Nonhomogeneous magnetic order in superconductor-ferromagnet multilayers

F. S. Bergeret,[1] K. B. Efetov,[1,2] and A. I. Larkin[1,2,3]
[1]Theoretische Physik III, Ruhr-Universität Bochum, 44780 Bochum, Germany
[2]L. D. Landau Institute for Theoretical Physics, 117940 Moscow, Russia
[3]Theoretical Physics Institute, University of Minnesota, Minneapolis, Minnesota 55455
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We study the possibility of a nonhomogeneous magnetic order [cryptoferromagnetic (CF) state] in heterostructures consisting of a bulk superconductor and a ferromagnetic thin layer, due to the influence of the superconductor. The exchange field in the ferromagnet may be strong and exceed the inverse mean free time. An approach based on solving the Eilenberger equations in the ferromagnet and the Usadel equations in the superconductor is developed. We derive a phase diagram between the cryptoferromagnetic and ferromagnetic states and discuss the possibility of an experimental observation of the CF state in different materials.

I. INTRODUCTION

Recently, the interest in experiments on superconducting-ferromagnet (S/F) hybrid structures has grown rapidly. Such structures show the coexistence of these two competing types of ordering but their mutual influence is still a controversial point.[1–6] In these experiments, the multilayers contained strong ferromagnets such as Fe or Gd with the Curie temperature up to 1000 K and superconductors with transition temperatures not exceeding 10 K, such as Nb or V.

Naturally, in most theoretical works only the influence of the ferromagnet on the superconductivity of S/F systems was considered.[7–9] One may argue that a modification of the magnetic ordering would need energies of the order of the Curie temperature, which is much larger than the superconducting transition temperature T_c. Therefore, any change of the ferromagnetic order would be less energetically favorable than the destruction of the superconductivity in the vicinity of the ferromagnet.

This simple argument was questioned in a recent experimental work,[10] where Nb/Fe bilayers were studied using different experimental techniques. Direct measurements using the ferromagnetic resonance showed that in several samples with thin ferromagnetic layers ($10-15$ Å) the average magnetic moment started to decay at the superconducting transition temperature T_c. The measurements were possible only in a limited range of the temperatures below T_c and the decrease of the magnetic moment in this interval reached 10% without any sign of a saturation. As a possible explanation of the effect, it was assumed in Ref. 10 that the superconductivity affected the magnetic order causing a domainlike structure.

A possibility of a domainlike magnetic structure in presence of superconductivity was first suggested by Anderson and Suhl long ago.[11] They argued that a weak ferromagnetism of localized electrons should not destroy the superconductivity in the conduction band. Instead, it may become more favorable energetically to build a domain structure called the cryptoferromagnetic state.[11] Later this state was investigated in detail for small concentrations of the magnetic moments both theoretically and experimentally (for review see, e.g., Ref. 12).

At the same time, any competition of the ferromagnetic ordering and superconductivity is hardly possible in a bulk material if the concentration of the magnetic moments is high because in this case the superconductivity is immediately destroyed. The effect of the magnetic moments should be somewhat reduced if they are distributed not everywhere in the bulk but are concentrated in certain regions of the sample. The S/F multilayers can be an example of such a system.

In this paper, we investigate theoretically the possibility of a cryptoferromagneticlike (CF) state in S/F bilayers with parameters corresponding to the structures used in the experiments.[1–6,10] The magnetic moments in the ferromagnetic materials such as Fe or Gd used in these works are quite strong and therefore one cannot apply directly approaches developed previously.[12] However, such a study is very important because it may allow to clarify the question about the cryptoferromagnetic state in the experiment[10] and to make predictions for other S/F multilayers. The large magnetic energies involved make the problem quite nontrivial and demand development of new approaches.

To the best of our knowledge, the possibility of a nonhomogeneous magnetic order in multilayers was considered only in Ref. 13. However, although the authors of Ref. 13 came to the conclusion that a first order phase transition from the homogeneous ferromagnetic state to the domainlike structure (DS) state due to the interaction with the superconductor may occur, the results obtained can hardly be used for quantitative estimates. For example, they assumed that the period of the structure b had to be not only much smaller than the size of the Cooper pair $\bar{\xi}$, but also than $\bar{\xi}\sqrt{T_c/h}$, where h is the energy of interaction of conduction electrons with the localized magnetic moments. These assumptions are not suitable for strong ferromagnets such as Fe or Gd. In addition, the authors of Ref. 13 used as a boundary condition the continuity of the superconducting order parameter Δ and of its derivative at the S/F boundary. In order to use this condition one had to assume that the electron-electron attrac-

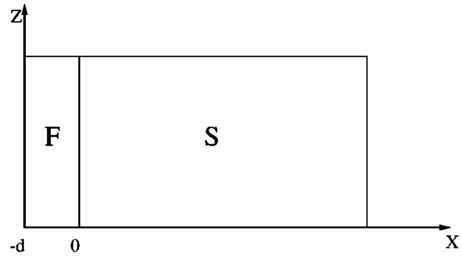

FIG. 1. Geometry of the F/S system. The magnetization of the F layer is parallel to the S/F interface, i.e., perpendicular to the plane of the figure.

tion was the same in the superconductor and ferromagnet, which is definitely not the case for Fe/Nb or Gd/Nb structures used in the experiments.[1–6,10]

In contrast, we present here a microscopic derivation of the phase diagram valid for realistic parameters of the problem involved. Our consideration is based on writing the Eilenberger equations[14] for the ferromagnetic material and the Usadel[15] equations for the superconductor. Of course, these equations are modified to include spin variables. Interaction of the magnetic moments with spins of electrons of Cooper pairs is the most important in the case involved. We will consider here a CF state with a magnetic moment that rotates in space. In the absence of a strong anisotropy this state is more favorable than the domain structure of Ref. 13. For such a cryptoferromagnetic state, spin variables do not separate in the Eilenberger and Usadel equations. The thickness of the ferromagnet is assumed to be small and this allows us to perform calculations explicitly. We will show that the phase transition between the CF and ferromagnetic (F) phases is of second order and the period of the structure b goes to infinity at the critical point. The restrictions we use explicitly are consistent with the parameters in Ref. 10 and can be written as

$$d \ll \xi_F = v_0/h, \quad T_c \ll h \ll \epsilon_0, \tag{1}$$

where d is the thickness of the ferromagnetic layer, v_0 and ε_0 are the Fermi velocity and Fermi energy. Even in such a strong ferromagnet as iron, ξ_F is of the order 10 Å. For weaker ferromagnets such as Gd, ξ_F is considerably larger and the inequalities (1) can be fulfilled rather easily. The phase diagram we derive below allows us to make definite predictions about a possibility of the cryptoferromagnetic state in different materials.

II. THE MODEL

We consider an S/F bilayer assuming that the superconductor occupies the half space $x>0$, while the ferromagnetic film is located in the region $-d<x<0$, as shown in Fig. 1. The Hamiltonian describing the system is chosen in the following form:

$$H(\gamma) = H' - \gamma \int_{-d<x<0} d\mathbf{r} \Psi_\alpha^+(\mathbf{r}) [\mathbf{h}(\mathbf{r}) \boldsymbol{\sigma}]_{\alpha\beta} \Psi_\beta(\mathbf{r}) + H_M, \tag{2}$$

where

$$H' = H_0 + H_{\mathrm{int}} \tag{3}$$

contains the one-particle electron energy H_0 (including interaction with impurities) and the interaction between the conduction electrons H_{int}. We assume that H_{int} has the form

$$H_{\mathrm{int}} = -\lambda_0 \int_{x>0} \Psi_\alpha^+(\mathbf{r}) \Psi_\beta^+(\mathbf{r}) \Psi_\beta(\mathbf{r}) \Psi_\alpha(\mathbf{r}) d\mathbf{r} \tag{4}$$

which means that there is no interaction between the conduction electrons in the ferromagnet. We assume that $\lambda_0 > 0$ such that without the ferromagnet one would have a conventional superconductor with s pairing.

The second term in Eq. (2) describes the interaction of the conduction electrons with the exchange field of the magnetic moments in the ferromagnet, where γ is a constant that will be put to 1 at the end. $\mathbf{h}$ is the exchange field and $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ is the vector containing the Pauli matrices as components. According to the geometry described above $\mathbf{h}$ is nonzero in the region $-d<x<0$. Writing Eq. (2) we neglect influence of the localized moments on the orbital motion of the conduction electrons since the exchange interaction is the dominant Cooper pair breaking mechanism for the problem involved.

This can be rather easily understood for the geometry used. If the magnet is in the ferromagnetic state and the exchange magnetic field $\mathbf{h}$ is directed along the interface, the corresponding vector potential $\mathbf{A}$ in the superconducting region is a constant and can be removed by a gauge transformation of the superconducting order parameter Δ. At the same time, in the ferromagnet one may consider the influence of the exchange field on the electron spins only. The situation cannot change if the cryptoferromagnetic state is formed unless the period of the oscillations of the magnetic moment is very small.

The last term H_M in Eq. (2) describes the interaction between the localized moments in the ferromagnet. Of course, this interaction can be very complicated and to determine it one should know a detailed band structure and different parameters of interaction. Such calculations would be too complicated for the present study and we write the term H_M phenomenologically.

Our aim is to obtain an expression for the free energy of the system for different magnetic structures in the F layer. To determine the contribution of an inhomogeneous alignment of magnetic spins to the total energy we use the limit of a *continuous* material and replace the spins by classical vectors. We want to study here structures with magnetic moments directed parallel to the interface between the ferromagnet and superconductor. A perpendicular component of the magnetization would induce strong Meissner currents in the superconductor, which would require greater additional energy.

Therefore, we write the energy H_M of a nonhomogeneous structure in the continuum limit as

$$H_M = \int J[(\boldsymbol{\nabla} S_x)^2 + (\boldsymbol{\nabla} S_y)^2 + (\boldsymbol{\nabla} S_z)^2] dV, \tag{5}$$

where the magnetic stiffness J characterizes the strength of the coupling between the localized moments in the F layer and S_i's are the components of a unit vector which is parallel to the local direction of the magnetization. Writing $\mathbf{S} = (0, -\sin\Theta, \cos\Theta)$ and minimizing the energy H_M we obtain the equation $\Delta\Theta=0$. We consider only the solutions of this equation that are of interest for us:

$$(\mathrm{a})\Theta=0, \quad (\mathrm{b})\Theta=Qy. \tag{6}$$

The solution (a) in Eq. (6) corresponds to the F state, whereas the solution (b) describes a CF state with a homogeneously rotating magnetic moment. The wave vector of this rotation is denoted by Q. The magnetization is chosen to be parallel to the FS interface. With all these assumptions the magnetic energy Ω_M (per unit surface area) is given by

$$\Omega_M=JdQ^2. \tag{7}$$

The corresponding energy of the F state equals zero. Assuming that T is close to T_c one can determine the lowering of the superconducting energy due to the suppression of the superconductivity in the S layer. Not very close to the interface, at distances exceeding $\bar{\xi}\sim\sqrt{\xi_0 l}$, where $\xi_0=v/T_c$ is the coherence length in the clean limit and l is the mean free path (the dirty limit is considered here), one can use the Ginzburg-Landau equations. The proper solution for the order parameter $\Delta(\mathbf{r})$ describing the loss of the superconducting energy can be written in the form[16–18]

$$\Delta(x)=\Delta(T)\tanh\left(\frac{x}{\sqrt{2}\xi(T)}+C\right), \tag{8}$$

where $\Delta(T)=\sqrt{8\pi^2/7\zeta(3)|\tau|}T_c\equiv\Delta_0\tau^{1/2}$ is the value of the order parameter in the bulk superconductor, $\xi(T)=\sqrt{\pi D/8T_c}|\tau|^{-1/2}$ is the characteristic scale of the spatial variation of $\Delta(\mathbf{r})$, D is the diffusion coefficient in the superconductor, and C is a constant. The solution (8) is valid at distances exceeding $\bar{\xi}$. At the distances of the order of $\bar{\xi}$ one needs to solve quasiclassical equations, which we will derive in the next section.

At the moment, we simply write the lowering of the superconducting energy for a given constant C. Substituting $\Delta(x)$, Eq. (8), into the Ginzburg-Landau free energy functional and integrating over x we evaluate the decrease of the superconducting energy at the F/S interface per unit surface area as (Ref. 17)

$$\Omega_S=\frac{\sqrt{\pi}}{6\sqrt{2}}|\tau|^{3/2}(2+K)(1-K)^2, \tag{9}$$

where $K=\tanh C$. The influence of the ferromagnet on the superconductivity is determined by the parameter K that will be found by minimizing the total energy.

The contribution $\Omega_{M/S}$ of the second term in Eq. (2) to the total energy has still to be determined. Differentiating the function $\Omega(\gamma)$

$$\Omega(\gamma)=-T\ln\left[\mathrm{Tr}\exp\left(-\frac{H(\gamma)}{T}\right)\right],$$

where $H(\gamma)$ is given by Eq. (2), one reduces the derivative of free energy to an averaged Green function. Then, reconstructing the free energy one obtains the following expression for $\Omega_{M/S}$:

$$\Omega_{M/S}=-i\pi T\nu_0\sum_{\omega}\int_0^1 d\gamma\int d^3\mathbf{r}(\mathbf{h}\sigma)_{\alpha\beta}\langle g_{\beta\alpha}\rangle_0. \tag{10}$$

Here ν_0 is the density of states and $\langle\hat{g}\rangle_0$ is the quasiclassical Green function averaged over all directions of the Fermi velocity. Its definition will be given in the next section. Once we know $\hat{g}$ we can calculate $\Omega_{M/S}$ using expression (10). In the next section we derive the equations for the Green functions.

III. QUASICLASSICAL EQUATIONS

In this section we derive from Eq. (2) the appropriate Eilenberger equations[14] for the quasiclassical Green functions in the superconductor and the ferromagnet and their matching conditions. In the S layer one can simplify the problem considering the ''dirty limit'' $l\ll\xi_0$, where l is the mean free path and $\xi_0=v/T_c$ is the coherence length of the superconductor in the clean limit, which allows to use more simple Usadel equations.[15] The condition $l\ll\xi_0$ is usually fulfilled for real superconductors and this allows to use the final results for a quantitative description of a wide number of experiments. If we assume that $|\tau|\ll 1$, $\tau=(T-T_c)/T_c$, the Usadel equations can be linearized.

Writing the Usadel equations in the ferromagnet is not always a good approximation because the exchange energy h in realistic cases is not necessarily smaller than $1/\tau_{tr}$, where τ_{tr} the mean free time, and so one should write in this region the Eilenberger equations. At the end one should match the solutions of all the equations using proper boundary conditions.

First, we introduce microscopic Green functions. Since we are dealing with a nonhomogeneous magnetic structure, the spin flips cannot be excluded and therefore averages of the form $\langle\Psi_\alpha(\mathbf{r},t)\Psi_\alpha(\mathbf{r}',t')\rangle$ or $\langle\Psi_\alpha^+(\mathbf{r},t)\Psi_{-\alpha}(\mathbf{r}',t')\rangle$, where $\alpha=\uparrow,\downarrow$, are not necessarily zero. So, we introduce the 4×4-matrix Green function $\check{G}_\omega(\mathbf{r},\mathbf{r}')$

$$\check{G}=\begin{pmatrix}\hat{G} & -\hat{F}\\ \hat{F}^+ & -\hat{G}^+\end{pmatrix},$$

where $\hat{G}$ and $\hat{F}$ are the normal and anomalous matrix Green functions in spin space, respectively, i.e.,

$$\hat{\mathcal{G}}=\begin{pmatrix}\mathcal{G}_{\uparrow\uparrow} & \mathcal{G}_{\uparrow\downarrow}\\ \mathcal{G}_{\downarrow\uparrow} & \mathcal{G}_{\downarrow\downarrow}\end{pmatrix}, \quad \hat{\mathcal{F}}=\begin{pmatrix}\mathcal{F}_{\uparrow\uparrow} & \mathcal{F}_{\uparrow\downarrow}\\ \mathcal{F}_{\downarrow\uparrow} & \mathcal{F}_{\downarrow\downarrow}\end{pmatrix}.$$

The matrix Green function $\check{G}_\omega(\mathbf{r},\mathbf{r}')$ satisfies the Gorkov equations that can be written in the spin $\otimes$ particle-hole space in the form

$$[-\check{\tau}_3\partial_\tau-\xi(\hat{\mathbf{p}})+\check{\Delta}(\mathbf{r})-\gamma\check{V}(\mathbf{r})-\check{\Sigma}_{\mathrm{imp}}]\check{G}(x,x')=\delta(x-x'), \tag{11}$$

where $\check{\tau}_i$, $i=1,2,3$, are Pauli matrices in the particle-hole space, $\xi(\hat{\mathbf{p}})=\hat{\mathbf{p}}^2/2m-\mu$, $\check{\Delta}=\check{\tau}_1\otimes i\sigma_y\Delta(\mathbf{r})$, $\check{V}=\mathrm{Re}[\mathbf{h}(\mathbf{r})\boldsymbol{\sigma}]\otimes\check{1}+\mathrm{Im}[\mathbf{h}(\mathbf{r})\boldsymbol{\sigma}]\otimes\check{\tau}_3$ and Δ is the pair potential, which should be determined self-consistently by

$$\Delta(\mathbf{r})=\lambda_0 T\sum_n F_{\uparrow\downarrow}(\mathbf{r},\mathbf{r},\omega), \tag{12}$$

where λ_0 is the constant of the electron-electron interaction. As we have mentioned, $\lambda_0=0$ and hence $\Delta=0$ in the ferromagnet. At the same time, $h=0$ in the superconductor. The term $i\check{\Sigma}_{\mathrm{imp}}$ in Eq. (11) describes the scattering by impurities.

Subtracting from Eq. (11) its complex conjugate and using the assumption that the quasiclassical Green function $\check{G}$ varies slowly as a function of $(\mathbf{r}+\mathbf{r}')/2$ one can derive in the usual way the Eilenberger equation[14] that can be written in a matrix form as

$$[\{\omega\check{\tau}_3-i\check{\Delta}+i\gamma\check{V}+i\check{\Sigma}_{\mathrm{imp}}\},\check{g}]+\mathbf{v}_0\nabla_{\mathbf{r}}\check{g}=0, \tag{13}$$

where $\mathbf{p}_0$ and $\mathbf{v}_0$ are the momentum and velocity at the Fermi surface and $\check{g}$ is the quasiclassical Green function, defined by

$$\check{g}(\mathbf{r},\mathbf{p_F})=\begin{pmatrix}\hat{g} & -\hat{f}\\ \hat{f}^{+} & -\hat{g}^{+}\end{pmatrix}=\frac{i}{\pi}\int d\xi\check{G}(\mathbf{r},\mathbf{p_F}).$$

For a short range interaction one can consider impurities in the self-consistent Born approximation, which gives $\check{\Sigma}_{\mathrm{imp}}=-i/2\tau\langle\check{g}\rangle_0$, where $\langle\cdots\rangle_0$ denotes averaging over the Fermi velocity. Equation (13) should be complemented by the normalization condition $\check{g}^2=\check{1}$. This condition follows as usual from the fact that $\check{g}^2$ is also a solution of Eq. (13).

Although Eq. (13) contains all the information, its solution is rather complicated. At the same time, the experiments[10] are performed not far from the superconducting transition temperature T_c. Moreover, calculations near T_c are considerably simpler and so we concentrate on this region.

Near T_c, the anomalous functions $\hat{f}$ and $\hat{f}^{+}$ are small and $\hat{g}\approx\mathrm{sgn}(\omega)$. This allows us to linearize equations for $\hat{f}$ and $\hat{f}^{+}$ in the ferromagnetic region $-d<x<0$ and in the region of the superconductor limited by the inequalities $0<x\ll\xi(T)$. Finding in this region the solution for $\hat{f}$ and using the self-consistency equation (12) we can find the order parameter $\Delta(x)$ and match this function with the expression for $\Delta(x)$, Eq. (8), valid for distances $x\gg\overline{\xi}$. This allows the possibility to determine the coefficient C and calculate the energy Ω, Eqs. (9), (10).

Such a procedure is simple conceptually but in practice very complicated. We did not manage to carry out the calculations for an arbitrary thickness of the ferromagnet. However, one can expect the cryptoferromagnetic state in thin layers only ($d\ll\xi_F$), where, fortunately, one can find the solution explicitly.

In the limit $T_c\ll h$, the off-diagonal component (1,2) in the particle-hole space of Eq. (13) in the region $-d<x<0$ is written as

$$\mathbf{v}_0\nabla\hat{f}=-i\hat{V}\hat{f}^{(F)}+i\hat{f}^{(F)}\hat{V}^{*}-\frac{\mathrm{sgn}(\omega)}{\tau}(\hat{f}^{(F)}-\langle\hat{f}^{(F)}\rangle),$$

$$\hat{V}=h(x)\sigma_z\exp(iQy\sigma_x), \tag{14}$$

where h is the strength of the exchange field in the F layer and Q denotes the wave vector of the cryptoferromagnetic state.

For thin ferromagnetic layers, the function $\hat{f}$ varies slowly between the boundaries. Assuming that $d\ll v_0/h$ we can relate the values of the function $\hat{f}^{(F)}(\mathbf{v_0},\mathbf{r})$ at the interface, i.e., at $x=0^-$ to the values at the boundary to the vacuum at $x=-d$, using the Taylor expansion

$$\hat{f}^{(F)}(\mathbf{v}_0,\mathbf{r}_0-\mathbf{r_d})\approx\hat{f}^{(F)}(\mathbf{v}_0,\mathbf{r}_0)-d\partial_x\hat{f}^{(F)}(\mathbf{v}_0,\mathbf{r}_0), \tag{15}$$

where $\mathbf{r}_0=(0,y,z)$ and $\mathbf{r_d}=(-d,y,z)$. Applying general boundary conditions[19] to the problem involved we conclude that for a perfectly transparent interface the function $\hat{f}$ is continuous at the interface. Assuming a specular reflection at the boundary with the vacuum $(x=-d)$ we write the following boundary condition for the function $\hat{f}$:

$$\hat{f}^{(F)}(v_x,\mathbf{r}_0-\mathbf{r_d})=\hat{f}^{(F)}(-v_x,\mathbf{r}_0-\mathbf{r_d}). \tag{16}$$

Using Eqs. (14)–(16) and the continuity of $\hat{f}$ at $\mathbf{r}=\mathbf{r}_0$ the problem is reduced to the solving of the Usadel equation in the superconductor with the following effective boundary condition at the interface between the superconductor and the ferromagnet

$$\eta D(\partial_x+d\partial_y^2)\hat{f}_0(\mathbf{r}_0)+i\,\mathrm{sgn}(\omega)d(-\hat{V}\hat{f}_0+\hat{f}_0\hat{V}^{*})_{\mathbf{r}_0}=0, \tag{17}$$

where $\eta=v_0^F/v_0^S$ and $\hat{f}_0$ is the zero harmonics of the function $\hat{f}$ in the superconductor. When deriving Eq. (17) we used the fact that the Usadel equation is applicable in the S layer at distances down to the mean free path l and extrapolated its solution to the interface. We also neglected the contribution of angles $\theta<dh/v_0$ to the average $\langle\hat{f}\rangle_0$, where θ is the angle between $\mathbf{v_0}$ and the x axis, i.e., $v_x=v_0\cos\theta$. Only the first two spherical harmonics $\hat{f}^{(s)}\approx\hat{f}_0+\mathbf{v_0}\hat{\mathbf{f}}_1$ were kept in the derivation. With this assumption one can derive from Eq. (13) the Usadel equation in the spin$\otimes$particle-hole space

$$-D\nabla_{\mathbf{r}}(\hat{g}_0\nabla_{\mathbf{r}}\hat{g}_0)+[\omega\hat{\tau}_3-i\hat{\Delta}(\mathbf{r})+i\hat{V}(\mathbf{r}),\hat{g}_0(\mathbf{r},\omega)]=0, \tag{18}$$

$$\hat{\mathbf{g}}_1=-\tau_{tr}\hat{g}_0\nabla_{\mathbf{r}}\hat{g}_0. \tag{19}$$

Here $\hat{g}_0$ and $\mathbf{v_0}\hat{\mathbf{g}}_1$ are the first two spherical harmonics of the function $\hat{g}=\hat{g}_0+\mathbf{v_0}\hat{\mathbf{g}}_1$.

Using the fact that $\tau=(T_c-T)/T_c\ll1$ one can linearize the Usadel equation. The off-diagonal component (1,2) in the particle-hole space of Eq. (18) can be written in the standard form

$$D\nabla^2\hat{f}_0-2|\omega|\hat{f}_0-2\Delta(x)\sigma_y=0. \tag{20}$$

Equation (20) is sufficiently simple that one can find the solution using the boundary condition, Eq. (17). This allows us to calculate the total energy and find the coefficient C in Eq. (8). Minimizing the energy in Q we can determine the boundary in parameter space of the cryptoferromagnetic state. Such calculations will be performed in the next section.

IV. CRYPTOFERROMAGNETIC STATE

With the above preparation we are in a position to find the solution of the equations derived in the previous section and calculate the energy. The general solution of Eq. (20) with the boundary condition, Eq. (17), and using Eq. (14) for $\hat{V}$ can be written as

$$\hat{f}_0(\mathbf{r},\omega)=\alpha_\omega(x)\sigma_x e^{-i\sigma_x Qy}+\beta_\omega(x)i\sigma_y, \tag{21}$$

where

$$\alpha_\omega(x)=C_\omega \exp\left(-\sqrt{Q^2+\frac{2|\omega|}{D}}x\right),$$

$$\beta_\omega(x)=i\frac{\Delta(x)}{|\omega|}+B_\omega \exp\left(-\sqrt{\frac{2|\omega|}{D}}x\right).$$

Equation (21) is applicable at distances much smaller than $\xi(T)$, where the solution for Δ can be approximated by a linear function [see Eq. (8)]. Substituting Eq. (21) into Eq. (12) one can find a rather complicated dependence of the order parameter $\Delta(x)$ on x. Matching this solution with the function determined by Eq. (8) is generally speaking not simple.

Fortunately, the exponentially decaying part of the solution given by Eq. (21) does not give a considerable contribution to $\Delta(x)$. One can check using the self-consistency equation, Eq. (12), that the relative correction to $\Delta(x)$ coming from the exponentially decaying part of Eq. (21) is of the order $(\ln \omega_D/T_c)^{-1}$, where ω_D is the Debye frequency, allowing its neglect. The coefficients C_ω and B_ω can be now determined from Eq. (17) and we obtain

$$B_\omega=\frac{i}{|\omega|}\frac{\eta^2 D\Delta'(0)\sqrt{D(DQ^2+2|\omega|)}-4h^2d^2\Delta(0)}{\eta^2 D\sqrt{2|\omega|(DQ^2+2|\omega|)}+4h^2d^2}, \tag{22}$$

$$C_\omega=\frac{2hd\eta D}{\omega}\frac{\Delta(0)\sqrt{\frac{2|\omega|}{D}}+\Delta'(0)}{\eta^2 D\sqrt{2|\omega|(DQ^2+2|\omega|)}+4h^2d^2}. \tag{23}$$

The condition $\check{g}^2=1$ allows us to find the function g which gives, on substitution into Eq. (10) finally the energy $\Omega_{M/S}$. Introducing the dimensionless parameters

$$a^2\equiv\frac{2h^2d^2}{DT_c\eta^2},\quad q^2\equiv\frac{DQ^2}{2T_c},\quad \tilde{\Omega}\equiv\frac{\Omega}{\nu_F\Delta_0^2}\sqrt{\frac{2T_c}{D}} \tag{24}$$

and using Eq. (8) we obtain

$$\tilde{\Omega}_{M/S}=\frac{\pi}{2}F_{3/2,1}K^2|\tau|+\sqrt{2}F_{2,1}K(1-K^2)|\tau|^{3/2}+\pi^{-1}F_{5/2,1}(1-K^2)^2|\tau|^2, \tag{25}$$

where

$$F_{m,l}=\eta\frac{4a^2}{\pi^{3/2-m}}\sum_{n>0}\alpha_n^{-m}[\sqrt{\alpha_n(\alpha_n+q^2)}+a^2]^{-l}, \tag{26}$$

$\alpha_n=\pi(2n+1)$ and ν_F is the density of states in the ferromagnet.

The total energy is given by $\tilde{\Omega}=\tilde{\Omega}_M+\tilde{\Omega}_S+\tilde{\Omega}_{M/S}$, Eqs. (7), (9), (25) and is a function of two parameters, K and q, that should be determined from the conditions $\partial\tilde{\Omega}/\partial K=\partial\tilde{\Omega}/\partial q=0$. The parameter q is in fact the order parameter for the cryptoferromagnetic state. Close to the CF-F transition this parameter is small and one can expand the energy $\tilde{\Omega}_{M/S}$, Eq. (25), in q^2. As concerns the value K_0 at the minimum, it can be found near the transition minimizing $\tilde{\Omega}_{M/S}$ at $q=0$. As a result, the first terms of the expansion of the energy $\tilde{\Omega}$ in q^2 near the CF-F transition can be written as

$$\begin{aligned}\tilde{\Omega}\approx{}&\tilde{\Omega}_s(K_0)+\tilde{\Omega}_{M/S}(K_0,q=0)\\&-\frac{q^2}{2}\Big[\frac{\pi}{2}F_{3/2,2}K_0^2|\tau|+\sqrt{2}F_{2,2}K_0(1-K_0^2)|\tau|^{3/2}\\&+\pi^{-1}F_{5/2,2}(1-K_0^2)^2|\tau|^2-2\lambda\Big]_{q=0}+\frac{q^4}{4}\Big[\frac{\pi}{2}H_{3/2}K_0^2|\tau|\\&+\sqrt{2}F_2K_0(1-K_0^2)|\tau|^{3/2}+\pi^{-1}H_{5/2}(1-K_0^2)^2|\tau|^2\Big],\end{aligned} \tag{27}$$

where we have defined

$$H_m=\frac{4a^2}{\pi^{3/2-m}}\sum_{n>0}\frac{1}{\alpha_n^m}\left[\frac{1}{(\alpha_n+a^2)^2}+\frac{1}{2(\alpha_n+a^2)^3}\right].$$

Since $0<K_0<1$ the term proportional to q^4 is positive, which means that the CF-F transition is of the second order. This is in contrast to the conclusion of Ref. 13, where a domain structure appeared with a finite period, which corresponded to a first order transition. The parameter λ in Eq. (27) is

$$\lambda\equiv\frac{Jd}{\nu\sqrt{2T_cD^3}}\frac{7\zeta(3)}{2\pi^2}. \tag{28}$$

According to the Landau theory of phase transitions the transition from the ferromagnetic state $(q=0)$ to the cryptoferromagnetic state $(q\neq0)$ should occur when the coefficient in the second-order term turns to zero. The phase diagram for the variables h and J, Eqs. (24), (28), is represented in Fig. 2. The curves are plotted for different values of $|\tau|$. The function $\tilde{\Omega}(q)$ has only one minimum at q_0 continuously going to zero as the system approaches the transition point. This demonstrates that the transition is of second order. Not close to the transition point the characteristic values of the wave number of the structure are of the order of $Q\sim\overline{\xi}^{-1}$. Figure 2 gives a possibility to determine explicitly the boundary of the cryptoferromagnetic state for any materials forming the multilayers.

V. DISCUSSION

In the previous sections we studied a possibility of the cryptoferromagnetic state in a ferromagnet-superconductor

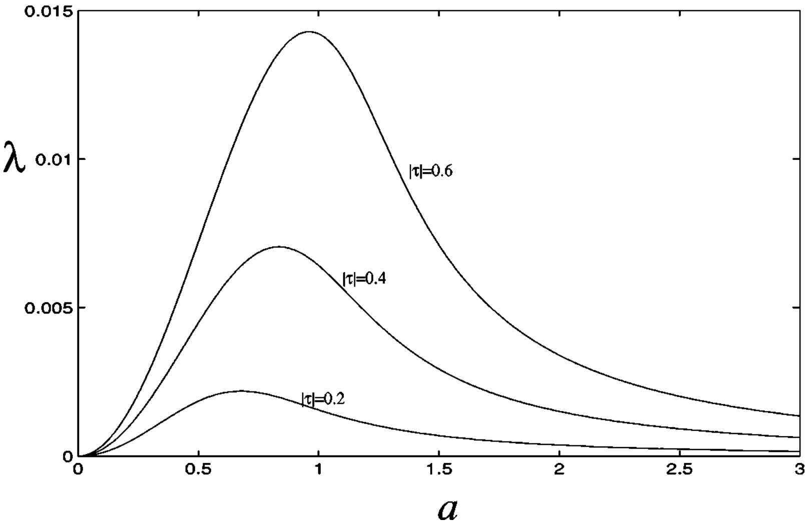

FIG. 2. Phase diagrams (λ,a) for different values of $|\tau|=(T_c-T)/T_c$. The area above (below) the curves corresponds to the *F* (CF) state.

bilayer. Matching the solutions of quasiclassical equations in the ferromagnet and superconductor we determined the phase diagram in the vicinity of the superconducting transition for given parameters of the materials forming the system. It is clear from our solution that the transition between the ferromagnetic and the cryptoferromagnetic states is of second order. At the transition point, the wave number Q characterizing the magnetic structure is equal to zero. The parameter Q grows smoothly when going into the cryptoferromagnetic state and its typical value can be of the order of the inverse size of the Cooper pair $\overline{\xi}^{-1}$.

Let us make estimates for the materials used in the experiment of Ref. 10. The stiffness J for materials such as Fe and Ni is ≈ 60 K/Å. The parameters characterizing Nb can be estimated as follows: $T_c=10$ Å, $v_F=1{,}37\times 10^8$ cm/s, and $l=100$ Å. The thickness of the magnetic layer is of order $d=10$ Å, and the exchange field $h=10^4$ K, which is proper for iron. Assuming that the Fermi velocities and energies of the ferromagnet and superconductor are close to each other we obtain $a\approx 25$ and $\lambda\sim 6\times 10^{-3}$. It is clear from Fig. 2 that the cryptoferromagnetic state is hardly possible in the Fe/Nb structure studied in Ref. 10.

How can one explain the decay of the average magnetic moment below T_c observed in that work? In reality, samples analyzed in Ref. 10 show a quite rough interface between the Nb and Fe layers. Thus, one can expect that there were ''islands'' in the magnetic layers with smaller values of J and/or h. A reduction of these parameters in the Fe/Nb bilayers is not unrealistic because proximity to Nb leads to formation of nonmagnetic ''dead'' layers,[4] and can affect the parameters of the ferromagnetic layers, too. If the cryptoferromagnetic state were realized only on the islands, the average magnetic moment would be reduced but remain finite, which would correlate with the experiment.[10] One can also imagine islands very weakly connected to the rest of the layer, which would lead to smaller energies of a nonhomogeneous state.

Another possibility to observe the cryptoferromagnetic state would be to use multilayers with a weaker ferromagnet. A good candidate for this purpose might be Gd/Nb. The exchange energy h in Gd is $h\approx 10^3$ K and the Curie temperature and, hence, the stiffness J is 3 times smaller than in Fe. So, one can expect $a\approx 2.5$ and $\lambda\approx 2\times 10^{-3}$. Using Fig. 2 we see that the cryptoferromagnetic phase is possible for these parameters. One can also considerably reduce the exchange energy h in $V_{1-x}Fe_x$/V multilayers[6] by varying the alloy composition. Hopefully, the measurements that would allow us to check the existence of the cryptoferromagnetic phase in these multilayers will be performed in the near future.

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[1] P. Koorevaar, Y. Suzuki, R. Coehoorn, and J. Aarts, Phys. Rev. B **49**, 441 (1994).

[2] C. Strunk, C. Sürgers, U. Paschen, and H.v. Löhneysen, Phys. Rev. B **49**, 4053 (1994).

[3] J.S. Jiang, D. Davidovi, Daniel H. Reich, and C.L. Chien, Phys. Rev. Lett. **74**, 314 (1995).

[4] Th. Mühge, N.N. Garif'yanov, Yu.V. Goryunov, G.G. Khaliullin, L.R. Tagirov, K. Westerholt, I.A. Garifullin, and H. Zabel, Phys. Rev. Lett. **77**, 1857 (1996).

[5] G. Verbanck, C.D. Potter, V. Metlushko, R. Schad, V.V. Moshchalkov, and Y. Bruynseraede, Phys. Rev. B **57**, 6029 (1998).

[6] J. Aarts, J.M.E. Geers, E. Brück, A.A. Golubov, and R. Coehoorn, Phys. Rev. B **56**, 2779 (1997).

[7] Z. Radovic, L. Dobrosavljevic-Grujic, A.I. Buzdin, and John R. Clem, Phys. Rev. B **38**, 2388 (1988); Z. Radovic, M. Ledvij, L. Dobrosavljevic-Grujic, A.I. Buzdin, and John R. Clem, *ibid.* **44**, 759 (1991); A.I. Buzdin, M.Yu. Kupriyanov, and B. Vujićić, Physica C **185**, 2025 (1991).

[8] O. Šipr and B.L. Györffy, J. Phys.: Condens. Matter **7**, 5239 (1995).

[9] C.A.R. Sa de Melo, Phys. Rev. Lett. **79**, 1933 (1997).
[10] Th. Mühge, N.N. Garif'yanov, Yu.V. Goryunov, K. Theis-Bröhl, K. Westerholt, I.A. Garifullin, and H. Zabel, Physica C **296**, 325 (1998).
[11] P.W. Anderson and H. Suhl, Phys. Rev. **116**, 898 (1959).
[12] L.N. Bulaevskii, A.I. Buzdin, M.L. Kulić, and S.V. Panjukov, Adv. Phys. **34**, 175 (1985).
[13] A.I. Buzdin and L.N. Bulaevskii, Zh. Eksp. Teor. Fiz. **94**, 256 (1988) [Sov. Phys. JETP **67**, 576 (1988)].
[14] G. Eilenberger, Z. Phys. **214**, 195 (1968); A.I. Larkin and Yu.N. Ovchinnikov, Zh. Eksp. Teor. Fiz. **55**, 2262 (1968) [Sov. Phys. JETP **28**, 1200 (1969)].
[15] K.D. Usadel, Phys. Rev. Lett. **25**, 507 (1970).
[16] A. Abrikosov, L. Gorkov, and I. Dzyaloshinski, *Methods of Quantum Field Theory in Statistical Physics* (Dover Publications, New York, 1963).
[17] P. G. de Gennes, *Superconductivity of Metals and Alloys* (Benjamin, New York, 1966).
[18] A.A. Abrikosov, *Fundamentals of the Theory of Metals* (North-Holland, Amsterdam, 1988).
[19] A.V. Zaitsev, Zh. Eksp. Teor. Fiz. **86**, 1742 (1984) [Sov. Phys. JETP **59**, 1015 (1985)].