Magnetic exchange coupling through superconductors: A trilayer study

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The possibility of magnetic exchange coupling between two ferromagnets (F) separated by a superconductor (S) spacer is analyzed using the functional integral method. For this coupling to occur three *prima facie* conditions need to be satisfied. First, an indirect exchange coupling between the ferromagnets must exist when the superconductor is in its normal state. Second, superconductivity must not be destroyed due to the proximity to ferromagnetic boundaries. Third, roughness of the F/S interfaces must be small. Under these conditions, when the superconductor is cooled to below its critical temperature, the magnetic coupling changes. The appearance of the superconducting gap introduces a new length scale (the coherence length of the superconductor) and modifies the temperature dependence of the indirect exchange coupling existent in the normal state. The magnetic coupling is oscillatory both above and below the the critical temperature of the superconductor, as well as strongly temperature-dependent. However, at low temperatures the indirect exchange coupling decay length is controlled by the coherence length of the superconductor, while at temperatures close to and above the critical temperature of the superconductor the magnetic coupling decay length is controlled by the thermal length.

I. INTRODUCTION

The problem of magnetic coupling in ferromagnetic metal/normal-metal multilayers have received considerable attention in recent years, both on the experimental side[1–3] and on the theoretical side.[4,5] The main features that have emerged from these experiments were associated with an indirect exchange coupling between the ferromagnetic layers via the normal-metal host. An oscillatory coupling as a function of the thickness of the normal-metal spacer was ubiquitously observed in several multilayered systems.[6]

The prevailing experimental evidence indicates that the exchange coupling with metal spacers is short ranged, i.e., the magnetic coupling can only be observed across a layer of thickness 10 to 130 Å,[7] i.e., in the nanometer range. Thus, the key question, for both theory and experiment, is the following. Can such a thin metallic layer survive pair breaking effects of the ferromagnetic layers from both sides and yet remain superconducting? It is the purpose of this paper to show that appropriate choices of superconductor and ferromagnet lead to the survival of superconductivity and to effects on the magnetic coupling. Presently there are no experimentally known multilayered systems that show magnetic coupling both above and below the critical temperature T_c of the superconductor. The main reasons for that are that the required conditions are difficult to achieve. The desirable superconductor should have high critical temperature and short coherence length, while the desirable ferromagnet should be metallic with not so large pair breaking effects. Furthermore, the desirable F/S interfaces should be atomically flat and well lattice matched to avoid the effects of roughness and strain.[8] Thus, ideal systems to study may be high-T_c superconductor/colossal magnetoresistance ferromagnet multilayers such as $Nd_{2-x}Ce_xCuO_4/La_{1-y}Sr_yMnO_3$ or $Nd_{2-x}Ce_xCuO_4/La_{3-y}Sr_yMn_2O_7$ (for the s-wave case) and $YBa_2Cu_3O_{7-\delta}/La_{1-y}Sr_yMnO_3$ or $YBa_2Cu_3O_{7-\delta}/La_{3-y}Sr_yMn_2O_7$ (for the d-wave case).[8] In addition to these, a possible class of systems for the observation of such effects may be the layered rutheno-cuprate family,[9–12] where the interplay between superconductivity and magnetism is very important. If the number of cuprate planes can be systematically changed then magnetic coupling between ruthenate planes and its consequences can be systematically studied as a function of the number of cuprate layers. Furthermore, if the number of ruthenate layers can be systematically changed, then the Josephson coupling between cuprate planes and its consequences can be systematically studied as a function of the number of ruthenate layers.

If the superconducting metal, in its normal state, allows an indirect exchange coupling between ferromagnetic layers, it is valid to ask the following questions. First, how does the presence of ferromagnetic layers affect the superconductivity of the spacer? Second, does anything dramatic happen to the magnetic coupling when the system is cooled through the superconducting transition temperature of the spacer? Third, what happens to the magnetic coupling at very low temperatures when the superconductivity of the spacer is well established? Fourth, what are the effects of interfacial roughness? These questions are the central topic of this paper, and they are intended to establish the conditions under which magnetic coupling in $F/S/F$ multilayers should exist.

The recent published literature on F/S multilayers has focused mostly on the changes of the critical temperature T_c of the superconductor[13–16] as a function of thickness of the ferromagnetic layers. The experimental reports have been mixed. In the cases of Nb/Gd multilayers[15,16] the observation of a nonmonotonic T_c has been attributted either to a change in the underlying pair breaking mechanism with increasing thickness of the Gd layer[15] or to evidence for the predicted π-phase coupling[17–19] as a function of thickness of the Gd layer.[16] More recently, there have been experimental attempts to observe magnetic coupling in $F/S/F$ multilayer systems $Fe_4N/NbN/Fe_4N$, Ni/Nb/Ni, and GdN/NbN/GdN.[20,21] No magnetic coupling was observed even when the superconductor was in its normal state. Thus indicating that the effects of interfacial roughness and strains in these systems may be strong, unlike in the more usual multilayers.[1–3,6,7] However, in GdN/W/NbN/W (100), coexistence of superconductivity and magnetism was reported down to layer thicknesses of 40 and 22 Å of NbN and GdN, respectively,[22] but again no magnetic interlayer coupling was observed. Very recently, however, oscillatory magnetic coupling and magnetoresistive effects were observed in the oxide based superlattice (TiN/Fe_3O_4) (Ref. 23) and injected new excitement into the question of magnetic coupling and magnetotransport in oxide based layered materials.

On the theoretical side, the questions raised in this paper have been only preliminarily addressed previously by the author.[8] The only other theoretical work relevant to these questions that has been published so far, to the best knowledge of the author, is the pioneering work of Sipr and Gyorffy.[24] They analyzed *numerically* the possibility of magnetic coupling through a superconductor at zero temperature, without solving for the gap equation self-consistently. The work presented here distinguishes itself from the work of Sipr and Gyorffy in the following ways. The present work shines light on the *temperature* dependence of the magnetic coupling through the superconductor (both at $T\approx 0$ and $T\approx T_c$), and the results obtained here are mostly *analytical* in contrast with the *numerical* work of Sipr and Gyorffy.[24] In addition, the spatially averaged self-consistent gap equation is solved in the asymptotic limit of larger spacer thickness, and the spatially averaged order parameter is used to estimate the magnetic coupling.[25–27]

The rest of the paper is divided as follows. In Sec. II, basic assumptions and model are described. In Sec. III, the functional integral method is discussed and the supression of the critical temperature T_c of the superconductor is analyzed. In Sec. IV, the formalism to calculate the magnetic coupling is illustrated. In Sec. V, analytical results both close to T_c and at $T=0$ are presented and discussed. The limitations of the approach and the effects of roughness are also mentioned. In Sec. VI, the main results are discussed and summarized. In Sec. VII, final comments and new directions are pointed out. In Appendix A details about the formal derivation of the magnetic coupling are given. In appendix B an outline of the explicit evaluation of the magnetic coupling is presented. Finally, in Appendix C an approximate form for the dependence of the averaged order parameter on spacker thickness is discussed.

II. BASIC ASSUMPTIONS AND MODEL

For the purpose of answering the questions raised in Sec. I, it is important to choose a physical system and outline the underlying assumptions. The system chosen here is a trilayer consisting of two identical ferromagnets of thickness d_F separated by a superconductor of thickness d_s. The ferromagnetic layer is labeled the F layer, and the trilayer system will be referred as $F/S/F$, when the spacer is superconducting (S layer) and $F/N/F$, when the spacer is in its normal state (N layer). In Fig. 1, the trilayer system is shown.

The underlying assumptions are as follows. It is assumed that $d_F \gg d_s$, such the ferromagnetic layers can be treated as semi-infinite. When the spacer is in its normal state (N layer), it is assumed that an indirect exchange coupling exists between the ferromagnetic layers (F layers). It is further assumed that the critical temperature T_c of the superconductor is much smaller than the Curie temperature T_f of the ferromagnet, such that fluctuation effects on the magnetism are negligible. Furthermore, the F layers are weak ferromagnets and good metals, the superconductor is assumed to be s-wave BCS type and the F/S interface is assumed to be smooth, i.e., atomically flat.

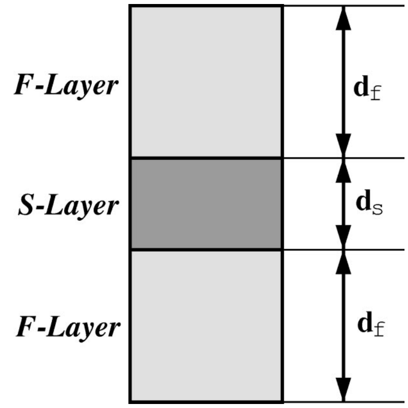

FIG. 1. Illustration of the trilayer system. The superconducting metal layer (S layer) is the spacer sandwiched between identical ferromagnetic layers (F layers). The thickness of the S layer is d_s, and the thickness of each F layer is $d_F \gg d_s$.

Under all these assumptions, the Hamiltonian density is written as

$$H = H_S + H_{SF}, \tag{1}$$

where H_S contains kinetic and potential energy terms for the trilayer system, and the attractive interaction term inside the superconductor, i.e.,

$$H_S = \overline{\Psi}_\alpha(r)\{[K + U(\mathbf{r})]\delta_{\alpha\gamma}\}\Psi_\gamma(r) + V, \tag{2}$$

and H_{SF} is the part of the total Hamiltonian describing the effects of the ferromagnetic boundaries,

$$H_{SF} = \overline{\Psi}_\alpha(r)\{(H_c)_{\alpha\gamma}\}\Psi_\gamma(r), \tag{3}$$

i.e, the coupling between ferromagnets and superconductor.

The indices α and γ indicate spin components and repeated indices indicate summation. The kinetic energy term is $K = (i\nabla)^2/2m - \mu$, while the potential energy is $U(\mathbf{r}) = U_0 F(r)$, where $F(r) = [\Theta(x - d_s/2) + \Theta(-x - d_s/2)]$. Here U_0 can be positive or negative.[28] The only exchange interaction considered in the ferromagnetic layers is between the spins of itinerant electrons, but a real space representation of the exchange interaction is used given that the $F/S/F$ system is inhomogeneous. Thus, the exchange interaction $(H_c)_{\alpha\gamma}$ is more transparently written as

$$(H_c)_{\alpha\gamma} = -[(\sigma_x)_{\alpha\gamma} h_x(r) + (\sigma_y)_{\alpha\gamma} h_y(r) + (\sigma_z)_{\alpha\gamma} h_z(r)], \tag{4}$$

where $h_i(r) = \int dr' J_i(r,r') S_i(r') F(r)$ is an effective exchange field felt only within the boundaries of the ferromagnet. Here, the spin variable $S_i(r') = \psi^\dagger_\mu(r')(\sigma_i)_{\mu\nu}\psi_\nu(r')$.[29] For $T_f \gg T_c$, and $|h_z| \gg \max(|h_y|,|h_x|)$, the F layers have neg-

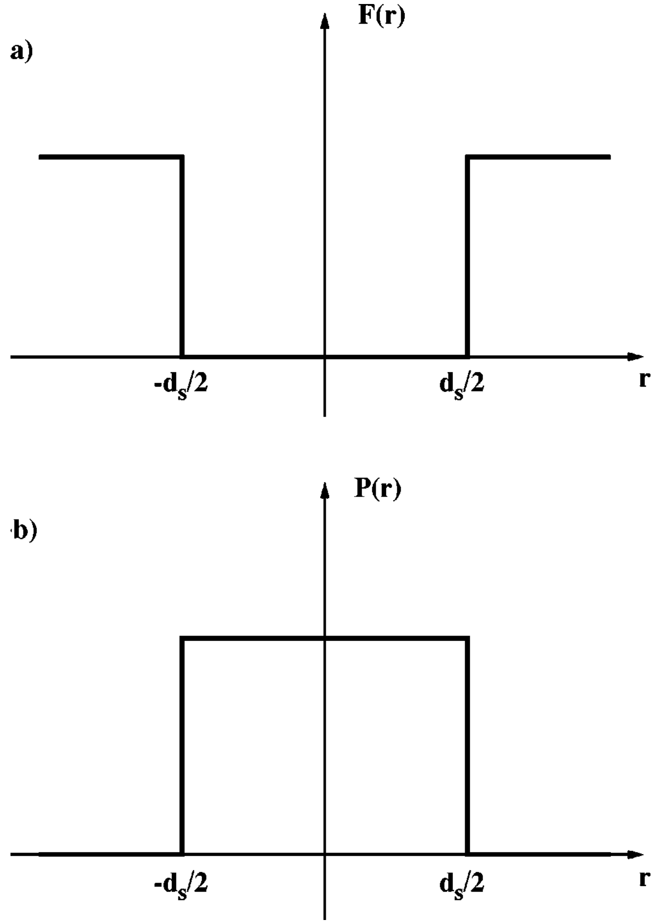

FIG. 2. (a) The function $F(r)=[\Theta(x-d_s/2)+\Theta(-x-d_s/2)]$ appearing in the single particle potential $U(r)$ is shown. (b) The function $P(r)=1-F(r)$ appearing in the interaction energy V is illustrated.

ligible magnetization fluctuations at low T, and an easy axis along the z axis. Thus, $h_z(r)$ can be replaced by its average value $\langle h_z(r)\rangle = J\langle S_z\rangle F(r)$. The interaction energy in the superconductor is

$$V=-g\bar{\Psi}(r)_\uparrow\bar{\Psi}(r)_\downarrow\Psi(r)_\downarrow\Psi(r)_\uparrow P(r) \tag{5}$$

resulting from a delta function contact attraction, where $P(r)=[1-F(r)]$. This attractive interaction leads to an s-wave superconductor. In Fig. 2 the functions $F(r)$ entering the single particle potential $U(r)$, and the function $P(r)$ entering the interaction energy V are illustrated.

III. METHOD AND T_c SUPPRESSION

The functional integral method is used here to estimate both the change in T_c due to the proximity effect to F layers and the indirect exchange coupling between the F layers. To estimate the change in T_c of the superconductor one needs to worry only about the *restricted* partition function Z at temperature $T=\beta^{-1}$, which can be written as a functional integral[30] with action

$$S=\int_0^\beta d\tau\int d\mathbf{r}[\bar{\Psi}_\alpha(r)\partial_\tau\Psi_\gamma(r)-H_S-H_{SF}], \tag{6}$$

where $r=(\mathbf{r},\tau)$ and $\hbar=k_B=1$. The Hubbard-Stratonovich pair field $\Delta(\mathbf{r},\tau)$ is introduced and upon integration over the fermionic variables the partition function $Z=\int D\Delta D\bar{\Delta}\exp(-S_{\rm eff}[\Delta,\bar{\Delta}])$. The effective action is

$$S_{\rm eff}[\Delta,\bar{\Delta}]=\int_0^\beta d\tau\left\{\frac{|\Delta(r)|^2}{g}-\frac{1}{\beta}\mathrm{Tr}\ln\beta G^{-1}[\Delta(r),\bar{\Delta}(r)]\right\}. \tag{7}$$

The inverse Nambu propagator is

$$G^{-1}=-\partial_\tau-[K+U(\mathbf{r})+(H_c)_\sigma]\sigma_z+\Delta(r)\sigma^+ +\bar{\Delta}(r)\sigma^-, \tag{8}$$

where $\sigma^\pm=(\sigma_x\pm i\sigma_y)/2$, with σ_j being the Pauli matrices. In a bulk superconductor the critical temperature is obtained by solving simultaneously the gap equation and the number equation. The gap equation is obtained from $\delta S_{\rm eff}/\delta\Delta=0$ leading to

$$1/g=\sum_{\mathbf{k}}\tanh(\xi_{\mathbf{k}}/2T_c)/2\xi_{\mathbf{k}}, \tag{9}$$

where $\xi_{\mathbf{k}}=\varepsilon_{\mathbf{k}}-\mu$ and $\varepsilon_{\mathbf{k}}=|\mathbf{k}|^2/2m$. Using the s-wave scattering length a_s defined by the two-body problem in vacuum via $m/4\pi a_s=-1/g+\Sigma_k(2\varepsilon_k)^{-1}$ the gap equation may be written as

$$-\frac{m}{4\pi a_s}=\sum_{\mathbf{k}}\left[\frac{\tanh(\xi_{\mathbf{k}}/2T_c)}{2\xi_{\mathbf{k}}}-\frac{1}{2\varepsilon_{\mathbf{k}}}\right]. \tag{10}$$

The number equation is $N=-\partial\Omega/\partial\mu$, where $\Omega=S_{\rm eff}[\Delta=0]$ in the weak coupling BCS limit, leading to $n=\Sigma_{\mathbf{k}}[1-\tanh(\xi_{\mathbf{k}}/2T)]$, where $n=N/V$ is the electron density. Take the volume $V=L_xL_yL_z$, and define the Fermi momentum via the relation $E_F=k_{F_s}^2/2m=(3\pi^2n)^{2/3}/2m$. Furthermore, choose $L_x=d_s$ and $\min\{L_y,L_z\}\gg d_s$, take the asymptotic limit $k_{F_s}d_s\gg 1$ and perform an expansion of the effective action (7) in powers Δ to obtain the static linearized Ginzburg-Landau equation

$$\epsilon\Delta(x)-\xi_{\rm GL}^2\frac{d^2}{dx^2}\Delta(x)=0, \tag{11}$$

where the coefficients $\epsilon=(T-T_c)/T_c$, correspond to a dimensionless deviation from T_c, and

$$k_{F_s}\xi_{\rm GL}=\frac{2e^{-2\gamma}}{\pi}\sqrt{\frac{7\zeta(3)}{48}\frac{E_F}{T_c}}$$

is the dimensionless coherence length of the superconductor. Notice that $k_{Fs}\xi_{\rm GL}\gg 1$ when $E_F\gg T_c$, as expected in the BCS limit.

The boundary conditions to Eq. (11) are $[d\Delta(x)/dx]_{x=\pm ds/2}=\mp\Delta(x)/b$ at the F/S interfaces. The coefficient b is the extrapolation length

$$k_{F_s}b\approx\lambda\frac{E_F}{2T_c}\frac{1+\gamma_0}{\gamma_0}, \tag{12}$$

where $\gamma_0=\sqrt{1-U_0/E_F}$ characterizes the nonmagnetic offset of the single particle potential of the ferromagnets, and λ

$=1-[1/4](J\langle S_z\rangle/\Lambda)^2$, with $J\langle S_z\rangle \ll \Lambda = \min\{2\pi T_c,(E_F-U_0)\}$ when $E_F > U_0$, characterizes the magnetic offset of the single particle potential of the ferromagnets.

This estimate is a result of the assumptions that the F layer is a good metal, but a weak ferromagnet. Notice two important features in the extrapolation length. The first feature is that b is independent of the sign of the magnetization $\langle S_z\rangle$ in the F layers. And the second feature is that b is independent of the sign of the exchange coupling J. Physically, this originates from the fact that an exchange field coupled to electrons forming Cooper pairs is always pair breaking irrespective to the sign of the coupling.

The solution of Eq. (11) is $\Delta(x)=\Delta_0\cos(kx)$ which leads to a correction to critical temperature of the bulk superconductor $\epsilon=-(k\xi_{\rm GL})^2$ where k satisfies the transcendental equation $\tan(kd_s/2)=(1/kb)$ which in the limit $k_{F_s}d_s \gg 1$, the parameter $k\approx \pi/d_{\rm eff}$ implies a correction

$$\epsilon=-\left(\frac{\pi\xi_{\rm GL}}{d_{\rm eff}}\right)^2 \tag{13}$$

to the critical temperature of the bulk superconductor $T_c(\infty)$. Here, $d_{\rm eff}=(d_s+2b)$ is the *effective* length of the superconductor, and the new critical temperature is $T_c(d_s)=T_c(\infty)(1+\epsilon)$. The supression of T_c from the bulk value is small provided that $\pi\xi_{\rm GL} \ll d_{\rm eff}$, and thus superconductivity survives. A strong suppression of T_c happens when $\pi\xi_{\rm GL}\sim d_{\rm eff}$. Notice that the choices of the superconductor and the ferromagnetic metal are very important in order to have a weak suppression of T_c. The choice of the spacer should be a superconductor with a *high* bulk T_c and a *short* coherence length $k_{F_s}\xi_{\rm GL}$. For the ferromagnet, it is important to be a *metal* ($E_F > U_0$), with $T_f \gg T_c$, but with *not so strong* pair breaking effects, i.e., $\alpha_{\rm PB}=|J\langle S_z\rangle|/2\pi T_c \ll 1$. Here $\alpha_{\rm PB}$ is the pair breaking parameter. It is important to emphasize that the pair breaking parameter $\alpha_{\rm PB}=|J\langle S_z\rangle|/2\pi T_c$ and the parameter $\alpha_0=J\mathcal{N}(E_F)$ that determines the Curie temperature of the ferromagnet are distinct quantities.[31] It is possible to have $T_f \gg T_c$ at the same time that the pair breaking effects are not so dramatic. A strong ferromagnet would produce strong suppression of T_c by bringing the order parameter to zero at the F/S interface due to strong pair breaking effects.

In Fig. 3, it is shown the suppression of the critical temperature of the superconductor from its bulk value. The plot if of $T_c(d_s)/T_c(\infty)=(1+\epsilon)$ versus $k_{F_s}d_s$. The superconductor is considered to have a short coherence length ($k_{F_s}\xi_{\rm GL}\approx 5$) and a not so small ratio $T_c/E_F\approx 0.05$ which can be achieved with a *high* T_c or *low* E_F. The solid line corresponds to the weak metallic ferromagnet discussed here, while the dotted line describes a strong ferromagnet which has the boundary conditions $\Delta(x)=0$ at the F/S interfaces, i.e., strong pair breaking effect. Notice that for a weak ferromagnet T_c is not as strongly suppressed at smaller values of $k_{F_s}d_s$, and thus chances of preserving superconductivity at smaller thickness of the spacer are more favorable. This is expected since weak ferromagnets have weaker pair breaking effects in comparison to strong ferromagnets.

Take the ideal case of $Nd_{2-x}Ce_xCuO_4/La_{1-y}Sr_yMnO_3$ multilayers, where the bulk T_c of $Nd_{2-x}Ce_xCuO_4$ is $T_c\approx 30$ K, and $\xi_{\rm GL}\approx 10$ Å,[32] and the ferromagnet $La_{1-y}Sr_yMnO_3$ with $T_f\approx 300$ K and $J\approx 10$ K (Ref. 33) to further emphasize the need for weak pair breaking ferromagnets. Ignoring anisotropy of the superconductor and complex spin structure effects of the ferromagnet, the pair breaking parameter is $\alpha_{\rm PB}\approx 0.027$, and thus Eq. (13) can still be used to estimate the reduction of T_c due to the ferromagnetic boundaries. A modest $E_F\approx 10^4$ K with $k_F^{-1}\approx 0.38$ Å leads to an extrapolation length $b\approx 60$ Å. The effective length of the superconductor becomes $d_{\rm eff}=d_s+2b$, and for $d_s=10$ Å, the reduction in T_c from its bulk value $T_c(\infty)$ is only 6%.[34] Thus, for spacer thickness in the range $d_s=10$ Å and $d_s=130$ Å (where magnetic coupling is observed in many systems[7]), the suppression of T_c is even smaller and superconductivity survives. Therefore, multilayers of high-T_c/colossal magnetoresistance (CMR) ferromagnets may be quite attractive to study experimentally. In addition to high-T_c/CMR ferromagnets, another system interesting to study experimentally are the newly discovered layered rutheno cuprates ($RuSr_2GdCu_2O_{8-\delta}$), where superconductivity and ferromagnetism coexist. These systems, unlike almost all other cases of coexisting superconductivity and ferromagnetism, becomes ferromagnetic first at $T_f=132$ K where the ruthenium ions order, and at a lower temperature $T_c\approx 35-40$ K superconductivity appears (most likely at the cuprate planes). These materials are very different from previously known ferromagnetic superconductors,[35] where the T_f and T_c were usually very close, and strong competition between superconductivity and magnetism lead to a very small coexistence region.[36]

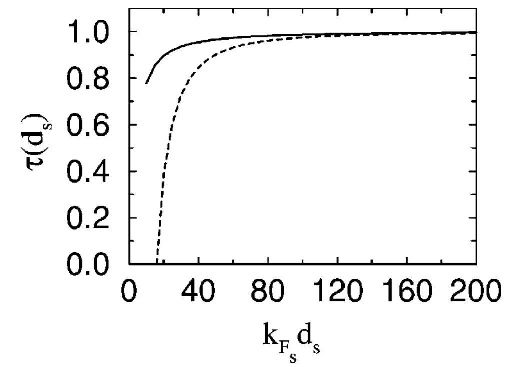

FIG. 3. The suppression of T_c expressed as $\tau=T_c(d_s)/T_c(\infty)$ is indicated for a weak ferromagnet ($\alpha_{\rm PB}\ll 1$) and for a strong ferromagnet [$\alpha_{\rm PB}\sim\mathcal{O}(1)$], i.e., $\Delta(x=\pm d_s/2)=0$. The solid line indicates the weak ferromagnet, the dotted line indicates the strong ferromagnet. The parameters used are $k_{F_s}\xi_{\rm GL}\approx 5$ and $T_c/E_F=0.05$.

Now that the suppression of T_c as a function of $k_{F_s}d_s$ has been estimated, it is important to analyze the possibility of magnetic coupling between the two weak ferromagnets through the superconducting spacer. This is discussed next.

IV. MAGNETIC COUPLING: FORMALISM

Provided that the transition temperature of the superconducting film is not strongly supressed by the ferromagnetic boundaries, the exchange coupling H_c appearing in Eq. (3) may be treated as a perturbation. For the purpose of calculating the magnetic coupling across a superconductor, it is important to analyze the effective action (7) in terms of the inverse Nambu propagator

$$G^{-1}=\mathcal{G}^{-1}+(H_c)_\sigma\sigma_z, \tag{14}$$

where an explicit separation between the inverse propagator

$$\mathcal{G}^{-1}=-\partial_\tau-[K+U(\mathbf{r})]\sigma_z+\Delta(r)\sigma^+ +\overline{\Delta}(r)\sigma^- \tag{15}$$

(in the absence of ferromagnetic boundaries) and the exchange contribution $(H_c)_\sigma\sigma_z$ (due to the ferromagnetic boundaries) is made. This separation is used to treat the exchange contribution as a perturbation. As a result, the effective action (7) can now be conveniently written as

$$S_{\text{eff}}=\int_0^\beta d\tau\left\{\frac{|\Delta(r)|^2}{g}-\frac{1}{\beta}\text{Tr}\ln\beta\mathcal{G}^{-1}-H_{\text{eff}}\right\}. \tag{16}$$

Minimization of the effective action without H_{eff}, i.e., $\delta S_{\text{eff}}(H_{\text{eff}}=0)/\delta\Delta^*(\mathbf{r})=0$ leads to the saddle point gap equation $\Delta(\mathbf{r})/g=\text{Tr}[G\sigma^-]/\beta$, which can be used to compute the effective Hamiltonian to second order in H_c,

$$H_{\text{eff}}\approx-\frac{1}{2\beta}\text{Tr}[(H_c)_\sigma\sigma_z\mathcal{G}]^2. \tag{17}$$

The effective magnetic Hamiltonian H_{eff} contains various terms, but the interest here is focused only on the magnetic coupling across the superconductor, which leads to

$$H_{\text{eff}}=-\frac{E_J}{\mathcal{N}_{3D}}\sum_{nm,k_\perp}\int\int_{\partial\Omega_{x_1,x_2}}dx_1dx_2\chi_{nmk_\perp}(x_1,x_2), \tag{18}$$

where the energy scale $E_J=\frac{1}{2}J_+J_-\langle S_z\rangle_+\langle S_z\rangle_-\mathcal{N}_{3D}V_s$ contains the density of states of the bulk superconductor $\mathcal{N}_{3D}$, and the domain of integration $\partial\Omega_{x_1,x_2}$ includes the entire sample. The details of the derivation of Eq. (18) can be found in Appendix A. The indices $\pm$ refer to the F layers. The matrix element

$$\chi_{nmk_\perp}(x_1,x_2)=2D_{nm}(x_1,x_2)Q_{nm}(k_\perp) \tag{19}$$

defines the *nonlocal* ''susceptibility'' of the system which appears under the double integration over x_1 and x_2. The weighting factor

$$D_{nm}(x_1,x_2)=w_{nk_\perp}(x_1)w_{nk_\perp}(x_2)w_{mk_\perp}(x_1)w_{mk_\perp}(x_2) \tag{20}$$

contains the eigenstates $w_{nk_\perp}(x)$, i.e.,

$$H_e(x,k_\perp)w_{nk_\perp}(x)=\xi_n(k_\perp),w_{nk_\perp}(x) \tag{21}$$

of the one-dimensional Hamiltonian

$$H_e(x,k_\perp)=-[1/2m]\partial^2/\partial x^2-\mu_{\text{eff}}+U(x), \tag{22}$$

where the *effective* chemical potential $\mu_{\text{eff}}=\mu-k_\perp^2/2m$ with $k_\perp^2=k_y^2+k_z^2$. The additional term

$$Q_{nm}(k_\perp)=C_T^{nm}(k_\perp,k_\perp)T_{nm}(k_\perp,k_\perp)+C_P^{nm}(k_\perp,k_\perp)P_{nm}(k_\perp,k_\perp), \tag{23}$$

contains the coherence factor $C_T^{nm}=[p_{u_n}p_{u_m}+p_{v_n}p_{v_m}]^2$, the thermal factor $T_{nm}=[f(\epsilon_m)-f(\epsilon_n)]/[\epsilon_m-\epsilon_n]$ in the quasiparticle-quasihole channel, and the coherence factor $C_P^{nm}=[p_{u_n}p_{v_m}-p_{u_m}p_{v_n}]^2$, the thermal factor $P_{nm}=[f(\epsilon_m)+f(\epsilon_n)-1]/[\epsilon_m+\epsilon_n]$ in the quasiparticle-quasiparticle channel. The coefficients p_{u_n} and p_{v_n} defined, respectively, by $|p_{u_n}|^2=[1+\xi_n/\epsilon_n]/2$ and $|p_{v_n}|^2=[1-\xi_n/\epsilon_n]/2$, reflect the simplification $\langle\Delta(x)\rangle\approx\Delta$ (see Appendix A), thus leading to the approximate eigenenergies $\epsilon_n^2\approx\xi_n^2+\Delta^2$. The averaged order parameter is dependent on d_s the thickness of the superconducting spacer (see Appendix C).

V. MAGNETIC COUPLING: ANALYTICAL RESULTS

Under the assumptions used to derive Eq. (18), the magnetic coupling can be obtained analytically only for large spacer thickness $k_{F_s}d_s\gg 1$ both at very low temperatures $T\approx 0$ and near the critical temperature $T\approx T_c$. Recall that $k_{F_s}=\sqrt{2mE_F}$ is the Fermi momentum, and assume that $E_F\gg\max\{\Delta,T\}$. In the asymptotic limit $k_{F_s}d_s\gg 1$, the single particle wave functions $w_n(x)$ are standing waves (which can be decomposed into plane waves), the discrete quantum number n becomes a continuous *momentum* index, and the sums over the indices (n,m) become integrals. Notice that the momentum $k_\perp$ is conserved across the interface. An outline of the procedure used to evaluate the effective coupling is given in Appendix B. At low temperatures $(\Delta/T\gg 1)$ the form of the coupling is

$$H_{\text{eff}}=-E_J\frac{\mathcal{F}}{2\pi^2}\frac{\cos(2k_{F_s}d_{\text{s}})}{(2k_{F_s}d_{\text{s}})^2}\exp(-k_{F_s}d_s\Delta/E_F), \tag{24}$$

while at temperatures close to T_c, $(\Delta/T\ll 1)$ the magnetic coupling becomes

$$H_{\text{eff}}=-E_J\frac{\mathcal{F}}{2\pi^2}\frac{\cos(2k_{F_s}d_{\text{s}})}{(2k_{F_s}d_{\text{s}})^2}\left[1-\frac{2}{3\pi^2}\left(\frac{\Delta}{E_F}\right)^2\right]\times\exp(-\pi k_{F_s}d_sT_c/E_F). \tag{25}$$

The averaged order parameter Δ depends weakly on the superconductor thickness d_s in the asymptotic limit $k_{F_s}d_s\gg 1$, and a discussion of this dependence can be found in Appendix C. In Fig. 4, two plots of H_{eff} are shown, the dotted curves correspond to temperatures near T_c, the solid curves to temperatures near zero, while the dashed curves correspond to $T=0$ in the absence of superconductivity. Formulas (24) and (25) are used to plot the curves appearing in Fig. 4, with the following parameters $U_0/E_F=0.1$ and $k_{F_s}\xi_{\text{GL}}=5$. Notice that the coupling in the superconducting state (at $T=0$) is small at large values of $k_{F_s}d_s$.

It is important to analyze the qualitative features of the previous expressions. First, notice that the period of oscillation $\ell_p=\pi/k_{F_s}$ of the magnetic coupling across the superconductor is entirely controlled by the Fermi momentum of the superconductor k_{F_s}. The appearance of superconductivity does not introduce any new periods. This is expected since no new momentum modulation in the spin degrees of free-

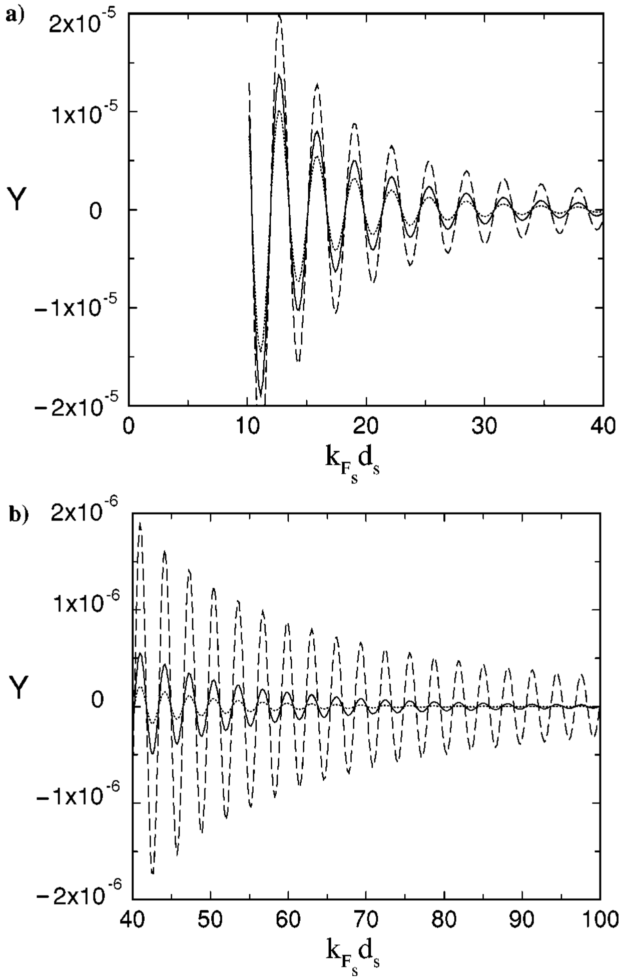

FIG. 4. The magnetic coupling $Y=H_{\mathrm{eff}}/E_J$ is plotted as a function of $k_{F_s}d_s$, for $k_{F_s}d_s \gg 1$. (a) Plot of Y versus $k_{F_s}d_s$ from $k_{F_s}d_s=0$ to $k_{F_s}d_s=40$. (b) Plot of Y versus $k_{F_s}d_s$ from $k_{F_s}d_s=40$ to $k_{F_s}d_s=100$. The dotted curves correspond to the limit $T\approx T_c$, while the solid curves correspond to $T\approx 0$. The dashed line indicates what the coupling would have been at $T=0$ in the absence of superconductivity.

dom occurs, through the appearance of the superconducting gap, at length scales comparable to $k_{F_s}^{-1}$. At low temperatures $(\Delta/T \gg 1)$ there is an energy cost to be paid. Almost all of the electrons that were easily polarized in the normal state of the superconductor are now paired into singlets. As a result the spin polarization of the superconductor is costly, i.e., when summing over all intermediate states (virtual quasiparticle states) there is a minimum energy required: the superconducting gap.[37] The gap introduces a new length scale ξ_{GL} which controls the decay of the coupling (see Fig. 4). Notice that $\Delta/E_F \approx 0.15/k_{F_s}\xi_{\mathrm{GL}}$ in the asymptotic limit $k_{F_S}d_s \gg 1$. At temperatures close to T_c, when $(\Delta/T \ll 1)$, there is no decay caused by the superconducting gap. The gap is so small that intermediate quasiparticle states are strongly thermally populated. Near T_c, these intermediate quasiparticle states resemble the normal state eigenfunctions, except for the presence of a small superfluid density controlled by Δ. As a result, the temperature dependence of the magnetic coupling is purely controlled by thermal effects, and only an overall reduction of the prefactor of the oscillations appears. The decay length for $T\approx T_c$ and $T>T_c$ is controlled by the thermal length $k_{F_s}\lambda_T = E_F/\pi T_c$ and the magnetic coupling prefactor is controlled by $\Delta/E_F = 0.26(1-T/T_c)^{1/2}/k_{F_s}\xi_{\mathrm{GL}}$, which vanishes at $T=T_c$, leading to a Ruderman-Kittel-Kasuya-Yoshida (RKKY)-type coupling for $T \geqslant T_c$. This behavior is consistent with the temperature dependence observed by Zhang *et al.*[38] in nonsuperconducting multilayers, and suggested by Edwards *et al.*[4] The interesting aspect is that there is an enhancement of the magnetic coupling at low temperatures $T\approx 0$ in comparison with values at $T \geqslant T_c$, but a reduction of the coupling in comparison with a normal-state system at $T=0$. So notice in Fig. 4 the differences in magnetic coupling when $T\approx 0$ in the presence or absence of superconductivity, and when $T\approx T_c$. Second, notice that the asymptotic decay is proportional to $(k_{F_s}d_s)^{-2}$ instead of the usual RKKY decay $(k_F r)^{-3}$ for magnetic impurities. The change in the form of the decay can be viewed as a geometrical effect since the magnetic ''impurities'' are now two semi-infinite ferromagnets and as a result the magnetic coupling has to be more effective. This behavior was already theoretically suggested in the context of nonsuperconducting spacers,[4,39] and experimentally observed.[6] Third, the magnetic coupling is strongly dependent on the type of ferromagnet, similar to the situation encountered experimentally when the spacer is in its normal metallic state.[6] This dependence on the ferromagnet is present in H_{eff} via the magnetic exchange J and the potential U_0. The dependence of H_{eff} on U_0 is explicit in Eqs. (24) and (25), since

$$\mathcal{F} = \gamma_0/(\gamma_0+1)^2, \tag{26}$$

where $\gamma_0 = \sqrt{1-U_0/E_F}$. The amplitude of the coupling is gradually reduced from its maximal value at $U_0=0$ until it vanishes at $U_0=E_F$. This reduction can be interpreted as the disapperance of spin-dependent states at the Fermi energy of the ferromagnets as $U_0/E_F \ll 1$ increases to $U_0/E_F \approx 1$, thus it is essentially a density of states effect. Furthermore, the dependence of H_{eff} on the exchange coupling J is explicit in the energy scale E_J, which is proportional to the product $J_+J_-\langle S_z\rangle_+\langle S_z\rangle_-$, and thus depends both on the strength of the coupling of the itinerant electrons (J_+,J_-) and on the magnetization of the ferromagnets $(\langle S_z\rangle_+,\langle S_z\rangle_-)$. Notice that $H_{\mathrm{eff}} \propto \mathcal{O}(J\langle S_z\rangle)^2$, thus if $J\langle S_z\rangle$ is too small, the magnetic coupling may be hard to measure even in the normal state of the spacer. However, for ferromagnets with moderate $J\langle S_z\rangle$ values and high critical temperature superconductors such that the pair breaking parameter $\alpha_{pb} = J\langle S_z\rangle/2\pi T_c$ is small favors the observation of magnetic coupling accross the superconductor spacer. Thus suggesting, that ideal systems for the observation of such effect may be high-T_c superconductor/colossal magnetoresistance ferromagnet multilayers such as $Nd_{2-x}Ce_xCuO_4/La_{1-y}Sr_yMnO_3$ or $Nd_{2-x}Ce_xCuO_4/La_{3-y}Sr_yMn_2O_7$ (for the *s*-wave case) and $YBa_2Cu_3O_{7-\delta}/La_{1-y}Sr_yMnO_3$ or $YBa_2Cu_3O_{7-\delta}/La_{3-y}Sr_yMn_2O_7$ (for the *d*-wave case). Other possible candidates include the newly discovered family of rutheno-cuprates[9–12] discussed in Sec. I.

A. Limitations of approach

It is also important to mention the limitations of the results obtained here. The magnetic coupling is calculated only perturbatively under the assumption of a weak ferromagnet ($\alpha_{pb} \ll 1$), given that one does not want superconductivity to be destroyed by the proximity to the ferromagnet. A strong ferromagnet [$\alpha_{\mathrm{PB}} \sim \mathcal{O}(1)$ or $\alpha_{\mathrm{PB}} \gg 1$] could destroy superconductivity and wash out completely the effects just discussed. In this case the superconductor would remain in its normal metallic state all the way down to $T=0$, and thus no modification to the RKKY-like coupling would occur. This is certainly not an interesting situation. The perturbative method used in this paper does not permit a nontrivial analysis of the dependence of the magnetic coupling H_{eff} on the exchange coupling J of the ferromagnet, since it was explicitly assumed that the ferromagnets produce small pair breaking effects ($\alpha_{\mathrm{PB}} \ll 1$) in the superconductor. Small pair breaking effects can be achieved either by having small $J\langle S_z\rangle$ or large $2\pi T_c$. A very small $J\langle S_z\rangle$ could seriously jeopardize the experimental observation of the magnetic coupling across the spacer either in its normal or in its superconducting state, since $H_{\mathrm{eff}} \propto \mathcal{O}(J\langle S_z\rangle)^2$. However, the choice of ferromagnets with moderate values of $J\langle S_z\rangle$ and high critical temperature superconductors should facilitate experimental observation.

Another limitation is that only the asymptotic limit ($k_{F_s} d_s \gg 1$) can be obtained analytically, the limit ($k_{F_s} d_s \approx 1$) can only be treated through numerical calculations, which are not presented here. Thus, the present results are valid only when $d_s \gg \ell_p/\pi$, where $\ell_p = \pi/k_F$ is the period of oscillation of the magnetic coupling. Thus, if $\ell_p = 10$ Å, then the calculations are valid only for $d_s \gg 3.18$ Å. This is not such a serious limitation, since as a general rule $d_s > 5\ell_p/\pi$ is practically already in the asymptotic limit.

The dependences of H_{eff}, and of T_c on thickness d_F of the ferromagnet are important experimental issues, which have not been addressed here. Only a trilayer structure was discussed, where the ferromagnets were considered to be semi-infinite, and thus the expressions obtained for H_{eff} do not contain any nontrivial dependence on the thickness d_F of the ferromagnetic layers. The most favorable experimental situation may be that of a multilayered structure (instead of the trilayer case discussed here) with thin ferromagnetic layers (instead of semi-infinite) to further reduce detrimental effects to the superconductivity of the spacer. For instance, if the ferromagnetic layers are thin enough, neighboring superconducting layers may couple and the superconductivity becomes more three-dimensional. In this case, the superconductivity of the spacer layers may be less harmed by the ferromagnetic layers. This situation will be discussed in a future publication.

It is also important to point out that lattice effects on both the spacer and the ferromagnet were not included in the present calculation of the magnetic coupling. For instance, the coupling is very sensitive to the lattice structure of nonsuperconducting spacers.[4,5,39] The inclusion of such effects are important since it is also known experimentally that the magnetic coupling depends strongly on the type of ferromagnet and on the type of spacer used (in the case of nonsuperconducting spacers).[6] In particular, the lattice structure is quite important because the magnetic coupling with nonsuperconducting spacers occurs at the nanometer scale (from a few Å to hundred Å or so),[7] and a similar range applies for superconducting spacers. However, lattice effects were not considered in this manuscript, since the ferromagnets and the spacer were treated in the continuum approximation.

In addition, this work cannot be directly applied to high-T_c/colossal magnetoresistance multilayers. For these systems, one should include the effects of anisotropy (both in the band structure and in the order parameter) for the superconductors, and the effects of complex spin structure (localized and itinerant spins) for the CMR materials. Furthermore, the inevitable effects of roughness have not yet been included and need to be analyzed. This analysis is done next.

B. Roughness effects

The effects of roughness on the critical temperature of the superconductor are negligible provided that the length scale of the roughness fluctuations $\ell_r \ll \xi_{\mathrm{GL}}$. On the other hand, the effects of roughness on the magnetic coupling are very important, since ℓ_r can be easily of the order of the magnetic oscillation period $\ell_p = \pi/k_F$, and thus average out the oscillatory behavior. These studies have been performed experimentally in Fe/Cr/Fe multilayers,[40] and also theoretically for several $F/N/F$ multilayers.[39,41,42] The case of $F/S/F$ multilayers is not so different, i.e., roughness is also expected to affect the magnetic coupling in two basic ways. The first one is that the magnetic coupling must be averaged over thickness fluctuations of the superconductor film. The second way is that the magnetic coupling is affected by lateral fluctuations, which break translational invariance, and thus conservation of momentum parallel to the F/S interface. Only the first case is discussed here. Assuming that the thickness fluctuations are Gaussian around the mean value thickness $\overline{d}_{\mathrm{s}}$ with variance σ, then in the limit that $\overline{d}_{\mathrm{s}} \gg \sigma$ the only modifications in Eqs. (24) and (25) consist in replacing the thickness d_s by the average thickness $\overline{d_s}$ and replacing $\cos(2k_{F_s} d_{\mathrm{s}})$ by $\exp[-(2k_{F_s}\sigma)^2/2]\{\cos(2k_{F_s}\overline{d}_{\mathrm{s}}) + 2(2k_{F_s}\sigma)(\sigma/\overline{d}_{\mathrm{s}})\sin(2k_{F_s}\overline{d}_{\mathrm{s}})\}$. Thus, provided that $\sigma < \sigma^* = \ell_p/\pi\sqrt{2}$, the magnetic coupling is not dramatically reduced. For instance, if $\ell_p = 10$ Å, then for $\sigma < \sigma^* = 2.25$ Å magnetic coupling should still be observed. But if $\ell_p = 3$ Å, then only for $\sigma < \sigma^* = 0.68$ Å magnetic coupling should be observed. This last condition is very stringent. This means that the overall effect of roughness is to suppress short period oscillations in much the same way as in $F/N/F$ multilayers.

VI. SUMMARY AND DISCUSSION

In this paper, the functional integral method was used to study the possibility of magnetic exchange coupling between two ferromagnets (F) separated by a superconductor. The system of choice was a trilayer made of two identical semi-infinite ferromagnets separated by a supeconductor spacer of thickness d_s. Provided that three *prima facie* conditions are satisfied, magnetic exchange coupling is predicted to exist at large spacer thickness ($k_{F_s} d_s \gg 1$). These important conditions are as follows. First, an indirect exchange coupling between the ferromagnets must exist when the superconductor is in its normal state . Second, superconductivity must not be destroyed due to the proximity to ferromagnetic boundaries. Third, roughness of the F/S interfaces must be small. Under these conditions, when the superconductor is cooled off below its critical temperature, the magnetic coupling is expected to change. The appearance of the superconducting gap causes a reduction of the indirect exchange coupling existent in a corresponding $T=0$ normal state. This reduction was predicted to occur within a length scale controlled by the coherence length of the superconductor. Furthermore, the coupling was predicted to be temperature dependent, i.e., the reduction of the coupling amplitude was smaller at zero temperature and larger near the critical temperature due to thermal effects.

It was emphasized that the prevailing experimental evidence indicates that the exchange coupling with metal spacers is short ranged, i.e., the magnetic coupling can only be observed across a layer of thickness 10 to 130 Å.[7] Thus, the key question, for both theory and experiment, was the following. Could such a thin metallic layer survive pair breaking effects of the ferromagnetic layers from both sides and yet remain superconducting? This question poses an important experimental challenge, given that presently there are no known multilayered systems that show magnetic coupling both above and below T_c. The main reason for the inexistence of such an experimental system is that the conditions that need to be satisfied are difficult to achieve. It was the purpose of this paper to show that appropriate choices of superconductor and ferromagnet lead to the survival of superconductivity and to new effects on the magnetic coupling. The desirable superconductor should have *high* critical temperature and *short* coherence length, while the desirable ferromagnet should be *metallic* with *not so large* pair breaking effects. Furthermore, the desirable F/S interfaces should be atomically flat and well lattice matched to avoid the effects of roughness and strain. Thus, ideal systems to study theoretically and experimentally are high-T_c superconductor/colossal magnetoresistance ferromagnet multilayers such as $Nd_{2-x}Ce_xCuO_4/La_{1-y}Sr_yMnO_3$ or $Nd_{2-x}Ce_xCuO_4/La_{3-y}Sr_yMn_2O_7$ (for the *s*-wave case) and $YBa_2Cu_3O_{7-\delta}/La_{1-y}Sr_yMnO_3$ or $YBa_2Cu_3O_{7-\delta}/La_{3-y}Sr_yMn_2O_7$ (for the *d*-wave case). In addition to these systems, the newly discovered layered rutheno cuprates (Ru1212) are also good candidates of magnetic coupling across the superconducting layers, since the superconducting state is not destroyed by the proximity to the FM layers and the ferromagnet/superconductor interface is atomically flat.[43]

Four basic questions were the central topic of this paper. First, how does the presence of ferromagnetic layers affect the superconductivity of the spacer? Second, does anything dramatic happens to the magnetic coupling when the system is cooled through the superconducting transition temperature of the spacer? Third, what happens to the magnetic coupling at very low temperatures when the superconductivity of the spacer is well established? Fourth, what are the effects of interfacial roughness?

The answer to the first question is that weak ferromagnets do not strongly suppress the critical temperature T_c of the short coherence length (ξ_{GL}) superconductor, because pair breaking effects are not large ($\alpha_{PB} \ll 1$). Thus, the superconducting state is expected to survive down to moderately small spacer thickness provided that $\pi\xi_{GL} \ll d_{eff} = d_s + 2b$), where b is the extrapolation length defined at the F/S interface. Thus, for a system with $\xi_{GL} \approx 10$ Å and $b \approx 60$ Å, superconductivity is expected to survive all the way down to spacer thickness $d_s = 10$ Å. The need for a short coherence length superconductor and a weak ferromagnetic metal are crucial for the survival of superconductivity down to small thickness.

The answer to the second question is that nothing dramatic happens to the magnetic coupling at T_c. Near T_c ($\Delta \ll T_c$) spin polarization is easily accessible at nearly zero energy cost (almost zero gap), thus the magnetic coupling is essentially controlled by the normal-state properties of the superconductor. The intermediate quasiparticle states resemble the normal-state eigenfunctions, except for the presence of small superfluid density controlled by Δ. This leads to a small reduction of the amplitude of the coupling controlled by $\Delta/E_F \approx 0.26(1-T/T_c)^{1/2}/(k_{F_s}\xi_{GL})$. Therefore, the magnetic coupling just crosses over from the RKKY-like oscillatory coupling in the normal state to a similar behavior just below T_c, with a slight reduction in the amplitude, but with an additional temperature dependence. Such an additional temperature dependence is absent when the superconducting spacer is in its normal state, since Δ vanishes. The only temperature dependence left in the normal state is due to the thermal length $k_{F_s}\lambda_T = E_F/\pi T$. Furthermore, the appearance of superconductivity does not introduce any new periods, since no new momentum modulation in the spin degrees of freedom occurs at length scales comparable with $k_{F_s}^{-1}$.

The answer to the third question is that at low temperatures ($\Delta/T \gg 1$) there is an energy cost to be paid, since almost all electrons that were easily polarized in the normal state of the superconductor are now paired into singlets. As a result the spin polarization of the superconductor is costly, i.e., when summing over all intermediate states (virtual quasiparticle states) there is a minimum energy required: the superconducting gap. The gap introduces a new length scale ξ_{GL} which controls the decay of the coupling amplitude. However, this length scale (ξ_{GL}) is responsible for a larger magnetic coupling at low temperature in comparison with the same coupling close to T_c (which is controlled by λ_T).

The answer to the fourth question is that the effects of roughness on the critical temperature of the superconductor are not so important provided that the length scale for roughness fluctuations $\ell_r \ll \xi_{GL}$. But the effects of roughness on the magnetic coupling are very important, since ℓ_r can be easily of the order of the magnetic oscillation period $\ell_p = \pi/k_F$ and average out the oscillatory behavior. Thus, provided that the variance of thickness fluctuations σ is much smaller than the mean value thickness $\overline{d_s}$, and that σ is small in comparison with the period ℓ_p, magnetic coupling should still be observed, both in the normal state and in the superconducting state of the spacer.

VII. FINAL COMMENTS

Here, only isotropic *s*-wave supercondutors with short coherence lengths have been discussed. But, allowing for anisotropy in the superconductor, and allowing for the existence of nodes in the gap function (say *d*-wave order parameter) seems to be the next natural (experimental and theoretical) steps to investigate magnetic coupling across superconductors. Specially because the conditions for the observation of magnetic coupling (which include *high* temperature and *short* coherence length superconductors) are easily met by high-T_c copper-oxide superconductors. In particular, multilayers of weak ferromagnets separated by high-T_c-short-coherence length copper-oxide superconductors are very natural candidates. After the discovery of colossal magnetoresistance materials which have a similar perovskite structure to the copper oxide superconductors, it may be possible to grow high T_c/CMR multilayers, such as $Nd_{2-x}Ce_xCuO_4/La_{1-y}Sr_yMnO_3$ or $Nd_{2-x}Ce_xCuO_4/La_{3-y}Sr_yMn_2O_7$ (for the *s*-wave case) and $YBa_2Cu_3O_{7-\delta}/La_{1-y}Sr_yMnO_3$ or $YBa_2Cu_3O_{7-\delta}/La_{3-y}Sr_yMn_2O_7$ (for the *d*-wave case). The advantage of using these multilayers over more conventional materials is clear. (a) The critical temperatures of these superconductors is one order of magnitude higher than conventional superconductors, the coherence lengths are extremely short, and superconductivity survives down to a single monolayer. (b) High-T_c and CMR materials are perovskites with nearly matched lattices, which should reduce roughness and strains at their interfaces. (c) CMR materials have high T_F (between 120 and 300 K) and may be weak ferromagnets, i.e., $\alpha_{\mathrm{PB}} \ll 1$. Additional complications do occur, however: high-T_c superconductors are layered, their gap function (in the hole doped superconductors) is anisotropic and has nodes at the Fermi surface, i.e., *d* wave.[37] The gap anisotropy makes the problem of magnetic coupling through these systems more complex, but also more interesting, since nodes may lead to quite nontrivial spatial, directional, and temperature dependences of H_{eff}. Lastly the newly discovered rutheno cuprates (Ru1212) are also good candidates of magnetic coupling across the superconducting layers, since the superconducting state is not destroyed by the proximity to the FM layers and the ferromagnet/superconductor interface is atomically flat.[43] If the number of cuprate planes can be systematically changed then magnetic coupling between ruthenate planes and its consequences can be systematically studied as a function of the number of cuprate layers.

Note added in proof. Nikolaev *et al.*[46] have observed oscillatory exchange coupling in epitaxial oxide heterostructures consisting of layers of the CMR material $La_{2/3}Ba_{1/3}MnO_3$ separated by the paramagnetic metal nickelate $LaNiO_3$. Although $LaNiO_3$ is not superconducting at low temperatures, the observed magnetic oscillatory coupling was damped and resembled the normal-state magnetic coupling plotted in Fig. 4 of this paper. Their damping coefficient may be due to thermal and/or interfacial roughness effects, as discussed here in Sec. V. The experimental results of Ref. 46 indicate that the possibility of making oxide heterostructures consisting of CMR materials and cuprate oxides is very real and that the possible observation of magnetic coupling through superconductors is very near.

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APPENDIX A

This appendix is dedicated to the derivation of Eq. (18), starting from equation (17) and from the saddle point gap equation

$$\Delta(\mathbf{r})/g = \mathrm{Tr}[G\sigma^-]/\beta. \tag{A1}$$

The effective Hamiltonian to second order, that involves coupling across the superconductor, is

$$H_{\mathrm{eff}} \approx -\frac{1}{2\beta}\sum_{i\omega} \int \frac{d\mathbf{r}_1}{V}\int \frac{d\mathbf{r}_2}{V}\Gamma[L_A+L_B], \tag{A2}$$

where

$$\Gamma = J_+J_-\langle S_z\rangle_+\langle S_z\rangle_-[\Theta(x_1-d_s/2)\Theta(-x_2-d_s/2) + \Theta(x_2-d_s/2)\Theta(-x_1-d_s/2)]$$

represents the *coupling* of the exchange fields from the two ferromagnets and

$$L_A = \mathcal{G}_{\uparrow\uparrow}(\mathbf{r}_1,\mathbf{r}_2;i\omega)\mathcal{G}_{\uparrow\uparrow}(\mathbf{r}_2,\mathbf{r}_1;i\omega) + \mathcal{G}_{\uparrow\downarrow}(\mathbf{r}_1,\mathbf{r}_2;i\omega)\mathcal{G}_{\downarrow\uparrow}(\mathbf{r}_2,\mathbf{r}_1;i\omega), \tag{A3}$$

$$L_B = \mathcal{G}_{\downarrow\downarrow}(\mathbf{r}_1,\mathbf{r}_2;i\omega)\mathcal{G}_{\downarrow\downarrow}(\mathbf{r}_2,\mathbf{r}_1;i\omega) + \mathcal{G}_{\downarrow\uparrow}(\mathbf{r}_1,\mathbf{r}_2;i\omega)\mathcal{G}_{\uparrow\downarrow}(\mathbf{r}_2,\mathbf{r}_1;i\omega), \tag{A4}$$

represent the propagators (Green's functions) that carry the spin information across the superconductor. The Green's function matrix $\mathcal{G}$ has elements

$$\mathcal{G}_{\uparrow\uparrow}(\mathbf{r}_1,\mathbf{r}_2;i\omega) = \sum_n \left\{\frac{u_n^*(\mathbf{r}_2)u_n(\mathbf{r}_1)}{i\omega+\epsilon_n} + \frac{v_n(\mathbf{r}_2)v_n^*(\mathbf{r}_1)}{i\omega-\epsilon_n}\right\}, \tag{A5}$$

$$\mathcal{G}_{\downarrow\uparrow}(\mathbf{r}_1,\mathbf{r}_2;i\omega) = \sum_n \left\{\frac{v_n^*(\mathbf{r}_2)u_n(\mathbf{r}_1)}{i\omega+\epsilon_n} - \frac{u_n(\mathbf{r}_2)v_n^*(\mathbf{r}_1)}{i\omega-\epsilon_n}\right\}, \tag{A6}$$

$$\mathcal{G}_{\uparrow\downarrow}(\mathbf{r}_1,\mathbf{r}_2;i\omega) = \sum_n \left\{\frac{u_n^*(\mathbf{r}_2)v_n(\mathbf{r}_1)}{i\omega+\epsilon_n} - \frac{v_n(\mathbf{r}_2)u_n^*(\mathbf{r}_1)}{i\omega-\epsilon_n}\right\}, \tag{A7}$$

$$\mathcal{G}_{\downarrow\downarrow}(\mathbf{r}_1,\mathbf{r}_2;i\omega) = \sum_n \left\{\frac{v_n^*(\mathbf{r}_2)v_n(\mathbf{r}_1)}{i\omega+\epsilon_n} + \frac{u_n(\mathbf{r}_2)u_n^*(\mathbf{r}_1)}{i\omega-\epsilon_n}\right\}, \tag{A8}$$

where ϵ_n are the eigenvalues of the Bogoliubov–de Gennes equations

$$\epsilon_n u_n(\mathbf{r}) = H_e u_n(\mathbf{r}) + \Delta(\mathbf{r})v_n(\mathbf{r}), \tag{A9}$$

$$\epsilon_n v_n(\mathbf{r}) = \Delta^*(\mathbf{r})u_n(\mathbf{r}) - H_e^* v_n(\mathbf{r}). \tag{A10}$$

Notice that these Bogoliubov–de Gennes equations do not carry spin indices since the ferromagnetic layers are treated as a perturbation. This fact simplifies the problem dramaticaly in comparison to the *numerical* calculations of Sipr and Gyorffy,[24] where the spin dependent Bogoliubov–de Gennes equations have been used.[44,45]

These Bogoliubov–de Gennes equation has to be solved together with the gap equation (A1), which can be expressed as

$$\Delta(\mathbf{r}) = \frac{1}{\beta}\sum_{i\omega} G_{\downarrow\uparrow}(\mathbf{r},\mathbf{r};i\omega), \tag{A11}$$

leading finally to

$$\frac{\Delta(\mathbf{r})}{g} = \sum_{n} u_n(\mathbf{r}) v_n^*(\mathbf{r})[1-2f(\epsilon_n)]. \tag{A12}$$

The expressions for Eqs. (A3) and (A4) can be rewritten after summation over frequency as

$$L_A = \sum_{nm} \{\lambda_{AT}^{(nm)} T_{nm} + \lambda_{AP}^{(nm)} P_{nm}\}, \tag{A13}$$

$$L_B = \sum_{nm} \{\lambda_{BT}^{(nm)} T_{nm} + \lambda_{BP}^{(nm)} P_{nm}\}, \tag{A14}$$

where T_{nm} and P_{nm} are just thermal factors for the quasiparticle-quasihole and quasiparticle-quasiparticle channels given by

$$T_{nm} = \frac{1}{\beta}\sum_{i\omega} \frac{1}{(i\omega \pm \epsilon_n)(i\omega \pm \epsilon_m)} = \frac{f(\epsilon_m)-f(\epsilon_n)}{\epsilon_m - \epsilon_n}, \tag{A15}$$

$$P_{nm} = \frac{1}{\beta}\sum_{i\omega} \frac{1}{(i\omega \pm \epsilon_n)(i\omega \mp \epsilon_m)} = \frac{f(\epsilon_m)+f(\epsilon_n)-1}{\epsilon_m + \epsilon_n}. \tag{A16}$$

The λ coefficients are products of coherence factors A, B, and C defined below, i.e.,

$$\lambda_{AT}^{(nm)} = [A_n(1,2)A_m(2,1) + B_n(1,2)B_m(2,1) + C_n(1,2)C_m^*(1,2) + C_n(2,1)C_m^*(2,1)], \tag{A17}$$

$$\lambda_{AP}^{(nm)} = [A_n(1,2)B_m(2,1) + B_n(1,2)A_m(2,1) - C_n(1,2)C_m^*(2,1) - C_n(2,1)C_m^*(1,2)], \tag{A18}$$

$$\lambda_{BT}^{(nm)} = [B_n(2,1)B_m(1,2) + A_n(2,1)A_m(1,2) + C_n^*(2,1)C_m(2,1) + C_n^*(1,2)C_m(1,2)], \tag{A19}$$

$$\lambda_{BP}^{(nm)} = [B_n(2,1)A_m(1,2) + A_n(2,1)B_m(1,2) - C_n^*(2,1)C_m(1,2) - C_n^*(1,2)C_m(2,1)], \tag{A20}$$

where the coherence factors, expressed in terms of the eigenfunctions $u_n(\mathbf{r})$ and $v_n(\mathbf{r})$, are

$$A_n(1,2) = u_n^*(\mathbf{r}_2)u_n(\mathbf{r}_1), \tag{A21}$$

$$B_n(1,2) = v_n(\mathbf{r}_2)v_n^*(\mathbf{r}_1), \tag{A22}$$

$$C_n(1,2) = u_n^*(\mathbf{r}_2)v_n(\mathbf{r}_1). \tag{A23}$$

The calculation of the exchange coupling defined in Eq. (A2) is very difficult, because it involves the knowledge of the eigenvalues and eingenvectors of the Bogoliubov–de Gennes (BdG) equations (A9) and (A10), a sum over all these intermediate states at finite temperatures and a double integration in real space. Thus, the general results can only be obtained through heavy numerical computations. Here, though, this path is not chosen. First, an understanding of a simplified version of the problem is attempted.

The solutions of the BdG equations are quite generally written as

$$u_n(\mathbf{r}) = h_{u_n}(x)\exp[i(k_y y + k_z z)], \tag{A24}$$

$$v_n(\mathbf{r}) = h_{v_n}(x)\exp[i(k_y y + k_z z)], \tag{A25}$$

where the translational invariance of the system along the *yz* was taken into account. The BdG equation reduces to the form

$$\epsilon_n \mathbf{h}(x) = H_e(x)\sigma_z \mathbf{h}(x) + \Delta(x)\sigma_x \mathbf{h}(x) = \hat{\Omega}\mathbf{h}(x), \tag{A26}$$

where the eigenvector $\mathbf{h}(x)$ has components $[h_{u_n}(x), h_{v_n}(x)]$. Here the single particle Hamiltonian is

$$H_e(x) = -\frac{1}{2m}\frac{\partial^2}{\partial x^2} - \mu_{\text{eff}} + U(x), \tag{A27}$$

with an *effective* chemical potential

$$\mu_{\text{eff}} = \mu - \frac{k_\perp^2}{2m}, \tag{A28}$$

where $k_\perp^2 = k_y^2 + k_z^2$. It is important to notice that the operator $\hat{\Omega}$ is not positive definite, i.e., ϵ_n and $-\epsilon_n$ are simultaneous eigenvalues of $\hat{\Omega}$. To circumvent this difficulty, it is useful to square the operator $\hat{\Omega}$ to obtain

$$\hat{\Omega}^2(x) = H_e^2(x) + \Delta^2(x) + i[H_e(x), \Delta(x)]\sigma_y, \tag{A29}$$

and it is also useful to write the corresponding eigenvector $\mathbf{h}(x)$ as

$$\mathbf{h}(x) = w_n(x)\mathbf{p}_n(x) \tag{A30}$$

where $\mathbf{p}_n(x)$ is a spinor state with components $[p_{u_n}(x), p_{v_n}(x)]$ and $w_n(x)$ is a scalar function satisfying the eigenvalue equation

$$H_e(x,k_\perp)w_n(x) = \xi_n(k_\perp)w_n(x), \tag{A31}$$

in the normal state of the superconductor. In addition, notice that the eigenfunctions $\mathbf{h}(x)$ can be classified in terms of even and odd solutions about the point $x=0$, given that the single particle potential $U(x)$ and the pair potential $\Delta(x)$ are even functions of x. In addition, the squared eigenergies can be written as

$$\epsilon_n^2 = \langle H_e^2(x)\rangle + \langle \Delta^2(x)\rangle + i\langle [H_e(x),\Delta(x)]\sigma_y\rangle, \quad \text{(A32)}$$

and since the operator $\hat{\Omega}$ is Hermitian and have *real* eigenvalues ϵ_n, the squared eigenergies ϵ_n^2 must be real and positive definite.

With these simplifications it is easy to obtain the eigenfunctions $w_n(x)$, but it is still very difficult to obtain the spinor $\mathbf{p}(x)$. If we assume that $\mathbf{p}(x)$ is a spinor with real elements, then the last term in (A32) vanishes. This can be assumed without loss of generality. If, in addition, $\mathbf{p}(x)$ is nearly independent of x a dramatic simplification occurs, i.e.,

$$|p_{u_n}|^2 = \frac{1}{2}[1+\xi_n/\epsilon_n], \quad \text{(A33)}$$

$$|p_{v_n}|^2 = \frac{1}{2}[1-\xi_n/\epsilon_n], \quad \text{(A34)}$$

where the eigenvalues are

$$\epsilon_n^2 = \xi_n^2 + \Delta^2, \quad \text{(A35)}$$

with $\Delta = \langle \Delta(x)\rangle$. This simplification is justified only when $\Delta(x)$ can be approximated by its spatially averaged value

$$\langle \Delta(x)\rangle = g\sum_{n,k_\perp} \langle w_n^2(x)\rangle \frac{\Delta}{2\epsilon_n(k_\perp)}\{1-2f[\epsilon_n(k_\perp)]\}. \quad \text{(A36)}$$

The analysis described in this appendix applied to Eq. (A2) leads to effective magnetic coupling indicated in Eq. (18) of Sec. IV.

APPENDIX B

In this appendix, the procedure to calculate the effective Hamiltonian is outlined. The effective Hamiltonian of Eq. (18) can be written as

$$H_{\text{eff}} = -\frac{E_J}{N_{3D}}\int\int_{\partial\Omega_{\xi,\xi'}} d\xi d\xi' N(\xi,\xi')Q(\xi,\xi',\Delta), \quad \text{(B1)}$$

where

$$N(\xi,\xi') = 2\sum_{n,m,k_\perp}\int\int_{\partial\Omega_{x_1,x_2}} dx_1 dx_2 D_{nmk_\perp}(x_1,x_2) \times \delta[\xi-\xi_n(k_\perp)]\delta[\xi'-\xi_m(k_\perp)] \quad \text{(B2)}$$

plays the role of a spatially averaged two-particle density of states, with $D_{nmk_\perp}(x_1,x_2)$ being the weighting factor defined in Eq. (20). The function

$$Q(\xi,\xi',\Delta) = C_T(\xi,\xi',\Delta)T(\xi,\xi',\Delta) + C_P(\xi,\xi',\Delta)P(\xi,\xi',\Delta) \quad \text{(B3)}$$

contains the coherence factors in the quasiparticle-quasihole sector

$$C_T(\xi,\xi',\Delta) = [p_u(\xi,\Delta)p_u(\xi',\Delta) + p_v(\xi,\Delta)p_v(\xi',\Delta)]^2 \quad \text{(B4)}$$

and quasiparticle-quasiparticle sector

$$C_P(\xi,\xi',\Delta) = [p_u(\xi,\Delta)p_v(\xi',\Delta) - p_v(\xi,\Delta)p_u(\xi',\Delta)]^2. \quad \text{(B5)}$$

The quasiparticle-quasihole thermal factor

$$T(\xi,\xi',\Delta) = \frac{f(\epsilon')-f(\epsilon)}{\epsilon'-\epsilon}, \quad \text{(B6)}$$

and the quasiparticle-quasiparticle thermal factor is

$$P(\xi,\xi',\Delta) = \frac{f(\epsilon')+f(\epsilon)-1}{\epsilon'+\epsilon}, \quad \text{(B7)}$$

where $\epsilon = \sqrt{\xi^2+\Delta^2}$. Using the definitions

$$p_u(\xi) = \frac{1}{\sqrt{2}}\sqrt{(1+\xi/\epsilon)}, \quad \text{(B8)}$$

$$p_v(\xi) = \frac{1}{\sqrt{2}}\sqrt{(1-\xi/\epsilon)}, \quad \text{(B9)}$$

the coherence factors can be rewritten as

$$C_T(\xi,\xi',\Delta) = \frac{1}{2}\left[1+\frac{\xi\xi'}{\epsilon\epsilon'}+\frac{\Delta^2}{\epsilon\epsilon'}\right], \quad \text{(B10)}$$

$$C_P(\xi,\xi',\Delta) = \frac{1}{2}\left[1-\frac{\xi\xi'}{\epsilon\epsilon'}-\frac{\Delta^2}{\epsilon\epsilon'}\right]. \quad \text{(B11)}$$

Using the fact that $\epsilon'^2-\epsilon^2 = \xi'^2-\xi^2$ (s-wave case, where Δ is independent of ξ), the effective Hamiltonian can be written as a sum of two contributions:

$$H_{\text{eff}}^{(1)} = -\frac{E_J}{N_{3D}}\int_{-\mu}^{\infty} d\xi G(\epsilon)\xi I_1(\xi), \quad \text{(B12)}$$

$$H_{\text{eff}}^{(2)} = -\frac{E_J}{N_{3D}}[2\Delta^2]\int_{-\mu}^{\infty} d\xi G(\epsilon) I_2(\xi), \quad \text{(B13)}$$

where the auxiliary function is given by

$$G(\epsilon) = \tanh(\epsilon/2T)/\epsilon \quad \text{(B14)}$$

and the auxiliary integrals are of the form

$$I_1(\xi) = \int_{-\mu}^{\infty} d\xi' \frac{N(\xi,\xi')}{\xi'-\xi}, \quad \text{(B15)}$$

$$I_2(\xi) = \int_{-\mu}^{\infty} d\xi' \frac{N(\xi,\xi')}{\xi'^2-\xi^2}. \quad \text{(B16)}$$

The dominant contribution to H_{eff} in the asymptotic limit $k_{Fs}d_s \gg 1$ comes from $H_{\text{eff}}^{(1)}$, where

$$I_1(\xi) = \frac{\mathcal{F}N_{3D}}{4\pi^2(2k_{Fs}d_s)^2} d_s \sqrt{\frac{2m}{\sqrt{\xi+\mu}}}\sin[2d_s\sqrt{2m(\xi+\mu)}]. \quad \text{(B17)}$$

The evaluation of $H_{\rm eff} \approx H_{\rm eff}^{(1)}$ proceeds as follows. Defining $z=\epsilon/2T$, integrating by parts over the variable ξ in $H_{\rm eff}^{(2)}$, and making the variable substitution $\xi=\eta^2/2m-\mu$ leads to

$$H_{\rm eff} \approx -\frac{E_J \mathcal{F}}{4\pi^2(2k_{Fs}d_s)^2}\Gamma(d_s), \tag{B18}$$

with

$$\Gamma(d_s)=\frac{1}{2mT}\mathrm{Re}\int_0^\infty d\eta\,\eta\,\exp(2i\eta d_s) \times\left[\frac{\tanh(z)}{z}+\xi\frac{d}{d\xi}\left(\frac{\tanh(z)}{z}\right)\right], \tag{B19}$$

where Re indicates real part. Using the series representation

$$\frac{\tanh(z)}{z}=8\sum_n \frac{1}{\pi^2(2n+1)^2+4z^2}, \tag{B20}$$

where n is a positive integer, setting $\beta=1/T$ and $\mu=E_F$, one can write

$$\Gamma(d_s)=\frac{4\beta}{m}\frac{\partial}{\partial\alpha}[\alpha\Lambda(\alpha,d_s)]|_{\alpha=1}, \tag{B21}$$

where the auxiliary function $\Lambda(\alpha,d_s)$ can be represented by the integral

$$\lambda(\alpha,d_s)=\mathrm{Re}\sum_n \int_0^\infty d\eta\frac{\eta\exp(2i\eta d_s)}{\pi^2(2n+1)^2+\beta^2\Delta^2+\alpha^2\beta^2\xi^2}. \tag{B22}$$

The poles of the integrand occur at $\eta=\pm\eta_n\exp(\pm i\theta_n)$ where $\alpha\beta\eta_n^2/2m=\sqrt{\beta^2E_F^2+y_n^2}$, $y_n^4=\pi^2(2n+1)^2+\beta^2\Delta^2$ and $\tan(2\theta_n)=y_n^2/(\beta E_F)$. Integration over the first quadrant of the complex plane leads to

$$\Lambda(\alpha,d_s)=\frac{\pi m}{\alpha\beta}\sum_n\left(\frac{1}{y_n^2}\exp[-2\eta_n d_s\sin(\theta_n)] \times\cos[2\eta_n d_s\cos(\theta_n)]\right). \tag{B23}$$

This expression can be estimated in the limit $\Delta \gg T$ leading to

$$\Gamma(d_s)=2\cos(2k_{F_s}d_s)\exp(-k_{F_s}d_s\Delta/E_F), \tag{B24}$$

and in the limit $\Delta \ll T$ leading to

$$\Gamma(d_s)=2\cos(2k_{F_s}d_s)\left[1-\frac{2}{3\pi^2}\left(\frac{\Delta}{E_F}\right)^2\right]\exp(-\pi k_{F_s}d_sT/E_F). \tag{B25}$$

Combining Eqs. (B24), (B25), and (B18) leads to Eqs. (24) and (25) of Sec. V.

APPENDIX C

The averaged gap equation indicated in Eq. A36 can be recast in the form

$$\langle\Delta(x)\rangle=g\int_{-\mu}^\infty d\xi\langle N_1(\xi,x)\rangle\frac{\Delta}{2}G(\epsilon), \tag{C1}$$

where $G(\epsilon)$ is the same function as in Eq. (B14), and

$$\langle N_1(\xi,x)\rangle=\sum_{n,k_\perp}\langle w_n^2(x)\rangle\delta[\xi-\xi_n(k_\perp)] \tag{C2}$$

is the spatially averaged normal density of states. In the asymptotic limit $k_{F_s}d_s \gg 1$, and for $U_0/E_F \ll 1$,

$$\langle N_1(\xi,x)\rangle=\frac{\alpha m}{2\pi^2}\left[1-\frac{\mathrm{Si}(\alpha d_s)}{\alpha d_s}\right], \tag{C3}$$

where $\alpha=\sqrt{2m(\xi+\mu)}$ and $\mathrm{Si}(\alpha d_s)$ is the complementary sine integral defined by

$$\mathrm{Si}(\alpha d_s)=\int_0^{\alpha d_s}dy\frac{\sin(y)}{y}. \tag{C4}$$

Since the main contribution to the integral in Eq. (C1) comes from contributions close to the Fermi energy, one can approximate $\langle\Delta(x)\rangle$ by

$$\langle\Delta(x)\rangle\approx\frac{\langle N_1(0,x)\rangle}{N_{3D}(0)}\Delta_\infty(T). \tag{C5}$$

Using the asymptotic forms of $\mathrm{Si}(z)$ one obtains

$$\langle\Delta(x)\rangle\approx\Delta_\infty(T)\left[1-\frac{\pi}{4k_{F_s}d_s}+\frac{\cos(2k_{F_s}d_s)}{(2k_{F_s}d_s)^2}+\frac{\sin(2k_{F_s}d_s)}{(2k_{F_s}d_s)^3}\right]. \tag{C6}$$

This result relies heavily on the assumption that the energy spectrum can be approximated by Eq. (A35), it is intrinsically perturbative in nature, and represents an average over the spacer thickness d_s. This result should be contrasted with more realistic calculations of Valls and collaborators,[26,27] where changes in the order parameter near a superconductor-insulator interface and in superconducting films were calculated self-consistently for both short and long coherence length superconductors. Since the estimate of Eq. (C6) is perturbative and involves the assumption of weak metallic ferromagnets, it is not clear how to compare the present results with those of Valls and collaborators,[26,27] where superconductor-insulator interfaces are considered. The same method employed in their nice work should be applied here for a more realistic estimate, however, this calculation is beyond the scope of the present manuscript.

The results of this appendix differ subtantially from the GL results with boundary conditions presented in Sec. III, where the intrinsic assumption that the only length scale relevant to the spatial variation of the order parameter $\Delta(x)$ was the coherence length $\xi_{\rm GL}$. In the present appendix, this assumption is relaxed by considering that the spatial variations of the order parameter are controlled by the averaged changes normal state density of states $\langle N_1(\xi,x)\rangle$, thus by changes of the order of the Fermi wavelength $\lambda_F=2\pi/k_F$. This approximation seems to be consistent with the limit where $k_F\xi_{\rm GL}\approx\mathcal{O}(1)$, and indicates that the averaged order parameter $\langle\Delta(x)\rangle$ approaches the bulk value from below.

Within the same aproximation to obtain Eq. (C1), the critical temperature can be calculated to be

$$T_c(d_s)=T_c(\infty)\exp(-\eta/gN_{3D}), \tag{C7}$$

where the parameter

$$\eta=\frac{\pi}{4k_{F_s}d_s}+\frac{\cos(2k_{F_s}d_s)}{(2k_{F_s}d_s)^2}+\frac{\sin(2k_{F_s}d_s)}{(2k_{F_s}d_s)^3}. \tag{C8}$$

Since $\eta\ll 1$, the critical temperature takes the form

$$T_c(d_s)=T_c(\infty)[1-\eta/gN_{3D}], \tag{C9}$$

which varies very little as a function of d_s for large $k_{F_s}d_s$, and thus superconductivity is not strongly suppressed. This seems also to be the case for short coherence length superconductors in a slab geometry with insulating boundaries.[26,27] Thus, all this suggests that the approximation used in this appendix is valid for $k_{F_s}\xi_{\rm GL}\approx\mathcal{O}(1)$.

The reason for the difference between the Ginzburg-Landau results of Sec. III and the approximation discussed in this appendix is the following. In the derivation of the Ginzburg-Landau theory it was implicitily assumed that the variation of the order parameter is very small over a range much larger than the interatomic distance, which means that the first derivative terms of the order parameter do not contribute to the free energy functional even near the surface. This seems to be microscopically correct for the case of long coherence length superconductors, but for short coherence length superconductors a significant variation of the order parameter near the boundaries is expected, as it is also the case near superconductor/insulator interfaces.[26,27]

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