

Proposal for a quantum Hall pump

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A device is proposed that is similar in spirit to the electron turnstile except that it operates within a quantum Hall fluid. In the integer quantum Hall regime, this device pumps an integer number of electrons per cycle. In the fractional regime, it pumps an integer number of fractionally charged quasiparticles per cycle. It is proposed that such a device can make an accurate measurement of the charge of the quantum Hall effect quasiparticles.

The basic idea of a parametric pump is that some parameters of a system are varied slowly and periodically such that after each full cycle the system returns to its initial state with the net effect being that some amount of a fluid is transferred from a source to a drain. There are many examples of such pumps in a very wide range of contexts — from the human heart to a firemen's bucket brigade. Over the past few years there has been increasing interest in parametric pumping of charge in mesoscopic systems both theoretically^{1,2} and experimentally.^{3–5} One particularly interesting example of a parametric pump is the electron turnstile—a device that transfers a single electron per cycle from a source to a drain. Such devices seem quite promising as metrological current and capacitance standards.^{4,5} In this paper, I propose a device very similar to the electron turnstile that operates in the quantum Hall regime. Similar to the electron turnstile, when operated adiabatically at low temperature in the integer quantum Hall regime, the number of electrons pumped in a single cycle is quantized. However, in the fractional quantum Hall regime, it is an integer number of *fractionally charged quasiparticles* that is pumped in each cycle. Thus, this device has the potential to make measurements of the fractional charge of quantum Hall quasiparticles.

Description of the Device. The structure of the proposed device, shown schematically in Fig. 1, is quite similar to the devices used in Refs. 6–9.

A full pumping cycle is shown in Fig. 2. Throughout the cycle, the source-drain voltage may be held at zero.¹⁰ The cycle can be described as the following steps:

(a) Begin in a state where the edges are far from the antidot. In this state, tunneling from the antidot to either the right or left edge is forbidden (i.e., the tunneling amplitude is very close to zero).

(b) Move the left edge state close to the antidot (by charging the left gate negatively) such that the tunneling amplitude between the left gate and the antidot becomes large (compared to the pumping frequency).

(c) Negatively charge the central gate such that the size of the antidot grows. Here, as the potential of the central gate increases, particles (or quasiparticles) that were occupying states near the edges of the antidot are shifted above the Fermi energy. As they cross through the Fermi energy, they tunnel out to the left edge (they cannot tunnel to the right edge because the right edge is insulated from the dot by a large region of quantum Hall fluid).

(d) Move the left edge state back to its original position far from the antidot (by uncharging the left gate) such that tunneling from the antidot to either the right or left edge is once again forbidden.

(e) Move the right edge state close to the antidot (by charging the right gate negatively) such that the tunneling amplitude between the right edge and the antidot becomes large.

(f) Uncharge the central gate such that the antidot becomes smaller. As the potential on the central gate decreases, the quasiparticles from the right edge tunnel back to the region near the edges of the antidot, filling states that were above the Fermi energy.

(a) Move the right edge back far away from the antidot (by uncharging the right gate) to return the system to the original state.

Similar to the electron turnstile, the charge pumped in this cycle is given by the difference between the charge on the antidots in steps (a) and (d). It is important to note that in stages (a) and (d), when the tunneling to both edges is turned off, the charge on the antidot is quantized either in units of the electron charge (in the integer regime) or in units of the quasiparticle charge (in the fractional regime). Thus, we expect that the charge pumped in a cycle will similarly be quantized, at least at low temperature. More rigorous arguments for this quantization will be made below.

If we then imagine that we fix the central gate voltage at stage (a) and measure the charge pumped per cycle as a function of the central gate voltage at stage (d), at zero temperature, we would obtain a step-like curve, illustrated as the solid line in Fig. 3.

Quantization of pumping: integer case. A general approach to understanding quantized charge pumping is reminiscent of Laughlin's argument for quantized Hall conductance.¹¹ Consider the Corbino geometry shown in Fig. 4. In the integer quantum Hall regime, at low temperature, the ground state of the system is unique and gapped at all times in the pumping cycle. If the deformation is made adiabatically, the system simply tracks the ground state.¹² ("Adiabatic" here is defined to mean that the system tracks the ground state). Thus, at the beginning and end of the cycle, the system is in the same state and the only net effect is that an integer number of electrons could have been transferred from the inside to the outside edge of the annulus (or vice versa).

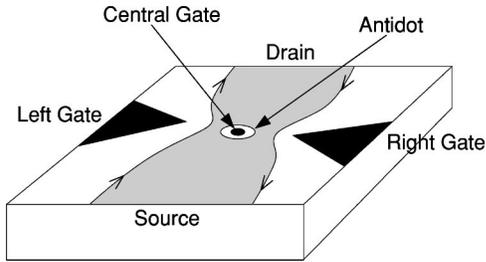


FIG. 1. Cartoon schematic of the proposed quantum Hall pump. The lightly shaded region in the center is quantum Hall fluid. The black areas are gates. Arrows at the edges of the fluid indicate edge state propagation direction. The side gates can push the edges of the fluid closer to or further from the central antidot. The small gate in the center can change the size of the antidot.

For the simple case of noninteracting electrons, one can write the dynamics in terms of a simple time dependent Schrodinger equation. This can be integrated explicitly (exactly, or perturbatively) to demonstrate the quantization of pumped charge as claimed above. This explicit approach is useful in that it allows us to study the effects of nonadiabaticity in detail. Such a study is a subject of current research and will be reported elsewhere.

Fractional case. In the case of the fractional quantum Hall effect, the Laughlin argument must be modified to account for fractionalization of charge.¹³ It now becomes possible to transfer a single fractionally charged quasiparticle across the system. (As usual, increasing the charge on the antidot by a fractional amount results in the decrease of the charge on the edges of the system by the same amount, being that the bulk is incompressible and the total charge of the system is conserved). The argument given in the above section—which would seem to require transfer of an integer number of electrons per cycle—fails in the fractional Hall effect case because the ground state becomes q -fold degenerate,¹³ with q a small integer related to the quasiparticle charge and the quantum numbers of the particular quantum Hall state. For example, for the simple case of a Jain $\nu = p/(2p+1)$ state, there are $q = 2p+1$ degenerate ground

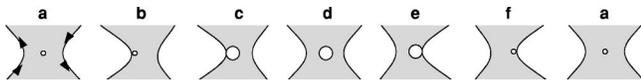


FIG. 2. A full cycle of pumping (a-b-c-d-e-f-a). Each frame is a top view of the device at a different point in the pumping cycle. In frame (a), the direction of edge state propagation is also shown. This pumping cycle transfers charge from the the source (bottom) to the drain (top) at zero applied source-drain voltage. Note that the antidot does not connect to both edges simultaneously, so at any moment during the cycle the quantized Hall fluid (shaded) connects the source to the drain and the source-drain conductance is quantized. Analogous to the electron turnstile, the antidot picks up charge (holes) from the left edge, moves over to the right edge, and then releases the charge (and then repeats the process). Since the amount of charge carried by the antidot is quantized, so is the resulting pumped current per cycle. In the integer regime, the charge on the antidot (and hence the pumped current per cycle) is quantized in units of the electron charge, whereas in the fractional regime it is quantized in units of the fractionally charged quasiparticle.

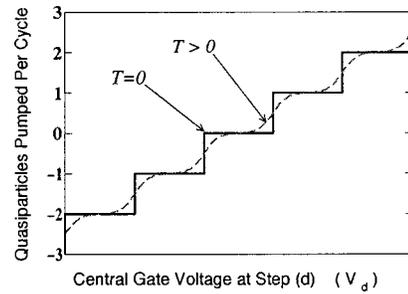


FIG. 3. Pumped charge as a function of central gate voltages V_d at step (d) in the pumping cycle (schematic). It is assumed that the other parameters of the pumping cycle—and in particular the central gate voltage at step (a) are held constant as V_d is changed. The solid line is zero temperature, whereas the dashed line is finite temperature with T roughly 10% of the single particle addition energy.

states [and the quasiparticle charge is $e/(2p+1)$]. Because of this ground state degeneracy, the system need not return to the same ground state after each pumping period, but may instead cycle through the q ground states. As a result, it is the number of electrons transferred across the system in q cycles that is quantized, rather than the number transferred in a single cycle. Thus, the average charge transferred in a single cycle is quantized in units of e/q , which is the quasiparticle charge. Indeed, it is known that adiabatic transfer of a quasiparticle across such a Corbino system does indeed cycle the degenerate ground states.¹³

Other than this minor modification of the above Laughlin-like argument, we expect that the same considerations as in the above integer case will apply for all fractional quantized Hall states. We also expect that, as above, the temperature scale at which the quantization is smeared out is roughly given by the single quasiparticle addition energy. For a more detailed calculation, we expect that chiral Luttinger liquid theory¹⁴ can be used to calculate the pumped current explicitly. This, too, is a subject of current research, and will be reported elsewhere.

Scattering matrix approach. A rather elegant, more formal, argument for quantization is based on the scattering matrix approach to adiabatic parametric pumping.² In this approach, one writes the charge pumped in one cycle (t varies from 0 to τ) as

$$Q = e \int_0^\tau \frac{dt}{2\pi} \sum_{\beta} \sum_{\alpha \in \text{source}} \text{Im} \left[S_{\alpha\beta}^*(t) \frac{d}{dt} S_{\alpha\beta}(t) \right], \quad (1)$$

where $S_{\alpha\beta}(t)$ is the scattering matrix at time t from channel α to channel β . Here $S(t)$ is to be calculated as if the pa-

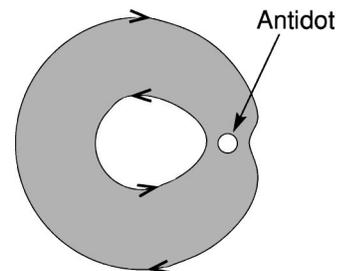


FIG. 4. Quantum Hall pump in a Corbino geometry.

rameters of the system are frozen at time t , and α is summed only over channels at the source. In the quantum Hall regime, so long as there is no direct tunneling across the quantum Hall bar (i.e., as long as the antidot is not simultaneously connected to both edges), the structure of the scattering matrix is trivial—anything that comes into the left edge at the source (bottom left of each frame of Fig. 2) must follow that edge all the way to the drain (upper left). If we have a quantum Hall state with only a single edge channel ($\nu=1$, for example) the scattering matrix has only two nonzero elements—each with unit magnitude (one element for the edge state leaving the source on the lower left side and ending up at the upper left, and one leaving the drain at the upper right and ending up at the source at the lower right). Only one of these two nonzero elements (the one representing the state leaving the source) enters into Eq. (1). We write this relevant unit magnitude [$U(1)$ valued] element as $e^{i\phi(t)}$, such that we have the charge pumped per cycle as $Q = e \int_0^T (dt/2\pi) (d\phi(t)/dt)$. In the integer quantum Hall regime, the system must return to its original state after a full cycle. Thus, $\phi(t)$ must return to its original value modulo 2π . The pumped charge is then just the number of times ϕ wraps by 2π per cycle. In this way we see that the pumped charge is quantized as a result of being a topological quantity!

This quantization argument can be generalized to the case of m copropagating channels per edge. In this case, the m edge channels can mix with each other as long as they all go directly along the edge from the source to the drain and do not cross the Hall bar. The relevant nonzero terms of the scattering matrix then form a $U(m) = U(1) \otimes SU(m)$ matrix. It can be shown that the $U(1)$ part is again the only important piece (representing the total charge) and the pumped charge per cycle is again quantized as described above.

This scattering matrix formalism is easily extended to finite temperature² (at least for the integer case). One needs only to define scattering matrices $S(E, t)$ as a function of incoming energy. Equation 1 becomes E dependent resulting in a charge transfer $Q(E)$ which is then smeared by a Fermi function to give the charge transfer: $Q = \int dE Q(E) [dn_F(E)/dE]$ with n_F the Fermi function. In Fig. 3 this smearing by a Fermi function is shown as the dashed line (in the figure T is taken to be 10% of the antidot single particle addition energy).

For the noninteracting electron (integer) case and for some simple interacting cases, it is possible to solve for the scattering matrix explicitly (given the energies of eigenstates on the antidot and the tunneling matrix elements as a function of time). Indeed, it can be established, as claimed above, that the charge pumped per cycle at $T=0$ is quantized and is equal to the difference in the charge on the antidot between steps (a) and (d).

To generalize this scattering matrix approach to the fractional quantum Hall regime, we imagine connecting a fractional quantum Hall sample to integer Hall leads in a smooth fashion¹⁵, so that one can still ask about the scattering matrix for electrons injected into the system. Here, due to the above mentioned ground state degeneracy,¹³ the system need not return to its original state after a single pumping cycle. In the case of having q degenerate ground states, the system can cycle through the ground states returning to the original state only after q full periods of pumping. Thus, the pumped charge Q in Eq. (1) need only be quantized in units of the electron charge after q cycles, so the pumped current per cycle is quantized in units of e/q .

Experiments. This experiment can thus be used as a measurement of the charge of the fractional quantum Hall quasiparticle. Although, a number of previous works have measured the fractional charge of quantum Hall quasiparticles,^{8,9,16,17} it is quite possible that the currently proposed pumping experiment will be the theoretically clearest measurement yet.

The main experimental problem in carrying out this experiment appears to be that temperature must be sufficiently low that the current steps (see Fig. 3) are not too smeared out. As discussed above, this temperature scale is mostly determined by the single (quasi)particle addition energy for the antidot. It is thus quite useful to note that this energy scale has in fact been measured for several similar experimental systems in both the integer and fractional regimes.⁶⁻⁹ Although the precise addition energy depends on the particular sample in question, the authors of Refs. 6-9 were able to achieve addition energies on the order of several hundred mK for both the $\nu=1$ and $\nu=1/3$ states. For the case of $\nu=2/5$, however, this energy seems to be somewhat lower,⁸ but may still be high enough to successfully perform the proposed pumping experiment.

Another experimental issue is how fast one can pump the system and expect to have the pumped charge quantized. This somewhat subtle issue is a subject of current research. However, as estimates, one can expect that the tunnelling time from the antidot to the edge should set one time scale, the single particle addition energy sets another time scale, and the dissipation time yet another time scale. It is quite safe to say that pumping at a rate slower than all of these time scales will remain quantized. The effects of pumping faster will be discussed in a forthcoming paper.

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- ⁹J. D. F. Franklin *et al.*, *Surf. Sci.* **361**, 17 (1996).
- ¹⁰If a nonzero voltage is maintained between the left and right edges (source-drain), then the central gate is no longer required—analogueous to many single electron turnstile experiments. However, in such an arrangement a (large) Hall current also flows due to this finite voltage and the (small) pumped current must be distinguished from it. This presumably can be done with an appropriate four point measurement technique. (The author thanks C. Chamon for pointing this out.)
- ¹¹R. B. Laughlin, *Phys. Rev. B* **23**, 5632 (1981); see also B. I. Halperin, *ibid.* **25**, 2185 (1982).
- ¹²Strictly speaking, there is spectral flow of the ground state, and the state with an electron transferred from the inner edge to the outer edge is not the true ground state, but is returned to the true ground state by transferring an electron through an external wire (or a current meter) back from the outer to the inner edge.
- ¹³See, for example, the discussion in *Quantum Hall Effect*, edited by M. Stone (World Scientific, Singapore, 1992), and references therein. The fractional quantum Hall ground state is truly q -fold degenerate for a torus geometry. For Corbino geometry, there are q sectors of the Hilbert space—distinguished by the number of quasiparticles that have been moved across the system. The lowest energy state of each these q sectors are not precisely degenerate, but differ by a small energy associated with the slope of the confining potential.
- ¹⁴See for example, C. L. Kane and M. P. A. Fisher, in *Perspectives In Quantum Hall Effects*, edited by S. Das Sarma and A. Pinczuk (Wiley, New York, 1997).
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- ¹⁶M. Reznikov *et al.*, *Nature (London)* **399**, 238 (1999); R. de Piccioto *et al.*, *ibid.* **389**, 162 (1997); L. Saminadayar *et al.*, *Phys. Rev. Lett.* **79**, 2526 (1997); S. I. Dorozhkin, R. J. Haug, K. von Klitzing, and K. Ploog, *Phys. Rev. B* **51**, 14 729 (1995).
- ¹⁷An important issue that arises is the question of whether the experiment measures the quasiparticle charge or just the quantum number ν (related to the quantized Hall conductance) (Ref. 16). For a $\nu=1/3$ state, for example, these two quantities are the same, so any measurement of this state is ambiguous in this sense. For $\nu=2/5$, however, the charge of the quasiparticle is $1/5$. The proposed pumping experiment for a $\nu=2/5$ state should properly pump charge quantized in units of $e/5$.