Anomalous sliding friction and peak effect near the flux lattice melting transition

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Recent experiments have revealed a giant ''peak effect'' in ultrapure high T_c superconductors. Moreover, the data show that the peak effect coincides exactly with the melting transition of the underlying flux lattice. In this work, we show using dynamical scaling arguments that the friction due to the pinning centers acting on the flux lattice develops a singularity near a continuous phase transition and can diverge for many systems. The magnitude of the nonlinear sliding friction of the flux lattice scales with this atomistic friction. Thus, the nonlinear conductance should diverge for a true continuous transition in the flux lattice or peak at a weakly first-order transition or for systems of finite size.

One of the central unsolved problems in type-II superconductivity concerns the so-called ''peak effect.'' When a current I is passed through the superconductor in the mixed phase, the flux-line lattice (FLL) moves in response to the Lorentz force, leading to dissipation and an induced voltage V. Naively, the nonlinear conductance $C=I/V$ is expected to decrease monotonically towards the H_{c2} phase boundary because of the diminishing order parameter and hence a reduced pinning strength. However, experimentally it was observed a long time ago that instead of a monotonic behavior, the conductance peaks sharply to a large value before dropping at the superconducting-normal transition.[1–4] It has also been established that this peak effect is not just limited to conventional superconductors, but shows up in a similar fashion in the high-T_c Y-Ba-Cu-O superconductors also.[5,6] To date, however, there has been no satisfactory explanation for this peak effect, although various possible mechanisms have been proposed as the origin of this phenomenon. One popular idea based on the collective-pinning theory[7] is that the FLL softens towards the H_{c2} boundary, leading to a smaller elastic coherence length (Larkin length) and enhanced stronger pinning by the impurities.[9] It is not clear though how this mechanism can give rise to the sharp peak in the conductance. Moreover, recent data have revealed giant peak effects in ultrapure high-T_c superconductors with as much as a 35-fold increase in C from onset to peak,[8] which is hard to explain with the collective pinning idea. It has also been widely suggested that the peak effect is either caused by or related to an underlying FLL melting transition.[6,9–11] Up until very recently, this idea has remained speculative because of the difficulty of a direct experimental observation of the FLL melting transition. Two recent studies have now established conclusively the relation of the peak effect with the underlying phase transition in the FLL.[8,12] In the study[8] involving ultrapure Y-Ba-Cu-O, the peak effect is shown to coincide exactly with the point at which there is a small magnetization jump ΔM or discontinuity in the slope of the magnetization. This behavior of ΔM is interpreted as the signature of either a continuous or very weak first-order transition. In another study of the conventional superconductor[12] Nb, an ac magnetic susceptibility (χ) measurement was made in conjunction with a small-angle neutron scattering (SANS) study of the structure of the underlying FLL. The peak effect (a dip in the real part of the ac susceptibility χ) is observed to occur precisely at the point where the diffraction peak in the SANS pattern of the FLL begins to broaden into ringlike features. In this case, the transition is more strongly first order, with a direct observation of hysteresis involving superheating and supercooling behavior.

We have, in an earlier paper,[13] suggested that the sliding friction for the FLL would be anomalously large near a continuous or very weak first-order melting transition due to the enhanced coupling of the pinning centers to the FLL through the critical fluctuations. The central idea is that the mobility of the FLL is not just controlled by the pinning strength of the impurities, which is an equilibrium property. It depends also on the nonadiabatic coupling of the pinning center to the dynamical excitations of the FLL, leading to a frictional damping η on the FLL. Recent theoretical developments in understanding nonlinear sliding friction of an adsorbed monolayer[14–17] in the boundary lubrication problem are particularly helpful in elucidating this problem. Aside from inertia mass effects, the behavior of these systems is very similar. In the FLL, the driving force F is the Lorentz force proportional to the current passing through the superconductor, and the moving FLL produces a changing flux and an induced voltage that is proportional to the average drift velocity $\langle v \rangle$ of the FLL. Thus, in the language of the boundary lubrication problem, the static friction of the adlayer corresponds to the critical current J_c in the superconductor, and

the nonlinear sliding friction $\bar{\eta}$ of the adlayer defined as $\bar{\eta} = F/\langle v \rangle$ is essentially the nonlinear conductance C for the superconductor. Note that $\bar{\eta}$ is just the inverse of the usual definition of the mobility for the adlayer. In the discussion of the peak effect problem, the nonlinear conductance related to the sliding friction $\bar{\eta}$ of the FLL is actually the more relevant quantity. Near the occurrence of the peak effect, the I-V curve is often of such a nature that there is a continuous rise of the voltage with increasing driving current such that the exact value of threshold critical current J_c is ill-defined, and the nonlinear conductance is a better measure of the anomaly for the mechanical response of the FLL.[18] In the ac magnetic susceptibility measurement, what determines the magnitude of the screening current and hence the magnitude of the susceptibility is clearly the nonlinear conductance and not so much a single threshold critical current density J_c. Results from various numerical studies of the boundary layer problem[14,15,17] have shown that both the static friction and the nonlinear sliding friction depend in a complicated manner on the interplay of the strength of the pinning potential, interactions among the particles (vortices for the FLL) and the bare frictional damping η from the environment. In this paper, we will quantify the concept that for the FLL, it is the variation of the nonadiabatic frictional damping η and not the adiabatic pinning strength that develops anomalous temperature and magnetic-field dependence near the melting transition. This anomalous behavior of the frictional damping then leads to the peak effect for the nonlinear sliding friction $\bar{\eta}$ for the FLL and hence the conductance C of the superconductor. We show below through general dynamical scaling arguments the explicit singularity of the friction η near the transition.

Let us first consider the random force acting on the pinning center at the position $\mathbf{r}$ by the flux lattice. In the simple pair interaction model, this can be expressed in terms of the linear displacement $u_{\mathbf{q},\alpha}$ of each vortex from its equilibrium position as

$$f_\alpha(\mathbf{r}) = \sum_{\mathbf{q}} W(\mathbf{r},\mathbf{q}) u_{\mathbf{q},\alpha}. \tag{1}$$

Here, $\mathbf{q}$ stands for the normal mode index of the FLL, α is the Cartesian component label, and W represents the coupling function. In response to this, there is an equal and opposite reaction force on the FLL by the pinning center. When correlations between the different pinning centers are neglected, the frictional damping (in the Markovian limit) on the FLL is then given by[19,20]

$$\eta = \sum_{\mathbf{q},\mathbf{r}} W^2(\mathbf{r},\mathbf{q}) S(\mathbf{q},\omega=0), \tag{2}$$

where $S(\mathbf{q},\omega=0)$ is the dynamic structure factor defined as $\int_0^\infty dt \langle u_{\mathbf{q},\alpha}(t)^* u_{\mathbf{q},\alpha}(0)\rangle$. Correlations between the random forces from pinning centers at different positions would lead to higher-order terms in the pinning center concentration n_p in Eq. (2), and are negligible in the limit $n_p \to 0$. According to general dynamical scaling arguments,[21,22] $S(\mathbf{q},\omega)$ should take the scaling form near T_c for a continuous phase transition as

$$N^d S(\mathbf{q},\omega) = \xi^{z+\gamma/\nu} g_\pm(q\xi, \omega\xi^z), \tag{3}$$

where $g_\pm$ is a scaling function, $\xi \propto |T/T_c - 1|^{-\nu}$ is the divergent correlation length, d is the system dimension, γ is the susceptibility exponent, and z is the dynamical critical exponent. Substitution of Eq. (3) back into Eq. (2) then leads to the conclusion that as one approaches T_c, the friction η has a singular part that goes as $\eta \propto |T/T_c - 1|^{-x}$ with $x = \nu(z-d)+\gamma$. The dimension d enters explicitly through the q integration in Eq. (2) where we have assumed a typical short ranged coupling potential $W(q)$ that is regular at $q=0$. Thus the friction η can either diverge if $x>0$ or be finite with a cusp only.[23] Similar anomaly has also been predicted for adatom diffusion near the surface reconstruction transition of the W(100) surface.[24] For this case, the exponent x has been explicitly evaluated for a model Hamiltonian and shown to have the value[24,25] $x \approx 1.8$. Thus the diffusion constant of adatoms on this surface is predicted to vanish at the transition.

The friction η calculated in Eq. (2) corresponds to the bare friction acting on the center of mass (CM) degree of freedom of the FLL. It is analogous to the friction exerted by the substrate on an adsorbed layer in the boundary lubrication problem. Experimentally, the mobility measurements of the FLL have all been performed in the nonlinear regime. In the presence of an external pinning potential, the CM motion of the FLL is coupled to the single vortex motion which depends in turn on the interactions with other vortices. Thus the nonlinear response of the flux lattice under a driving current can only be determined by solving the coupled Langevin equations. In general, the nonlinear sliding friction $\bar{\eta}$ of the FLL depends on the details of the vortex interaction, strength of the pinning potentials, and the driving force. However, in various recent studies of the nonlinear sliding friction of an adsorbed overlayer on an substrate,[14,17] it has been shown that the magnitude of $\bar{\eta}$ is determined by the bare friction η as given in Eq. (2), with $\bar{\eta}$ approaching the bare friction η in the limit of large driving force. Even for a system with a positive exponent x leading to a divergent behavior for η and $\bar{\eta}$ near the transition, the conductance peak at the transition in practice will be significantly rounded by crossover effects due to the nonzero driving current. It has been argued, in general terms, that the current density J sets an additional length scale in resistance measurements[30] $L_J^{d-1} \sim kT/J$ due to thermal fluctuations. The divergent critical fluctuations at T_c will be then cut off by this length when $\xi \sim L_J$, giving rise to a nonlinear resistance behavior $R \sim I^{x/\nu(d-1)}$. Experimentally,[8,10] a strong nonlinearity is indeed observed for the conductance maxima which decreases for increasing I. Thus, we conclude that for a FLL system with a positive exponent x, its nonlinear sliding friction $\bar{\eta}$ has a peak at the melting transition, its origin being the strong critical fluctuation near the melting transition. This then leads to the peak in the conductance C. In the case of a weak first-order transition or finite-size system, the divergence or the cusp singularity of η would be rounded off even in the linear regime and thus we expect the peak effect for these systems to be much weaker.

Now we come back to the recent experimental data on peak effect and discuss them in light of the above theoretical

considerations. Much of the difficulty associated with understanding the FLL dynamics starts with the fact that we do not even have a very detailed understanding of the ground state. The accepted picture now for the weak pinning limit is that of a Bragg glass with quasitranslational long-range order (LRO) and true orientational LRO,[27] due to the presence of random pinning centers. Similarly, we do not have a clear picture of how or whether the FLL melts just before the superconducting-normal transition.[28–30] The recent data[8,12] strongly support the existence of a phase transition in the FLL just before the H_{c2} phase boundary. For an ultrapure sample of the high-T_c Y-Ba-Cu-O superconductor, static magnetization measurements show a very small discontinuity at high magnetic field (5 T) and no discernible jump but only a discontinuity in the slope of the magnetization at lower fields. This is identified as the melting transition, the transition being continuous at low fields and weakly first order at high fields. The ''peak effect,'' identified by the dip in the real part of the ac magnetic susceptibility χ occurs at precisely the same temperature and magnetic field as this ''melting'' transition. This peak effect is ''gigantic'' involving a 35-fold increase in the nonlinear conductance C through a narrow range of change of temperature or the magnetic field. This is much stronger than all the previously observed peak effects which typically show a peak to onset ratio of 3 to 4. According to the present theory, this sharp peak behavior in C can be understood as arising from the sharp rise in the friction acting on the FLL due to the coupling of the pinning centers to the strong critical fluctuations near the continuous or weakly first order melting transition. According to our scaling arguments, the existence of a peak effect require that the exponent $x=\nu(z-d)+\gamma$ be positive. At the moment, there exists no detailed information on any of these exponents for the FLL melting transition in the presence of pinning centers. However, existing calculations of the dynamical exponent z for disordered systems[26] give results which are generally larger than $z=4$. Thus, it is entirely plausible that the corresponding exponent x for the FLL can be positive. In practice, the divergence of the critical fluctuations will be cut off by the length scale set by the current $L_J^{d-1}\sim kT/J$. In addition, imperfections in the crystalline order of the sample also provides a cutoff. This explains then the gigantic peak effect for the ultrapure Y-Ba-Cu-O as opposed to the much smaller peak effect for the poorer quality samples. Another feature of the data that supports the present theory is the large width of the $\chi'(T)$ dip. At $H=5.0$ T, the width of the $\chi'(T)$ dip is about 1 K while the width of the ΔM discontinuity is only about[8] 0.08 K. This can be understood from the fact that $\chi'(T)$ measures the critical fluctuations through its dependence on the friction while ΔM is just an order-parameter measurement connected with the density of the vortices. In addition to this study for Y-Ba-Cu-O, there is also a recent study for the conventional superconductor Nb involving simultaneous small-angle neutron scattering (SANS) as well as ac magnetic susceptibility measurements.[12] The melting transition here can be clearly identified as the point where the sharp Bragg-like peak in the ordered FLL phase first begins to broaden into ringlike features. By contrast with the high-T_c Y-Ba-Cu-O material, the stronger first-order nature of the melting transition in Nb is clearly evidenced by the observation of superheating and supercooling below and above the melting transition.[12] Again, the peak effect as determined from the magnetic susceptibility measurements coincides with the melting transition. However, the peak effect in this case is much weaker, and the conductance C only shows a fourfold increase from onset to the peak value. Since the transition here is of first order, the critical fluctuations are much weaker and the correlation length does not diverge at the melting transition point. In fact, the situation here is similar to the poorer quality sample of Y-Ba-Cu-O where impurities and imperfections cut off the divergent critical fluctuations. As a result, the friction acting on the FLL has only a weak maximum instead of a divergent behavior at the transition point, and the corresponding peak effect is much weaker.

In conclusion, we have presented here a general scaling argument that the frictional damping exerted by the pinning centers on the flux lattice has a singularity (or a cusp) near a continuous melting transition in the lattice. While most previous theoretical considerations of the peak effect focus on the adiabatic pinning strength, the present work identifies the origin of the peak effect through the nonadiabatic coupling of the pinning centers to the strong critical fluctuations near the transition point. This leads to a vanishing linear mobility for the flux lattice at the transition. In the nonlinear regime, the finite driving current provides a cutoff for the divergent critical fluctuations, and this leads to a finite peak in the nonlinear sliding friction for the FLL and hence the conductance for the superconductor, with the strength of the peak dependent on the magnitude of the driving current. The recently observed gigantic peak effect in high-T_c superconductors and the strong correlation between the peak effect and the observed melting transition provide strong support for the mechanism proposed here.

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