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Reduction of energy absorption by phonons and spin waves in a disordered solid due to localization

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It is argued on the basis of the Chirikov overlap of resonances criterion for the development of chaos and ergodicity that Anderson localization of the phonon modes or the spin-wave modes (in the case of ferromagnetic resonance) can result in a large reduction in the rate of energy absorption by a solid from an oscillating external field. This phenomenon is illustrated by numerical calculations on simple model systems.

I. INTRODUCTION

The excitations of a solid, e.g., phonons or spin waves (for a magnetic solid), are usually approximated by harmonic oscillators. When we are interested in how energy is absorbed by a solid interacting with an external field, however, it is crucially important to include anharmonic contributions to these excitations and to the coupling between different excitations. First of all, anharmonicity makes the frequency of a given mode depend on its amplitude. As a consequence, the resonant frequency of the mode gets spread into a range of resonant frequencies. Since the normal modes of a solid (in the harmonic approximation) are independent harmonic-oscillator modes, one must include the coupling of modes resulting from anharmonicity in order to include spreading of energy among the modes. It was shown by Chirikov that a single resonance in a nonlinear system will not absorb energy.[1–3] It is only when a collection of such resonances are coupled and overlap sufficiently in frequency that they are able to absorb energy from an external force acting on one or more of the resonances.[1–3] Here it is argued that Anderson localization[4] of the phonon or spin-wave modes of a solid can suppress the exchange of energy among the various modes, and thus by Chirikov's overlap of resonances criterion for the occurrence of ergodicity and chaos, the energy absorbtion by these modes from an external field will be suppressed.[1–3]

In previous work by the present author,[5] it was argued on the basis of the Chirikov overlap of resonances criterion for the occurrence of chaos[1–3] that very small particles (of the order of a few atoms) will absorb practically no energy on the average from an external oscillating force acting on some of the atoms at sufficiently low temperature. This argument was proposed to show that such small particles might be able to slide with respect to each other with practically no kinetic friction. It will be argued here on the basis of the Chirikov idea that the rate of energy absorption by a larger solid from an external oscillating field (e.g., an electromagnetic field) can be drastically reduced if the solid is sufficiently disordered for its phonon modes to be localized. I will also consider the case of an oscillating magnetic field interacting with a classical Heisenberg model ferromagnet, which should describe ferromagnetic resonance.

The transition from energy absorbing to nonenergy absorbing behavior as a function of decreasing system size, which was found in the model studies of Ref. 5, can be explained in terms of the overlap of resonances criterion for the occurrence of ergodic and/or chaotic behavior of the system. The basic idea is that when weak anharmonicity is included, the vibrational degrees of freedom can be modeled as a collection of nonlinear resonances of the system, i.e., modes whose resonance frequency depends on the amplitude of vibration.[1] As a result, a nonlinear resonance, has a range of resonance frequencies, rather than a single one. If such a resonance is acted on by a periodic driving force, it will not gain energy from the force forever, because even if the resonant frequency is initially equal to the frequency of the external driving force, since it depends on the vibrational amplitude, it will not remain at that value. Consequently, the energy of the system will oscillate as a function of time. In contrast, if there are two or more such resonances, the system might heat up if the driving force is sufficiently large. When these ranges overlap, the resonances can exchange energy with each other, but when they do not, they behave as independent resonances which, as was stated earlier, do not continuously absorb energy from the driving force.

In the case of small solids,[2,5] the mode spacing is just the phonon mode spacing resulting from the fact that the solid is finite. As a result the solid undergoes a transition from nonenergy absorbing behavior when it is sufficiently small to energy absorbing behavior when it is larger (and hence the modes are closer together).[5] In this paper, I wish to propose an alternative way to get nonabsorbing behavior, which can occur for larger solids. If the solid is sufficiently disordered, the phonon modes might be localized.[4] In this case it is statistically improbable that a mode localized around a particular lattice site will be nearly in resonance with modes localized around neighboring sites. Then, the Chirikov criterion tells us that the phonon modes might behave as nearly uncoupled nonlinear resonances. If these modes are acted on by a time-dependent driving force, they might not absorb energy from it.

II. ENERGY ABSORPTION BY PHONONS

In order to illustrate effects of localization on energy absorption by phonon modes, let us consider a simple one-dimensional model, consisting of a chain of atoms, taken to represent a solid film, interacting with each other and with a rigid substrate with anharmonic interactions of random strength. Each of these atoms is then acted on by a sinusoidally oscillating force, which represents the external field. The equation of motion for this model is

$$\ddot{q}_\alpha = -\gamma \dot{q}_\alpha - g_\alpha f(q_\alpha) - K_\alpha f(q_\alpha - q_{\alpha-1}) + K_{\alpha+1} f(q_{\alpha+1} - q_\alpha) + F_\alpha \sin(\omega t), \tag{1}$$

where q_α represents the displacement of an atom with respect to the substrate and g_α represents the force constant which gives the coupling of the atom to the substrate. We will use this equation to study two models of a disordered film. In both models K_α, the force constant between the αth and $(\alpha+1)$st atoms, is chosen to be a random number between 0 and K_{max}. In one model called the substitutional impurity model, g_α has a value g_{max} on all except a few sites chosen at random with probability c, on which it is equal to zero. In the second model, which will be called the continuous random model, g_α is chosen to be a random number between 0 and g_{max}, ω is the frequency of the driving force and $f(x)$ is a nonlinear function given by

$$f(x) = x - Ax^2 + Bx^3. \tag{2}$$

The coefficients A and B in Eq. (2) are arbitrarily chosen to be equal to 9.354 434 7 and 49.007 713 9, respectively. These are the values of the coefficients for the expansion up to third order of the Lennard-Jones interaction, but this particular choice is not relevant for the present discussion. For the periodic film that we study for comparison to the disordered models, g_α and K_α are chosen to be independent of α and equal to g_{max} and K_{max}, respectively. (This choice makes the mode spacing larger for the periodic system than if the average values were used, which will if anything suppresses the energy absorption.) A linear in velocity damping term with damping coefficient γ can be included to simulate the cooling effect of the coupling of the chain to the substrate. For the substitutional impurity model, we will take the amplitude of the driving force F_α to be independent of α and equal to F_{max}, but for the continuous random model and the periodic comparison system described above, F_α is chosen to be randomly distributed with a square distribution between 0 and F_{max}, because if it were not random, it would only couple to the zero wave-vector mode of the periodic comparison system, making it an inappropriate comparison system. The total energy of the chain was calculated as a function of time both for a disordered chain and for an ordered one.

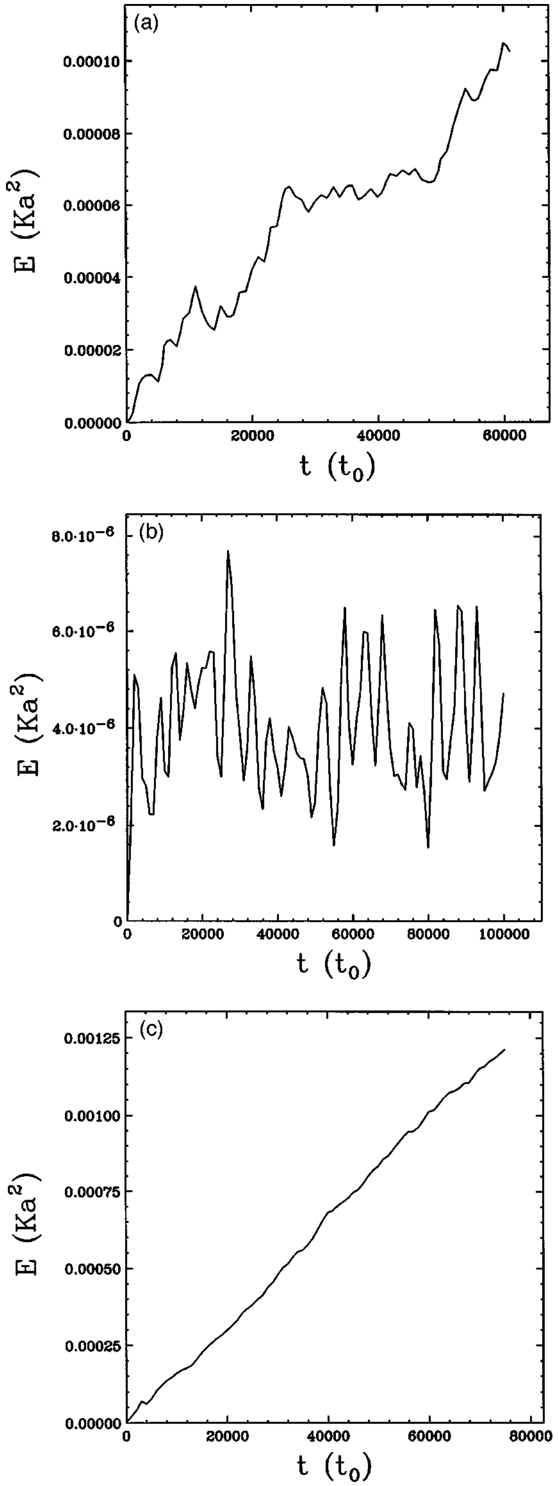

FIG. 1. E in units of Ka^2 versus t in units of t_0 for a 1000 atom chain for the substitutional impurity model with (a) an impurity concentration $c = 0.05$ with no restriction on the impurity locations, (b) with $c = 0.1$ with the restriction that no two impurities can be less than a distance $5a$ apart, (c) a periodic lattice with the same values of ω and F but with the applied force restricted to 1% of the lattice sites.

In Fig. 1(a), the energy of the chain is plotted as a function of time for the substitutional impurity model with concentration c equal to 0.05. The frequency of the driving force was chosen to be $(0.43K_{max})^{1/2}$, which puts it in the frequency range of the vibrational states which are localized around the impurity sites. In these runs the average values of g_α and F_α were taken to be $5.0K$ and $0.002Ka$, respectively, in order to insure that the low-frequency modes (‘‘impurity modes’’) are well localized, and a is the interparticle spacing in the chain, because our goal here is to study the effects of localization on energy absorption. In this run, the energy was found to increase with increasing time. In Fig. 1(b), the energy is shown as a function of time for the same values of g_{max}, K_{max}, and F_α, but with a concentration c equal to 0.1 but with the restriction placed on the impurity sites that there

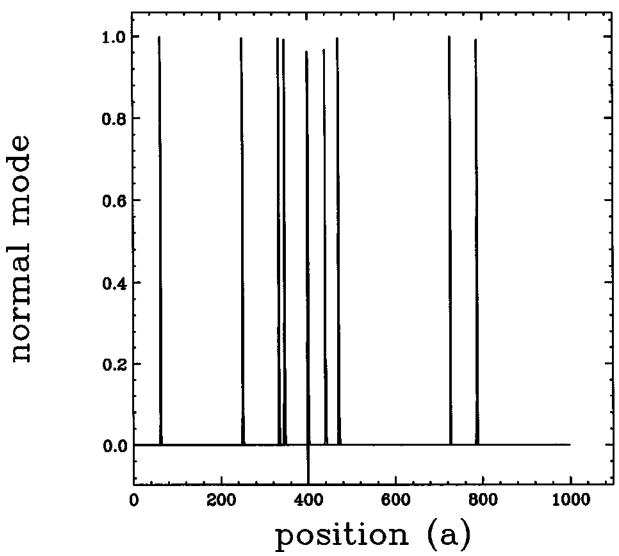

FIG. 2. The 10 lowest modes for the model presented in Fig. 1. The square frequencies and positions of the peaks of each of the modes are, respectively $0.135\,72t_0^{-2}$ peaked at $400a$, $0.255\,33t_0^{-2}$ peaked at $727a$, $0.364\,10t_0^{-2}$ peaked at $63a$, $0.495\,28t_0^{-2}$ peaked at $347a$, $0.552\,27t_0^{-2}$ peaked at $401a$, $0.603\,92t_0^{-2}$ peaked at $471a$, $0.610\,15t_0^{-2}$ peaked at $788a$, $0.627\,32t_0^{-2}$ peaked at $251a$, $0.675\,77t_0^{-2}$ peaked at $440a$, and $0.677\,13t_0^{-2}$ peaked at $334a$.

must be a minimum spacing between impurity sites of five lattice sites. In this case, we see that despite the fact that the concentration of impurities is larger, the energy does not increase with increasing time, implying that this system does not absorb energy from the driving force. This implies that the absorption that took place in the other case was due to the overlap of the localized modes (since in the present case, the restriction placed on the impurity site spacing guarantees that the modes do not overlap). This is expected because on the average, the number of pairs of impurity sites which are a distance n or fewer sites apart, where n is the number of sites to which a localized phonon state extends, is Nnc^2, where N is the number of sites in the chain. For comparison, in Fig. 1(c), the energy is shown as a function of time for a periodic version of this model with all values of g_α equal to zero and all values of K_α equal to K_{max}. The frequency of the driving force is the same as in the models of Figs. 1(a) and 2(b), but F_α is chosen to be zero on all sites except for 1% of the sites chosen at random where it is set equal to F_{max}. Here, we find that in contrast to the random case, the periodic system does absorb energy from the driving force, despite the fact that it acts only on a smaller fraction of the sites than it acts on in the calculations presented in Fig. 1(b). Disordered chains with 400 and 200 atom, without any restriction on the closeness of sites on which g_α is zero, generally did not absorb energy.

It is expected that the initial energy absorption can be described in the harmonic approximation, since the anharmonic terms in Eq. (1) will only become important when the amplitude of the driven motion of the chain becomes sufficiently large. Therefore, it is expected that one can gain some insights into how the nonlinear system absorbs energy by studying the exact harmonic approximation solution [i.e., when A and B are set equal to zero in Eq. (1)] and comparing it to the numerical calculations on the nonlinear system. Differences between the behavior of the two systems will tell us something about the mechanisms for energy absorption. Following Ref. 1, the harmonic approximation solution of Eq. (1), for times sufficiently long for the transient solutions to die out, can be written as

$$q_\alpha(t)=\sum_{\alpha'}\int dt' G_{\alpha,\alpha'}(t-t')f_{\alpha'}\sin(\omega_0 t'), \tag{3}$$

where ω_0 is the driving frequency and the Green's function

$$G_{\alpha,\alpha'}(t-t')=(2\pi N)^{-1}\sum_\sigma \int d\omega \frac{S_{\sigma,\alpha}S^*_{\sigma,\alpha'}e^{-i\omega(t-t')}}{\omega_\sigma^2-\omega^2+i\gamma\omega}, \tag{4}$$

where $S_{\sigma,\alpha}$ is the σth normal mode of the system, ω_σ is its frequency, and γ is the damping constant for the modes. For $t \gg \gamma^{-1}$ the transients will have died out. Several of the low-frequency normal modes are plotted in Fig. 2 for the above choice of force constants for the substitutional impurity model 1000 atom chain with $c=0.05$ and no restriction on the spacings of the randomly chosen impurity sites. The modes are clearly highly localized. Consistent with our hypothesis that the energy absorption by the system is caused by mode overlap, we see that there is a mode localized at the site at locations $400a$ and one at $401a$. In order to illustrate that this is precisely the place where almost all of the energy absorption takes place in this system let us consider the quantity $\dot{q}_\alpha^2+\omega_0^2 q_\alpha^2$ called the amplitude function, which is a measure of the amplitude of the vibrations of the αth atom. We find using Eqs. (3) and (4) that it is given by

$$\dot{q}_\alpha^2+\omega_0^2 q_\alpha^2=\omega_0^2\left|\sum_\sigma \frac{S_{\sigma,\alpha}\sum_{\sigma'} S_{\sigma',\alpha}f_{\alpha'}}{\omega_\sigma^2-\omega_0^2-i\gamma\omega_0}\right|^2, \tag{5}$$

which is independent of time. This quantity is plotted in Fig. 3(a) in order to compare it with the same quantity found for the full nonlinear system described by Eq. (1) for the substitutional impurity model with $c=0.05$ and no restriction on the location of the impurity sites, which is plotted at $t=61\,000t_0$ (where $t_0=K^{-1/2}$, and where K is the average value of the intersite force constant) in Fig. 3(b). [For comparison, the amplitude was also calculated for the periodic comparison model and and plotted in Fig. 3(c). It is clearly much more spread over the entire lattice than the function plotted in Fig. 3(b).] It appears as if the mode excited at position $400a$, as shown in Fig. 3(a), is now transferring its vibrational energy to modes localized around neighboring sites, consistent with the application of the Chirikov criterion to the energy absorption problem.

Calculations were also done of similar quantities for the continuous random model. It was found that whereas (for 200 and 400 atom chains), the disordered system was found to not absorb energy on the average, the corresponding periodic chain did. The 1000 atom disordered chain, in contrast did absorb energy, but the absorption was found to occur only in a small region of the chain.

Let us now consider the question of whether mode localization is likely to lead to reduced energy absorption in the long chain limit. For the periodic chain described by Eq. (1)

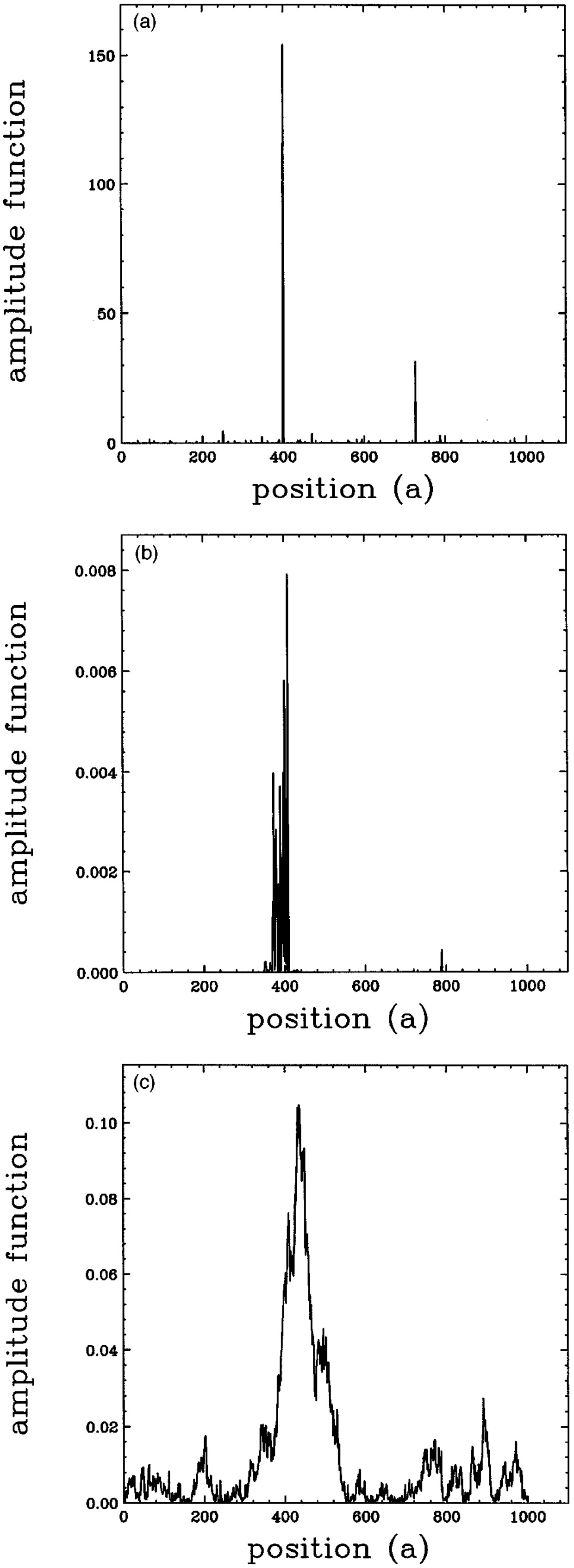

FIG. 3. The amplitude function versus position in units of a for the substitutional impurity model, calculated for $\omega^2=0.1849t_0^{-2}$ (a) in the harmonic approximation, (b) from the solution of the nonlinear problem governed by Eq. (1), and (c) from the solution of the nonlinear problem for the periodic comparison model described in the text.

with K_α and g_α independent of α, it is useful to expand q_α in terms of its harmonic approximation normal modes (Q_k) (i.e., $q_\alpha=N^{-1/2}\Sigma_k e^{ik\alpha a}Q_k$). Then the resulting equation of motion for Q_k is

$$\ddot{Q}_k=-\gamma\dot{Q}_k-\omega^2(k)Q_k+N^{-1/2}A\sum_{k'}Q_{k'}Q_{k-k'}-N^{-1}B\sum_{k',k''}Q_{k'}Q_{k''}Q_{k-k'-k''},\qquad(6)$$

where $\omega(k)$ is the normal mode frequency. This equation results from Hamilton's equations with the Hamiltonian

$$H=(1/2)\sum_k[|\dot{Q}_k|^2+\omega^2(k)|Q_k|^2]+(1/3)A\sum_{k'}Q_kQ_{-k'}Q_{k'-k}-N^{-1}(1/4)B\sum_{k',k''}Q_kQ_{-k'}Q_{k''}Q_{k'-k-k''}.$$

The $k'=k''=k$ term in the third summation gives the nonlinearity of the mode Q_k. (The second summation does not have a term containing only a single k value.) From the discussion in Sec. 3.2 in Chirikov's review article,[1] this term gives the inverse mass defined in Eq. (3.18) in that section, implying that the inverse mass is proportional to N^{-1}. The largest contribution to the coupling between modes (which plays the role of the coupling of a mode to an external force, represented by V_{mn} in Sec. 3.2) comes from the first summation, and hence is proportional to $N^{-1/2}$. Thus, from Eq. (3.22) in that section, we conclude that the width of the nonlinear mode represented by Q_k is proportional to $N^{-3/4}$. Since the mode spacing is proportional to N^{-1}, the number of coupled modes that fall within the width of a single mode is proportional to $N^{1/4}$. Thus, since the total number of modes is N, the number of overlapping mode pairs for a periodic chain is proportional to $N^{5/4}$. Since we saw earlier that the number of overlapping mode pairs for the disordered chain is approximately equal to nNc^2 (where n represents the spatial extent of a localized mode), we see that for the large-N limit, the number of overlapping mode pairs (which according to the Chirikov criterion can absorb energy) should be much larger for the periodic than for the random chain. (Although the number of overlapping modes for the periodic chain grows faster than proportional to N, coupling to other degrees of freedom will likely result in the rate of energy absorption being only proportional to N.) The proportionality to $N^{5/4}$ is only indicative of the fact that the mode overlap should be much larger for the periodic chain.) For the continuously disordered case, the density of localized modes is proportional to N. Let us write it as $N\rho_0$, where ρ_0 is an intensive variable. Thus if the width of a single mode due to nonlinearity is denoted by ΔE, the number of modes within a mode width is $N\rho_0\Delta E$. Thus from the above arguments, we again conclude that a macroscopic random chain should be less dissipative than a periodic one.

In the calculations presented here, the damping constant γ in Eq. (1) was assumed to be small compared to the mode spacing. Clearly, such an assumption cannot be made for an infinite solid with a continuous distribution of force constants and/or atomic masses. The damping γ was included to account for coupling of the modes of the solid under consideration due to anharmonicity with a substrate. Therefore, if instead of including a parameter γ in the calculation, we think in terms of coupling to the substrate as being due to anharmonic coupling of the solid under consideration to the modes of the substrate, if the substrate is also a disordered solid, the arguments based on the Chirikov criterion can also be applied to the coupling of the modes of the the solid to those of the substrate. Applying the arguments presented in this article to the suppression of the transfer of energy with the substrate because of the Chirikov argument, we are justified in setting γ (which is included in the model to represent this energy transfer) equal to zero.

The spatial extent of a given localized mode should depend only on the ratio of typical force constants connecting neighboring sites to the typical spread in the random on-site force constants, which would not depend significantly on dimensionality. It is expected, however, that in higher than one dimension the nonlinear terms in the equation corresponding to Eq. (1) will include contributions from coupling to more localized modes because there are more directions in which they can be found, in contrast to one dimension for which there are only two directions. Thus we would expect that in a higher than one-dimensional model, the parameters will have to be chosen such that the modes are slightly more localized than is necessary in one dimension for this phenomenon to occur. The main qualitative results should, however, not be changed.

We will discuss in the next section ferromagnetic resonance in highly disordered systems, where it will be proposed that a similar phenomenon to that described in this section but due to spin-wave localization could occur.

III. REDUCTION IN FERROMAGNETIC RESONANCE RELAXATION RESULTING FROM SPIN-WAVE LOCALIZATION

Ferromagnetic resonance is an area in which the effects of localization of the excitations of a solid on absorption of energy when the solid is subject to an external field can be easily studied. In ferromagnetic resonance, a ferromagnet is subject to an electromagnetic field, usually in the microwave range. When the field's frequency is close to that of one of the spin-wave modes, there is a resonant peak in the absorption of the radiation. For a crystalline solid, the radiation only couples to the zero wave-vector spin wave because of wave-vector conservation. Each spin wave acquires a width due to its interaction with the other spin waves, phonons, and sometimes electrons.[6] If the spin waves were not broadened, the radiation could only be absorbed if its frequency was exactly equal to that of one of the spin waves. This statement is only true in the linear spin-wave approximation (where the spin waves are treated as a collection of noninteracting harmonic oscillators).[6] Once one goes beyond the linear approximation in the classical equations of motion for the spin waves, each of the spin waves can be thought of as a nonlinear oscillator which acquires a width, as the phonons did, because the spin-wave frequencies will shift with increasing amplitude. The Chirikov overlap of resonances criterion again implies for the spin waves, as was found for the phonons, that even if the frequency of the radiation is slightly off resonance, the zero-wave-vector spin wave will absorb energy from the field if the nonlinear resonance of this spin wave overlaps with other spin-wave resonances.

Sufficient localization of the spin waves can lead to a situation in which the spin-wave modes localized around neighboring sites are far from being in resonance with each other, and therefore do not absorb energy on the average (although they might heat up slightly initially).

Our discussion will be based on the classical equations of motion for the spins, e.g., the Landau-Lifshitz equation of motion,[6]

$$d\vec{m}/dt=-\Gamma\vec{m}\times\vec{H}-\gamma\vec{m}\times(\vec{m}\times\vec{H})/m^2, \tag{7}$$

where γ is the damping parameter, to account for loss of energy from the spin degrees of freedom to the vibrational degrees of freedom or from the magnetic material to a substrate on which it is deposited[6] and Γ is the gyromagnetic ratio. The spin on each lattice site is assumed to obey this equation of motion, with $\vec{H}$ taken to include the sum of all external fields applied to the system, the exchange field due to neighboring spins and anisotropy field, except in the damping (i.e., the second) term where the ac field is not included. In the calculations to be presented here, the field acting on the ith spin $\vec{m}_i$ is given by

$$\vec{H}_i=\vec{H}_{\rm ext}+\sum_j J_{i,j}\vec{m}_j+2A_i m_j^z\hat{z}, \tag{8}$$

where $J_{i,j}$ is the exchange interaction acting between the ith and jth lattice sites, A_i is the single-ion anisotropy parameter and $\hat{z}$ is a unit vector in the z direction, which is assumed to be the symmetry axis of the crystal. This expression for the field follows from the fact that

$$H_i^\alpha=-\partial H/\partial m_i^\alpha, \tag{9}$$

where m_i^α is the αth component of the spin on the ith lattice site and H is the Hamiltonian given by

$$H=-\sum_{i,j} J_{i,j}\vec{m}_i\cdot\vec{m}_j-\sum_i A_i(m_i^z)^2-\vec{H}_{\rm ext}\cdot\sum_i \vec{m}_i. \tag{10}$$

It was shown by Srivastava and Muller[7] that Eq. (7) represents Hamilton's equations for this system if we write $\vec{m}$ in spherical coordinates as $\vec{m}_i=m_i(\sin\theta_i\cos\phi_i,\sin\theta_i\sin\phi_i,\cos\theta_i)$, and identify ϕ_i and $m_i\cos\theta_i$ with the coordinate and canonical momentum, respectively. We will take $\vec{H}_{{\rm ext},i}$ to include both a static dc field $H_0\hat{z}$ and an ac magnetic field $h_i\hat{x}\sin\omega t$, due to an external radiation field of frequency ω. The quantities $J_{i,j}$, A_i, and h_i were taken to be random variables with square distributions ranging from 0 to J_{max}, A_{max}, and h_{max}, respectively. For comparison, a nondisordered system with $J_{i,j}$ and A_i chosen equal to the average value for these quantities for the disordered crystal was studied as well. Since we wish to study the system at temperatures low compared to the Curie temperature, which is where ferromagnetic resonance is usually per-

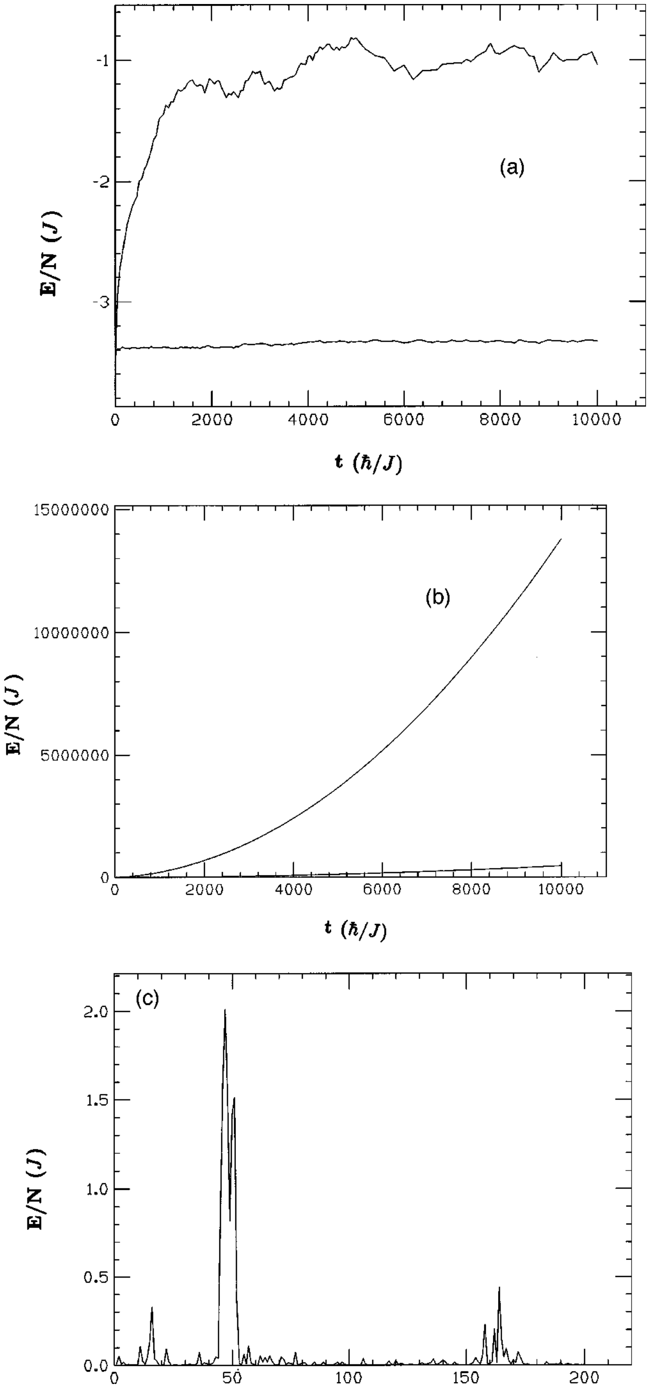

FIG. 4. Energy (in units of J) as a function of time [in units of $(\hbar/J)$] for a 200 spin magnetic chain with $A=2.5J$ and $\omega^2=5.1J$. The upper curve is for a periodic and the lower one is for a random chain in the continuous random model with the same mean values of K and g. (a) For $\gamma=0.0$ and (b) $\gamma=0.02J/\hbar$. (c) Deviation of the anisotropy energy from N for the disordered 200 spin magnetic chain considered in a as a function of position in the lattice (in units of a).

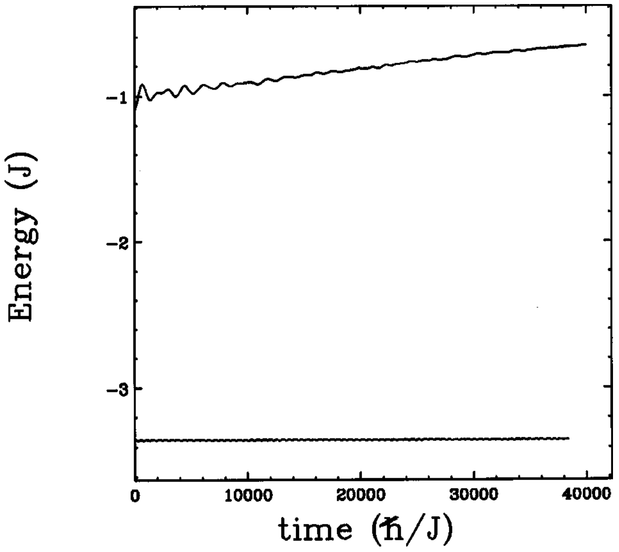

FIG. 5. E in units of J versus time in units of $\hbar/J$ for a 1000 atom Heisenberg model with a 10% impurity concentration with the restriction that no two impurities can be closer than $5a$ with $A_{max}=2.5J, h=H=0.1J$ (the lower curve), and for comparison, the energy for a periodic chain with the same values of A_{max}, h and H (the upper curve) is shown.

formed, our starting spin configuration will have almost all of the spins aligned with the dc field, but with 4 out of 200 spins chosen at random having their direction changed from being lined up with the z axis to making an angle of 0.1 radians with the z axis (lying in the x-z plane). The results of solving Eq. (3) are shown in Fig. 4(a) for a 200 atom chain with γ set equal to zero and for $\gamma=0.02J/\hbar$ in Fig. 4(b). The energy of the magnet increases as a function of time for the nondisordered crystal until the energy of the system is near zero, but for the disordered crystal it does not continue to increase but levels off and fluctuates about a finite value which is not far from its initial value, implying that the disorder suppresses the ability of the magnet to absorb energy from the ac field. In order to support this premise, the deviation of the mean anisotropy energy for the disordered system $N^{-1}A_j\Sigma_j(S_j^z)^2$ from its maximum value of A at the end of the run is plotted as a function of position in the lattice in Fig. 4(c). Again, we find that whatever energy (mostly anisotropy energy) is absorbed, it seems to be absorbed only in localized regions in the crystal. The calculations were repeated with a small nonzero value of γ. The spin system's energy increased as a function of time for both random and periodic systems but the rate of heating was again much larger for the periodic case. This is analogous to the behavior found for the nonlinear vibrational mode model.

We have also done some runs with the substitutional impurity model defined as follows: The distribution of the exchange interaction J is the same as that in the model for disorder described earlier in this section. The anisotropy parameter A takes on the value A_{max} on all sites except for a few randomly chosen impurity sites where it is zero. Since we are interested in studying this system at temperatures low compared with the Curie temperature, we begin the run with all of the spins lined up with the dc field $H\hat{z}$ except for 20 spins, chosen at random, which make an angle of 0.1 radians with the dc field. For these calculations, h_i has the value h_{max} on all sites. The results are given in Fig. 5 for a 1000 spin chain with A_{max} equal to 2.5 J, H and h_{max} equal to 0.1 J, and driving frequency equal to $0.1J/\hbar$, with 10% impurity sites. The distribution of the impurity sites was random but with the restriction that no two impurity sites be less than five spin sites apart as in the vibrational case. For comparison, calculations are also shown for a periodic Heisenberg model with A equal to zero on all sites and h_i chosen to be nonzero only on sites corresponding to the impurity sites in the disordered model, since these were the only sites to which the field coupled in the case of the disordered model that we just discussed (because the field is only close to being in resonance with modes localized on these sites). The frequency is again chosen to be $0.1J/\hbar$. As can be seen, the disordered Heisenberg model does not absorb energy from the field, while the corresponding periodic model does. Again, the behavior shown in Fig. 5 is analogous to that found in Fig. 2 for the analogous vibrational mode study in Sec. II.

The mechanism of energy absorption from the field that we are speaking of corresponds to the energy absorption by spin waves which is known as transverse pumping, which occurs at high power.[6] At lower incident power, the primary mechanisms of energy absorption by the solid are due to damping of the spin waves resulting from coupling of the mode in question with a continuum of spin wave, phonon or electron excitations.[6] The concept being put forward here, which should apply in the low power regime as well, is that localization of such modes in a disordered solid can suppress damping mechanisms, by not allowing modes to exchange energy with each other. It will certainly be the case if the primary mechanism for spin-wave damping is due to spin-wave–spin-wave (and phonon-phonon) interaction resulting from the nonlinear nature of the equations of motion.

We have not included the dipole-dipole interaction because it is generally considerably weaker than the interactions that were included. (Similarly, the dipole-dipole interaction due to elasticity has not been included in the phonon case.) Since this interaction falls off slowly with distance (i.e., as r^{-3}), it can couple states localized around distant lattice sites. Although this interaction leads to diffusion in the Anderson localization problem,[4] because it is very weak, the additional energy diffusion that it introduces in the present problem is expected to be too small to affect our results.

IV. CONCLUSION

Models consisting of a force field acting on a model disordered nonmagnetic solid and a magnetic field acting on a model disordered magnetic solid are studied in order to demonstrate that Anderson localization of the excitations of the solid can lead to a suppression of the rate of energy absorption from an external field by a disordered solid. The model could describe absorption of electromagnetic waves by a solid or kinetic friction. The magnetic system can describe ferromagnetic resonance by a highly disordered solid.

In a solid that is sufficiently disordered the vibrational modes (or spin-wave modes for a magnetic solid) are highly localized. As a consequence, mode resonances that are close to each other in frequency and to the frequency of the external field do not overlap significantly with each other, and as a result, energy absorption from the external field will be highly suppressed, in the sense that there might be no absorption at all except by modes localized on close lying sites. Only coupling of the film to the substrate will result in broadening of these modes, which makes them able to absorb energy, but this coupling can often be reduced to very small values. As the size of the system increases, however, there is often a non-negligible probability that in large systems some modes that are close in frequency will be localized on neighboring lattice sites, resulting in considerable energy absorption by these modes, as has been seen in some of the calculations presented here. It was shown, however, that in the large system limit, the number of overlapping mode pairs which are close in frequency can be much larger for a periodic than for a disordered solid, which, according to the Chirikov criterion, implies that there will be much more energy absorption for a periodic than for a disordered solid.

The arguments used in this paper are based on classical mechanics. The justification for this is that it is expected that coupling of the system to the outside world should lead to decoherence of the quantum system, allowing classical chaos to occur.[8]

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[1] F.M. Israelev and B.V. Chirikov, Dokl. Akad. Nauk. USSR **166**, 57 (1966) [Sov. Phys. Dokl. **11**, 30 (1966)]; B.V. Chirikov, Phys. Rep. **52**, 263 (1979).

[2] E. Fermi, J. Pasta, and S. Ulam, in *Collected Papers of Enrico Fermi 2* (Chicago University Press, Chicago, 1955), p. 978; J. Ford, Phys. Rep. **213**, 271 (1992); J. Ford and G.H. Lunsford, Phys. Rev. A **1**, 59 (1970); S.D. Stoddard and J. Ford, *ibid.* **8**, 1504 (1973); G.H. Walker and J. Ford, Phys. Rev. **188**, 416 (1969).

[3] A.N. Kolmogorov, Dokl. Akad. Nauk SSSR **98**, 527 (1954); V.I. Arnold, Usp. Mat. Nauk **18**, 91 (1963); V. I. Arnold and A. Avez, *Ergodic Problems of Classical Mechanics* (Benjamin, New York, 1968); J. Ford (private communication).

[4] P.W. Anderson, Phys. Rev. **109**, 1402 (1958).

[5] J.B. Sokoloff, Phys. Rev. Lett. **71**, 3450 (1993); Wear **167**, 59 (1993); Phys. Rev. B **52**, 7205 (1995); in *Physics of Sliding Friction*, edited B. N. J. Person and E. Tosatti, NATO ASI Series, Series E: Applied Sciences, Vol. 311 (Kluwer, Dordrecht, 1996), p. 217.

[6] M. Sparks, *Ferromagnetic Relaxation Theory* (McGraw Hill, New York, 1964).

[7] N. Srivastava and G. Muller, Z. Phys. B: Condens. Matter **81**, 137 (1990).

[8] W.H. Zurek, Phys. Today **44** (10), 36 (1991); W.G. Unruh and W.H. Zurek, Phys. Rev. D **40**, 1071 (1989); W.H. Zurek, S. Habib, and J.P. Paz, Phys. Rev. Lett. **70**, 1187 (1993).