Staggered magnetization and realization of Jahn-Teller-like effects

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A realization for the cooperative Jahn-Teller-like effect in low-dimensional quantum critical systems is proposed. We show that at low temperatures these systems are unstable with respect to the spontaneous appearance of alternating effective *g* factors, which also pertain to nonzero alternating distortions of ligands, surrounding magnetic ions. The effect is magnetic field-induced. The proposed effect is discussed in connection with recent experiments on effectively low-dimensional electron systems.

During the past years interest in electron systems with strong interactions between charge, spin and orbital degrees of freedom of the electrons, on the one hand, and the elastic subsystem of a crystal, on the other hand, has grown considerably. Recent important manifestations of such cooperative effects include the colossal magnetoresistance of manganites,[1] spin-Peierls and charge ordering behaviors in nonorganic systems,[2] non-Fermi-liquid behavior of some heavy fermion compounds[3] and unconventional superconductivity, e.g., in ruthenates and organic salts.[4] Probably the oldest known quantum manifestation of such a connection is the Jahn-Teller effect.[5] There the degeneracy of orbital degrees of freedom of a molecule is lifted due to the distortion of the latter. The *cooperative* Jahn-Teller effect (which reveals itself as a structural phase transition) was observed later in a number of compounds.[6] In fact the spin-Peierls transition can also be considered as belonging to the class of Jahn-Teller-like transitions, in the sense that the degeneracy of an electron (spin, not orbital) subsystem of a one-dimensional (1D) spin chain is removed due to the coupling to the longitudinal phonon of a 3D lattice. In this case the gap is open for the low-lying spin excitation, while the corresponding phonon mode possesses softening (Kohn anomaly).

Some magnetic compounds with a strong coupling between the spin, orbital and elastic subsystems exhibit a paramagnetic spin behavior with two inequivalent magnetic centers at low temperatures. (The temperature is higher, though, than that of the phase transition to a magnetically ordered 3D state. The latter has often not been observed.[7–12]) For higher temperatures the inequivalence between two magnetic centers smears out. Two inequivalent magnetic centers in low-dimensional quantum spin systems usually pertain to slightly different local surrounding of two types of magnetic ions. On the one hand, they are connected with staggered *g* factors of magnetic ions.[7] (Another explanation corresponds to two anisotropic *g* tensors canting with respect to the main axis.[11]) On the other hand, they pertain to Dzyaloshinskii-Moriya couplings in crystals without reflection magnetic symmetry (with an odd magnetic structure with respect to the main axis[13]). In fact in all of the mentioned examples[14,8–12] the spin subsystems are effectively low-dimensional. Low-lying spin excitations are gapless for those systems without an external magnetic field. However in an external field, the low-temperature specific heat of some of these systems reveals the emergence of a spin gap (governed by an external magnetic field).[14,15,11] To explain that gapped behavior it was necessary to consider a 1D spin subsystem with staggered effective *g* factors.[16,8,17] Low temperature low field electron spin paramagnetic resonance (ESR) measurements in copper benzoate[7] and some rare-earth molibdates[9–12] revealed two inequivalent magnetic centers, while higher-field ESR treatments manifested a single magnetic center.[18] Optical (Raman and infrared) measurements[19] revealed an anomalous behavior of some lattice modes, and ultrasonic measurements also reported anomalies in the behavior of sound velocities for some phonon modes in such crystals.[20,21] In this work we propose a new mechanism for the possible explanation of the observed anomalous behavior of spin, orbital, and elastic subsystems of this group of magnetic low-dimensional systems. Namely, we point out that the dimerization of effective *g* factors in a quantum antiferromagnetic spin chain can be caused by the interaction of the spin and orbital subsystems with an elastic subsystem. This has to be followed by a distortion of the local surrounding of magnetic ions. The effect can be called ''Jahn-Teller-like,'' because the degeneracy of the orbital degrees of freedom of magnetic ions is changed due to distortions in the elastic subsystem. Note, though, that we will discuss mostly the magnetic manifestation of the effect.

It is well known that the values of effective *g* factors in paramagnets become different than 2 because of the effect of the (crystalline) electric field of nonmagnetic ligands, through the spin-orbit interaction. Suppose that the configuration of ligands, which surround two neighboring magnetic ions along the chain direction, possesses small shifts (distortions) of opposite signs (antiferrodistortions[11]). Such distortions will immediately produce a change of the crystalline electric fields of the ligands. Hence, the orbital moment of the magnetic ions will be (alternately) affected. Then the spin-orbit interaction yields (a) two different values of the effective *g* factors of the magnetic ions and (b) a staggered Dzyaloshinskii-Moriya interaction (because of the odd magnetic symmetry with respect to the main axis). The effect of the latter can also be transfered to the effective staggered *g* factor by means of a nonuniform unitary transformation (al-

ternating rotation of spins).[22] Hence, the staggered distortion of the nonmagnetic surroundings (ligands) of the magnetic ions produces two inequivalent magnetic (spin) centers for the spin subsystem. The elastic subsystem loses its energy due to the alternating distortions of ligands, while the electron subsystem reveals an energy gain. The steady-state configuration of the total crystal is determined by the competition between these two processes. This effect is *collective*, because the spins interact with each other. On the other hand, the inequivalence of magnetic centers can be seen only through a *nonzero* magnetic field, so that the effect is field induced.

Let us illustrate the above scheme for a concrete system. Consider the low-temperature behavior of a magnetic crystal, which reveals the low dimensionality (1D). This means that the spin-spin coupling along one direction is larger than the interactions along the other crystallographic directions. Suppose that the quasi-1D spin subsystem can be described by a Hamiltonian of a gapless spin $\frac{1}{2}$ antiferromagnetic (AF) chain. The fact that the low-lying excitation of a 1D AF spin chain is gapless implies that the system is *critical*, i.e., the ground state correlation functions decay in a power-law manner. Suppose also that due to the reasons explained above some components of the effective g factors of the spins are alternating, i.e., $g_{1,2}=g(1\pm\delta)$, where $\delta\ll 1$ is proportional to small alternating distortions of the local surrondings.[23,24] This is equivalent to the mean field approximation for phonons, which is justified for the 3D elastic subsystem. As for the 1D spin-spin interactions, we take them into account nonperturbatively. The parameter δ describing the inequivalence of the g factors and the distortions of the ligands is not fixed. It is to be determined by minimizing the total energy (including the magnetic and elastic energies) within the adiabatic approximation. The total Hamiltonian reads

$$\mathcal{H}_{sp}=J\sum_j\ (\vec{S}_j\vec{S}_{j+1}+(\Delta-1)S_j^zS_{j+1}^z)$$

$$-\mu_BH\sum_j\ (g_1S_{2j}^z+g_2S_{2j+1}^z), \qquad (1)$$

where J is the exchange constant, $0\leqslant\Delta\leqslant 1$ is the parameter of the ''easy-plane'' magnetic anisotropy, H is an external magnetic field and μ_B is the Bohr's magneton.

For rare-earth ions (with a Kramers-doublet or non-Kramers-doublet crystalline field ground state) for sufficiently low temperatures T compared with the crystalline field splitting D, one can use the two lowest levels of the ion as an effective spin $\frac{1}{2}$ with magnetically anisotropic behavior. It turns out that no change of the g factors can be produced by distortions of ligands for the groundstate doublets. They do not have orbital degrees of freedom, and are therefore unaffected by distortions. However the excited crystalline electric field states are effectively included. This is why a distortion of the ligands can produce a change of the g factors, through the off-diagonal matrix elements of the relevant electric-multipole operators. This means that the conditions for the applicability of our effective spin-$\frac{1}{2}$ description for rare-earth compounds are the following. First, $T\ll D$ i.e., there is no thermal population of the crystalline field-excited levels. Second, off-diagonal matrix elements (between the ground state doublet and excited levels) of the Zeeman term have to be small compared with D. Third, matrix elements between the ground state doublet and crystalline field-exited levels of the operator describing the distortion must not be too small compared to D. (Otherwise distortions would produce a negligible change of the ground state g factors.) Finally, distortions of the ligands must not affect J (with $J\ll D$).

It is easy to show that because of nonzero δ the application of an external field to this system yields two effects, namely homogeneous magnetization (m^z) and staggered magnetization. The former one changes the ground state filling of the Dirac sea for the low-lying spin excitations (H plays the role of the Fermi energy). Notice that for large enough values of the field H, the spin subsystem exhibits trimerization (i.e., for the one-third filled Dirac sea of spinons), quadrimerization (for one-forth filling) etc. For those values of the field k_F is equal to $\pi/2n$, with n being integer and $2k_F=\pi(1+2m^z)$. Thus we expect a series of transformations towards an inhomogeneous (incommensurate) magnetic structure due to umklapp processes rather than due to dimerization.[25] Here we limit ourselves to small enough values of H. As for the staggered magnetization, it is the relevant perturbation from the renormalization group (RG) viewpoint, and it produces the gap for low-lying magnetic excitations of the system (spinons). Unfortunately the behavior of the spin subsystem with the Hamiltonian Eq. (1) cannot be described exactly. However we can calculate that behavior using the response of our (critical) spin chain to the relevant perturbation (staggered field). We perform such a calculation in an RG framework. Our study shows that the exponents are nonintegral in general, in contrast to simple perturbation or mean-field theories.[11,19] The latters of course are not legitimate for low-dimensional interacting quantum spin systems. An application of scaling relations provides a simple tool to understand some essential aspects of the behavior of a critical chain under a relevant perturbation.

Recall that the response of the free energy f_{cl} and the correlation function ξ_{cl} of a classical critical d-dimensional system perturbed by a relevant operator $\delta\mathcal{H}'$ with RG eigenvalue $\nu^{-1}>0$ is $\Delta f_{cl}\propto\delta^{d\nu}$ and $\xi_{cl}\propto\delta^{-\nu}$. A quantum critical d-dimensional system (which is in our case the spin 1D subsystem of the crystal) formally behaves in the scaling regime equivalently to a $(d+z)$-dimensional classical system, where z is the dynamical critical exponent. Hence, the ground state energy and the gap of the low-lying spin excitations of the d-dimensional quantum critical system are formally proportional to the free energy and the inverse correlation function of the effective ''$(d+z)$''-dimensional classical critical system, respectively. The RG eigenvalue ν^{-1} is related to the scaling dimension x of the particular operator by $x+\nu^{-1}=d+z$. For the (conformally invariant) AF spin chain we have $d=z=1$, i.e., $\nu=(2-x)^{-1}$. Hence the renormalization of the ground state energy per site of the quantum critical chain, and the low-lying spin excitation gap (which is equal to zero at the unperturbed point) due to the staggered magnetic field are $\Delta E_g\propto-(g\delta\mu_BH)^{2/(2-x_e)}$, and $\Delta\propto(g\delta\mu_BH)^{1/(2-x_e)}$, respectively. Here x_e is the minimal scaling exponent for energy-energy correlations. [We have

ignored logarithmic corrections; they are present due to the marginal operators in the RG sense and are essential for the $SU(2)$-symmetric case.]

To find the scaling dimension for our critical spin chain we use the results of the conformal field theory (CFT). According to the CFT approach,[26] the asymptotics of correlation functions of primary fields in the ground state are known to be $\langle \phi_{\Delta^\pm}(r,t)\phi_{\Delta^\pm}(0,0)\rangle = \exp(2iDP_F r)(r-iv_F t)^{-2\Delta^+}(r+iv_F t)^{-2\Delta^-}$. Here v_F and P_F are the Fermi velocity and the Fermi momentum, respectively. A (half)integer D measures the momentum of the primary field in units of the Fermi momentum. The scaling dimension and spin for each primary field are determined by $x_\phi = \Delta^+ + \Delta^-$ and $s_\phi = \Delta^+ - \Delta^-$. Parameters $\Delta^\pm$ can be calculated according to the finite-size analysis of the low energy physics of the critical spin chain.

Combining all of the effects, we can write for the ground state energy of the spin subsystem with nonzero δ

$$E_{sp} = -v_F\left(\frac{g\delta\mu_B H}{v_F}\right)^{2/(2-x_e)}, \tag{2}$$

where for our AF spin chain $v_F = \pi J\sqrt{1-\Delta^2}/2\cos^{-1}\Delta$ is the Fermi velocity of spinons for $H=0$. We point out that the scaling approach is only valid in the vicinity of the critical point. This means that in principle the values of the magnetic field and δ are small. Note that the Fermi velocity monotonically decreases with growing H and becomes zero at the spin-saturation point (for $H \geqslant H_c \sim J\Delta/4g\mu_B$. The exponent is equal to $x_e = \pi/2(\pi - \cos^{-1}\Delta)$ for $H=0$. It increases monotonically with increasing H and becomes 1 at $H=H_c$.[27] Obviously $E_{sp}<0$; hence one has an energy gain due to nonzero δ. On the other hand, in the lowest order in δ the elastic subsystem loses energy proportional to $C\delta^2/2$, where C is the elastic constant. Whether the groundstate steady-state configuration corresponds to zero or nonzero δ depends on the scaling exponent x_e. For $x_e<1$ the ground state steady-state configuration of the spin and elastic subsystems corresponds to nonzero δ, and hence, to two inequivalent spin centers (with two different g factors) and to the nonzero alternating distortions (antiferrodistortions) of the local surroundings of the magnetic ions (ligands). (Here we take into account that $|\delta|<1$.) The steady-state δ is equal to

$$\delta_0 = \left(\frac{v_F}{C}\right)^{(2-x_e)/2(1-x_e)}\left(\frac{g\mu_B H}{v_F}\right)^{1/(1-x_e)}. \tag{3}$$

Clearly $\delta_0 = 0$ for $H=0$ and for $H \geqslant H_c$. For high temperatures (much higher than J), the steady-state δ is zero, naturally. Hence, there has to be a phase transition between the low-temperature and high-temperature phases. The former is characterized by a nonzero staggered magnetization, gapped low-lying spin excitations and nonzero alternating distortions of ligands, surrounding the magnetic ions. The latter has zero staggered magnetization, gapless low-lying spin excitations and no distortions of ligands. We have studied this effect by assuming that the initial frequencies ω_0 of the phonons, which are interacting with the spin chain are small $\omega_0 \ll J$ (adiabatic approximation). It is in principle possible to calculate the effect more precisely for any ω_0.[28,29] The conditions for the steady-state configuration of the total system correspond to the cross section of the respective phonon modes $\omega(q)$ (at $q=2k_F$) connected to the alternating distortions (antiferrodistortions) of ligands with the electron orbital modes.

Let us consider the interesting limiting case, namely $\Delta = 0$. It is marginal with $x_e = 1$. This corresponds to the XX spin $\frac{1}{2}$ AF chain with alternating g factors. The Hamiltonian can be exactly mapped by means of the non-local Jordan-Wigner transformation onto the Hamiltonian of noninteracting lattice fermions.[30] The groundstate energy of the spin subsystem and the elastic subsystem can be written in the form

$$E_{gs} = \frac{C\delta^2}{2} - g\mu_B H\left[1 - \frac{\lambda_c}{\pi}\right] - \frac{1}{\pi}\int_0^{\lambda_c} d\lambda\sqrt{(\delta g\mu_B H)^2 + [J\cos(\lambda/2)]^2}, \tag{4}$$

where $\lambda_c = \cos^{-1}[2(1-\delta^2)(g\mu_B H/J)^2 - 1]$. Minimizing the ground state energy with respect to δ, we obtain two possible solutions. The first, $\delta = 0$, corresponds to zero distortions. As for the second, it satisfies the equation

$$\frac{\pi C\sqrt{J^2 + (\delta g\mu_B H)^2}}{(2g\mu_B H)^2} = F\left(\frac{\lambda_c}{2}, k\right), \tag{5}$$

where $F(\lambda_c/2,k)$ is the incomplete elliptic integral of first order, with $k^{-1} = \sqrt{1 + (\delta g\mu_B H/J)^2}$. Notice that for $H \geqslant H_c = J/g\mu_B\sqrt{1-\delta^2}$ one has $\lambda_c = \pi$ and the integral is zero. Hence for $H \geqslant H_c$ there is only one $\delta_0 = 0$ solution in the ground state, as should be the case. We emphasize again that we study the effect of sufficiently weak magnetic fields. Here, on the one hand, the scaling approach is valid. On the other hand, we limit ourselves to dimerization only. This means that the field is not strong enough to produce the one-third filling of the Dirac sea, etc., or a series of transformations into inhomogeneous magnetic phases.

Equation (5) can be rewritten in the form

$$\sqrt{1 - (1-\delta^2)(g\mu_B H/J)^2} = \mathrm{sn}(u,k), \tag{6}$$

where $\mathrm{sn}(u,k)$ is the elliptic sinus (Jacoby) function with $u = \pi C\sqrt{J^2 + (\delta g\mu_B H)^2}/(2g\mu_B H)^2$. This equation can be solved numerically for any values of field, exchange constant and elastic constant. For $k \sim 1$ (which corresponds to small δ), the asymptotic of the elliptic function is known to be $\mathrm{sn}(u,k) \approx \tanh u + (1-k^2)(\sinh u\cosh u - u)/4\cosh^2 u$. [For large k one has $\mathrm{sn}(u,k) \approx \sin u - k^2\cos u(u - \sin u\cos u)/4$.] It is possible to write the analytic asymptotic expression for the ground state steady-state nonzero δ_0 as

$$\delta_0 \approx (2J/g\mu_B H)\sqrt{\frac{A\tanh u_0 - A^2}{2 - A\tanh 2u_0 - (Au_0/\cosh^2 u_0)}}, \tag{7}$$

where $A = \sqrt{1-(g\mu_B H/J)^2}$ and $u_0 = \pi CJ/(2g\mu_B H)^2$. It is clear that a nonzero δ_0 can appear only for $\tanh u > A$. Equation (7) is plotted in Fig. 1. The absence of a physical solu-

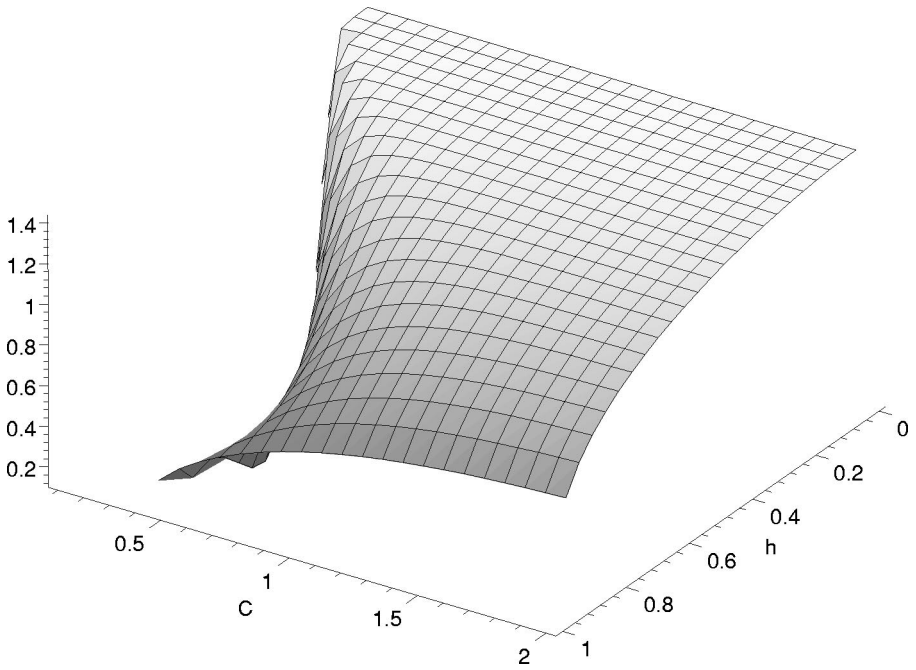

FIG. 1. The steady-state lattice (antiferro)distortion of ligands δ_0 as a function of the applied magnetic field $h=g\mu_B H$ and elastic constant C at $T=0$ for the $J=1$ XX spin $S=\frac{1}{2}$ AF chain with two inequivalent g factors. Notice that $\delta_0>1$ corresponds to the nonphysical solution.

tion ($\delta_0<1$) for small values of H is an artifact of the approximation (i.e., the replacement of the elliptic function by its asymptotics).

For nonzero temperatures we calculate the free energy of our system. We assume that the Debye energy of the phonons is large. Therefore the elastic subsystem effectively remains in the groundstate. The free energy is

$$F=\frac{C\delta^2}{2}-\frac{T}{\pi}\sum_{\pm}\int_0^{\pi}d\lambda\,\ln\left[2\cosh\left(\frac{\varepsilon_{\pm}(\lambda)}{2T}\right)\right], \qquad (8)$$

where $\varepsilon_{\pm}(\lambda)=g\mu_B H\pm\sqrt{(\delta g\mu_B H)^2+[J\cos(\lambda/2)]^2}$. Then the minimization of the free energy with respect to δ yields

$$C\delta=\frac{1}{2\pi}\sum_{\pm}\int_0^{\pi}d\lambda\,\tanh\frac{\varepsilon_{\pm}(\lambda)}{2T}\,\frac{\partial\varepsilon_{\pm}(\lambda)}{\partial\delta}. \qquad (9)$$

For $T\gg J$ we can replace the hyperbolic tangent by its argument and observe that there exists only one solution to Eq. (9): $\delta_0=0$. We performed a numerical analysis of this equation, and it revealed that a $\delta_0\neq 0$ solution exists for low temperatures. The critical temperature T_c of the transition between the low temperature phase (with two inequivalent paramagnetic centers) and uniform high temperature phase is obtained by equating $\delta_0=0$ for the second order phase transition, or the free energies of the phases with $\delta_0=0$ and $\delta_0\neq 0$ for the first order case. The critical temperature can be estimated (with the main input to the integral given by the van Hove singularities) as

$$T_c\sim(J-g\mu_b H)/\ln(\pi CJ/2g^2\mu_B^2H^2). \qquad (10)$$

The analysis shows that $\delta_0=0$ pertains to the minimum of the free energy for $T>T_c$ and to the maximum for $T<T_c$. Hence the phase transition is of second order. We again emphasize that the phase transformations studied here are magnetic field induced—they are absent without the field. We also point out that the order of the phase transformation in the general case depends on the orientation of the direction of the magnetic field with respect to crystal axes and on the mutual ratios of the exchange constant, anisotropy, magnetic field and elastic constant. Naturally, in a pure 1D system the temperature of the phase transition must be zero. However in our case the system is *not* pure 1D: the magnetic subsystem is 1D, however the elastic subsystem is 3D. That is why the critical value of the temperature of the phase transformation is nonzero. This reflects the fact that the order parameter δ describes both the inequivalence of the g factors and the *3D distortions* of nonmagnetic ligands.

It is clear that the effect studied here can be generalized with ease to other quasi-1D quantum critical systems of spins and correlated electrons. The possibility of such a field-induced Jahn-Teller-like effects (nonzero staggered magnetization) is determined by the (minimal) scaling exponent x_e. This exponent can be calculated using the conformal (finite-size) corrections for energies of low-lying spin excitations of critical quantum spin and electron chains. For example, for the $SU(2S+1)$-symmetric spin S chain[31] the exponent can be calculated as $x_e=2S/(2S+1)$ (i.e., it is less than 1 for any S except of the quasiclassical situation $S\gg 1$). For the $SU(2)$-symmetric case, it is $x_e=r(r+2)/4(S+1)$ ($r=1,\ldots,S$).[32] It is possible to calculate these minimal scaling exponents for correlated electron models (i.e., when a charge movement of electrons along the chains is taken into account) like the Hubbard model, t-J model, etc. In the simplest case of one electron per site those electron models correspond to the above mentioned quantum spin chains. Analytic expressions can be obtained, e.g., for Hubbard models with the Hubbard constant being much larger than the hopping integral, $U\gg t$. In that case for $H=0$ we have, e.g., for metallic phases of the spin-S Hubbard chain $x_e=[1+2S(2S+1)]/(2S+1)^2$.[33] Notice that $S/2$ can be considered as the number of channels. For all those models with $x_e<1$, we expect the possibility of a cooperative transition to the low-temperature state. There the staggered magnetization is nonzero and low-lying spin excitations are gapped. In that state the local alternating distortions of ligands surrounding

the neighboring magnetic ions will be nonzero and of different signs. It turns out that the exponent x_e is the smallest (hence the effect is the most strong) for the isotropic AF Heisenberg spin $S=\frac{1}{2}$ chain, and it increases with S (or with the number of channels).

We suppose that the cooperative effects similar to the one studied in our work have likely been observed in some rare-earth molibdates. There are low temperature ESR studies which observed two inequivalent magnetic centers, see, e.g., Refs. 9–12. For higher temperatures[10,11] or higher magnetic fields (higher frequencies)[18] only one magnetic center was seen. Ultrasonic measurements revealed anomalies in the low temperature dependencies of sound velocities for acoustic[20] and optical branches[21] of the phonon spectra. Optical and magnetic treatments observed the cooperative effect in nonzero magnetic field, which was classified as one of the Jahn-Teller type.[9,19,34] Notice that the transition to the magnetically ordered state was observed at much lower temperatures (see, e.g., Refs. 11, 34, 21). Hence the cooperative effect discussed here was observed in the *paramagnetic* (magnetically disordered) phase. It turns out that magnetic and elastic characteristics of some of those systems manifested the mentioned features only for *nonzero* external magnetic fields. The order of the transition was determined by the direction of the field. (It is determined as usual by checking which derivative of the thermodynamic potential possesses a singular behavior.) For zero field for some compounds there were no observations of the Jahn-Teller-like cooperative effect.[35]

It turns out that (alternating) distortions of the positions of *ligands* (they result in the inequivalence of effective g factors of neighboring spins in quasi-1D chains) in real compounds can be supplemented by alternating distortions of the positions of *magnetic ions themselves*. The latter leads to spin-Peierls instabilities and was observed, e.g., in $CuGeO_3$. There ESR studies reported an alternating Dzyaloshinskii-Moriya coupling.[36] Inequivalent surroundings of Cu ions were observed very recently in nuclear quadrupole resonance.[37] In such a situation both effects are additive in the formation of a spin gap for low-lying spin excitations. The magnitudes of the effects are naturally different. Also, the effect studied in this work manifests itself only in some domain of the magnetic field values. On the other hand, the spin-Peierls period doubling is *not* caused by a magnetic field. The critical temperatures are determined by which instability takes place first. For instance, in $CuGeO_3$ the alternating positions of Cu ions exist at higher temperatures than the softening of the phonon mode, corresponding to the distortion of the magnetic ions themselves.

In conclusion, we have proposed a realization of the Jahn-Teller-like cooperative effect in the low-dimensional quantum critical electron or spin systems. Alternating local distortions of ligands, surrounding the neighboring magnetic ions (antiferrodistortions), lift the degeneracy of orbital degrees of freedom of the magnetic ions. This, in turn, leads to the appearance of *different g* factors on neighboring magnetic ions (inequivalent magnetic centers). The result is nonzero staggered magnetization and field-induced gaps in the spectra of low-lying spin excitations. We suppose that the effects theoretically studied in this work were probably observed in low temperature experiments in some quasi-low-dimensional magnetic compounds, in which spin, charge and orbital characteristics of electron subsystem were strongly coupled to the elastic subsystems.

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