PHYSICAL REVIEW B VOLUME 61, NUMBER 12 15 MARCH 2000-II

Calculated temperature-dependent resistance in low-density two-dimensional hole gases in GaAs heterostructures

S. Das Sarma and E. H. Hwang
Department of Physics, University of Maryland, College Park, Maryland 20742-4111
(Received 6 January 2000)

We calculate the low temperature resistivity in low-density two-dimensional hole gases in GaAs heterostructures by including screened charged impurity and phonon scattering in the theory. Our calculated resistance, which shows striking temperature-dependent nonmonotonicity arising from the competition among screening, nondegeneracy, and phonon effects, is in excellent agreement with recent experimental data.

A number of recent density-dependent low temperature transport measurements in dilute two-dimensional (2D) *n*-Si metal-oxide-semiconductor field-effect transistor (MOSFET) and *p*-GaAs heterostructure systems have attracted a great deal of attention[1] because the experiments nominally exhibit a metal-insulator-transition (2D MIT) as a function of 2D carrier density (n). In addition to this unexpected 2D MIT phenomenon [at this stage it is unclear whether the transition represents a true $T=0$ quantum phase transition (QPT) or a finite temperature crossover behavior] these measurements reveal a number of intriguing transport properties[1] in dilute 2D systems, such as a remarkable temperature dependence of the low-density resistivity in the nominally metallic phase, which deserve serious theoretical attention in their own rights irrespective of whether the 2D MIT phenomenon is a true QPT or not.

In this paper we provide a quantitative theory for one such recent experiment[2] carried out in a low-density GaAs-based 2D hole gas. In our opinion, Ref. 2 represents a particularly important experiment in relation to the 2D MIT phenomenon (although ironically no MIT is actually observed in Ref. 2 — even the lowest density data in Ref. 2 are entirely in the nominally metallic phase) because the ultrapure samples used in Ref. 2 explore the 2D ''metallic'' regime of the highest mobility (i.e., the best quality or equivalently the lowest disorder), the lowest carrier density, and the lowest temperature so far studied in the context of the 2D MIT phenomenon. More specifically, there have been suggestions and speculations[1] that the 2D MIT phenomenon is an interaction-driven QPT (the scaling theory of localization[3] rules out a true localization transition in 2D disordered system) with the dimensionless r_s parameter, which is the ratio of the interaction energy to the noninteracting kinetic energy of the 2D electron system, being the tuning parameter which drives the QPT. It is important to emphasize that r_s increases as n decreases ($r_s \propto n^{-1/2}$), and therefore the 2D systems of Ref. 2 represent the highest (lowest) r_s (n) and consequently the most strongly interacting 2D systems experimentally studied so far in the context of the 2D MIT phenomenon. To be precise, r_s values of the nominally ''metallic'' 2D hole regime explored in Ref. 2 go down to as low as $r_s=26$ (corresponding to the lowest hole density $n=3.8\times10^9$ cm^{-2} studied in Ref. 2) with no sign of an MIT whereas the other systems studied in the literature exhibit the 2D MIT transition[1] at critical r_s values as low as $r_s \sim 8-12$ (Si MOSFET's) and $10-20$ (GaAs hole systems). The experimental results presented in Ref. 2 thus compellingly demonstrate that interaction (i.e., the r_s parameter) is by no means the only (or perhaps even the dominant) variable controlling the physics of 2D MIT; disorder (and perhaps even temperature) also plays an important role.

Our transport theory for the 2D hole system employs the finite temperature Boltzmann equation technique, which has earlier been successful in *n*-Si MOSFET's (Ref. 4) and *n*-GaAs systems.[5,6] We *include* the following effects in our calculation: (i) Subband confinement effects (i.e., we take into account the extent of the 2D system in the third dimension and do not assume it to be a zero-width 2D layer); (ii) scattering by screened charged random impurity centers; (iii) finite temperature and finite wave vector screening through random phase approximation (RPA) (actually we employ a slightly modified version[7] of RPA, the so-called 2D Hubbard approximation, which approximately and rather crudely incorporates the electron-electron interaction-induced vertex correction in the screening function which may be important at the low carrier densities[2] being investigated—it turns out that our calculated resistance with the Hubbard approximation is within 30% of the corresponding RPA results); (iv) phonon scattering.[6] The effects we *neglect* in our theory are (i) all localization and multiple scattering corrections; (ii) inelastic electron interaction effects—in fact, all effects of electron-electron interaction are neglected in our theory except for the long-range screening through RPA and (approximate) short-range vertex correction through Hubbard approximation.

Our calculations are similar to the ones[8] we recently carried out for electron inversion layers in *n*-Si MOSFET's with two important differences: (i) we include the *full* hole density in the current calculations without subtracting out any critical density as done in Ref. 8—this is, in fact, consistent with our Si MOSFET calculations since the critical density in Ref. 2 must be extremely small, and in any case SdH measurements carried out in Ref. 2 show that *all* the carriers are ''free'' and participating in the conduction process; (ii) we include phonon scattering effects in the current calculations because phonon scattering is significant for GaAs holes already in the $T=1-10$ K temperature range whereas phonon scattering is negligibly small in *n*-Si MOSFET's in the

1–10 K temperature range. Details of phonon scattering calculations are given in Ref. 6. The essential point is that the phonon resistivity is proportional to T for $T>1$ K and is negligibly small in the low temperature Bloch-Grüneisen regime.

Our calculated resistivity for 2D holes in GaAs structures is shown in Figs. 1 and 2 for two different types of 2D quantum confinement: Square well (Fig. 1) and heterojunction (inversion layer type approximately ''triangular'') confinement (Fig. 2). The qualitative results for the two kinds of confinement are, as expected, very similar (although the actual quantitative resistance values depend on the nature of confinement since the scattering and screening matrix elements are strongly confinement dependent[4] through the wave function spread normal to the 2D confinement plane). The resistivity can be written as $\rho(T)=\rho_0+\rho_{imp}(T)+\rho_{ph}(T)$, where $\rho_0\equiv\rho(T\rightarrow 0)$ is the residual resistivity arising entirely from (screened) charged impurity scattering in our theory (for a weakly localized system ρ_0 diverges logarithmically as $T\rightarrow 0$, our theory is valid above the crossover temperature scale for weak localization to set in—no indication for the expected $\ln T$ weak localization divergence is seen in the experimental data of Ref. 2 down to the lowest reported measurement temperature, 35 mK). $\rho_{ph}(T)$ is the resistivity contribution by phonon scattering[6] which could be quite significant for 2D holes in GaAs already in the $1-10$ K temperature range. Finally, $\rho_{imp}(T)$ is the *temperature-dependent part* of the charged impurity (i.e., random disorder) scattering contribution to the resistivity, i.e., $\rho_0+\rho_{imp}(T)\equiv\rho_i$ is the total impurity contribution to the resistivity. We note that ρ_0, which sets the overall resistivity scale [by definition, both $\rho_{imp}(T)$ and $\rho_{ph}(T)$ vanish as $T\rightarrow 0$] in the problem, is determined by the amount of the random disorder in the system which is in general unknown. The amount of random disorder (and consequently ρ_0) depends on the strength and the spatial distribution of all the impurity scattering centers in the system. We parametrize the charged impurity density, assuming them to be randomly distributed static Coulomb charged centers interacting with the 2D carriers via the screened Coulomb interaction. We adjust the charged impurity density (assumed to be randomly distributed in our calculations) to get agreement between theory and the experimental data—thus the scale ρ_0 is essentially an adjustable parameter in our theory since the actual impurity distribution in the 2D systems of interest is simply not known. We emphasize, however, that the charged impurity density needed in our theory to obtain agreement between our calculations and the experimental data for ρ_0 are reasonable.

Before discussing our results we make three salient remarks about our calculation and model. First, we neglect scattering by interface roughness, alloy disorder, etc., in our calculation (including only charged impurity scattering in the theory) since it is well-known that the dominant low temperature resistive mechanism in high quality GaAs structures arises essentially from charged impurity scattering (it is straightforward to include additional scattering mechanisms in our calculations with the unpleasant complication of having additional unknown parameters, such as the interface roughness strength, in the theory; our choice is to keep the number of unknown adjustable parameters at a minimum by assuming that all of the random disorder scattering is caused by randomly distributed charged impurity scattering which should be an excellent approximation for the extreme high quality GaAs samples used in Ref. 2). Second, the Matthiessen's rule, which is implicitly assumed in separating out $\rho_i(T)$ and $\rho_{ph}(T)$, is known to be not strictly valid at finite temperatures[4] because different scattering rates do not simply add in the total resistivity. It is important to emphasize, however, that we do not assume the Matthiessen's rule in our theoretical calculations, and the total resistivity and $\rho(T)$ is written down simply as a rough guide for qualitative discussion. In any case, the deviation from Matthiessen's rule is of the order of 30% or less, which is not of much consequence for our discussion. Finally, the third remark we make is regarding our use of the single scattering Born approximation in our Boltzmann theory (neglecting all multiple scattering effects), which can be justified by noting that our calculated resistivity (and the corresponding experimental resistivity measured in Ref. 2) always satisfies the weak scattering condition of $k_F l\gg 1$—in fact, our results are restricted to $k_F l>3$ even in the worst situation (for our highest resistance results). We therefore believe that the Born approximation may not be a poor approximation for our problem.

In Fig. 1 we show our calculated 2D hole resistivity for symmetric square well systems corresponding to the sample of Ref. 2. The actual sample configuration is shown schematically as an inset in Fig. 1. We also show some representative experimental[2] results (from Fig. 2 of Ref. 2). We emphasize that the quantitative agreement with the data of Ref. 2, while being certainly indicative of the essential validity of our theoretical approach, should not be taken too seriously—it is certainly not the feature of our theory we would focus on, particularly since the random impurity distribution in the experimental samples is unknown. It is the overall striking qualitative similarity between our microscopic theory and the experimental data[2] which deserves attention. This is particularly so because the density and temperature dependence of the measured resistance in Ref. 2 shows a thoroughly nontrivial nonmonotonic behavior which is completely reproduced in our calculations. This striking nonmonotonicity in $\rho(T)$, at lower carrier densities, arises from a competition among three mechanisms: Screening, which is particularly important at lower T; nondegeneracy and the associated quantum-classical crossover for $T\geqslant T_F$ ($\equiv E_F/k_B$, the Fermi temperature) which was discussed in Ref. 8 in the context of *n*-Si MOSFET's; and phonon scattering effect which is negligible below 1 K, but starts becoming quantitatively increasingly important for $T>1$ K. The Fermi temperature for the 2D hole system can be expressed as $T_F=0.64(n/10^{10})$ K where n is the 2D hole density measured in units of 10^{10} cm^{-2}. Thus for $n=4.8\times 10^{10}$ cm^{-2} between $n=0.65\times 10^{10}$ cm^{-2} in Fig. 1, T_F varies between 3 and 0.4 K. This makes the quantum-classical crossover physics particularly significant for the results of Ref. 2 as was already noted by the authors in Ref. 2.

At higher densities (the bottom two curves in Fig. 1) the quantum-classical crossover effects are not particularly important because phonon scattering becomes important before the classical behavior[8] $\rho\sim T^{-1}$ can show up, and the system makes a transition from the quantum regime to the phonon scattering dominated regime. The fast rise in $\rho(T)$ at high T

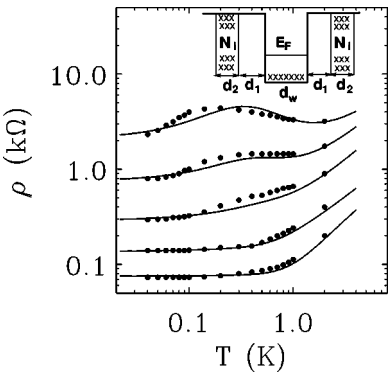

FIG. 1. The calculated 2D hole resistivity $\rho(T)$ for symmetric square well systems corresponding to the hole densities $n=0.65,1.07,1.63,3.26,4.80\times10^{10}$ cm^{-2} (from top to bottom) with random impurity densities $N_i=0.7,0.75,0.8,2.0,3.5\times10^{15}$ cm^{-3}, respectively. In the inset, the sample configuration is shown schematically. In this calculation we use the parameters $d_w=300$ Å, $d_1=200$ Å, and $d_2=50$ Å. We use a very small random impurity density, $N_w=3\times10^{12}$ cm^{-3}, in the GaAs layer itself which is consistent with the extreme high quality of the samples in Ref. 2. Some representative experimental data points from Ref. 2 are shown (the actual random disorder in the experimental samples is unknown).

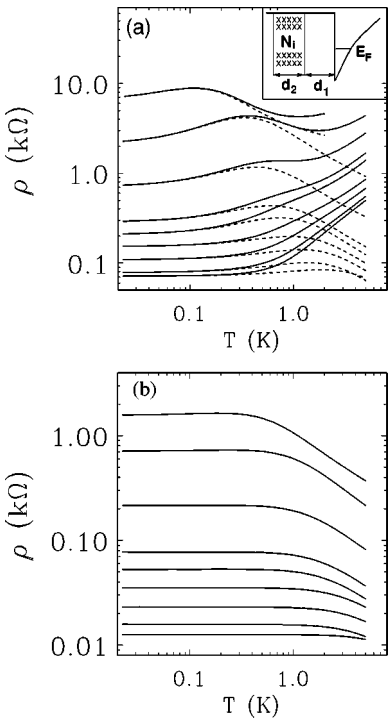

FIG. 2. The calculated resistivity for a heterostructure inversion-layer-type ''triangular'' confinement 2D (a) hole gas and (b) electron gas for carrier densities $n=0.38,0.65,1.07,1.63,1.93,3.26,4.15,4.80,8.66\times10^{10}$ cm^{-2} (from top to bottom) with random impurity densities $N_i=0.3,0.3,0.3,0.3,0.3,0.9,1.1,1.1,5.0\times10^{15}$ cm^{-3}, respectively. The dashed curves in (a) and the solid curves in (b) represent the pure impurity scattering contribution without any phonon scattering. The inset in (a) show the sample configuration. We use $d_1=250$ Å and $d_2=100$ Å. The impurity density in GaAs is $N_w=3\times10^{12}$ cm^{-3}.

in Fig. 1 is the phonon scattering effect. At low enough densities, however, phonon scattering effects are absent (because phonons are frozen out in the low temperature Bloch-Grüneisen range[6]) *at the quantum-classical crossover point* which occurs at very low temperatures around $T<T_F<1$ K (the top two curves in Fig. 1). In these low-density results one can see $\rho(T)$ increasing with T at lower temperatures due to screening effects,[8] then the quantum-classical crossover occurs at the intermediate temperature regime around T_F where nondegeneracy effects make resistivity decrease[8] as $\rho\sim T^{-1}$; eventually at higher temperatures ($T\geqslant1$ K) phonon scattering takes over and $\rho(T)$ increases with T again. At higher densities, T_F is pushed up to the phonon scattering regime, and the quantum-classical crossover physics is pre-empted by phonons so that nonmonotonicity effects are not manifest.

The nonmonotonic behavior of $\rho(T)$ as a function of n and T is made more explicit in Fig. 2(a) where we show our calculated resistivity for the same density and temperature range as in Fig. 1 for a heterostructure inversion-layer-type ''triangular'' confinement 2D hole gas, separating out the pure impurity scattering contribution (i.e., the dashed curves in Fig. 2(a) leave out the phonon scattering contribution completely). First, we note that the resistivity results in Fig. 2(a) are very similar to those in Fig. 1, indicating that the transport behavior seen in Ref. 2 is the generic behavior of a low density 2D GaAs hole system, and does not arise from any particular feature of the square well samples used in Ref. 2. Second, the interplay of screening (low temperature), phonons (high temperature), and nondegeneracy (high temperature and low density) is manifestly obvious in Fig. 2(a): the intriguing low-density nonmonotonicity in the observed $\rho(T)$ clearly arises from the fact that both screening and phonon scattering mechanisms give rise to a $\rho(T)$ monotonically increasing with T (at low temperature for screening, and at high temperatures for phonons), but nondegeneracy effects produce a $\rho(T)$ decreasing with T for $T\geqslant T_F$. Since phonon scattering is the dominant temperature-dependent scattering mechanism in GaAs holes for $T>1$ K, the nonmonotonicity can show up in any significant way only if $T_F\leqslant1$ K, which is precisely the experimental observation.

As an interesting comparison, we show in Fig. 2(b) the calculated $\rho(T)$, without any phonon scattering, for the same densities (and impurity scattering parameters) as in Fig. 2(a) for 2D *electron* inversion layers confined in a GaAs heterostructure [i.e., the only difference between the results for Fig. 2(a) and Fig. 2(b) is that the GaAs electron mass has been used in the calculations corresponding to Fig. 2(b) rather than the hole mass]. The neglect of phonon scattering is justified by the fact that phonons contribute[6] significantly to GaAs 2D electron resistivity only for $T>10$ K—in fact, inclusion[6] of appropriate phonon scattering would produce results indistinguishable from the results shown in Fig. 2(b) (i.e., up to 5 K). The difference between the results of Figs. 2(a) (holes) and 2(b) (electrons) is striking: there is essentially no observable (on log scale) temperature dependence at low temperatures in the 2D electron resistivity in GaAs heterostructure down to 2D densities as low as $n=0.38\times10^{10}$ cm^{-2}. This essential temperature independence of low temperature electronic resistance in high quality GaAs heterostructures, which is a well-known[9] experimental fact, arises from the weak screening property (associated with its low effective mass and the associated small electronic density of states) of 2D electrons in GaAs heterostructures com-

pared with higher mass 2D holes in GaAS or 2D electrons in Si MOSFET's. This weak screening behavior of GaAs electrons precludes any strong temperature-dependent $\rho(T)$ even at very low carrier densities (and temperatures). The quantum-classical crossover phenomenon, however, still occurs around $T \sim T_F$, leading to a $\rho(T) \sim T^{-1}$ for $T \geqslant T_F$, which is manifestly obvious in Fig. 2(b), particularly for lower densities. Note that the Fermi temperature in Fig. 2(b) corresponds to $T_F = 4.1(n/10^{10})$ K with n being the 2D electron density in Fig. 2(b) measured in units of 10^{10} cm^{-2}. Thus the Fermi temperature in Fig. 2(b) ranges from 1.5 K (top curve) to 35.5 K (bottom curve). We note that the decreasing $\rho(T)$ at higher T in Fig. 2(b) arises not only from a quantum to classical crossover (which is the dominant effect at lower densities when T_F is low), but also from the finite temperature Fermi surface averaging in a degenerate quantum system. It is easy to show that the Fermi surface averaging effect at finite temperatures, by itself, always leads to a finite temperature resistivity which decreases weakly with temperature (even in the $T \to 0$ limit)—in fact, this effect by itself leads to $\rho(T) \approx \rho_0[1 - O(T/T_F)^2]$, and can only be observed if the temperature-dependent screening effects are unimportant. This effect was first observed in 2D electrons in GaAS heterostructures more than fifteen years ago.[10]

To conclude, we have developed a theory for the low temperature transport properties of 2D holes and electrons confined in low-density and high mobility GaAs heterostructures. Our theory includes temperature-dependent screening of impurity scattering and phonon scattering effects. Agreement between our theory and experiment suggests that screening and impurity scattering effects play an essential role in determining much of the intriguing temperature and density dependent transport properties in 2D systems, and that random disorder (mostly arising from charged impurity scattering) is an important ingredient in the physics of low-density 2D systems.

This work was supported by the U.S. ARO. and the U.S. ONR.

[1] E. Abrahams, Physica E (Amsterdam) **3**, 69 (1998); Ann. Phys. (Leipzig) **8**, 539 (1999); and references therein.

[2] A.P. Mills, Jr., A.P. Ramirez, L.N. Pfeiffer, and K.W. West, Phys. Rev. Lett. **83**, 2805 (1999).

[3] E. Abrahams, P.W. Anderson, D.C. Licciardello, and T.V. Ramakrishnan, Phys. Rev. Lett. **42**, 673 (1979).

[4] T. Ando, A.B. Fowler, and F. Stern, Rev. Mod. Phys. **54**, 437 (1982); F. Stern, Phys. Rev. Lett. **44**, 1469 (1980); F. Stern and S. Das Sarma, Solid-State Electron. **28**, 211 (1985).

[5] F. Stern, Appl. Phys. Lett. **43**, 974 (1983); K. Hirakawa and H. Sakaki, Phys. Rev. B **33**, 8291 (1986).

[6] T. Kawamura and S. Das Sarma, Phys. Rev. B **42**, 3725 (1990); **45**, 3612 (1992); W. Walukiewicz, J. Appl. Phys. **59**, 3577 (1986).

[7] M. Jonson, J. Phys. C **9**, 3055 (1976).

[8] S. Das Sarma and E.H. Hwang, Phys. Rev. Lett. **83**, 164 (1999).

[9] H.L. Störmer, L.N. Pfeiffer, K.W. Baldwin, and K.W. West, Phys. Rev. B **41**, 1278 (1990). See also F.W. van Keuls *et al.*, *ibid.* **56**, 13 263 (1997).

[10] M.A. Paalanen, D.C. Tsui, A.C. Gossard, and J.C.M. Hwang, Phys. Rev. B **29**, 6003 (1984).