Pseudo-ε expansion of six-loop renormalization-group functions of an anisotropic cubic model

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Six-loop massive scheme renormalization group functions of a $(d=3)$-dimensional cubic model [J. M. Carmona, A. Pelissetto, and E. Vicari, Phys. Rev. B **61**, 15 136 (2000)] are reconsidered by means of the pseudo-ε expansion. The marginal order parameter components number $N_c=2.862\pm0.005$ as well as critical exponents of the cubic model are obtained. Our estimate $N_c<3$ leads in particular to the conclusion that all ferromagnetic cubic crystals with three easy axis should undergo a first-order phase transition.

I. INTRODUCTION

Progress in the qualitative understanding and the quantitative description of critical phenomena to a great extent was achieved by the ideas of renormalization group (RG) theory.[1] Only global features of a many-body system such as the range of interparticle forces, the space dimensionality d, as well as the dimension N and the symmetry of an order parameter were suggested to be responsible for long-distance and abrupt behavior of matter in the critical region. As a final step the role of the relevant parameters in microscopic Hamiltonians of various nature was represented adequately by effective Hamiltonians used in field theories. While already a vector field theory with an isotropic rotationally symmetrical order parameter allowed unified and correct description of a large spectrum of critical phenomena, an extension of theories is of special interest since in real substances anisotropies are always present.[2] For instance, in cubic crystals one expects the spin interaction to react to the lattice structure (crystalline anisotropy), suggesting additional terms in the Hamiltonian, invariant under the cubic group. The anisotropy breaks rotational symmetry of the Heisenberg-like ferromagnet and makes the order parameter to point either along edges or along diagonals of a cube. The corresponding field theory is defined by a Landau-Ginzburg-Wilson (LGW) Hamiltonian with two ϕ^4 terms of $O(N)$ and cubic symmetry and can exhibit a second order phase transition characterized either by spherical or cubic critical exponents. Varying the number of components of the order parameter N a new crossover phenomenon between these two scenarios takes place at the marginal value N_c.

In the framework of RG theory the critical point corresponds to the stable fixed point of the RG transformation.[1] In a model with competing fixed points the study of domains of their attraction as well as the crossover phenomenon is a fundamental problem for universality comprehension. Apart from the academic interest the determination of N_c can lead to decisive conclusions about phase transition order in a certain class of cubic crystals. For instance, $d=3$ cubic crystals with three easy axis should undergo either a second or a weak first-order phase transition provided N_c is greater or less than 3.[3] This argumentation states that the existence of the stable fixed point of a field theory is a necessary but not a sufficient condition for a model to exhibit a second-order phase transition. If the parameters of an microscopic Hamiltonian are mapped in the plain of the LGW Hamiltonian couplings to a point which lies outside the domain of attraction of the stable fixed point, the Hamiltonians will flow away to infinitely large values of couplings. Such a behavior might serve as evidence of a weak first-order phase transition and this is confirmed in some experiments (see Ref. 3, and references therein). If $N_c<3$ for a $d=3$ ferromagnet, the new cubic fixed point governing the critical regime appears to be inaccessible from the initial values of couplings which correspond to the ferromagnetic ordering with three easy axis. It appears that N_c is very close to $N=3$ and the critical exponents in both regimes are indistinguishable experimentally. In order to calculate the value of N_c within field theory one has to treat a complex model of two couplings. This is different from a Heisenberg-like N-component ferromagnet with weak quenched disorder, where the Harris criterion answers the question about the type of critical behavior.[4]

The description of the crossover and the precise determination of its numerical characteristics has been a challenge for many RG studies of the anisotropic cubic model. High orders of perturbation theory were obtained for this model in successive approximations either in ε expansion with dimensional regularization in the minimal subtraction (MS) scheme[5] or within the massive $d=3$ scheme.[6] The expressions are available now in the five-loop[7] and in the six-loop approximations,[8] respectively. However, divergent properties of the series did not allow their straightforward analysis and called for the application of various resummation procedures. For instance, N_c calculated in five-loop ε expansion[7] yielded depending on the resummation procedure $N_c=2.958$,[7] $N_c=2.855$,[9] and $N_c=2.87(5)$.[8] Alternative approaches on the basis of the ε expansion lead to $N_c=2.97(6)$ (Ref. 8) and to $N_c=2.950$.[10] On the other hand the massive $d=3$ scheme RG functions extended for arbitrary N to four loops,[11] yielded $N_c=2.89(2)$ (Ref. 11) (see Ref. 8 for a recent extended review of theoretical determination of N_c). These results suggest that the most reliable theoretical estimate is $N_c<3$.

However, recent MC simulations[12] questioned the values for N_c obtained so far. There, considering the finite size corrections of a cubic invariant perturbation term at the critical

$O(N)$-symmetric point, the eigenvalues ω_i of the stability matrix were extracted. From the estimate $\omega_2 = 0.0007(29)$ a value of $N_c = 3$ was concluded. This disagreement as well as the crucial influence of the value of N_c on the order of the phase transition makes an implementation of an alternative method for calculation of N_c to be hardly overrated. Recently the massive $d=3$ scheme RG functions of the cubic model were extended to five-loop order[13] and very recently the six-loop series[8] were obtained. The traditional analysis of these series, including an information on large order behavior of the RG functions,[8] yielded $N_c = 2.89(4)$. However let us note that the most accurate estimates of the critical exponents of a $d=3$ $O(N)$-symmetric ϕ^4 model in a massive field theoretical RG scheme are based on a pseudo-ε expansion technique.[14,15] Up to our knowledge the last method has never been applied to the cubic model.[17] Therefore the main aim of the present paper is to apply the pseudo-ε expansion to the up-to-date most precise massive scheme RG function[8] of the cubic model.

The setup of the article is as follows. After a brief consideration of the model and renormalization procedure, we present the pseudo-ε expansion for N_c and discuss its properties. Applying Padé and Padé-Borel analysis we obtain precise estimates of N_c and compare the result with the corresponding ε-expansion. Finally we evaluate the critical exponents of a $d=3$ cubic system belonging to the new universality class for different values of $N > N_c$ and discuss the weakly diluted Ising model case $N=0$.

II. PSEUDO-ε SERIES AND NUMERICAL RESULTS

We start from a $d=3$ effective LGW Hamiltonian with two couplings at terms of spherical and cubic symmetry:

$$\mathcal{H}(\varphi) = \int \mathrm{d}^3R \left\{ \frac{1}{2}\sum_{\alpha=1}^{N} \left[|\nabla \varphi_\alpha|^2 + m_0^2 \varphi_\alpha^2 \right] + \frac{u_0}{4!}\left(\sum_{\alpha=1}^{N} \varphi_\alpha^2 \right)^2 + \frac{v_0}{4!}\sum_{\alpha=1}^{N} \varphi_\alpha^4 \right\}, \tag{1}$$

where $\varphi_\alpha(R)$ are components of a bare N-component vector field; $u_0 > 0$, v_0 are bare couplings, m_0^2 is a squared bare mass being a linear function of temperature. The vicinity of a critical point corresponds to a long-distance behavior of the model (1), while ultraviolet divergences of the theory are dealt with by means of an appropriate renormalization procedure.[1] In particular, the renormalization of the bare couplings leads to $\beta_u(u,v)$ and $\beta_v(u,v)$—the so-called β functions; a renormalization of the bare field and square-field insertion produces $\gamma_\phi(u,v)$ and $\overline{\gamma}_{\phi^2}(u,v)$—the so-called γ functions. All these functions depend on renormalized couplings u and v.[18] The critical behavior of the model is determined by the infrared stable fixed point u^*, v^*. It is given by the condition that both β functions are zero and all the real parts of the stability matrix eigenvalues are positive. The pair correlation function critical exponent η and the correlation length critical exponent ν are obtained via the relations $\eta = \gamma_\phi(u^*,v^*)$, $1/\nu = 2 - \overline{\gamma}_{\phi^2}(u^*,v^*) - \gamma_\phi(u^*,v^*)$. The correction-to-scaling exponent ω is given by the largest stability matrix eigenvalue in the stable fixed point.

In the present study we reconsider RG functions of the model (1) as they are obtained within massive fixed $d=3$ scheme[19]

$$-\frac{\beta_u(u,v)}{u} = 1 - u - \frac{2}{3}v + \frac{4}{27}\frac{(190+41N)}{(8+N)^2}u^2 + \frac{400}{81}\frac{uv}{8+N} + \frac{92}{729}v^2 + \beta_u^{(3LA)} + \cdots + \beta_u^{(6LA)}, \tag{2}$$

$$-\frac{\beta_v(u,v)}{v} = 1 - 12\frac{u}{8+N} - v + \frac{4}{27}\frac{(370+23N)}{(8+N)^2}u^2 + \frac{832}{81}\frac{uv}{8+N} + \frac{308}{729}v^2 + \beta_v^{(3LA)} + \cdots + \beta_v^{(6LA)}, \tag{3}$$

$$\gamma_\phi(u,v) = \frac{8}{27}\frac{(2+N)}{(8+N)^2}u^2 + \frac{16}{81}\frac{uv}{8+N} + \frac{8}{729}v^2 + \gamma_\phi^{(3LA)} + \cdots + \gamma_\phi^{(6LA)}, \tag{4}$$

$$\overline{\gamma}_{\phi^2}(u,v) = \frac{(2+N)u}{8+N} + \frac{1}{3}v - 2\frac{(2+N)}{(8+N)^2}u^2 - \frac{4uv}{3(8+N)} - \frac{2}{27}v^2 + \overline{\gamma}_{\phi^2}^{(3LA)} + \cdots + \overline{\gamma}_{\phi^2}^{(6LA)}, \tag{5}$$

where $\beta_u^{(3LA)} \cdots \overline{\gamma}_{\phi^2}^{(6LA)}$ denote the three-loop contributions obtained in Ref. 20, the four-loop, the recent five-loop and the very recent six-loop contributions obtained in Refs. 11, 13, and 8, respectively. Furthermore, the large-order behavior was established for the RG functions (2)–(5) which allowed the application of the refined resummation procedure based on Borel transformation with conformal mapping.[8] In this way a convergent sequence of approximations for N_c as well as critical exponents within the cubic universality class were obtained.[8]

One possible way of analysis of the massive RG functions (2)–(5) consists in solution of a system of equations for the (resummed) β functions (2),(3)

$$\beta_u(u^*,v^*) = 0, \qquad \beta_v(u^*,v^*) = 0 \tag{6}$$

to get numerical values of a stable fixed point coordinates u^*, v^*. Then these numerical values are substituted into (resummed) series for γ functions (4),(5) which lead to the numerical values for critical exponents. As a result the final errors for the critical exponents are the sum of the errors of the series for exponents and of the errors coming from u^*, v^*. To avoid such errors accumulation it is standard now in the analysis of field theories with one coupling[14,15] to use a pseudo-ε expansion.[16] Here, we will apply the pseudo-ε expansion for a cubic model (1).[17] The procedure is defined in the following way. Let us introduce the functions

$$\beta_u(u,v,\tau) = -u\left(\tau - u - \frac{2}{3}v + \cdots \right),$$

$$\beta_v(u,v,\tau) = -v\left(\tau - 12\frac{u}{8+N} - v + \cdots\right), \qquad (7)$$

where the ''pseudo-ε'' auxiliary parameter τ has been introduced into the β-functions (2),(3) instead of the zeroth order term. Obviously, $\beta_u(u,v) \equiv \beta_u(u,v,\tau=1)$, $\beta_v(u,v) \equiv \beta_v(u,v,\tau=1)$. Then a fixed point coordinates are obtained as series in τ. The series for the stable fixed point coordinates $u^*(\tau), v^*(\tau)$ are then substituted into series (4),(5) for γ functions leading to the pseudo-ε expansion for critical exponents. In the resulting series the expansion parameter τ collects contributions from the loop integrals of the same order coming from both the series of β and γ functions. Finally, one puts $\tau=1$. In such a way one gets a self-consistent perturbation theory and avoids cumulation of errors originating from different steps of calculation.

With the above described method we obtain the marginal value N_c and the critical exponents in the cubic universality class for the model (1). The series for N_c reads

$$N_c = 4 - 4/3\tau + 0.29042005\tau^2 - 0.18967704\tau^3 + 0.19951035\tau^4 - 0.22465150\tau^5. \qquad (8)$$

One notes that at least up to the presented number of loops the series does not behave as an asymptotic one with factorial growth of coefficients. This can be seen by considering a Padé table (9) for N_c series (8)

$$\begin{bmatrix} 4 & 3 & 2.9158 & 2.8411 & ^{2.8922} & 2.8298 \\ 2.6667 & 2.9051 & ^{2.0643} & 2.8711 & 2.8638 & 0 \\ 2.9571 & 2.8423 & 2.8616 & 2.8616 & 0 & 0 \\ 2.7674 & 2.8646 & 2.8616 & 0 & 0 & 0 \\ 2.9669 & 2.8613 & 0 & 0 & 0 & 0 \\ 2.7423 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \qquad (9)$$

Here, a result of an approximant $[M/N]$ is represented as an element of a matrix with usual notation. The approximants $[0/4]$ and $[1/2]$ have poles at values of τ of the order 1 (at points $\tau_1=3.7$ and $\tau_2=1.1$, respectively) and thus the estimates of N_c on their basis are considered as unreliable (they are noted in the table by small numbers). The values in the first column of the table are merely the sums of the corresponding number of terms in the expansion (8) and do not diverge. However, the most prominent property of the table is the perfect convergence of the values within main diagonals. In particular, the six-loop result of the $[3/2]$ and the $[2/3]$ approximants and the five-loop result of $[2/2]$ approximant coincide within the fourth digit and lead to an estimate $N_c=2.8616$. Though the next order terms in Eq. (8) could spoil such convergence, it is worth to compare the pseudo-ε expansion (8) for N_c with corresponding ε-expansion series which is one loop order shorter[7]

$$N_c = 4 - 2\varepsilon + 2.58847559\varepsilon^2 - 5.87431189\varepsilon^3 + 16.82703902\varepsilon^4. \qquad (10)$$

The obvious worse convergence properties of the series (10) lead to a corresponding bad convergence of the values in the Padé table (obtained in Ref. 7). In particular, mere summation of several first terms leads now to diverging result

$$\begin{bmatrix} 4 & 2.667 & ^{3.627} & 1.952 & ^{-5.772} \\ 2 & 3.128 & 2.893 & 2.972 & 0 \\ 4.589 & 2.792 & 2.958 & 0 & 0 \\ -1.286 & 3.068 & 0 & 0 & 0 \\ 15.540 & 0 & 0 & 0 & 0 \end{bmatrix}. \qquad (11)$$

Here, the approximants $[0/2]$, $[0/4]$ have poles close to $\tau=1$ (at $\tau_1=2.3$, $\tau_2=0.9$) respectively, and thus are unreliable.

In order to take into consideration possible factorial divergence of the pseudo$-\varepsilon$ expansion (8), as a next step we apply to the series (8) the Padé-Borel resummation procedure. The Padé-Borel resummation of the initial sum $N_c(\tau)$ consists of the following steps: (i) construction of Borel-Leroy image of $N_c(\tau)$; (ii) its extrapolation by a rational Padé-approximant $[M/N](\tau t)$; and (iii) definition of a resumed $N_c^{\rm res}(\tau)$ by the integration $\int_0^\infty dt \exp(-t) t^p [M/N](\tau t)$, where p is an arbitrary parameter entering the Borel-Leroy image.[21] One possibility to fix p is to require fastest convergence of the resulting values, given by the diagonal approximant resummation similar to Padé analysis (9). However, the convergence of these values appears to be almost independent of p. On the other hand, approximants possessing poles on the positive real axis are considered as unreliable and we can equally well choose p to provide a minimal number of such divergent approximants. For instance, for $p \geqslant 4$ the imaginary part is smaller then 10^{-10} in the ''bad'' $[1/4]$ and $[3/2]$ approximants and therefore can be neglected. Processing the series (8) for $p=4$ as described we obtain the results presented in Eq. (12). Here, one encounters only one unreliable approximant $[1/2]$ which is again denoted with small numbers. This analysis yields $N_c = 2.862 \pm 0.005$, where error bar stems from the maximal deviation between the six and the five-loop results for arbitrary p between 0 and 10.

$$\begin{bmatrix} 4 & 3.0353 & 2.9245 & 2.8634 & 2.8763 & 2.8561 \\ 2.6667 & 2.8995 & ^{2.7173} & 2.8737 & 2.8685 & 0 \\ 2.9571 & 2.8461 & 2.8595 & 2.8631 & 0 & 0 \\ 2.7674 & 2.8617 & 2.8645 & 0 & 0 & 0 \\ 2.9669 & 2.8641 & 0 & 0 & 0 & 0 \\ 2.7423 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \qquad (12)$$

It is obvious that other values of interest such as fixed point coordinates and critical exponents can also be obtained within the pseudo-ε expansions. For instance, for different values of N we obtain the following expressions for the critical exponents γ of the susceptibility, ν of the correlation length, and ω of the correction-to-scaling

TABLE I. Our data for the critical exponents of the cubic model (last column) in comparison with other results. See the text for a full description.

N		Ref. 22	Ref. 8	Ref. 11	Ref. 24	Ref. 8′	This study
	γ	1.3746±0.0020	1.377(6)	1.3775	1.3850±0.0050	1.390(12)	1.387±0.001
3	ν	0.6997±0.0024	0.701(4)	0.6996	0.7040±0.0040	0.706(6)	0.705±0.001
	ω	0.8061 (Ref. 23)	0.799(14)	0.7786	0.7833±0.0054	0.781(4)	0.777±0.009
	γ	1.4208±0.0030	1.419(6)	1.4028	1.4074±0.0030	1.405(10)	1.416±0.004
4	ν	0.7225±0.0022	0.723(4)	0.7131	0.7150±0.0050	0.714(8)	0.719±0.002
	ω		0.790(8)		0.7887±0.0090	0.781(44)	0.777±0.002
	γ	1.4305±0.0040		1.4076			1.417±0.006
5	ν	0.7290±0.0016		0.7154			0.720±0.004
	ω						0.773±0.003
	γ	1.4322±0.0040		1.4082			1.417±0.009
6	ν	0.7301±0.0016		0.7157			0.718±0.003
	ω						0.771±0.005
	γ		1.422(6)	1.4074	1.4068±0.0030	1.404(10)	1.407±0.008
8	ν		0.723(2)	0.7153	0.7143±0.0035	0.712(6)	0.715±0.003
	ω		0.786(6)		0.7955±0.0150	0.775(88)	0.770±0.009
	γ	1.3993±0.0020	1.399(8)		1.3962±0.0040	1.396(14)	1.395±0.006
∞	ν	0.7108±0.0010	0.711(2)		0.7094±0.0030	0.708(8)	0.708±0.001
	ω		0.802(18)		0.7986±0.0200	0.790(18)	0.775±0.020

$$
\begin{aligned}
\gamma_{N=3} &= 1 + 2/9\tau + 0.10157666\tau^2 + 0.03325297\tau^3 + 0.02024452\tau^4 + 0.00312386\tau^5 + 0.00905558\tau^6,\\
\gamma_{N=4} &= 1 + 1/4\tau + 0.11188272\tau^2 + 0.03494088\tau^3 + 0.01575673\tau^4 - 0.00023288\tau^5 + 0.00322125\tau^6,\\
\gamma_{N=5} &= 1 + 4/15\tau + 0.11314861\tau^2 + 0.03107333\tau^3 + 0.00939269\tau^4 - 0.00376555\tau^5 - 0.00055733\tau^6,\\
\gamma_{N=\infty} &= 1 + 1/3\tau + 0.06675812\tau^2 + 0.00726155\tau^3 - 0.00746706\tau^4 - 0.00082309\tau^5 - 0.00713623\tau^6,\\
\nu_{N=3} &= 1/2 + 1/9\tau + 0.05383664\tau^2 + 0.01993814\tau^3 + 0.01227945\tau^4 + 0.00300477\tau^5 + 0.00535272\tau^6,\\
\nu_{N=4} &= 1/2 + 1/8\tau + 0.05902778\tau^2 + 0.02074731\tau^3 + 0.01004792\tau^4 + 0.00133632\tau^5 + 0.00248012\tau^6,\\
\nu_{N=5} &= 1/2 + 2/15\tau + 0.05964701\tau^2 + 0.01877086\tau^3 + 0.00688561\tau^4 - 0.00041380\tau^5 + 0.00060891\tau^6,\\
\nu_{N=\infty} &= 1/2 + 1/6\tau + 0.03612254\tau^2 + 0.00709205\tau^3 - 0.00142535\tau^4 + 0.00103317\tau^5 - 0.00285768\tau^6,\\
\omega_{N=3} &= \tau - 0.39042829\tau^2 + 0.29428918\tau^3 - 0.25565542\tau^4 + 0.31134025\tau^5 - 0.43957722\tau^6,\\
\omega_{N=4} &= \tau - 0.36419753\tau^2 + 0.24511892\tau^3 - 0.20419925\tau^4 + 0.21874431\tau^5 - 0.27962773\tau^6,\\
\omega_{N=5} &= \tau - 0.35129140\tau^2 + 0.21196053\tau^3 - 0.16985912\tau^4 + 0.17369321\tau^5 - 0.19948859\tau^6,\\
\omega_{N=\infty} &= \tau - 0.42249657\tau^2 + 0.34513141\tau^3 - 0.32006198\tau^4 + 0.44947688\tau^5 - 0.67842170\tau^6.
\end{aligned}
\tag{13}
$$

One can determine the cubic model critical exponents of the new universality class on the basis of the expansions (13) in the same manner as for the expansion (8) of N_c. However, the expansions for the combination $1/\gamma$ and $1/\nu - 1$ appear to have better convergence properties and all values are obtained on their basis. The Padé- and Padé-Borel analysis lead to the critical exponents values as they are given in the table I in the last column [here, we do not show intermediate results similar to Eqs. (9)–(12)]. The error bars for the critical exponents given in the Table I were obtained from the maximal deviation between the six- and the five-loop results among all deviations for the parameter p value $0 \leqslant p \leqslant 10$. The error bars within the pseudo-ε expansion are typically much smaller than those based on other methods. The reason is that β and γ functions contribute in a self-consistent way into the pseudo-ε expansion series for critical exponents.

To compare our results we represent in the Table I the values obtained from the five-loop ε expansion[7] by means of a modified Borel summation[22] and of the Borel transformation with conformal mapping[8] (the corresponding citation in the table is primed). Critical exponents from the four-loop fixed massive $d=3$ scheme with an application to the RG functions of Padé-Borel resummation[11] and the results from the six-loop RG functions resummed by Borel transformation with conformal mapping[8] are given in the table. Recently, the modified Padé-Borel resummation[22] has been applied[24] to the six-loop RG functions[8] of the cubic model. These data are also displayed in the table.

The error bars for the values of the Refs. 8 and 24 were obtained from the condition of the resulting stability in successive approximation orders. However, the numerical value of the fixed point coordinates were substituted into the expansions for the γ functions (4),(5). To this end the most reliable numerical values of the stable fixed point coordinates were substituted into the resummed γ functions and then an optimal value of the fit parameter in modified Padé-Borel resummation[24] and two fit parameters in the conformal mapping procedure[8] were chosen. The deviations between five- and six-loop results obtained within the resummation procedure with optimal fit parameter(s) value gave the error interval. To complete the list, we show the value of ω for $N=3$ obtained in Ref. 23 on the basis of Borel transformation involving knowledge on RG functions large-order behavior. Let us note that for finite values of N our data for γ and ν interpolate the results of the minimal subtraction[22,8] and the massive scheme,[11,8] though the values are closer to the last. On the other hand, our method gives smaller values for ω in comparison with other methods. We note as well that passing from the four-loop[11] to the six-loop[24] approximation in frames of a massive $d=3$ approach shifts the numerical values of critical exponents towards our data.

In the limit $N\to\infty$ the critical properties of the cubic model reconstitute those of the annealed diluted Ising model[25] where Fisher renormalization for critical exponents holds.[26] In particular, based on the recent RG estimates for the critical exponents of the pure Ising model $\alpha=0.109\pm0.004$, $\nu=0.6304\pm0.0013$, $\gamma=1.2397\pm0.0013$,[15] one obtains the values $\nu=0.708\pm0.005$, $\gamma=1.391\pm0.008$ for the Ising model with annealed disorder. The last values agree very well with our results of the last row of the Table I. Moreover, they are in very good agreement with other data of the table.

It is worth noting here that the RG series for the cubic model allow to reconstitute the functions which describe the Ising model with the other type of randomness. By substitution $N=0$ one reconstitutes the weakly diluted quenched Ising model (RIM).[27] In this case, however, the pseudo-ε expansion in τ degenerates into a $\sqrt{\tau}$ expansion for the same reasons as the ε expansion for the RG functions degenerates into a $\sqrt{\varepsilon}$ expansion.[27] Moreover, our calculation show that an expansion in $\sqrt{\tau}$ is numerically useless as this was shown for $\sqrt{\varepsilon}$ expansion.[9,10,28] This can be regarded as an evidence of the Borel nonsummability of the RIM RG functions. Since the asymptotic properties of the series still are not proven despite of noticing their divergent character,[29,30] the RG functions of RIM as series in renormalized couplings used to be treated by means of Padé-Borel or Chisholm-Borel resummations (see, e.g., Refs. 10,13,31). The first of the stated methods was recently applied to study the five-loop RG functions of RIM.[13] The analysis allowed the authors to obtain the five-loop estimates for the RIM critical exponents. Extending the analysis of Ref. 13 to the six-loop order reveals the wide gap between five- and six-loop fixed point coordinates. This leads to an inconsistency of the six-loop values of critical exponents compared with the five-loop results of Ref. 13. However, the analytical solution of a toy $d=0$ RIM showed its free energy to be Borel summable provided that resummation is done asymmetrically: resumming first the series in the coupling u and subsequently the series in v.[30] The corresponding resummation applied to $d=3$ RIM massive scheme RG functions allowed precise determination of the critical exponents.[32]

III. CONCLUSIONS

In the present paper we studied the critical properties of a cubic model associated with ϕ^4 terms of spheric and cubic symmetry of the LGW Hamiltonian. In particular, we were interested in the crossover between $O(N)$-symmetric and cubic behavior which occurs at a certain value N_c of order parameter components number. Recently, five-[13] and six-loop[8] order RG functions were obtained for the cubic model within massive $d=3$ scheme.[6] We applied the pseudo-ε expansion[16] to their analysis. This method is known as a standard one for the $O(N)$-symmetric model analysis and leads to the most accurate values of critical exponents.[15] Here, to our knowledge, it has been applied to the cubic model for the first time.[17]

The pseudo-ε expansion for N_c appears to have much better convergence properties then the corresponding ε expansion [see Padé tables (8) and (10)]. This provides very good convergence of its Padé analysis (9). The last together with the refined Padé Borel analysis yields the best estimate $N_c=2.862\pm0.005$ of the paper. Our conclusion $N_c<3$ means in particular, that all ferromagnetic cubic crystals with three easy axis should undergo a first-order phase transition.[3]

We obtained the values of cubic model critical exponents in the new universality class in pseudo-ε expansions with the results given in Table I. In the $N\to\infty$ limit our data reproduce the critical behavior of an annealed weakly diluted Ising model.[25] The $N\to0$ limit, corresponding to a quenched weakly diluted Ising model,[27] however, does not yield reliable results in pseudo-$\sqrt{\varepsilon}$ expansion. Within a traditional $d=3$ massive technique the resummation of the RG functions by means of the convenient Padé-Borel analysis reveals a gap between five- and six-loop fixed point coordinates. This leads to an inconsistency of the obtained critical exponents values compared to the declared in Ref. 13. Let us note, however, that recently reliable values have been obtained[32] by a resummation method which treats the couplings of the RIM model asymmetrically.[30]

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