}K_0(2d), \tag{41}$$

where we again have cut off the divergence at a distance κ^{-1}. The surface term is simply an integral along the y axis and we find

$$\begin{aligned}\oint d\mathbf{S}\cdot\mathbf{B}\times(\nabla\times\mathbf{B})&=-\int dy\, B\partial_x B\\ &=\frac{2Hd}{\kappa}\int dy\,\frac{K_1(\sqrt{d^2+y^2})}{\sqrt{d^2+y^2}}\\ &=\frac{2\pi H}{\kappa}e^{-d}.\end{aligned} \tag{42}$$

The integral was computed using the substitution $y=d\sinh(x)$ and Eq. (6.6648) in Ref. 11. Adding all the parts we find energy

$$\mathcal{F}_{BL}=\frac{4\pi}{\kappa}He^{-d}+\frac{2\pi}{\kappa^2}K_0(\kappa^{-1})-\frac{2\pi}{\kappa^2}K_0(2d). \tag{43}$$

Dropping irrelevant terms, we arrive at the well-known Bean-Livingston barrier,

$$U_{BL}(d)=\frac{4\pi}{\kappa}He^{-d}-\frac{2\pi}{\kappa^2}K_0(2d), \tag{44}$$

which we have normalized so that it vanishes for large d. This well-known result can be found in many textbooks on superconductivity (see, e.g., Ref. 12).

VI. INFINITELY LARGE COLUMN RADIUS

We now turn to the case of an infinitely large cavity and show that this reproduces the Bean-Livingston result above. We want to have a finite induction in the cavity and we therefore need a large number of flux quanta, $q\propto b^2$. Furthermore, we want the distance of the vortex from the edge of the cavity to be finite and write $r_0=b+d$. Inserting this into Eq. (36) and discarding irrelevant terms we find

$$U_{VC}(b+d)\approx\frac{8\pi q}{\kappa^2b^2}\frac{K_0(b+d)}{K_0(b)}+\frac{2\pi}{\kappa}B_2^c(b+d,0). \tag{45}$$

Using the fact that we have $B_0\approx 2q/\kappa b^2$, we can write this as

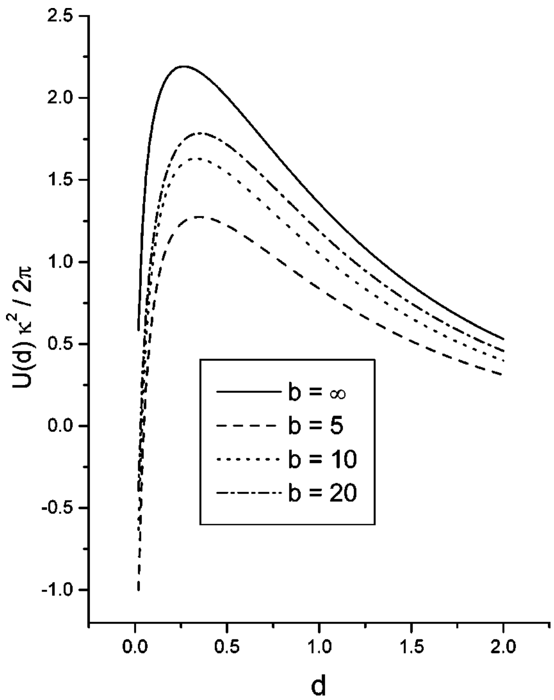

FIG. 2. Comparison of the Bean-Livingston barrier (solid line) with the vortex-column interaction for large columns ($b=5,10,20$). We always have $q=b^2$ flux quanta in the column (in dimensionless quantities), producing a magnetic induction of $B \approx q\Phi_0/\pi b^2$ in the limit of large columns. The convergence to the BL result is slow, but already the smallest column shows the correct characteristic shape.

$$U_{VC}(b+d) \approx \frac{4\pi}{\kappa} B_0 \frac{K_0(b+d)}{K_0(b)} + \frac{2\pi}{\kappa} B_2^c(b+d,0). \tag{46}$$

For the first term we have the approximation

$$\frac{4\pi}{\kappa} B_0 \frac{K_0(b+d)}{K_0(b)} \approx \frac{4\pi}{\kappa} B_0 e^{-d} \sqrt{\frac{b}{b+d}} \approx \frac{4\pi}{\kappa} B_0 e^{-d}, \tag{47}$$

in agreement with Eq. (44). It thus remains to show that

$$\lim_{b\to\infty} \kappa B_2^c(b+d,0) = -K_0(2d). \tag{48}$$

We did not find a simple proof of Eq. (48), but the result can be understood in the following manner: We begin by observing that

$$K_0(2d) = \sum_n I_n(b) K_n(b+2d) \tag{49}$$

for any b. It then follows that

$$K_0(2d) + \kappa B_2^c(b+d,0) = \sum_n I_n(b) K_n(b+2d) \left\{ 1 - \frac{K_n(b+d)^2}{K_n(b) K_n(b+2d)} \right\}. \tag{50}$$

Inserting the lowest order asymptotic expansion we find

$$1 - \frac{K_n(b+d)^2}{K_n(b) K_n(b+2d)} \approx \frac{1}{2} \left(\frac{d}{b} \right)^2, \tag{51}$$

which shows that the right-hand side of Eq. (50) vanishes in the limit $b \to \infty$, a result which we have also verified numerically. In Fig. 2 we plot the Bean-Livingston barrier (43) compared to the column-vortex interaction (36) for a number of different column sizes. The convergence is fairly slow, but already the column of the size $b=5$ shows the typical Bean-Livingston shape.

VII. CONCLUSION

We have derived the interaction between a vortex and an arbitrarily large cylindrical cavity in a type-II superconductor using the London approximation. In the limit of an infinitely large cavity, the interaction reduces to the well-known Bean-Livingston result. Our work extends an earlier work on the same subject.

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*Present address: ABB Corporate Research Ltd., 5405 Baden-Dättwil, Switzerland.

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