

Thermokinetic approach of the generalized Landau-Lifshitz-Gilbert equation with spin-polarized current

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In order to describe the recently observed effect of current induced magnetization reversal in magnetic nanostructures, the thermokinetic theory is applied to a metallic ferromagnet in contact with a reservoir of spin-polarized conduction electrons. The spin-flip relaxation of the conduction electrons is described thermodynamically as a chemical reaction. In the two-current approximation, the diffusion equation of the chemical potential, the giant magnetoresistance at the interface, and the usual Landau-Lifshitz-Gilbert (LLG) equation is obtained from the entropy variation in the absence of current. The description of the conservation laws, including spin dependent scattering and spin injection, leads to the derivation of a generalized LLG equation. The equation is applied to the measurements obtained on single magnetic Ni nanowires.

An unexpected and spectacular effect due to spin polarization of conduction electrons in metallic ferromagnets, the giant magnetoresistance (GMR), appeared with transport studies on magnetic nanostructures.¹ The spin-diffusion length of conduction electrons being of some few tens of nm, the relaxation of the conduction electron spins becomes observable when the magnetization can be controlled over this typical length. Some predictions about the inverse effect, namely the influence of spin-polarized current on the dynamics of the magnetization, were also proposed. Berger predicted the existence of some surprising phenomena due to the action of spin-polarized conduction electrons on domain walls² or spin waves³ in magnetic thin films. Slonczewski predicted the rotation of the magnetization due to polarized current in multilayered systems,⁴ and Bazaliy, Jones, and Zhang derived from microscopic considerations a generalized Landau-Lifshitz-Gilbert equation.⁵ All the above-mentioned approaches are microscopic and based on the ballistic approximation.

From an experimental point of view, Freitas and Berger, Hung and Berger,⁶ and Salhi and Berger⁷ show the action of a high current density on domain walls in thin films. Recent experiments on nanostructured samples bring interesting evidence for the interpretation in terms of the action of the spin of the conduction electrons. Tsoi *et al.*⁸ show an effect of a high current density on spin-wave generation in Co/Cu multilayers, Sun reported on current-driven magnetic switching in manganite,⁹ and Myers *et al.* reported an effect of current induced switching in magnetic multilayer device.¹⁰ In a recent work, we have evidenced an effect of current induced magnetization reversal in magnetic nanowires,^{11,12} where the reversal of the magnetization is induced by a high current at an applied field 20% smaller than the normal reversal field. The ballistic approximation is, however, difficult to justify in all these experiments.

A phenomenological approach based on the thermokinetic theory^{13,14} of a metallic ferromagnet in contact with a reservoir of spin-polarized conduction electrons is proposed. In contrast to the pioneering works of Johnson and Silsbee¹⁵ on spin-polarized current, the spin-flip scattering is introduced here as a chemical reaction. This formulation allows the con-

servation laws of the spin polarized conduction electrons and the ferromagnetic order parameter to be treated (Sec. I). This phenomenological approach models the different effects able to take place in the system by a set of coupled transport equations (Sec. II). The differential equation of the chemical potential is derived leading to the known formula of the GMR (Sec. III). For a closed ferromagnetic system without electric current, the common Landau-Lifshitz-Gilbert equation is derived (Sec. IV). The open system with both ferromagnetization and spin polarized conduction electrons leads to the description of the generalized Landau-Lifshitz-Gilbert equation with polarized current (Sec. V). The consequences in terms of current induced magnetization reversal are deduced. The model is applied on the data obtained on Ni nanowires.

I. CONSERVATION LAWS

Spin dependent transport processes in layered structures are described on the basis of the following simple picture. After entering in the k th layer Σ^k , the incident current (which was spin polarized along the axis described by the unit vector $\pm \vec{u}_{k-1}$ in the layer Σ^{k-1}) first aligns¹⁶ along the axis $\pm \vec{u}_k$ (see Fig. 1). Inside the ferromagnetic layer, the population of spin up (N_+) and spin down (N_-) is then not conserved due to spin-flip scattering, and some of the down spins relax to the up direction.

In this picture, the states of the subsystem Σ^k are described by the variables

$$(S^k, \vec{M}^k, N_+^k, N_-^k, \dot{M}_0^k), \quad (1)$$

where S^k is the entropy, $\vec{M}^k = M^k \vec{u}_k$ is the magnetization, and N_{\pm}^k are the number of conduction electrons with spin parallel to the unit vector $\pm \vec{u}_k$.

If \vec{M}^k and N_{\pm}^k are independent variables, the conservation of the magnetic momentum reads

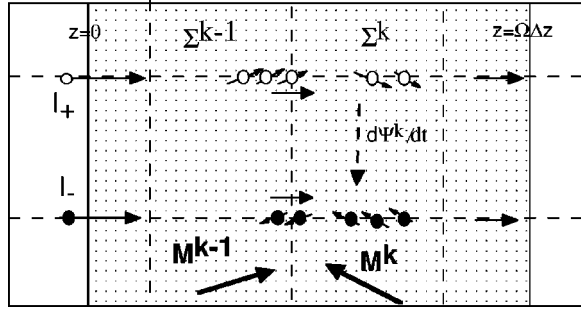


FIG. 1. Modelization of the system with the two-channel approximation. After entering in the k th layer Σ^k , the incident current (which was spin polarized along the axis $\pm \vec{M}_{k-1}$ in the layer Σ^{k-1}) first aligns along the axis $\pm \vec{M}_k$. The population of spin up (N_+) and spin down (N_-) are then not conserved due to spin-flip scattering, and some of the down spins relax to the up direction in the layer.

$$\frac{d\vec{M}^k}{dt} = \vec{M}_0^k + g\mu_B(\dot{N}_+^k - \dot{N}_-^k)\vec{u}_{k-1}, \quad (2)$$

where μ_B is the Bohr magneton and g is the Landé factor.

In order to write the conservation laws, the spin-flip scattering mechanism is described as a chemical reaction transforming a spin down into a spin up along the axis $\pm \vec{u}_k$. In this context, the reaction rate $\dot{\Psi}^k$ is introduced as the number of chemical events per unit of time.^{17,18} Let $I_+^{k \rightarrow k+1}$ and $I_-^{k \rightarrow k+1}$ be the current of particles flowing from the layer Σ^k to the layer Σ^{k+1} due, respectively, to the electrons with spin in the direction \vec{u}_k and to the electrons with spin in the direction $-\vec{u}_k$. The conservation of the particles is then described by

$$\begin{aligned} \frac{dN_+^k}{dt} &= \alpha(k-1;k)I_+^{k-1 \rightarrow k} + [1 - \alpha(k-1;k)]I_-^{k-1 \rightarrow k} \\ &\quad - I_+^{k \rightarrow k+1} - \dot{\Psi}^k \\ \frac{dN_-^k}{dt} &= [1 - \alpha(k-1;k)]I_+^{k-1 \rightarrow k} + \alpha(k-1;k)I_-^{k-1 \rightarrow k} \\ &\quad - I_-^{k \rightarrow k+1} + \dot{\Psi}^k, \end{aligned} \quad (3)$$

where α is the spin-flip probability of the alignment process.¹⁹ In the case of ballistic alignment $\alpha(k-1;k) = \cos^2[\frac{1}{2}\theta(k-1;k)]$ with θ the angle between \vec{u}_{k-1} and \vec{u}_k . Introducing the polarized current I_p , and the normal current I_N defined by

$$\begin{aligned} I_p^{k-1 \rightarrow k} &= I_+^{k-1 \rightarrow k} - I_-^{k-1 \rightarrow k}, \\ I_N^{k-1 \rightarrow k} &= I_+^{k-1 \rightarrow k} + I_-^{k-1 \rightarrow k} \end{aligned} \quad (4)$$

Eq. (3) can be put into the form

$$\frac{dN_+^k}{dt} = I_+^{k-1 \rightarrow k} - I_+^{k \rightarrow k+1} - [\gamma(k-1;k)I_p^{k-1 \rightarrow k} + \dot{\Psi}^k]$$

$$\frac{dN_-^k}{dt} = I_-^{k-1 \rightarrow k} - I_-^{k \rightarrow k+1} + [\gamma(k-1;k)I_p^{k-1 \rightarrow k} + \dot{\Psi}^k] \quad (5)$$

with $\gamma(k-1;k) = 1 - \alpha(k-1;k) = \sin^2[\frac{1}{2}\theta(k-1;k)]$.

Inserting Eq. (5) into Eq. (2) the conservation of the magnetization reads

$$\begin{aligned} \frac{d\vec{M}^k}{dt} &= \vec{M}_0^k + g\mu_B\vec{u}_{k-1}\{I_p^{k-1 \rightarrow k} - I_p^{k \rightarrow k+1} \\ &\quad - 2[\gamma(k-1;k)I_p^{k-1 \rightarrow k} + \dot{\Psi}^k]\}. \end{aligned} \quad (6)$$

The problem of the spin transfer between the polarized current and the magnetic layer is hence solved if the polarized current I_p and the reaction rates $\dot{\Psi}$ can be described as functions of the experimentally accessible parameters, the current I_N , the electric field E_0 , and the kinetic coefficients. This task is typically performed by the application of the first and second laws of the thermodynamics.

II. KINETIC EQUATIONS

The system Σ^k is open to heat transfer, to chemical transfer, and to mechanical work due to the magnetization and magnetic fields. Let us define the heat and chemical power by P_ϕ and the mechanical power by P_W . The first law of the thermodynamics applied to the layer Σ^k gives

$$\frac{dE^k}{dt} = P_\phi^{k-1 \rightarrow k} - P_\phi^{k \rightarrow k+1} + P_W^{ext \rightarrow k}, \quad (7)$$

where $P_W^{ext \rightarrow k} = -\vec{H}^{ext} \cdot \dot{\vec{M}}^k$. Furthermore, with using the canonical definitions $T^k = \partial E^k / \partial S^k$, $\mu_\pm^k = \partial E^k / \partial N_\pm^k$, $\vec{H}^k = \partial E^k / \partial \vec{M}^k$ the energy variation is

$$\frac{dE^k}{dt} = T^k \frac{dS^k}{dt} + \mu_+^k \frac{dN_+^k}{dt} + \mu_-^k \frac{dN_-^k}{dt} - \vec{H}^k \frac{d\vec{M}^k}{dt} + \frac{\partial E^k}{\partial \dot{\vec{M}}_0^k} \frac{d\dot{\vec{M}}_0^k}{dt}. \quad (8)$$

In the present work we limit our analysis to the isothermal case, $T^k = T$. The entropy variation of the sublayer is deduced from the two last equations, after introducing the conservation laws:

$$\begin{aligned} T \frac{dS^k}{dt} &= P_\phi^{k-1 \rightarrow k} - P_\phi^{k \rightarrow k+1} + (\vec{H}^k - \vec{H}^{ext}) \dot{\vec{M}}_0^k \\ &\quad - \frac{1}{2}[A^k - 2g\mu_B(H^{k-1} - H^{ext,k-1})] \\ &\quad \times [I_p^{k-1 \rightarrow k} - I_p^{k \rightarrow k+1} - 2\gamma(k-1;k)I_p^{k-1 \rightarrow k} - 2\dot{\Psi}^k] \\ &\quad - \frac{1}{2}\mu_0(I_N^{k-1 \rightarrow k} - I_N^{k \rightarrow k+1}) - \frac{\partial E^k}{\partial \dot{\vec{M}}_0^k} \frac{d\dot{\vec{M}}_0^k}{dt}, \end{aligned} \quad (9)$$

where the total chemical potential is $\mu_0^k \equiv \mu_+^k + \mu_-^k$. We have furthermore defined $H^{k-1} \equiv \vec{H}^k \cdot \vec{u}_{k-1}$ and $H^{ext,k}$

$\equiv \vec{H}^{ext} \cdot \vec{u}_k$. The chemical affinity of the reaction, defined by $A^k \equiv \partial E^k / \partial \Psi = \mu_+^k - \mu_-^k$, has also been introduced.

The entropy being an extensive variable, the total entropy variation of the system is obtained by summation over the layers 1 to Ω where the layer 1 is in contact to the left reservoir R^l and the layer Ω is in contact to the right reservoir R^r . Letting

$$\bar{A}^k \equiv A^k - 2g\mu_B(H^{k-1} - H^{ext,k}), \quad (10)$$

the total entropy variation is

$$\begin{aligned} T \frac{dS}{dt} = & [\dots]^{R^l \rightarrow 1} - [\dots]^{\Omega \rightarrow R^r} + \sum_{k=1}^{\Omega} (\bar{H}^k - \bar{H}^{ext}) \dot{M}_0^k \\ & + \sum_{k=1}^{\Omega} \left(-\frac{\partial E^k}{\partial \dot{M}_0^k} \right) \frac{d\dot{M}_0^k}{dt} \\ & + \sum_{k=2}^{\Omega} \frac{1}{2} [\bar{A}^{k-1} - \bar{A}^k + 2\gamma(k-1;k)\bar{A}^k] I_p^{k-1 \rightarrow k} \\ & + \sum_{k=2}^{\Omega} \frac{1}{2} (\mu_0^{k-1} - \mu_0^k) I_N^{k-1 \rightarrow k} + \sum_{k=1}^{\Omega} \bar{A}^k \dot{\Psi}^k, \quad (11) \end{aligned}$$

where the two first terms in the right-hand side of the equality stand for the heat and chemical transfer from the reservoirs to the system Σ .

The variation of entropy takes the form

$$T \frac{dS}{dt} = \sum_i F_i \dot{X}^i + P^{ext}(t), \quad (12)$$

where F_i are generalized forces and \dot{X}^i are the conjugated generalized fluxes. The variation of entropy is composed by an external entropy variation $P^{ext}(t)/T$ and by an internal entropy variation dS^{int}/dt .

By applying the second law of thermodynamics $dS^{int}/dt \geq 0$ we are leading to introduce the kinetic coefficients $l_{\alpha\beta}$ such that $dS^{int}/dt = \sum_i F_i (\sum_j l_{ij} F^j)$. By identification with the expression (11), the kinetic equations are obtained:

$$\begin{bmatrix} I_N^{k-1 \rightarrow k} \\ I_p^{k-1 \rightarrow k} \\ \dot{\Psi}^k \\ \dot{M}_0^k \\ \frac{d\dot{M}_0^k}{dt} \end{bmatrix} = \begin{bmatrix} l_{NN} & l_{Np} & l_{Nc} & l_{NM} & l_{N\dot{M}} \\ l_{pN} & l_{pp} & l_{pc} & l_{pM} & l_{p\dot{M}} \\ l_{cN} & l_{cp} & l_{cc} & l_{cM} & l_{c\dot{M}} \\ l_{MN} & l_{Mp} & l_{Mc} & l_{MM} & l_{M\dot{M}} \\ l_{\dot{M}N} & l_{\dot{M}p} & l_{\dot{M}c} & l_{\dot{M}M} & l_{\dot{M}\dot{M}} \end{bmatrix} \times \begin{bmatrix} \frac{1}{2}(\mu_0^{k-1} - \mu_0^k) \\ \frac{1}{2}[\bar{A}^{k-1} - \bar{A}^k + 2\gamma(k-1;k)\bar{A}^k] \\ \bar{A}^k \\ \bar{H}^k - \bar{H}^{ext} \\ -\left(\frac{\partial E^k}{\partial \dot{M}_0^k}\right) \end{bmatrix}. \quad (13)$$

The indices N and p stand, respectively, for the normal and polarized transport processes (see Sec. III), the indices c stands for the spin-flip scattering chemical reaction and the indices M and \dot{M} account for the dynamics of the magnetization (see Sec. IV).

The kinetic coefficients are state functions: $l_{ij} = l_{ij}(S^k, \vec{M}^k, N_+^k, N_-^k, \dot{M}_0^k)$ and the symmetrized matrix is positive: $\frac{1}{2}\{l_{ji} + l_{ij}\}_{\{ij\}} \geq 0$. Furthermore, according to Onsager relations, the kinetic coefficients are symmetric or antisymmetric, $l_{ij} = \pm l_{ji}$.

We assume in the following that cross effects between electronic transport and ferromagnetic transport are negligible, so that $l_{ij} = 0$ if $i = \{N, p, c\}$ and $j = \{M, \dot{M}\}$. Note that a polarized current is directly produced by a nonuniform magnetization state, through the coefficient $\gamma(k-1, k)$ in Eq. (13). The physical meaning of the kinetic coefficients is described in the two following sections.

III. GIANT MAGNETORESISTANCE

We focus in this section on the first three equations of Eq. (13) which describes the electric transport with spin polarization. After performing the continuum limit, we have

$$\begin{bmatrix} J_N \\ J_p \\ \dot{\Psi} \end{bmatrix} = \begin{bmatrix} -L_{NN} & -L_{Np} & L_{Nc} \\ -L_{pN} & -L_{pp} & L_{pc} \\ -L_{Nc} & -L_{pc} & L_{cc} \end{bmatrix} \begin{bmatrix} \frac{\partial \mu_0}{\partial z} \\ \frac{\partial A}{\partial z} - 2\tilde{\gamma}A \\ A \end{bmatrix}, \quad (14)$$

where $\tilde{\gamma} = \lim_{dz \rightarrow 0} (d\gamma/dz)$ and the choice of symmetric coefficients $L_{pN} = L_{Np}$ and $L_{Nc} = L_{cN}$ is motivated by the fact that we neglect in this work the direct effects of the magnetic field on the charge carriers.²⁰

In the framework of the two-channel approximation,²¹ the coupling between the two conduction bands is neglected (i.e., there is no cross effect²² between the currents J_+ , J_- , and $\dot{\Psi}$):

$$L_{Nc} = L_{pc} = 0; \quad L_{NN} = L_{pp} = \frac{2\sigma_0}{2}, \quad (15)$$

where $\sigma_0 > 0$ is the mean conductivity of the two spin channels. The conductivity asymmetry β of the two channels is given by

$$L_{Np} \equiv -\frac{2\sigma_0}{e}\beta. \quad (16)$$

Equation (14) leads then to the set of equations

$$\begin{bmatrix} J_N \\ J_p \\ \dot{\Psi} \end{bmatrix} = \frac{\sigma_0}{e} \begin{bmatrix} 1 & -\beta & 0 \\ -\beta & 1 & 0 \\ 0 & 0 & \frac{e}{\sigma_0} L_{cc} \end{bmatrix} \begin{bmatrix} -\frac{\partial \mu_0}{\partial z} \\ -\frac{\partial A}{\partial z} + 2\tilde{\gamma}A \\ A \end{bmatrix} \quad (17)$$

with $1 \geq \beta^2$ and $L_{cc} \geq 0$.

In the stationary state, $\partial J_N/\partial z=0$, and assuming that β , σ_0 , and γ are approximately independent of z the diffusion equation of the chemical affinity is deduced (see the Appendix):

$$\frac{\partial^2 A}{\partial z^2} = \left(\frac{1}{l_{sf}^2} + \frac{1}{l_{DW}^2} \right) A + k \frac{\partial A}{\partial z}, \quad (18)$$

where the spin diffusion length l_{sf} is given by

$$l_{sf} \equiv \sqrt{\frac{\sigma_0(1-\beta^2)}{2eL_{cc}}}, \quad (19)$$

the ‘‘domain-wall’’ diffusion length l_{DW} is given by

$$l_{DW} \equiv \sqrt{\frac{(1-\beta^2)}{4\tilde{\gamma}^2}}, \quad (20)$$

and the parameter k is given by

$$k \equiv \tilde{\gamma} \frac{2\beta^2}{(1-\beta^2)}. \quad (21)$$

Note that the chemical affinity A is equal to the difference of the chemical potentials of the two conduction bands $A = \mu_+ - \mu_-$. If we assume no rotation of the spin-polarization axis, $\tilde{\gamma}=0$ (which implies antiparallel magnetic configuration at the interface), then Eq. (18) is the well-known diffusion equation describing the so-called ‘‘spin accumulation’’ or ‘‘spin depletion’’ effect responsible for the giant magnetoresistance.^{15,23–25} A straightforward calculation (see the Appendix) leads to the giant magnetoresistance of the interface:

$$R^{GMR} = 2 \frac{\beta^2}{\sigma_0(1-\beta^2)} l_{sf}. \quad (22)$$

IV. LANDAU-LIFSHITZ-GILBERT (LLG) EQUATION

In this section we focus on the magnetic transport equation without electric current: $J_N = J_p = 0$. From Eq. (9) the entropy variation reduces to

$$T \frac{dS}{dt} = P_{\phi}^{ext \rightarrow in} - P_{\phi}^{in \rightarrow ext} + (\vec{H} - \vec{H}^{ext}) \frac{d\vec{M}_0}{dt} + \left(- \frac{\partial E}{\partial \dot{\vec{M}}_0} \right) \frac{d\dot{\vec{M}}_0}{dt} \quad (23)$$

so that the application of the second law of thermodynamics yields

$$(\vec{H} - \vec{H}^{ext}) = \tilde{l}_{MM} \frac{d\vec{M}_0}{dt} + \tilde{l}_{M\dot{M}} \frac{d\dot{\vec{M}}_0}{dt} + \left(- \frac{\partial E}{\partial \dot{\vec{M}}_0} \right) = \tilde{l}_{MM} \frac{d\vec{M}_0}{dt} + \tilde{l}_{M\dot{M}} \frac{d\dot{\vec{M}}_0}{dt}, \quad (24)$$

where the kinetic coefficients $\tilde{l}_{\alpha\beta}$ are the coefficients of the inverse matrix $\{l_{\alpha\beta}\}_{\{\alpha\beta\}}^{-1}$.

Note that in adiabatically closed systems, $(-\partial E/\partial \dot{\vec{M}}_0)$, H , and $d\vec{M}_0/dt$ are state functions [i.e., depend only of the state variables $(S, \vec{M}_0, \dot{\vec{M}}_0)$, and not on \vec{H}^{ext}]. Since the kinetic coefficients are also state functions, the first equation in Eq. (24) shows hence that $d\dot{\vec{M}}_0/dt$ depends on \vec{H}^{ext} . We are then leading to impose $\tilde{l}_{M\dot{M}}=0$ in order to satisfy the second equation in Eq. (24), which gives the magnetic kinetic energy.²⁶ The coefficient \tilde{l}_{MM} can be identified to the magnetic mass, and the first equation in Eq. (24) gives the total magnetic force \vec{F}^{mag} acting on the system:

$$\vec{F}^{mag} \equiv \tilde{l}_{M\dot{M}} \frac{d^2 \vec{M}_0}{dt^2} = (\vec{H} - \vec{H}^{ext}) - \tilde{l}_{MM} \frac{d\vec{M}_0}{dt}. \quad (25)$$

Equation (25) rewrites

$$\vec{F}^{mag} = \frac{\partial}{\partial \dot{\vec{M}}_0} (-E - \vec{H}^{ext} \cdot \vec{M}_0) - \eta \frac{d\vec{M}_0}{dt}, \quad (26)$$

where we have identified the Gilbert friction coefficient^{27,28} to $\eta = \tilde{l}_{MM}$.

The theorem of the kinetic momentum gives the equation of the dynamics:

$$\frac{d\vec{M}_0}{dt} = \Gamma (\vec{M}_0 \times \vec{F}^{mag}) = \Gamma \vec{M}_0 \times \left\{ - \frac{\partial V}{\partial \vec{M}_0} - \eta \frac{d\vec{M}_0}{dt} \right\}, \quad (27)$$

where Γ is the gyromagnetic ratio and the magnetic Gibbs potential²⁹ is defined by $V = E + \vec{M}_0 \cdot \vec{H}^{ext}$. Equation (27) is the well-known Gilbert equation,^{27,29} and can be put into the following Landau-Lifshitz form. In the case of uniform magnetization we have $\vec{M}_0 = M_s \vec{u}_0$, where M_s is the saturation magnetization. Equation (27) rewrites

$$\dot{\vec{u}}_0 = -g' (\vec{u}_0 \times \vec{\nabla} V) - h' \vec{u}_0 \times (\vec{u}_0 \times \vec{\nabla} V), \quad (28)$$

where $\vec{\nabla}$ is here the gradient operator on the surface of a unite sphere. The phenomenological parameters h' and g' are linked to the gyromagnetic ratio Γ and the Gilbert damping coefficient η by the relations³⁰

$$h' = \frac{\Gamma \alpha}{(1 + \alpha^2) M_s}$$

$$g' = \frac{\Gamma}{(1 + \alpha^2) M_s}$$

$$\alpha = \eta \Gamma M_s.$$

V. LLG EQUATION WITH SPIN-POLARIZED CURRENT

Let us assume an interface composed by an incident current I_p^i of conductivity asymmetry $-\beta$, polarized in the direction \vec{e}_p which enters in a ferromagnetic layer (F) polar-

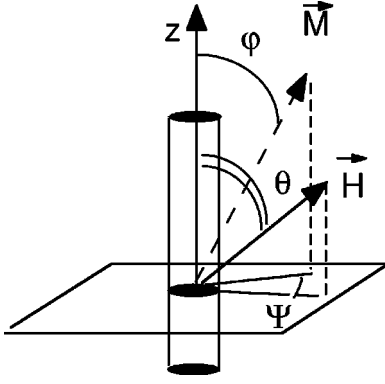


FIG. 2. Uniform magnetization and the magnetic field in the case of uniaxial anisotropy.

ized in the direction \vec{u}_0 with conductivity asymmetry β . The transfer of magnetic moments is described by the term $dN_p^F/dt = \dot{N}_+^F - \dot{N}_-^F$. Equation (5) rewrites

$$\frac{dN_p^F}{dt} = I_p^i - I_p^F - 2\gamma I_p^F = \beta_{eff} I_N, \quad (29)$$

where $\beta_{eff} = 2\beta[1 - 2\gamma(1 - \gamma)]$ and the expressions of I_p^F and I_p^i are derived in the Appendix.

The change of the magnetic moment of the layer due to the polarized current is given by Eqs. (6) and (28):

$$\dot{\vec{u}} \approx -g'(\vec{u}_0 \times \vec{\nabla} V) - h'\vec{u}_0 \times (\vec{u}_0 \times \vec{\nabla} V) + \frac{g\mu_B}{M_0} \beta_{eff} I_N \vec{e}_p, \quad (30)$$

where the first, second, and third term in the right-hand side are, respectively, the precession term (or transverse relaxation), the longitudinal relaxation term, and the spin transfer due to spin-polarized conduction electrons.

In order to estimate the effect of the injection of spin-polarized current, Eq. (30) is applied to the case of monodomain ferromagnet with applied field oriented at the angle θ from a single anisotropy axis (see Fig. 2). If the vector \vec{u} makes an angle φ from the anisotropy axis, the Gibbs energy density can be written in the following form:³⁰

$$V(\varphi, \psi) = KS\{-\cos^2\varphi - 2h[\cos(\theta)\cos(\varphi) + \sin(\theta)\sin(\varphi)\cos(\psi)]\}, \quad (31)$$

where $h = H^{ext}/H_a$ is the reduced applied field defined with the anisotropy field H_a , $K = H_a M_s$ is the anisotropy constant, S is the section, and ψ is the out-of-plane coordinate of the vector \vec{u} . Before injecting the current, the angle φ_0 is given by the equilibrium condition $\vec{\nabla} V = 0$. The precessional term can be neglected in Eq. (31) (low-frequency response and/or high damping limit³⁰), and $M \approx M_0$.

Experiments and samples are described in Refs. 12, 32, and 33. Ni nanowires are obtained by the method of electrodeposition in track etched membrane templates. A microcontact is realized, and the magnetoresistance of a single nanowire is measured. The wires are about 80 nm diameter and 6000 nm length and the magnetic energy is dominated by the Zeeman energy term and the shape anisotropy (or

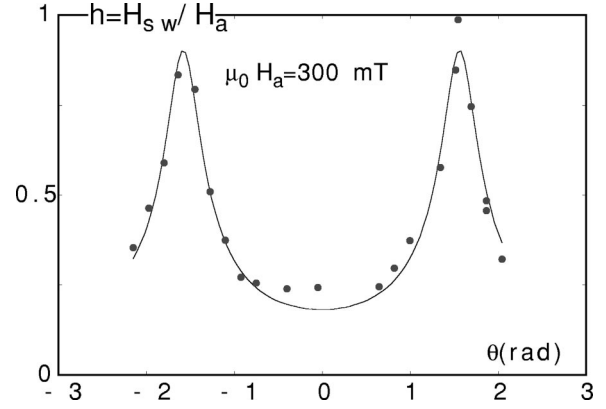


FIG. 3. Circle: measured position of the switching field H_{sw} for different angle of the applied field. Line: one-parameter fit with the curling formula Eq. (35).

magnetostatic term), very close to that of an infinite cylinder. The anisotropy field is calculated to be $\mu_0 H_a \approx 0.3$ T.

The effect of the spin-polarized current was evidenced experimentally by injecting a strong current of about 2×10^7 A/cm² at a fixed value of the external field $h = h_{sw} - \Delta h$ smaller than the field h_{sw} where the switching occurs without current. The magnetization switch occurs at the angle $\varphi^c(\theta)$. The maximum distance Δh where the jump of the magnetization can still be observed corresponds then to the variation of the angle $\Delta\varphi = \varphi^c - \varphi_0$ needed to shift the magnetization up to the unstable state.

For steady states, inserting $h = h_{sw} - \Delta h$, $\varphi = \varphi_c$, Eq. (30) leads to

$$\Delta h = h_{sw}(\theta) - \frac{2cI_e(\vec{e}_p \cdot \vec{v}) - \sin(2\varphi^c)}{\sin(\varphi^c - \theta)}, \quad (32)$$

where \vec{v} is the polar vector perpendicular to \vec{u} and I_e is the electric current. The parameter c is defined by the relation

$$c = \frac{\beta_{eff} \hbar}{eKv_a\alpha}, \quad (33)$$

where the activation volume v_a of magnetization M_s was estimated to be $v_a \approx 10^{-22}$ m³, $K \approx 10^5$ J/m³,³² and $\beta_{eff} \approx \beta \approx 0.3$,³⁴ $\alpha \approx 0.15$.³⁵ We obtain $c \approx 200$ A⁻¹.

All parameters in Eq. (32) are known if the magnetization reversal mode, which describes the irreversible jump, is known. In a few theoretical models of magnetization reversal,³¹ the functions $H_{sw}(\theta)$ and $\varphi_c(\theta)$ are analytical. In the framework of the present empirical approach, the experimental data are fitted by the relation deduced from a curling reversal mode in an infinite cylinder:^{36,32}

$$h_{sw}(\theta) = \frac{a(a+1)}{\sqrt{a^2 + (2a+1)\cos^2(\theta)}}. \quad (34)$$

The single adjustable parameter $a = -k(R_0/r)^2$ is defined by the geometrical parameter k ,³¹ by the exchange length $R_0 = 20$ nm,¹¹ and by the radius of the wire r . The experimental points $H_{sw}(\theta)$ are fitted in Fig. 3.

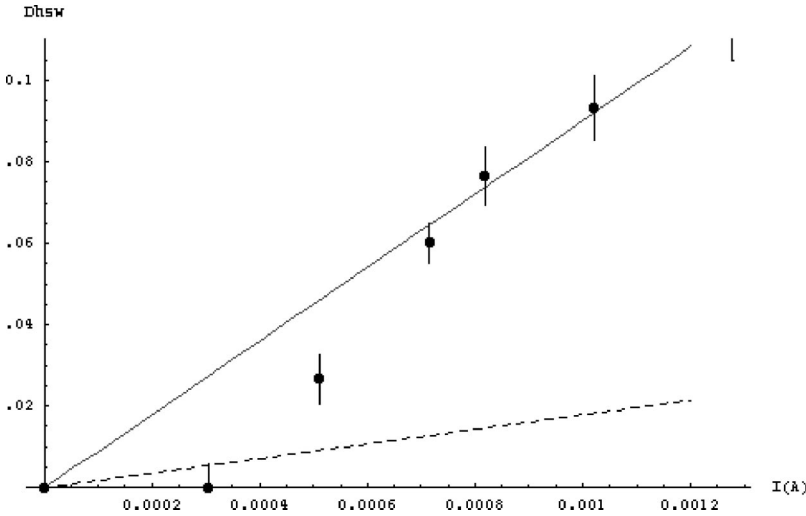


FIG. 4. Parameter $\Delta h = \Delta H_{max}(I_e)/H_a$ as a function of the pulsed current amplitude. $\mu_0 H_a = 300$ mT is the anisotropy field. The linear fit (continuous line) gives $c \approx 190$ [see Eq. (32)]. The dashed line is the maximum magnetic field induced by the pulsed current.

We obtained $a = -0.15$ (which corresponds to r of about 60 nm). The relation between the angle of the applied field θ and the angle of the magnetization φ^c is

$$\tan(\theta) = \frac{a+1}{a} \tan(\varphi^c). \quad (35)$$

The curve Δh , evaluated from Eq. (32) by numerical resolution with a polarization in the direction of the wire axis $\vec{e}_p \cdot \vec{v} = \sin(\varphi_0)$, is plotted in Figs. 4 and 5, together with the experimental data. A strong discrepancy from the linear curve of $\Delta h(I_N)$ at small current pulses can be observed. Above a critical current corresponding to about 10^7 A/cm² the linear fit gives a parameter $c = 190$, which is in accordance with the rough evaluation of Eq. (33). This critical current below which the linear regime failed in Fig. 5 could be interpreted following Refs. 4 and 5 as the current needed in order to excite spin waves or other magnetization inhomogeneities.³⁷ The curve given by Eq. (32) can then be plotted without adjustable parameter (Fig. 5). The divergence at 90° is due to the numerical resolution of Eq. (32) (numerator and denominator tend to zero at $\theta = 90^\circ$ angle).

VI. CONCLUSION

A systematic thermokinetic description of a metallic ferromagnetic layer open to electronic spin-polarized reservoirs

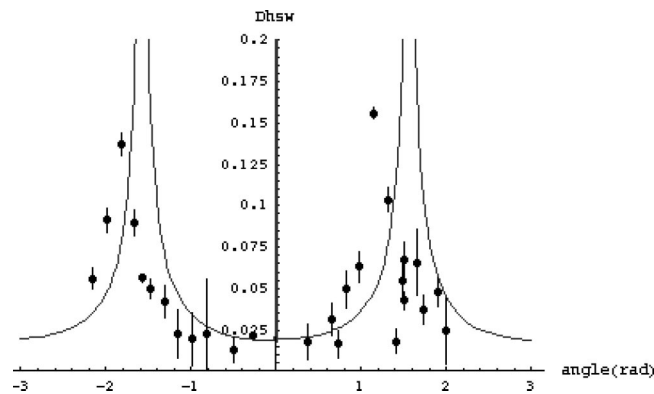


FIG. 5. Angular dependence of the parameter $\Delta h = \Delta H_{max}(\theta)/H_a$. The curve is given by the Eq. (33) of the text, with $c = 190$ A⁻¹.

has been performed. At constant temperature, assuming the two current approximation and neglecting direct action of the magnetic field on charge carriers, five coupled transport equations account for the complexity of the system. The approximation of the explicit uncoupling of the transport processes leads to the known results about GMR and Landau-Lifshitz-Gilbert equations for magnetization dynamics. Within this approximation and on the basis of the conservation equation of the magnetic moment, the description of both polarized current and magnetization dynamics leads to a generalized Landau-Lifshitz-Gilbert equation. The application of this model to experimental data about current induced magnetization reversal is performed. The existence of a critical current indicates that the kinetics of magnetization inhomogeneity also plays an important role. However, the comparison with experimental data shows that the derived thermokinetic generalized Landau-Lifshitz-Gilbert equation provides a description of the basic mechanism responsible for the effect of polarized current induced magnetization reversal.

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APPENDIX

The appendix is structured in three parts. In the first part (Sec. 1), the equation of the difference of chemical potentials $A = \Delta\mu$ is derived from Eq. (17) in the case of steady states. In Sec. 2 the equation is applied to the simplified case of GMR or spin accumulation, where the polarization axis is assumed constant through the interface. The GMR of the interface is deduced. In Sec. 3, the equation is applied in the framework of the experimental study of polarized current induced magnetization reversal, where an abrupt change of the polarization axis occurs at the interface.

1.

Assuming that the kinetic coefficients which coupled the dynamics of the magnetization and the electric currents vanish, we obtained the following set of kinetic Eqs. (17):

$$\begin{bmatrix} J_N \\ J_p \\ \dot{\Psi} \end{bmatrix} = \frac{\sigma_0}{e} \begin{bmatrix} 1 & -\beta & 0 \\ -\beta & 1 & 0 \\ 0 & 0 & \frac{e}{\sigma_0} L_{cc} \end{bmatrix} \begin{bmatrix} -\frac{\partial \mu_0}{\partial z} \\ -\frac{\partial A}{\partial z} + 2\tilde{\gamma}A \\ A \end{bmatrix}. \quad (\text{A1})$$

In the stationary state $(\partial/\partial z)J_N(z)=0$, and assuming that β , σ_0 , and γ are approximately independent of z , the diffusion equation of the chemical affinity is deduced:

$$\frac{\partial^2 \mu_0}{\partial z^2} = \beta \left(\frac{\partial^2 A}{\partial z^2} - 2 \frac{\partial(\tilde{\gamma}A)}{\partial z} \right). \quad (\text{A2})$$

Inserting Eq. (A2) into Eq. (A1) yields

$$\frac{\partial J_p}{\partial z} = \frac{\sigma_0}{e} \left((\beta^2 - 1) \frac{\partial^2 A}{\partial z^2} + 2 \frac{\partial(\tilde{\gamma}A)}{\partial z} \right) \quad (\text{A3})$$

and by integration,

$$J_p = \frac{\sigma_0}{e} \left((\beta^2 - 1) \frac{\partial A}{\partial z} + 2\tilde{\gamma}A \right), \quad (\text{A4})$$

where we assumed that $J_p(\infty)=0$.

On the other hand, from the conservation equations we have

$$\frac{d}{dt}(N_+^k - N_-^k) = I_p^{k-1 \rightarrow k} - I_p^{k \rightarrow k+1} - 2\dot{\Psi}^k - 2\gamma(k-1,k)I_p. \quad (\text{A5})$$

At the continuum limit, we obtain the following relation:

$$\frac{dn_p}{dt} = -\frac{\partial J_p}{\partial z} - 2L_{cc}A - 2\tilde{\gamma}J_p, \quad (\text{A6})$$

where n_p is the density of spin-polarized conduction electrons.

Equation (A6) rewrites

$$\frac{\partial J_p}{\partial z} = -2L_{cc}A - 2\tilde{\gamma}J_p - \frac{dn_p}{dt}, \quad (\text{A7})$$

where dn_p/dt is constant for steady states. Furthermore, inside the ferromagnet and far away from the interface, J_p is constant, whence

$$\frac{dn_p}{dt} = -2L_{cc}^\infty A_\infty = 0, \quad (\text{A8})$$

where we assumed for simplicity that $L_{cc}^\infty=0$. Together with Eqs. (A7), (A4), and (A3) the differential equation for $A(z)$ is obtained:

$$\frac{\partial^2 A}{\partial z^2} = \left(\frac{1}{l_{sf}^2} + \frac{1}{l_{DW}^2} \right) A + k \frac{\partial A}{\partial z}, \quad (\text{A9})$$

where the spin-diffusion length l_{sf} is given by

$$l_{sf} \equiv \sqrt{\frac{\sigma_0(1-\beta^2)}{2eL_{cc}}}, \quad (\text{A10})$$

the domain-wall diffusion length l_{DW} is given by

$$l_{DW} \equiv \sqrt{\frac{(1-\beta^2)}{4\tilde{\gamma}^2}}, \quad (\text{A11})$$

and the parameter k is given by

$$k \equiv \tilde{\gamma} \frac{2\beta^2}{(1-\beta^2)}. \quad (\text{A12})$$

2. Application to GMR

Assuming that the polarization axis is the same for all sublayers, we have $l_{DW}=0$ and the last term in Eq. (A9) vanishes. The chemical affinity obeys the diffusion equation

$$\frac{\partial^2 A}{\partial z^2} = \frac{1}{l_{sf}^2} A. \quad (\text{A13})$$

The chemical affinity and the total chemical potential are then

$$A(z) = ae^{\frac{z}{l_{sf}}} + be^{-\frac{z}{l_{sf}}}, \quad (\text{A14})$$

$$\mu_0(z) = d + cz + \beta A(z), \quad (\text{A15})$$

where a , b , c , and d are constants. The electric field $E(z)$ is defined by $-eE(z) \equiv \partial \mu_0 / \partial z$ so that $c = -eE(\infty) = -eJ_N / \sigma_0$. Under the condition of continuity of the currents of the two spin channels at the interface (no surface scattering), $J_\pm(0^-) = J_\pm(0^+)$, we have $a = [el_{sf}\beta / \sigma_0(1-\beta^2)] \times J_N$. The spin-polarized current on the left side of a single interface ($b=0$) is deduced:

$$J_p(z) = \frac{\sigma_0}{e} \left(\frac{\partial A}{\partial z} - \beta \frac{\partial \mu_0}{\partial z} \right) = J_N \beta (e^{-|z|/l_{sf}} - 1). \quad (\text{A16})$$

The electric field $\partial \mu_0 / e \partial z \equiv -E(z)$ is

$$E(z) = \frac{eJ_N}{\sigma_0} \left(1 + \frac{\beta^2}{1-\beta^2} e^{-|z|/l_{sf}} \right) \quad (\text{A17})$$

and the supplementary potential due to the spin-polarized current is

$$\Delta V = \int_{-\infty}^{+\infty} \left(E(z) - \frac{eJ_N}{\sigma_0} \right) dz = 2 \frac{\beta^2}{1-\beta^2} l_{sf} \frac{eJ_N}{\sigma_0} \quad (\text{A18})$$

from which the GMR resistance (22) is deduced.

3. Application to spin transfer

In the case of an interface composed by an incident current of conductivity asymmetry β , polarized in the direction \vec{e}_p , entering in a ferromagnetic layer polarized in the direction \vec{u} with conductivity asymmetry $-\beta$, Eqs. (A14) and (A15) still hold in the left- and right-hand sides of the interface. However, the change of the polarization axis at the interface leads to modify the continuity equation of the cur-

rent of the two spin channels $J_{\pm}(0^-) = J_{\pm}(0^+) \mp \gamma J_p$. The integration constant now reads

$$a = \frac{el_{sf}\beta(1-\gamma)}{\sigma_0(1-\beta^2)} J_N \quad (\text{A19})$$

and the expression of the polarized current is

$$J_p(z) = J_N \beta (1-\gamma) (e^{-|z|/l_{sf}} - 1). \quad (\text{A20})$$

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