PHYSICAL REVIEW B VOLUME 61, NUMBER 4 15 JANUARY 2000-II

Waveguide diffusion modes and slowdown of D'yakonov-Perel' spin relaxation in narrow two-dimensional semiconductor channels

A. G. Mal'shukov
Institute of Spectroscopy, Russian Academy of Sciences, 142092 Troitsk, Moscow Region, Russia

K. A. Chao
Department of Theoretical Physics, Lund University, S-223 62 Lund, Sweden

(Received 18 June 1999)

We have shown that in narrow two-dimensional semiconductor channels the D'yakonov-Perel' spin relaxation rate is strongly reduced. This relaxation slowdown appears in special waveguide diffusion modes which determine the propagation of spin density in long channels. Experiments are suggested to detect the theoretically predicted effects. A possible application is a field effect transistor operated with injected spin current.

In connection to the possible use of electron spin for storage and information transfer in quantum computers,[1] there have been many recent studies on spin transport in semiconductor nanostructures. Among various materials for the so-called *spintronic devices*, a favorite candidate is III-V semiconductors, because their spin-orbit split conduction bands have unusual electron spin dynamics. The spin-orbit interaction (SOI) in such materials has the form $H_{so}=\mathbf{h}(\mathbf{k})\cdot\mathbf{s}$, where $\mathbf{s}$ is the electron spin, and the direction and magnitude of the vector $\mathbf{h}(\mathbf{k})$ depend on the electron momentum $\mathbf{k}$. When an electron propagates, its spin precesses around the direction of $\mathbf{h}(\mathbf{k})$. In materials with a narrow band gap, the magnitude $|\mathbf{h}(\mathbf{k})|$, and hence the angle of spin rotation can be varied by simply applying a gate voltage.[2] While this effect is valuable to those devices which make use of ballistically propagating spin-polarized electrons,[3] it requires a high degree of coherency in electron propagation, because scattering from an impurity or the boundary changes the direction of $\mathbf{h}(\mathbf{k})$. Scatterings thus randomize spin precession. If the mean scattering time τ is shorter than the precession frequency $|\mathbf{h}(\mathbf{k})|^{-1}$, the spin dynamics is more like a diffusive relaxation in the angular space with the D'yakonov-Perel' relaxation rate of the order of $1/\tau_s=\frac{1}{2}\tau h^2(\mathbf{k})$.[4] However, the evolution of an inhomogeneous spin polarization at a spatial region is determined not only by the diffusive randomization of the local spin orientation, but also by a balance of the incoming and outgoing spin currents. These spin currents depend on the polarization gradient, and contain a component associated to the spin precession. Hence, it is impossible to determine unambigously which part of the spin relaxation is due to the diffusion of spin orientation in the angular space, and which is due to the spin diffusion in the coordinate space. On the other hand, one can examine the evolution of a given spin polarization by studying the dynamics of individual eigenmodes of the spin diffusion equation. After a sufficiently long time, only those modes with the lowest spin relaxation rate survive, and this rate will be characteristic to the spin relaxation of the system under consideration.

In this paper we will study the spin diffusion in a 2D channel made from a quantum well with growth direction along the y axis. We choose our coordinate system to have x axis along the channel, and the boundaries of the channel are marked at $z=\pm d/2$. The channel width d is much shorter than the spin precession length $L_s=v_F/|h(\mathbf{k}_F)|$, where v_F is the electron Fermi velocity. On the other hand, the width d is much longer than the Fermi wavelength, so that the electron motion perpendicular to the channel (along z axis) is semiclassical. We will show that the long-time evolution of spin polarization in such a channel is dominated by a waveguide *diffusion* mode. The spin relaxation time of this mode is slowed down dramatically by a factor L_s^2/d^2 with respect to the D'yakonov-Perel' relaxation time τ_s, which is the typical spin relaxation time for bulk materials and quantum wells. Furthermore, the diffusion mode produces a periodic rotation of the spin polarization from point to point along the 2D channel. Such a phenomenon is similar to the spin precession of ballistic electrons considered in Ref. 3, but with a difference: in our case the rotating spin has the spin quantum number 1 instead of 1/2. This difference is due to the fact that our nonequilibrium spin density is represented by two-particle excitations with electrons above the Fermi level and holes below it. As an example, we will demonstrate the oscillations in the resistance of a diffusive channel when the spin polarization is injected and probed via ferromagnets at both ends of the channel. We will also discuss qualitatively the effects of spin relaxation slowdown in a channel on the weak localization behavior of transport parameters.

In terms of the creation and destruction operators $c_{\mathbf{k}}^{\dagger}(t)$ and $c_{\mathbf{k}}(t)$, the spin density $\mathbf{S}(\mathbf{r},t)$ of a two-dimensional (2D) electron gas is defined as

$$\mathbf{S}(\mathbf{r},t)=\sum_{\mathbf{k},\mathbf{q}} e^{i\mathbf{q}\mathbf{r}}\langle Tr[c_{\mathbf{k}+\mathbf{q}}^{\dagger}(t)\mathbf{s}c_{\mathbf{k}}(t)]\rangle,$$

where the average is taken over the ground state of the electron system. Taking into account the SOI and the less important Coulomb effects on spin density excitations, from the quasiclassical kinetic equation for $\langle Tr[c_{\mathbf{k}+\mathbf{q}}^{\dagger}(t)\mathbf{s}c_{\mathbf{k}}(t)]\rangle$, or by means of the standard perturbation theory,[7,5,6] one can derive the diffusion equation for the spin density $\mathbf{S}(\mathbf{r},t)$.

It is convenient to represent $\mathbf{S}(\mathbf{r},t)$ in the basis set of three eigenstates of the z component J_z of the angular momentum

operator **J**, which has the angular momentum quantum number $J=1$. Accordingly, we introduce $\psi_1=(S_x-iS_y)/\sqrt{2}$, $\psi_0=S_z$, and $\psi_{-1}=-(S_x+iS_y)/\sqrt{2}$, where the indices 1, 0, and -1 are the three eigenvalues of J_z. In term of this basis set, the diffusion equation is expressed as

$$\frac{\partial\psi}{\partial t}+\tau\langle(-i\mathbf{v}_F\cdot\boldsymbol{\nabla}_\mathbf{r}+\mathbf{h}_\mathbf{k}\cdot\mathbf{J})^2\rangle_{dir}\psi=I(\mathbf{r},t), \tag{1}$$

where $\langle\ldots\rangle_{dir}$ is an angular average over the Fermi line and $I(\mathbf{r},t)$ represents a possible source of spin oriented electrons inside the channel. The corresponding eigenmode equation is simply

$$\tau\langle(-i\mathbf{v}_F\cdot\boldsymbol{\nabla}_\mathbf{r}+\mathbf{h}_\mathbf{k}\cdot\mathbf{J})^2\rangle_{dir}\psi=\Gamma\psi. \tag{2}$$

The eigenvalue Γ is equal to the relaxation rate of the corresponding diffusion eigenmode. Equation (1) is valid if $v_F|\boldsymbol{\nabla}_\mathbf{r}\psi|\ll 1/\tau$ and $|h(k_F)|\ll 1/\tau$. The second inequality can not be satisfied for some high mobility InAs based quantum wells with a strong SOI, because in these quantum wells $v_F/|h(k_F)|\leqslant 500$ nm, according to Refs. 2 and 3. Equation (1) can be generalized to the region $|h(k_F)|\geqslant 1/\tau$, a situation outside the scope of the present work.

In a semiconductor quantum well, there are two contributions to the SOI $\mathbf{h}(\mathbf{k})$: the Dresselhaus term has its origin in the bulk crystal spin splitting,[8] and the Rashba term is due to the asymmetric potential profile in the quantum well.[9] The Rashba term is the dominating one in quantum wells made of narrow gap semiconductors, such as InAs.[2] Therefore, for the systems of our interest, we will retain only the Rashba term of SOI, which can be expressed as $h_z=\alpha k_x$, $h_x=-\alpha k_y$, and $h_y=0$. After taking the average over the direction of $\mathbf{k}_F$, Eq. (2) becomes

$$D\left(i\frac{\partial}{\partial x}+m^*\alpha J_z\right)^2\psi+D\left(i\frac{\partial}{\partial z}-m^*\alpha J_x\right)^2\psi=\Gamma\psi, \tag{3}$$

where $D\equiv v_F^2\tau/2$ is the diffusion constant, and m^* is the electron effective mass. If we replace D by $1/2m^*$, the above equation is quite similar to the Schrödenger equation for an electron. However, an important difference is that ψ is not a two-component spinor, but a three-component vector in the three-dimensional Hilbert space of the angular momentum $J=1$.

From Eq. (3), the spin flux can be expressed as

$$\mathbf{F}=-D(\boldsymbol{\nabla}_\mathbf{r}+im^*\alpha\mathbf{J}\times\mathbf{y})\psi,$$

where $\mathbf{y}$ is the unit vector along y axis. The boundary conditions at $z=\pm d/2$ are no spin flux across each boundary. Hence,

$$D\left(-i\frac{\partial}{\partial z}+m^*\alpha J_x\right)\psi\,|_{z=\pm\frac{d}{2}}=0. \tag{4}$$

For an extended 2D electron gas with $d\to\infty$, it is easy to see from Eq. (3) that the relaxation rates of homogeneous spin density are given by the eigenvalues of the operator $Dm^{*2}\alpha^2(J_z^2+J_x^2)$. The two eigenvalues correspond to the transverse (spin polarized in xz plane) D'yakonov-Perel' spin relaxation rate $1/\tau_s\equiv Dm^{*2}\alpha^2$, and the longitudinal (spin polarized in y direction) D'yakonov-Perel' spin relaxation rate $2/\tau_s$. In a 2D channel of electron gas, such a homogeneous solution of Eq. (3) with spin polarized along the channel, which satisfies the boundary conditions Eq. (4), also exists in the form of $\psi_{\pm1}=\pm1/\sqrt{2}$ and $\psi_0=0$. This state relaxes with the rate $1/\tau_s$ as in an extended 2D electron gas.

However, we will prove in this paper that in a narrow 2D channel there exist inhomogeneous diffusion modes with much lower spin relaxation rates. To derive these modes we will perform a canonical transformation $\psi=U(z)\tilde{\psi}$, where $U(z)=\exp(-iJ_xz/L_s)$ with $L_s=1/m^*\alpha$. The operator $U(z)$ transforms Eq. (3) to

$$D\left[-i\frac{\partial}{\partial x}-m^*\alpha\tilde{J}_z(z)\right]^2\tilde{\psi}+D\frac{\partial^2}{\partial z^2}\tilde{\psi}=\Gamma\tilde{\psi}, \tag{5}$$

with the boundary condition $(\partial\tilde{\psi}/\partial z)|_{z=\pm d/2}=0$, where $\tilde{J}_z(z)=U^{-1}(z)J_zU(z)$. If $d\ll L_s$, $U(z)$ can be expanded in powers of the small parameter z/L_s. To the second order we get $\tilde{J}_z(z)=J_z+J_yz/L_s-J_z(z/L_s)^2/2$ in which the last two terms will be treated perturbatively. The lowest order perturbation result gives the eigensolutions of Eq. (5):

$$\tilde{\psi}_{M,k,m}(x,z)=\exp(ikx)\chi_m(z)\Psi_M, \tag{6}$$

where M and Ψ_M are eigensolutions of J_z with $M=\pm1$ and 0. The $\chi_m(z)$ functions are $\chi_{2n}(z)=\cos(2\pi zn/d)$, and $\chi_{2n+1}(z)=\sin[\pi z(2n+1)/d]$ with integer n.

The unperturbed relaxation rates are readily obtained from Eq. (5) as

$$\Gamma^0_{M,k,m}=D(\pi m/d)^2+D(k-ML_s^{-1})^2. \tag{7}$$

Hence, the modes with $m\neq0$ relax very fast. On the other hand, the modes with $m=0$, which correspond to $\chi_0(z)=$constant, can have very low spin relaxation rates, which become zero for $k=ML_s^{-1}$. However, corrections due to terms depending on dL_s^{-1} in Eq. (5) make these rates finite. Using the standard perturbation theory to the second order of dL_s^{-1}, we find

$$\Gamma_{M,k,0}=\Gamma^0_{M,k,0}+\frac{(2-M^2)d^2}{24\tau_sL_s^2} \tag{8}$$

for $|k-ML_s^{-1}|\ll L_s^{-1}$. The above equation indicates that although the relaxation rates of the modes with $k=ML_s^{-1}$ become finite, their relaxation times retain much longer than the D'yakonov-Perel' relaxation time τ_s.

There are three slowly relaxing diffusion modes with $m=0$, $M=1$, 0, -1, and $k=ML_s^{-1}$. The two modes for $M=\pm1$ correspond to inhomogeneous spin distributions along the channel (x axis) with wave vectors $k=\mp L_s^{-1}$. Linear combinations of these modes give periodic spin density distributions $\mathbf{S}(x)$ with electron spins rotating in the plane perpendicular to z axis. Hence, such a spin density has a finite projection along the channel. The third mode for $M=0$ corresponds to electron spins pointing along z axis. The lowest relaxation rate is then achieved by an inhomogeneous spin density distribution along the channel. We notice that

although for $m=0$ the eigenfunctions given by Eq. (6) are independent of z, their corresponding spin distributions $\mathbf{S}(\mathbf{r})$ depend on z. This is because $\mathbf{S}(\mathbf{r})$ is expressed via the ψ function, which is obtained by applying the z-dependent unitary transformation $U(z)$ to $\tilde{\psi}_{M,k,0}(x)$.

We consider a stationary spin distribution in a channel of finite length $L \gg d$, with an applied electric voltage V between $x_l=-L/2$ at the left and $x_r=L/2$ at the right. This voltage can produce a magnetization in the channel, as was first proposed by Aronov.[10] The channel is connected with two ferromagnetic contacts, and a spin flux F_l is injected into the channel at $x=x_l$. At $x=x_r$ a spin flux F_r is then collected. The corresponding boundary conditions are

$$D\left(-\frac{\partial}{\partial x}+im^*\alpha J_z\right)\psi|_{x=x_{l,r}}=F_{l,r}. \tag{9}$$

The stationary spin density in the channel can be derived from Eq. (5) with the above boundary conditions. However, our goal is to calculate the change of dc resistance associated with the spin transport, which can be measured experimentally. To tackle this problem, we will use the formalism developed by Johnson and Silsbee.[11] The semiconductor-ferromagnet contacts are replaced by symmetric tunnel junctions with identical conductances $G=Ae^2N(E_F)v_F t/2$, where A is the channel cross section area, $t \ll 1$ the transmission probability, and $N(E_F)$ the density of states at the Fermi energy.

A dc current through the system will cause interfacial voltage drops ΔV_l across the left junction, and ΔV_r across the right one. Gradients $\Delta S_{l,r}$ of nonequilibrium magnetization across the tunnel junctions are also generated. Following Ref. 11, the spin fluxes through the junctions can be written as

$$F_{l,r}=-G[(\eta_{l,r}/2e)\Delta V_{l,r}-(\xi/e^2)\Delta S_{l,r}], \tag{10}$$

where parameters η_l and η_r with $|\eta_{l,r}|<1$ depend on the magnetizations of the ferromagnetic contacts. η_l and η_r have same sign if the magnetic polarizations in contacts are parallel, and opposite signs if antiparallel. The first term in Eq. (10) represents the spin injected by the applied voltage, and the second term with $\xi \simeq 1$ is the spin transport driven by gradients $\Delta S_{l,r}$. If in ferromagnets this magnetization relaxes sufficiently fast, $\Delta S_{l,r}$ are determined mainly by the spin polarizations in the channel at x_l and x_r, and so $\Delta S_l=-\psi(x_l)$ and $\Delta S_r=\psi(x_r)$. Under stationary condition, the dc current is given by[11]

$$I=-G[\Delta V_{l,r}-(\eta_{l,r}/2e)\Delta S_{l,r}]. \tag{11}$$

In this paper we will investigate the two cases in which the magnetizations of the ferromagnet contacts are either polarized along the z axis (perpendicular to the channel) or along the x axis (parallel to the channel). For each case, the magnetizations of the two contacts may be parallel or antiparallel. In terms of the momentum operator $\mathbf{J}$, the magnetization is represented by the $M=0$ state if it is along the z axis, but by a linear combination of the $M=\pm 1$ states if it is along the x axis. Hence, the corresponding injected fluxes are represented by $F_{l,r}=f_{l,r}\psi_0$ and $F_{l,r}=f_{l,r}(\psi_1-\psi_{-1})$, respectively. Since $L \gg$ d, the dominating contribution to the stationary spin distribution comes from the modes with low relaxation rates. The corresponding exponents of the stationary solutions of Eq. (5) can be derived from the equation $\Gamma_{M,k,0}=0$. In this way, from Eq. (8) we obtain $k=k_M \equiv ML_s^{-1} \pm il_M^{-1}$, where $l_M^{-1}=\sqrt{(2-M^2)/24}d/L_s^2$. In order to satisfy the boundary conditions Eq. (9), we have to make a proper linear combination of the $\exp(x/l_M)$ and $\exp(-x/l_M)$ solutions. Furthermore, to demonstrate the spin dynamics along the channel, we average Eq. (9) over z. It is easy to see that the corrections which are linear in d/L_s vanish after the averaging. If we ignore the higher order corrections, we can represent ψ as a linear combination of the zero order eigenfunctions Eq. (6) with $m=0$ and $k=k_M$. The spin density distribution is thus represented by

$$\psi=\sum_M \psi_M e^{iMx/L_s}[A_M \cosh(x/l_M)+B_M \sinh(x/l_M)]. \tag{12}$$

From Eq. (9), we derive $A_0=(f_l-f_r)L_s/[2D\cosh(L/2l_0)]$ and $B_0=-(f_l+f_r)L_s/[2D\sinh(L/2l_0)]$ for the magnetization along the z axis. Similarly, for the magnetization along the x axis, the coefficients $A_{\pm 1}$ and $B_{\pm 1}$ are obtained as

$$A_M=\frac{ML_s}{2D\cosh\dfrac{L}{2l_M}}\left[f_l\exp\left(\frac{iML}{2L_s}\right)-f_r\exp\left(-\frac{iML}{2L_s}\right)\right],$$

$$B_M=-\frac{ML_s}{2D\sinh\dfrac{L}{2l_M}}\left[f_l\exp\left(\frac{iML}{2L_s}\right)+f_r\exp\left(-\frac{iML}{2L_s}\right)\right].$$

It is important to point out that although the boundary conditions require the spin polarization at $x=\pm L/2$ to be along the x axis, within the channel the spin polarization rotates in the xy plane due to the exponential factors $\exp(\pm iMx/L_s)$ in Eq. (12). Since $L_s \ll l_M$, the polarization makes many complete rotations within the spin relaxation length L_s. As it will be shown below, this can lead to an oscillation of the channel resistance.

Substituting the expressions of A_M and B_M into Eq. (12), one can find the spin densities at $x=\pm L/2$, and hence the nonequilibrium magnetizations $\Delta S_{l,r}$. Together with Eqs. (10) and (11), we have a closed set of equations to solve numerically. However, it is important to illustrate in analytical form how the resistance depends on the spin transport through the 2D channel. For this purpose we consider a sample of length $L \simeq l_M$, and having a low enough transmission probability t such that the parameter $\kappa \equiv tl_M/l \ll 1$, where l is the electron mean free path. From Eqs. (10)–(12), we readily derive the spin-transport correction to the dc resistance as

$$\Delta R=(2\kappa/G)(\eta_l^2+\eta_r^2)\coth(L/l_M) -(2\kappa/G)\eta_l\eta_r\cos(ML/L_s)/\sinh(L/l_M). \tag{13}$$

The factor $\eta_l\eta_r$ in the second term at the right-hand side is positive if the magnetizations of the two ferromagnetic contacts are parallel, but negative if antiparallel. Furthermore, when the magnetization is along the channel, $M=\pm 1$, and

so this second term oscillates as a function of L/L_s. The amplitude of oscillation decreases with increasing L/l_M. However, due to the slowdown of the spin relaxation, we can manipulate the sample parameters such that $l_M \simeq L > L_s$. Then, our theoretically predicted resistance oscillation can be observed experimentally with a gate to change the value of L_s.

In our analysis above we have neglected the Dresselhaus contribution to the SOI, because in narrow gap systems which we are interested in, the Rashba contribution dominates the SOI. In quantum wells, the Dresselhaus contribution contains a linear term and a cubic term in electron momentum. It can be shown that the cubic term gives rise to an additional spin relaxation which is independent of the channel width d. Hence this term imposes a limit on the slowdown of spin relaxation. The corresponding relaxation rate is estimated to be insignificant for narrow gap quantum wells. However, in GaAs based quantum wells the cubic term is not negligible,[6] and can wash out the slowdown of spin relaxation.

Besides the classical spin diffusion, the spin relaxation slowdown also affects the weak localization corrections to transport parameters. In systems with strong enough SOI, the sign of such a correction to conductance is determined by the competition between the triplet and the singlet component of the Cooperon propagator.[13] The lifetime of the triplet is equal to the spin relaxation time. If the temperature is not very low, in a sufficiently narrow channel[12] this time can increase and becomes comparable to the dephasing time of the singlet. Hence, the sign of the correction can change from positive to negative.

The weak localization corrections to the spin diffusion coefficient and the D'yakonov-Perel' relaxation rate also contain contributions from the triplet and the singlet components of Cooperon.[7,14] The spin diffusion coefficient is included in the factor κ in Eq. (13). Therefore, by measuring the change of channel resistance ΔR with a controlled gate voltage or a weak magnetic field, the variations of spin diffusion coefficient can be investigated experimentally.

We acknowledge the support of the Royal Swedish Academy of Science under the Research Cooperation Program between Sweden and the former Soviet Union, Grant No. 12527.

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