Magnetotransport in nearly superconducting Fermi liquids

R. Hlubina
Department of Solid State Physics, Comenius University, Mlynská Dolina F2, 842 15 Bratislava, Slovakia
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The quasiparticle contribution to the conductivity of a nearly superconducting two-dimensional Fermi liquid in a perpendicular magnetic field B is studied. The Boltzmann equation for the case of scattering on the pair fluctuations is shown to be the same as for forward scattering on a particle-hole collective mode. It is shown that in both cases, the Jones-Zener expansion of the conductivity tensor in powers of B is severely modified. Implications for some theories of the normal state in the cuprates and also for nearly ferromagnetic systems are discussed.

The nature of the electron dynamics in the normal state of the cuprates remains an open subject.[1] Anderson has proposed that on the phenomenological level the in-plane magnetotransport data can be analyzed in terms of two lifetimes, one of which determines the response to the electric field and the other one to the magnetic field.[2] Soon after this proposal it has been suggested that these two different lifetimes can be realized in a quite conventional way: different parts of the Fermi surface can support different lifetimes.[3] Such anisotropic situations appear quite naturally in models with large-momentum scattering.[4,5] However, it has been pointed out that even the description of the simplest quantity of interest, ρ_{xx}, is not free of problems in such theories, since such scattering can affect only a small (hot) part of the Fermi surface. The remaining (cold) Fermi-surface electrons do not experience singular scattering and short circuit the contribution of the hot electrons at low temperatures.[6]

Some of the problems of the hot-spot models have been overcome in a phenomenological model due to Ioffe and Millis, in which the whole Fermi surface of the cuprates is supposed to be hot, except for a small number of electrons in the vicinity of the $(\pm 1, \pm 1)$ directions, which are supposed to be cold.[7] Ioffe and Millis argue that this so-called cold-spot model may result from the scattering of electrons on the pair fluctuations of a d-wave superconductor.

Anderson has proposed different microscopics.[2] The origin of his holon-spinon picture lies in a singular forward scattering of the electrons. Unfortunately, Anderson's suggestion has not been studied in much detail so far. We are aware only of Ref. 8 which shows that strong forward scattering can lead in a layered system to a divergent resistivity anisotropy ρ_{zz}/ρ_{xx} as the temperature is lowered, in agreement with experiment.

In this paper we elaborate on the following observation: an electron with momentum $\mathbf{k}$ scatters on a superconducting fluctuation in that it annihilates another electron with momentum $\mathbf{k}' \approx -\mathbf{k}$. Since the total momentum of the annihilated pair of electrons is close to zero, the electric current changes only little due to such scattering. We show that the Boltzmann equation for scattering on superconducting fluctuations is equivalent to the forward scattering case. We find the expansion of the conductivity tensor in powers of the magnetic field for a general forward-scattering mechanism. We argue that, on the level of the Boltzmann equation, forward scattering (and, hence, also superconducting fluctuations) do not lead to the two-lifetime phenomenology of the cuprates.

Boltzmann equation. Let us start by reviewing the case of electrons scattering on a bosonic mode which can be a phonon or a particle-hole collective mode of the electron system. Throughout this paper we assume that the bosons relax sufficiently fast so that they can be assumed to be in equilibrium even in an applied electric field $\mathbf{E}$ and magnetic field $\mathbf{B}$. Let us write the electron distribution function $f_{\mathbf{k}}$ of the electron gas in the form $f_{\mathbf{k}} = f^0_{\mathbf{k}} - \Phi_{\mathbf{k}} \partial f^0_{\mathbf{k}} / \partial \varepsilon_{\mathbf{k}}$, where $f^0_{\mathbf{k}}$ is the equilibrium distribution function. $\Phi_{\mathbf{k}}$ is a function to be determined from the Boltzmann equation,[9] which to linear order in $\mathbf{E}$ reads

$$e\left[\mathbf{E} + \frac{\partial \Phi}{\partial \mathbf{k}} \times \mathbf{B}\right] \cdot \mathbf{v}_{\mathbf{k}} \delta(\varepsilon_{\mathbf{k}}) = \sum_{\mathbf{k}'} W^{\rm ph}_{\mathbf{k},\mathbf{k}'} (\Phi_{\mathbf{k}} - \Phi_{\mathbf{k}'}).$$

The function $W^{\rm ph}_{\mathbf{k},\mathbf{k}'}$ describes the scattering of electrons on the bosonic mode[6] and $\mathbf{v}_{\mathbf{k}}$ is the electron group velocity.

From now on, let us specialize to a two-dimensional system with square symmetry and lattice constant a. We assume that the electric field $\mathbf{E}$ (magnetic field $\mathbf{B}$) is parallel (perpendicular) to the electron system.

The sum over $\mathbf{k}'$ in the collision term of the Boltzmann equation can be written $\Sigma_{\mathbf{k}'} = (a/2\pi)^2 \oint (dk'/v_{k'}) \int d\varepsilon_{\mathbf{k}'}$. Let us take the integral $\int d\varepsilon_{\mathbf{k}}$ of both sides of the Boltzmann equation and define a dimensionless scattering function $A^{\rm ph}_{\mathbf{k},\mathbf{k}'} = (a^2/2\pi v_{\mathbf{k}} v_{\mathbf{k}'}) \int d\varepsilon_{\mathbf{k}} \int d\varepsilon_{\mathbf{k}'} W^{\rm ph}_{\mathbf{k},\mathbf{k}'}$. Making use of the well-known result for the function $W^{\rm ph}_{\mathbf{k},\mathbf{k}'}$ (see, e.g., Ref. 6) we have

$$A^{\rm ph}_{\mathbf{k},\mathbf{k}'} = \frac{|g^{\rm ph}_{\mathbf{k},\mathbf{k}'}|^2 a^2}{4\pi v_{\mathbf{k}} v_{\mathbf{k}'}} \int_{-\infty}^{\infty} \frac{d\omega\, \omega \,{\rm Im}\chi_{\rm ph}(\mathbf{k}' - \mathbf{k}, \omega)}{T \sinh^2(\omega/2T)},$$

where ${\rm Im}\chi_{\rm ph}(\mathbf{q},\omega)$ is the spectral function of the boson mode and $g^{\rm ph}_{\mathbf{k},\mathbf{k}'}$ describes the coupling of electrons and bosons. We have assumed that the scattering is quasielastic and therefore we can neglect the dependence of ${\rm Im}\chi_{\rm ph}(\mathbf{k}' - \mathbf{k}, \omega)$ on $\varepsilon_{\mathbf{k}}, \varepsilon_{\mathbf{k}'}$. All momenta are taken at the Fermi surface.

Let us consider an electron-like Fermi line and define a "Fermi wave vector" $k_F = \oint dk/2\pi$, where the integration is taken along the Fermi line. The Fermi points shall be labeled by a new angular variable φ defined by $dk = k_F d\varphi$, and we set $\varphi = 0$ along one of the crystal axes. We assume furthermore that $\mathbf{E}$ is parallel to the $\varphi = 0$ direction. Since $\Phi_{\mathbf{k}}$ is proportional to $E = |\mathbf{E}|$, we can replace it by a new dimensionless function $g_{\mathbf{k}}$ by setting $\Phi_{\mathbf{k}} = (eE/k_F) g_{\mathbf{k}}$. With the above definitions, the Boltzmann equation can be written

$$\cos\psi(\varphi) + \beta \frac{dg}{d\varphi} = \oint \frac{d\varphi'}{2\pi} A_{\mathrm{ph}}(\varphi,\varphi')[g(\varphi) - g(\varphi')], \tag{1}$$

where $\cos\psi = \mathbf{E}\cdot\mathbf{v}_{\mathbf{k}}/(E v_{\mathbf{k}})$ and $\beta = e|\mathbf{B}|/\hbar k_F^2$ is a dimensionless magnetic field.

Scattering on superconducting fluctuations. Following Ref. 7, we consider the case when the quasiparticle contribution to the conductivity dominates over the paraconductivity. We assume that the interaction of the electrons with the superconducting fluctuations can be described by the Hamiltonian

$$H = \frac{1}{\sqrt{2}} \sum_{\mathbf{k},\mathbf{q}} g_{\mathbf{k}}^{\mathrm{pp}} [(c_{-\mathbf{k}+\mathbf{q}/2\downarrow} c_{\mathbf{k}+\mathbf{q}/2\uparrow} - c_{-\mathbf{k}+\mathbf{q}/2\uparrow} c_{\mathbf{k}+\mathbf{q}/2\downarrow}) \times (a_{\mathbf{q}}^{\dagger} + b_{-\mathbf{q}}) + \mathrm{H.c.}],$$

where $g_{\mathbf{k}}^{\mathrm{pp}}$ is a form factor which depends on the symmetry of the pairing state and $a_{\mathbf{q}}^{\dagger}$ and $b_{\mathbf{q}}^{\dagger}$ create particlelike and holelike pair fluctuations, respectively.

In order to stress the similarity to the case of scattering on phonons, for the moment being, let us assume that the particle- and hole-like pair fluctuations have sharply defined spectra $\omega_{a,b}$. Then the collision terms for scattering of the electrons on the a and b bosons are, respectively,

$$\left(\frac{\partial f_{\mathbf{k}}}{\partial t}\right)_a = 2\pi \sum_{\mathbf{q}} |g_{\mathbf{k}-\mathbf{q}/2}^{\mathrm{pp}}|^2 [(1-f_{\mathbf{k}})(1-f_{-\mathbf{k}+\mathbf{q}}) n_{a,\mathbf{q}} - f_{\mathbf{k}} f_{-\mathbf{k}+\mathbf{q}} (n_{a,\mathbf{q}}+1)] \delta(\varepsilon_{\mathbf{k}} + \varepsilon_{-\mathbf{k}+\mathbf{q}} - \omega_{a,\mathbf{q}}),$$

$$\left(\frac{\partial f_{\mathbf{k}}}{\partial t}\right)_b = 2\pi \sum_{\mathbf{q}} |g_{\mathbf{k}-\mathbf{q}/2}^{\mathrm{pp}}|^2 [(1-f_{\mathbf{k}})(1-f_{-\mathbf{k}+\mathbf{q}}) (n_{b,\mathbf{q}}+1) - f_{\mathbf{k}} f_{-\mathbf{k}+\mathbf{q}} n_{b,\mathbf{q}}] \delta(\varepsilon_{\mathbf{k}} + \varepsilon_{-\mathbf{k}+\mathbf{q}} + \omega_{b,\mathbf{q}}),$$

where $n_{a,b}$ is the Bose-Einstein distribution function, since we assume again that the collective modes are in equilibrium.

If we write $f_{\mathbf{k}} = f_{\mathbf{k}}^0 - \Phi_{\mathbf{k}} \partial f_{\mathbf{k}}^0/\partial\varepsilon_{\mathbf{k}}$, then to linear order in the deviation from equilibrium, $\Phi_{\mathbf{k}}$, the Boltzmann equation can be written

$$e\left[\mathbf{E} + \frac{\partial\Phi}{\partial\mathbf{k}} \times \mathbf{B}\right] \cdot \mathbf{v}_{\mathbf{k}} \delta(\varepsilon_{\mathbf{k}}) = \sum_{\mathbf{k}'} W_{\mathbf{k},\mathbf{k}'}^{\mathrm{pp}} (\Phi_{\mathbf{k}} + \Phi_{\mathbf{k}'}),$$

where

$$W_{\mathbf{k},\mathbf{k}'}^{\mathrm{pp}} = \frac{2}{T} |g_{(\mathbf{k}'-\mathbf{k})/2}^{\mathrm{pp}}|^2 f_{\mathbf{k}}^0 f_{\mathbf{k}'}^0 [n(\varepsilon_{\mathbf{k}} + \varepsilon_{\mathbf{k}'}) + 1] \times \mathrm{Im}\chi_{\mathrm{pp}}(\mathbf{k}+\mathbf{k}', \varepsilon_{\mathbf{k}} + \varepsilon_{\mathbf{k}'}).$$

Here we have relaxed our requirement of sharply defined energies of the pair fluctuations which are now described by the spectral function $\mathrm{Im}\chi_{\mathrm{pp}}(\mathbf{q},\omega)$.

Assuming quasielastic scattering, the Boltzmann equation can be cast in the form

$$\cos\psi(\varphi) + \beta \frac{dg}{d\varphi} = \oint \frac{d\varphi'}{2\pi} A_{\mathrm{pp}}(\varphi,\varphi')[g(\varphi) + g(\varphi')], \tag{2}$$

where

$$A_{\mathbf{k},\mathbf{k}'}^{\mathrm{pp}} = \frac{\left|g^{\mathrm{pp}}\left(\frac{\mathbf{k}'-\mathbf{k}}{2}\right)\right|^2 a^2}{4\pi v_{\mathbf{k}} v_{\mathbf{k}'}} \int_{-\infty}^{\infty} \frac{d\omega\,\omega\,\mathrm{Im}\chi_{\mathrm{pp}}(\mathbf{k}+\mathbf{k}',\omega)}{T\sinh^2(\omega/2T)}$$

and the notation is the same as for scattering on a particle-hole-like collective mode.

Let us comment on the symmetries of $A_\alpha(\varphi,\varphi')$ ($\alpha = pp, ph$). We require $A_\alpha(\varphi,\varphi') = A_\alpha(\varphi',\varphi)$, $A_\alpha(\varphi,\varphi') = A_\alpha(\varphi+\pi/2, \varphi'+\pi/2)$, and $A_\alpha(\varphi,\varphi') = A_\alpha(-\varphi,-\varphi')$. In addition to the functions $A_\alpha(\varphi,\varphi')$ it is convenient to consider also the functions $\mathcal{A}_\alpha(\overline{\varphi},\theta) = A_\alpha(\varphi,\varphi')$, where $\theta = \varphi' - \varphi$ and $\overline{\varphi} = (\varphi'+\varphi)/2$. The above symmetries allow us to write $\mathcal{A}_\alpha(\overline{\varphi},\theta) = \sum_{m,n=0}^{\infty} C_{m,n}^{\alpha} \cos 4m\overline{\varphi} \cos n\theta$. As regards $\psi(\varphi)$, we require $\psi(\varphi) = -\psi(-\varphi)$ and $\psi(\varphi+\pi/2) = \psi(\varphi) + \pi/2$. Therefore the deviation from a circular Fermi surface, $\alpha(\varphi) = \psi(\varphi) - \varphi$, can be expanded as $\alpha(\varphi) = \sum_{n=1}^{\infty} a_n \sin 4n\varphi$.

Making use of the symmetries of $\psi(\varphi)$ and $A_\alpha(\varphi,\varphi')$, one can show readily that Eqs. (1) and (2) are consistent with $g(\varphi) = -g(\varphi+\pi)$. That is why, in the general case when both the pp and the ph scattering is present, the Boltzmann equation can be written as

$$\cos\psi(\varphi) + \beta \frac{dg}{d\varphi} = \oint \frac{d\varphi'}{2\pi} A(\varphi,\varphi')[g(\varphi) - g(\varphi')], \tag{3}$$

where $A(\varphi,\varphi') = A_{\mathrm{ph}}(\varphi,\varphi') + A_{\mathrm{pp}}(\varphi,\varphi'+\pi)$. Note that both for dominant forward scattering and for scattering on the superconducting fluctuations, $A(\varphi,\varphi')$ is substantial only for $\varphi' \approx \varphi$.

Once we have solved for $g(\varphi)$, the conductivity tensor can be calculated from

$$\sigma = \frac{2e^2}{h} \int_0^{2\pi} \frac{d\varphi}{2\pi} g(\varphi) \begin{pmatrix} \cos\psi(\varphi), & \sin\psi(\varphi) \\ -\sin\psi(\varphi), & \cos\psi(\varphi) \end{pmatrix}. \tag{4}$$

Equations (3) and (4) solve (in principle) the magnetotransport problem.

Let us return to the symmetry analysis. It is easy to see that $A(\varphi,\varphi')$ has the same symmetries as $A_\alpha(\varphi,\varphi')$. Moreover, if we explicitly take into account the dependence of $g(\varphi)$ on the parameter β, we find that $g(\varphi,\beta) = g(-\varphi, -\beta)$. In weak applied magnetic fields it is convenient to expand the function $g(\varphi)$ in powers of β, $g = g_0 + g_1 + g_2 + \ldots$, where $g_n \propto \beta^n$. This is the so-called Jones-Zener expansion. From Eq. (4) it follows that g_n with even (odd) n determine the diagonal (off-diagonal) components of the conductivity tensor.

Approximate solution for dominant forward scattering. Let us introduce $\theta=\varphi'-\varphi$ so that we can write $A(\varphi,\varphi')=\mathcal{A}(\varphi+\theta/2,\theta)$. For dominant forward scattering, the scattering function $\mathcal{A}(\overline{\varphi},\theta)$ is non-negligible only for $|\theta|$ smaller than a characteristic angle $\theta_0 \ll 1$. We shall assume furthermore that $\mathcal{A}(\overline{\varphi},\theta)$ is a weak function of $\overline{\varphi}$ on the scale θ_0. In this case we can expand in powers of θ the functions entering the collision integral on the right-hand side of Eq. (3), namely $\mathcal{A}(\varphi+\theta/2,\theta)$ (as a function of its first argument) and $g(\varphi+\theta)$. Keeping only terms $\propto\theta^2$, the Boltzmann equation simplifies to

$$-(G_{\rm tr}^{-1}(\varphi)g')'=\cos\psi+\beta g', \tag{5}$$

where $G_{\rm tr}(\varphi)$ is a dimensionless ''transport mean free path,'' $G_{\rm tr}^{-1}(\varphi)=\oint(d\theta/2\pi)\mathcal{A}(\varphi,\theta)(1-\cos\theta)$. The solution of Eq. (5) can be expanded in powers of β as follows:

$$g_0(\varphi)=-\int_{\pi/2}^{\varphi}d\varphi' G_{\rm tr}(\varphi')\int_0^{\varphi'}d\varphi''\cos\psi(\varphi''),$$

$$g_{n+1}(\varphi)=-\beta\int_{n\pi/2}^{\varphi}d\varphi' G_{\rm tr}(\varphi')g_n(\varphi'),\quad n=0,1,\ldots. \tag{6}$$

Using the above solution for the electron distribution function in Eq. (4) we find that the conductivity in the absence of the magnetic field is

$$\sigma_{xx}=\frac{2e^2}{h}\int_0^{2\pi}\frac{d\varphi}{2\pi}G_{\rm tr}(\varphi)S^2(\varphi),$$

where $S(\varphi)=\int_0^{\varphi}dt\cos\psi(t)$. For a circular Fermi surface, e.g., $S(\varphi)=\sin\varphi$ and σ_{xx} is given by the standard formula $\sigma_{xx}=(e^2/h)\int_0^{2\pi}d\varphi G_{\rm tr}(\varphi)/2\pi$. Contributions to σ which are of a higher order in β are given by multiple angular integrations.

Cold-spot model. Ioffe and Millis[7] have proposed recently that the anomalous in-plane magnetotransport properties of the cuprates can be understood in the framework of a nearly superconducting Fermi liquid. In Ref. 7 the relaxation-time approximation is adopted, i.e., it is implicitly assumed that for all φ,

$$\oint d\varphi' A(\varphi,\varphi')g(\varphi')\approx 0. \tag{7}$$

In this case Eq. (3) simplifies to

$$G^{-1}(\varphi)g=\cos\psi+\beta g', \tag{8}$$

where the prime denotes a derivative with respect to φ, $g'=dg/d\varphi$, and $G(\varphi)$ is a dimensionless ''single-particle mean free path,'' $G^{-1}(\varphi)=\oint(d\varphi'/2\pi)A(\varphi,\varphi')$. Equation (8) can be solved by variation of constants, but here we shall discuss only the Jones-Zener expansion which reads

$$g_0=G\cos\psi,$$

$$g_{n+1}=\beta G g_n',\quad n=0,1,\ldots. \tag{9}$$

Note the difference of this standard result with respect to the Zener-Jones expansion for forward scattering, Eq. (6). From Eq. (9) it follows that (within the relaxation-time approximation) the conductivity can be written, to order β^2, in the form

$$\sigma_{xx}=\frac{e^2}{h}\oint\frac{d\varphi}{2\pi}G\{1-\beta^2[(G\psi')^2+(G')^2]+\cdots\},$$

$$\sigma_{xy}=\frac{e^2}{h}\oint\frac{d\varphi}{2\pi}G\{-\beta G\psi'+\cdots\}. \tag{10}$$

In calculating $G(\varphi)$, Ioffe and Millis take into account the d-wave symmetry of the superconducting fluctuations, $g_{\rm pp}(\varphi)=g_{\rm pp}^{(0)}\cos 2\varphi$, and for the spectral function of the superconducting fluctuations they take $\mathrm{Im}\chi_{\rm pp}(\mathbf{q},\omega)=F[\omega^2/u^2(q^2+\xi^{-2})]/(q^2+\xi^{-2})$, where $u\ll v_F$ is the velocity and $\xi\sim u/T\gg k_F^{-1}$ is the correlation length of the pair fluctuations. $F(x)$ is a fairly general scaling function.

A straightforward calculation leads then to the inverse mean free path $G^{-1}(\varphi)\sim\lambda\cos^2 2\varphi$, where λ is a T-independent constant. Taking into account the finite angular resolution, $\theta_0\sim 1/k_F\xi$, Ioffe and Millis obtain $G^{-1}(\varphi)\sim\lambda(T/uk_F)^2$ in the cold spots (i.e., for φ such that $\cos 2\varphi=0$). As the temperature is lowered, the mean free path becomes progressively more and more anisotropic and, as Ioffe and Millis have shown, the angular integrations in Eq. (10) lead to transport coefficients in agreement with experimental data (except for the magnetoresistance).

Unfortunately, for scattering on superconducting fluctuations, the criterion Eq. (7) for the applicability of the relaxation-time approximation is not satisfied. Instead, one should use the scheme for dominant forward scattering. A straightforward calculation shows that, due to the factor $(1-\cos\theta)\sim\theta_0^2\sim(T/uk_F)^2$, the inverse transport mean free path for the model spectral function of Ioffe and Millis is $G_{\rm tr}^{-1}(\varphi)\sim\lambda(T/uk_F)^2\cos^2 2\varphi$, i.e., Fermi-liquid like even in the ''hot'' region!

The above discussion shows clearly that the microscopic picture proposed in Ref. 7 does not lead to agreement with the cuprate in-plane transport data. In what follows we shall argue that, quite generally, neither the scattering on superconducting fluctuations, nor the dominant forward scattering, lead to the two-lifetime phenomenology observed in the cuprates.

Let us discuss the transport properties for the case of dominant forward scattering assuming a temperature-dependent anisotropy of the transport mean free path $G_{\rm tr}(\varphi)$. In order to present a closed-form analytic solution, we restrict ourselves to the case of a circular Fermi surface, which is however not such a bad approximation to the actual Fermi surface observed experimentally in the cuprates (if a simple particle-hole transformation is performed). Moreover, we discuss only two extreme cases of $G_{\rm tr}(\varphi)$: (i) a completely isotropic mean free path $G_{\rm tr}(\varphi)=\overline{G}_{\rm tr}$, corresponding to a high-temperature state and (ii) a simple soluble example of an extremely anisotropic low-temperature mean free path $G_{\rm tr}(\varphi)=(\pi/2)\overline{G}_{\rm tr}\Sigma_n\delta[\varphi-(2n+1)\pi/4]$, which corresponds to a Fermi line with cold spots at $\varphi=(2n+1)\pi/4$ and an average transport mean free path $\oint d\varphi G_{\rm tr}(\varphi)/2\pi=\overline{G}_{\rm tr}$.

Making use of Eqs. (6) we find that both the high-T and the low-T mean free paths lead to the same resistivity tensor $\rho_{xx}=\rho_{yy}=\rho_0\overline{G}_{\rm tr}^{-1}$, $\rho_{xy}=-\rho_{yx}=\rho_0 c(T)\beta$, where $\rho_0=h/e^2$.

The only difference between the two cases is that at high temperatures (for an isotropic lifetime) $c(T)=1$, whereas at low temperatures (in the anisotropic case) $c(T)=\pi/4$. Thus the Hall number $R_H=\rho_{xy}/B$ increases slightly with temperature (if we assume that the electron concentration does not change with T). This suggests that the strange magnetotransport behavior of the cuprates (in which $R_H \propto T^{-1}$ has been observed[1]) is inconsistent with forward scattering.

We believe that the near T independence of R_H is a generic property of systems with dominant forward scattering. To illustrate this point, consider an example of a hot-spot model with a temperature-dependent anisotropy of the mean free path and a circular Fermi surface. For a system with dominant forward scattering and $G_{\rm tr}(\varphi)=G_0+G_1\cos 4\varphi$, we find

$$\rho_{xx}=\rho_0 G_0^{-1}[1+(\beta G_1/30)^2(34-G_1^2/G_0^2)],$$

$$\rho_{xy}=\rho_0\beta[1-(1/30)(G_1/G_0)^2].$$

Note that since $|G_1/G_0|<1$ [in order that $G_{\rm tr}(\varphi)>0$], R_H is nearly independent of temperature. Within the relaxation-time approximation, the same anisotropy of the mean free path $G(\varphi)=G_0+G_1\cos 4\varphi$ leads to a much larger deviation of ρ_{ij} from the isotropic case:

$$\rho_{xx}=\rho_0 G_0^{-1}[1+(\beta G_1/2)^2(34-G_1^2/G_0^2)],$$

$$\rho_{xy}=\rho_0\beta[1+(1/2)G_1^2/G_0^2].$$

Conclusions. The recent ARPES data of Valla *et al.*[10] suggest that the single-particle scattering rate in the cuprates exhibits a linear temperature dependence over most of the Fermi surface, including the $(\pm 1,\pm 1)$ directions. Our analysis shows that since $G(\varphi)\neq G_{\rm tr}(\varphi)$ for superconducting fluctuations, the results of Ref. 10 by themselves do not exclude the cold-spot picture.

Nevertheless, within standard transport theory we have shown that the in-plane magnetotransport in two-dimensional systems with dominant forward scattering and/or scattering on superconducting fluctuations, is different from the phenomenology observed in the cuprates. Although ρ_{xx} may deviate from the canonical Landau Fermi-liquid result $\rho_{xx}\propto T^2$, the Hall number depends only weakly on temperature.

Thus it appears that none of the proposed theories of the normal state of the cuprates which invoke singular scattering of the electrons on a collective mode is free of problems when applied to the in-plane magnetotransport. In fact, this seems to be the case if the exchanged boson is a pair fluctuation,[7] spin fluctuation,[4] charge fluctuation,[5] a mode leading to singular forward scattering[2], or a collective excitation leading to the marginal Fermi-liquid phenomenology (for a recent formulation, see Ref. 11).

Finally, let us point out that the results obtained in this paper should directly apply to nearly ferromagnetic two-dimensional systems. In that case one expects in the critical fluctuation-dominated region an anomalous resistivity $\rho_{xx}\propto T^{4/3}$ (see Ref. 12), whereas we *predict* that in the same region there is no temperature dependence to the Hall effect which would be caused by the orbital effects of the magnetic field.

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