

Confinement and Bose condensation in gauge theory of high- T_c superconductors

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(Received 25 June 1999; revised manuscript received 11 October 1999)

The issue of confinement and Bose condensation is studied for gauge models of high- T_c superconductors. First the Abelian-Higgs model in $(2+1)D$, i.e., XY model coupled to lattice gauge field a_μ with coupling g , is studied taking into account both the instantons and vortices. This model corresponds to integer filling of the bosons, and can be mapped to a dual superconductor. Our main result is that the instantons introduce a term which couples linearly to the dual superconductor order parameter, and tend to pin its phase. As a result the vortex condensation always occurs due to the instantons, and the Meissner effect for the gauge field a_μ is absent, although a_μ is massive. This state is essentially the same as the confining phase of the pure gauge model. Away from integer filling, a “magnetic field” μ (the chemical potential of the bosons) is applied to this dual superconductor. Then the Higgs phase revives in the case of weak g and large boson density x , where vortices do not condense in spite of the instantons. In the opposite case, i.e., strong g and small x , phase separation occurs, forming either microscopic patches or macroscopic stripe domains of the Mott insulating state.

I. INTRODUCTION

It has been established that the strong Coulomb repulsion between electrons is the key issue in the physics of high- T_c cuprates. Anderson proposed that this strong correlation gives rise to resonating valence bond (RVB) state, where the spin and charge are carried by spinons and holons, respectively.¹ This phenomenon is called spin-charge separation, and has been subject to intensive studies. One way of formulating the strong electron repulsion is to exclude the double occupancy of the electrons on each site, and study the effective Hamiltonian within that restricted Hilbert space. Slave boson method is a useful tool to implement this constraint and fits the idea of spin-charge separation, where two species of particles, i.e., spinons (fermions $f_{i\sigma}^\dagger, f_{i\sigma}$) and holons (bosons b_i^\dagger, b_i), are introduced to represent electron operator $C_{i\sigma}^\dagger, C_{i\sigma}$ as²⁻⁴

$$\begin{aligned} C_{i\sigma}^\dagger &= f_{i\sigma}^\dagger b_i, \\ C_{i\sigma} &= f_{i\sigma} b_i^\dagger. \end{aligned} \quad (1)$$

Here an electron is represented as the composite particle of spinon and holon. At the mean field level, these two species of particles are supposed to be independent of each other. The mean field phase diagram is determined by two phase transition lines, i.e., the spinon pairing transition and holon condensation characterized by the order parameters $\Delta = \langle f_{i\sigma} f_{j\sigma} \rangle$ and $B = \langle b_i \rangle$, respectively.²⁻⁴ In strange metal state both Δ and B are zero, while only Δ is nonzero in the underdoped “spin gap state” and only B is nonzero in the overdoped “Fermi liquid state.” The superconductivity is realized only when both Δ and B are nonzero, and the onset of the superconductivity is identified as the holon condensation in the underdoped region. However this simple-minded

picture of spin-charge separation needs to be critically studied because the constraint is replaced by the average one in the mean field theory and the more appropriate treatment of this constraint might change the whole picture. This constraint can be taken care of by gauge field which corresponds to the fluctuation of the phase of RVB order parameter and Lagrange multiplier to impose the constraint.⁵⁻⁷ Therefore the effective model is that of the spinons and holons coupled to the gauge field. It should be noted here that the gauge field represents the constraint and does not have its own dynamics. The effective action and the inverse of the coupling constant $1/g$ for the gauge field is generated only after integrating over spinons and holons. Therefore it is a highly nontrivial and crucial issue if the weak coupling perturbative analysis with respect to g makes sense or not.

This issue is closely related to the confinement/deconfinement of the gauge field.^{5,8,9} The original model is defined on a lattice and the gauge field is compact, and it is well known that the gauge field is confining in the strong coupling limit, i.e., large g .¹⁰ A simplified and rough picture of the confinement follows. On the lattice one can define the gauge flux $b(p)$ penetrating each plaquette p , and the action is periodic with respect to $b(p)$ with period 2π . The simplest potential energy for $b(p)$ is $-g^{-1} \cos b(p)$, and the kinetic energy of $b(p)$ is given by $(g/2) e^2$ where e is the electric field canonical conjugate to $b(p)$. A phase transition is possible between two states. One is the extended “Bloch wave state” of $b(p)$ where the tunneling events between different minima of the periodic potential are driven by the large kinetic energy, i.e., large g . The conjugate field e is localized on the other hand, and the string of the electric field is formed when positive and negative gauge charges are inserted with a separation R . This costs an energy proportional to R because of the finite string tension of the electric field. This phenomenon is called confinement. For small g , on the

other hand, the periodic potential is large and $b(p)$ is confined within one minimum. One can replace the periodic potential by the quadratic one $(1/2g)b^2$, which corresponds to the usual Maxwell Lagrangian. The Coulomb law is reproduced in this case, and the periodicity is irrelevant. The gauge field is deconfining in this case. In $(2+1)D$ these tunneling events are represented by the point singularities of the gauge field configuration called magnetic monopoles or instantons.¹¹ In the pure gauge model, the interaction between the magnetic monopoles is $1/r$, i.e., the Coulomb gas. Because the Coulomb gas in $(2+1)D$ is in the screening phase, the gauge field is always confining due to these monopoles or instantons.

When coupled to the holons and spinons, however, the gauge field becomes dissipative when these particles are integrated over, and the deconfining phase becomes possible in the strange metal normal state.¹² However, in the presence of gapless fermion or boson excitations, the integration over them is in general not justifiable. The fermion part is better controlled because of the presence of a large energy scale, the Fermi energy, and techniques such as large N can be used to control the expansion. The boson part is much more problematic because bosons tend to condense in the bottom of their band and we are faced with a strong coupling problem of bosons and gauge fields.

It has been argued that the strong inelastic scattering due to gauge fluctuation suppresses the coherency and hence the ordering temperature, but the effects of the quantum fluctuation in the strong coupling limit is a difficult problem which remains unsolved. In this paper we hope that a duality mapping of a simplified version of the boson-gauge field problem can shed some light on the issue. Summarizing the above, there are two crucial and related issues in gauge models of high- T_c superconductors, i.e., the Bose condensation and the confinement or deconfinement of the gauge field.

In order to clarify these two pictures, we study in this paper a simplified model, i.e., a Higgs model coupled to $U(1)$ gauge field defined on a $(2+1)D$ lattice, which is an important model of broad interests both in condensed matter physics and high-energy physics.^{13,14} The action is given as

$$S = -\kappa \sum_{\text{link}} \cos[\Delta_\mu \theta(i) - qa_\mu(i)] - \frac{1}{g} \sum_{\text{plaquette}} \cos[\Delta_\mu a_\nu(i) - \Delta_\nu a_\mu(i)], \quad (2)$$

where i is the lattice point and μ, ν specify the direction in the $(2+1)D$ lattice. The difference operator Δ_μ is defined as $\Delta_\mu f(i) = f(i + \mu) - f(i)$. Both $\theta(i)$ and $a_\mu(i)$ are compact and defined in the interval $[0, 2\pi]$. q is the (integer) charge of the Higgs bosons and $q = e$ (fundamental) for the holons while $q = 2e$ for the spinon pairing in the $U(1)$ gauge model of high- T_c cuprates.⁷ Hereafter we take the unit $e = 1$ except for Eq. (27) in Sec. IV. The amplitude of the bose field has been fixed, but the vortex excitations are allowed due to the lattice and the compactness. Note that, in contrast to the high T_c problem, the gauge field dynamics is Maxwellian in the continuum limit. This model has been studied extensively, and the essential features are as follows:

(1) In the limit $g \rightarrow 0$, the gauge fields are frozen out and we recover the XY model. For $d+1 \geq 2$, we expect a phase transition at $\kappa = \kappa_c$. The ordered phase in $d+1 \geq 3$ is characterized by an order parameter $\langle e^{i\theta} \rangle \neq 0$. For g nonzero, however, this object is not gauge invariant and can no longer serve as an order parameter.¹⁵ Instead we may fix the gauge to be, for instance, the unitary gauge $\theta = 0$ and consider small gauge fluctuations. Then we have the Higgs mechanism where a term $\rho_s a^2$ appears in the action (ρ_s : the superfluidity density). The gauge flux correlation has the form

$$\langle b_\mu b_\nu \rangle = \frac{k^2}{\rho_s} \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right). \quad (3)$$

(2) The limit $\kappa = 0$ yields the compact QED model in $(2+1)$ dimensions, which is known to be confining due to the appearance of instantons.¹¹ The instantons are singular configurations of \vec{a} which act as magnetic monopoles, i.e., sources and sinks of magnetic fields. Writing $\vec{a} = \vec{a}_0 + \vec{a}_{\text{inst}}$ as the sum of nonsingular and singular configurations, we have

$$\vec{b}_{\text{inst}} = \vec{\nabla} \times \vec{a}_{\text{inst}}. \quad (4)$$

For a given instanton density ρ_M , we have the analog of Poisson's equation

$$\vec{\nabla} \cdot \vec{b}_{\text{inst}} = \vec{\nabla} \cdot \vec{\nabla} \times \vec{a}_{\text{inst}} = 4\pi\rho_M. \quad (5)$$

If the instantons form a gas, the density-density correlation function is given by

$$\langle \rho_M(\vec{k}) \rho_M(-\vec{k}) \rangle = \frac{1}{(4\pi)^2} \frac{M_0^2 k^2}{M_0^2 + k^2} \quad (6)$$

in analogy with the more familiar density-density correlation function of a Coulomb gas, where M_0 plays the role of the inverse screening length. Combining Eq. (5) and Eq. (6) we find

$$\langle b_{\text{inst}}^\mu b_{\text{inst}}^\nu \rangle = \frac{k_\mu k_\nu}{k^2} \frac{M_0^2}{M_0^2 + k^2}. \quad (7)$$

When combined with the standard transverse correlation from the nonsingular part \vec{a}_0 , we have

$$\langle b_\mu b_\nu \rangle = \delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} + \frac{k_\mu k_\nu}{k^2} \frac{M_0^2}{M_0^2 + k^2} = \delta_{\mu\nu} - \frac{k_\mu k_\nu}{M_0^2 + k^2}. \quad (8)$$

Equation (8) shows that the electromagnetic field acquires a mass due to the gas of instantons.¹¹

Thus we see that in both the $g \rightarrow 0$ limit and the $\kappa = 0$ limits, the gauge field is massive. This inspired Fradkin and Shenker¹³ to consider whether the two limits are smoothly connected to each other. Their conclusion is as follows:

(1) When the charge is fundamental ($q = 1$), the strong coupling expansion argument shows that the Higgs phase with large κ and small g continues smoothly to the confinement phase with large g and small κ .

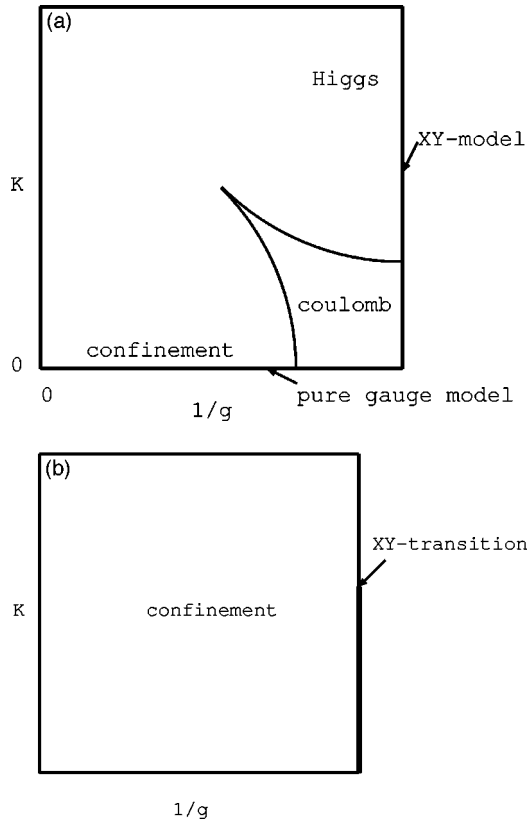


FIG. 1. Phase diagrams for Abelian Lattice Higgs model with fundamental charge in (3+1)D (a) and (2+1)D (b). In (b) the phase transition is isolated along the XY-model line, i.e., $g=0$. In the confinement phase, the gauge field is massive due to instantons, although no Meissner effect occurs.

(2) When the charge is not fundamental, e.g., $q=2$, the Higgs and the confinement are different phases and can be distinguished by the forces between the test charges $q_{\text{test}} = \pm 1$.

In the bulk of this paper we will focus on the $q=1$ case. The phase diagram for (3+1)D and $q=1$ case has been determined at least qualitatively, and consists of two phases, i.e., the Higgs-confinement and Coulomb phases as shown in Fig. 1(a). In the Coulomb phase, the gauge field is deconfining and massless and the Bose field remains disordered. However, the phase diagram for (2+1)D and $q=1$, which is the most relevant case to high- T_c cuprates, remains controversial.^{13,14} One proposal is that the XY transition becomes first order once the coupling g to the gauge field is turned on, due to the Weinberg-Coleman mechanism.¹⁶ This first order transition line terminates at some critical point. This resembles the vapor-liquid phase diagram. The other one is that a finite region of Coulomb phase exists.¹⁴

Since instantons dominate the physics for $\kappa=0$, the key question is whether instantons play an important role for general κ and g . Vortex lines in the Bose liquid carry unit flux quanta and these can originate and terminate at instantons and anti-instantons. This is illustrated in Fig. 2. Einhorn and Savit discussed the free energy of the finite vortex segments, and concluded that the XY transition remains at least for weak g .¹⁴ This leads to a picture of a finite region of the Coulomb phase mentioned earlier. Apart from the phase diagram, another issue is the behavior of the magnetic field

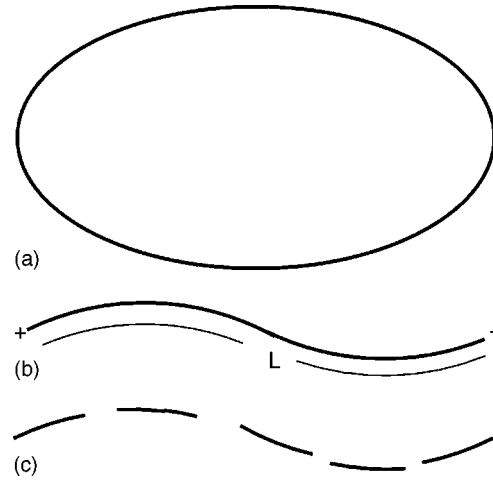


FIG. 2. (a) Vortex loop, (b) a vortex segment, and (c) vortex segment broken into pieces by the instantons. A vortex segment terminates at the instanton and anti-instanton.

correlation function in the region commonly labeled as confinement-Higgs after the work of Fradkin and Shenker.¹³ The point we want to raise in this paper is that even though the gauge field is massive, the correlation function in the Higgs phase and the confinement phase are very different, being given by Eq. (3) and Eq. (8), respectively. The question is then whether the correlation function is closer to Eq. (3) or Eq. (8) in the confinement-Higgs region. As far as we know, this question has not been addressed. In this paper we include the effect of instantons for general κ and g but we come to quite a different conclusion than Einhorn and Savit.¹⁴ The phase diagram we propose is shown in Fig. 1(b). The line $g=0$ is an isolated line with an isolated XY transition. We also calculate the magnetic field correlation function and show that it takes the confinement form of Eq. (8). Thus we conclude that the Higgs-confinement phase is better described as the confinement phase. This is the main result of this paper which we discuss in Sec. II.

It is known that the model given by Eq. (1) describes a Bose gas on a lattice where the density is an integer per a lattice site. Thus the confinement phase may be understood as a Bose Mott insulator. In Sec. III we extend the discussion to the incommensurate case, when the density is not an integer. We find two possibilities. The instantons may become irrelevant and the system becomes a superfluid (Higgs phase), or the system may break up into domains. In Sec. IV we briefly address the case $q=2$ and also the case of two kinds of bosons with opposite gauge charges, which arises in the SU(2) formulation of the t - J model.

II. THE ABELIAN-HIGGS MODEL

We start with the continuum action for Eq. (2):

$$S = \int d^3x \left[\frac{1}{2} \kappa [\nabla \theta(x) - \vec{a}(x) - \vec{A}(x)]^2 + \frac{1}{2g} (\nabla \times \vec{a})^2 \right]. \quad (9)$$

Here we have introduced the external electromagnetic field A_μ , which is put to be zero for the moment. We allow the singular configurations of θ and \vec{a} corresponding to the vor-

text and instanton, respectively. As for the vortex, $\theta(x)$ is divided into the single-valued part θ_0 and the multivalued vortex part θ_V as $\theta = \theta_0 + \theta_V$, and $\nabla \times \nabla \theta_V / 2\pi = \vec{j}_V$ is the vortex current. As for the instantons, $\vec{a} = \vec{a}_0 + \vec{a}_{\text{inst}}$ and $\nabla \cdot \nabla \times \vec{a}_{\text{inst}} / 4\pi = \rho_M$ is the instanton density as defined in Eqs. (4) and (5). We next derive a continuum version of the duality representation of this problem. The duality representation is a powerful tool which allows us to discuss the strong coupling limit (large g) and is particularly useful for the present problem. Introducing the Stratonovich-Hubbard field \vec{J} representing the boson current, the action becomes

$$S = \int d^3x \left[\frac{1}{2\kappa} \vec{J}^2 + i\vec{J}[\nabla \theta(x) - \vec{a}(x)] + \frac{1}{2g} (\nabla \times \vec{a})^2 \right] \quad (10)$$

and after integrating over θ_0 , we obtain $\nabla \cdot \vec{J} = 0$ corresponding to the conservation of the boson current. To enforce this conservation law, we introduce the vector potential \vec{c} to represent \vec{J} as $\vec{J} = \nabla \times \vec{c}$. Then after partial integration the action becomes

$$S = \int d^3x \left[\frac{1}{2\kappa} (\nabla \times \vec{c})^2 + i\vec{c}[2\pi \vec{j}_V - \vec{b}(x)] + \frac{1}{2g} (\vec{b})^2 \right] \quad (11)$$

with $\vec{b} = \nabla \times \vec{a}$. After integrating over \vec{b} , we obtain

$$S = \int d^3x \left[\frac{1}{2\kappa} (\nabla \times \vec{c})^2 + \frac{g}{2} (\vec{c})^2 + i2\pi \vec{c} \cdot \vec{j}_V \right]. \quad (12)$$

This is essentially equivalent to the action obtained by Einhorn-Savit in terms of the lattice formulation.¹⁴ It is noted that if we view \vec{c} as a gauge field coupled to the vortex current, the gauge symmetry is broken in Eq. (12), which corresponds to the nonconservation of the vortex current, i.e., $\nabla \cdot \vec{j}_V$ can be nonzero, due to the instantons. After integrating over the field \vec{c} in Eq. (12), the partition function Z is given by the integral over the vortex configurations $\{\vec{j}_V\}$ including both the vortex loop and open vortex segments the two ends of which are instanton and anti-instanton.¹⁴

$$Z = Z_0 \sum_{\{\vec{j}_V\}} \exp \left[\sum -4\pi^2 \kappa j_{V\mu}(j) D_{\mu\nu}(j-k, m^2) j_{V\nu}(k) \right], \quad (13)$$

where $D_{\mu\nu}(j-k, m^2)$ is the propagator of the field \vec{c} with the mass $m^2 = \kappa g$. This propagator is given as

$$D_{\mu\nu}(j-k, m^2) = \left[\delta_{\mu\nu} - \frac{\Delta_\mu \Delta_\nu}{m^2} \right]_j D(j-k, m^2), \quad (14)$$

where $D(j-k, m^2)$ satisfies

$$(-\Delta_\mu^2 + m^2) D(j-k, m^2) = \delta_{jk}. \quad (15)$$

The propagator decays exponentially as $D(j-k, m^2) \sim e^{-m|j-k|}$, and the interaction between the vortex segments are short range.

Let us start with a qualitative estimate of the free energy of the vortex loop segment regarding the partition function as

that of a classical statistical mechanics. We first repeat the argument of Einhorn and Savit.¹⁴ Let L be the length of the vortex loop segment. Then the free energy is the sum of the energy cost and the entropy as¹⁴

$$F_{\text{loop}} = 4\pi^2 \kappa D(0; m^2) L - \ln \tilde{\mu}^L,$$

$$F_{\text{segment}} = 4\pi^2 \kappa D(0; m^2) L - \ln(\tilde{\mu}^L L^\gamma) + 2S_{\text{inst}}, \quad (16)$$

where $S_{\text{inst}} = 4\pi^2 g^{-1} D(0; m^2)$ is the action for an instanton, and $\tilde{\mu}$ is a number of order unity which depends on lattice and dimension. The main difference between closed loop and open segment is that the two instanton action is added and entropy is enhanced by the factor L^γ ($\gamma > 0$) in the case of open segment.¹⁷ However the leading L -linear terms are the same for both of closed loop and open segment, and Einhorn-Savit concluded that the proliferation of the vortices, i.e., the appearance of the infinite length loop/segment, occurs at $4\pi^2 \kappa D(0; m^2) = \ln \tilde{\mu}$. This is the XY-like phase transition viewed in the duality picture. However the above consideration neglects the possibility that the long vortex loop segment are cut by the instantons into small pieces, as shown in Fig. 2(c). Let us consider that the total length L is cut into n pieces of open segments. The free energy in this case is

$$F(L, n) = [4\pi^2 \kappa D(0; m^2) - \tilde{\mu}] L - \gamma n \ln(L/n) + 2S_{\text{inst}} n. \quad (17)$$

Minimizing this with respect to n , we obtain the length of the pieces as $L/n = e^{1+2S_{\text{inst}}/\gamma}$, which is finite as $L \rightarrow \infty$. Then due to the finite density of instantons, the infinite length loop or segment does not appear. Instead the vacuum is full of finite size open segments and closed loops of vortex. The total length of these loops and segments are infinite as the sample size $L \rightarrow \infty$ when $4\pi^2 \kappa D(0; m^2) - \mu < 0$, but they are all finite size. Therefore we conclude that the phase diagram of (2+1)D Abelian Higgs model is given by Fig. 1(b), i.e., all the interior constitutes a single phase. Analogy of the phase diagram with the Ising model is helpful here. In the duality picture, i.e., regarding the vortex field as the order parameter, κ can be regarded as the temperature T , and the instanton density $e^{-\text{const}/g}$ is the magnetic field H . Then it is natural that the ordered state at $1/g = \infty$, i.e., $H = 0$, is the isolated line while all the other phase diagram is connected to the high temperature symmetric phase. Actually this analogy becomes more clear when one consider the path integral formulation of the the Ising model in a magnetic field H .^{17,18} The partition function is represented by the sum over the closed loop and the open segment ended at the magnetic field vertex H . This is exactly similar to the present case where the vortex segment terminates at instantons except that the vortex loop/segment has a direction and instanton has the \pm topological charge.

Now what is the nature of this single phase? One crucial question is whether the Meissner effect for \vec{a} remains or not. It should be noted that the magnetic field \vec{b} is tied to the vortices. In the case of closed loop the net magnetic field \vec{b} is zero. The open segment, on the other hand, is a magnetic dipole which has net \vec{b} . Therefore once the instanton fugacity is nonzero and there are vortex open segments, the magnetic

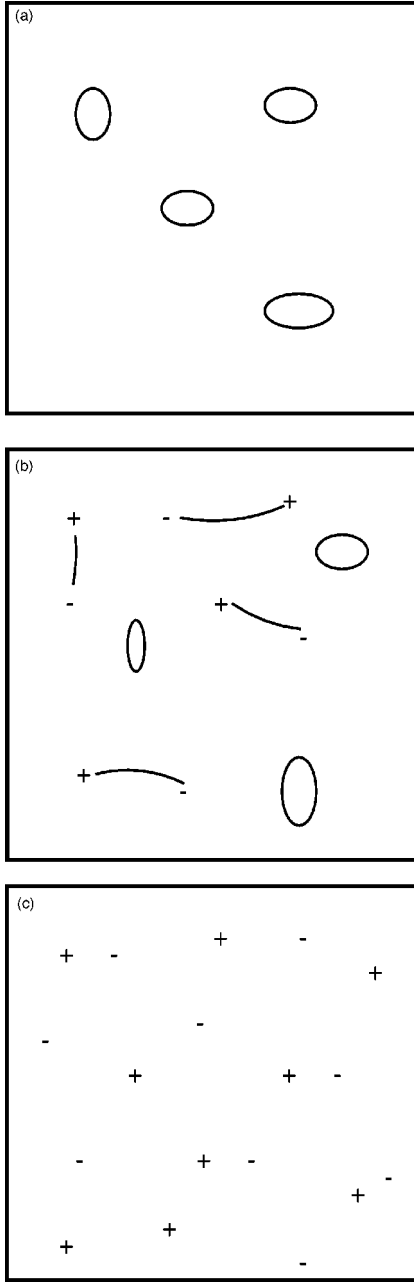


FIG. 3. Three possible states for the magnetic charges. (a) The vacuum state where only vortex loops of finite size exist. The gauge flux b cannot exist inside the sample, i.e., the Meissner effect occurs, and this is the superfluid state of the original bosons. (b) The dielectrics with finite size vortex segments and loops. The magnetic field b can penetrate into the sample, and the Meissner effect vanishes. The string tension of the vortex then becomes zero, and the confinement of the instanton and anti-instanton disappears. Therefore this state is unstable to (c). (c) The metallic state of the magnetic charges. The metallic screening prevents the gauge field from penetrating into the sample. This is the confining state of the gauge field.

field \vec{b} can penetrate into the sample, i.e., the Meissner effect disappears and the gauge field \vec{b} becomes massless. This corresponds to the “dielectrics” of the magnetic charge and is illustrated in Fig. 3(b). The instantons and anti-instantons appear to be bound into pairs. However, this state is not stable because once the Meissner effect is gone, the confine-

ment between the instanton and anti-instanton also disappears, and the magnetic dipole is liberated into free magnetic charges. This is the “metal” of the magnetic charges, and again the magnetic field \vec{b} cannot penetrate into the sample due to the screening. This is shown in Fig. 3(c). This massive gauge field \vec{b} corresponds to the confining phase of the pure gauge model discussed by Polyakov.¹¹ Therefore the state in the interior of the phase diagram in Fig. 1(b) continues smoothly not to the Higgs phase but to the pure gauge model, and should be called “confinement.”

In order to substantiate the above consideration, we now go to the second quantization formulation of the vortex system. Let us first divide the gauge field \vec{c} into the transverse and longitudinal parts, i.e., $\vec{c} = \vec{c}_\perp + \nabla\phi$. Then Eq. (11) is written as

$$S = \int d^3x \left[\frac{1}{2\kappa} (\nabla \times \vec{c}_\perp)^2 + \frac{g}{2} (\vec{c}_\perp)^2 + \frac{g}{2} (\nabla\phi)^2 + i2\pi \vec{c}_\perp \cdot \vec{j}_V + i\phi \cdot \rho_M \right], \quad (18)$$

where we have performed a partial integration and identified $\nabla \cdot \vec{j}_V$ with the instanton density ρ_M . We can view Eq. (18) as describing world-lines of vortex-particles which are coupled to the gauge field by the $i2\pi \vec{c}_\perp \cdot \vec{j}_V$ term. The vortices may be created and annihilated at instantons located at $x_1 \cdots x_n$ and anti-instantons located at $y_1 \cdots y_m$. Alternatively, we can write the action in terms of the second quantized vortex field ψ_V . The action is given by

$$S_V = \int d^3x \left[\frac{1}{2\kappa} (\nabla \times \vec{c}_\perp)^2 + \frac{g}{2} (\vec{c}_\perp)^2 + \frac{g}{2} (\nabla\phi)^2 + \psi_V^\dagger \frac{1}{2} \times [-K(\nabla + i\vec{c}_\perp)^2 + M^2] \psi_V + u(\psi_V^\dagger \psi_V)^2 + i\phi \cdot \rho_M \right], \quad (19)$$

where $M^2 = 4\pi^2 \kappa D(0; m^2) - \ln \tilde{\mu}$, and u represents the short range repulsion between the vortex segments. The gradient term comes from the extra cost of the action when the vortex line deviates from the straight line. This step is standard in the duality mapping. The novel feature of creation and annihilation of vortices can be included by summing over all instanton configurations as follows:

$$Z = \int D\psi_V^\dagger D\psi_V D\vec{c}_\perp D\phi \sum_{n,m=0}^{\infty} \int \frac{dx_1 \cdots dx_n}{n!} \int \frac{dy_1 \cdots dy_m}{m!} \times [z\psi_V^\dagger(x_1)e^{i\phi(x_1)}] \cdots [z\psi_V^\dagger(x_n)e^{i\phi(x_n)}] \times [z\psi_V(y_1)e^{-i\phi(y_1)}] \cdots [z\psi_V(y_m)e^{-i\phi(y_m)}] e^{-S_V}, \quad (20)$$

where z is the fugacity of the instantons which is roughly given as $z \sim e^{-S_{\text{inst}}}$. n, m are the number of instantons and anti-instantons, but only the term $n=m$ survives when one integrates over ψ_V, ψ_V^\dagger . As in the usual Coulomb gas mapping, the summation over n, m in Eq. (20) can be done, and our final result for the action, after reversing the step between Eqs. (11) and (12) to restore the original gauge field \vec{a} , is given as

$$\begin{aligned}
S = \int d^3x & \left[\frac{1}{2\kappa} (\nabla \times \vec{c}_\perp)^2 + \frac{1}{2g} (\nabla \times \vec{a})^2 - i\vec{c} \cdot \nabla \times \vec{a} \right. \\
& + \frac{1}{2} \psi_V^\dagger [-K(\nabla + i\vec{c}_\perp)^2 + M^2] \psi_V + u(\psi_V^\dagger \psi_V)^2 \\
& \left. - z(\psi_V^\dagger e^{i\phi} + \psi_V e^{-i\phi}) \right] \quad (21)
\end{aligned}$$

where it should be noted again that $\vec{c} = \vec{c}_\perp + \nabla\phi$ and $\vec{a} = \vec{a}_0 + \vec{a}_{\text{inst}}$. The gauge transformation $\psi_V \rightarrow \psi_V e^{i\phi}$, $\psi_V^\dagger \rightarrow \psi_V^\dagger e^{-i\phi}$ eliminates the exponential factor in the z -term, and also replaces \vec{c}_\perp by \vec{c} in the minimal coupling term. We have

$$\begin{aligned}
Z &= \int D\psi_V^\dagger D\psi_V D\vec{c} D\vec{a} e^{-S}, \quad (22a) \\
S &= \int d^3x \left\{ \frac{1}{2\kappa} (\nabla \times \vec{c})^2 + \frac{1}{2g} (\nabla \times \vec{a})^2 - i\vec{c} \cdot \nabla \times \vec{a} \right. \\
& + \frac{1}{2} \psi_V^\dagger [-K(\nabla + i\vec{c})^2 + M^2] \psi_V + u(\psi_V^\dagger \psi_V)^2 \\
& \left. - z(\psi_V^\dagger + \psi_V) \right\}. \quad (22b)
\end{aligned}$$

Except for the last term, Eq. (22b) is the standard duality representation of the Abelian Higgs model. The last term in Eq. (22b) represents the effect of the instantons and is the main result of this paper. We note that it takes the form of an external field coupled to the vortex field ψ_V . This is analogous to the Josephson coupling to an external superfluid with an order parameter z . The external order parameter will induce a nonzero order parameter $\langle \psi_V \rangle \neq 0$, even when $M^2 > 0$. (Note we have fixed the gauge and the nonzero order parameter is in the particular gauge choice.) It is useful to recall that vortices in the field ψ_V correspond to world lines of the original bosons. Condensation of the vortex field ψ_V means the absence of Bose condensation and vice versa. In the first quantized picture of vortex loops and segments, this can also be understood as follows. Consider the correlation function $C(\vec{x}, \vec{x}') = \langle \psi_V^\dagger(\vec{x}) \psi_V(\vec{x}') \rangle$. In the absence of instantons, the points \vec{x} and \vec{x}' are connected by a vortex line and long-range order is possible only when an infinite vortex line has zero energy, i.e., $M^2 < 0$. However, with finite z , an instanton and an anti-instanton appear near \vec{x}' and \vec{x} and create two finite segments, so that $C(\vec{x}, \vec{x}')$ reaches a finite value even as the separation between \vec{x} and \vec{x}' goes to infinity.

Recall that in the duality picture, $\langle \psi_V \rangle \neq 0$ means that the original boson is not Bose condensed. Thus we expect that the effect of the $z(\psi_V + \psi_V^\dagger)$ term is to destroy the Meissner effect of the original Abelian-Higgs theory. We check this by an explicit calculation of the gauge field correlation function. Let us represent ψ_V as $\psi_V = \psi_0 e^{i\phi}$. Then the action for the vortex field becomes

$$S_{\text{vortex}} = \int d^3x \frac{1}{2} [K\psi_0^2 (\nabla\phi + \vec{c})^2 - 4z\psi_0 \cos\phi] \quad (23)$$

which is the sine-Gordon model in $(2+1)D$. According to the analysis of the pure gauge model by Polyakov,¹¹ the fugacity z is always relevant and the ϕ -field is massive. Replacing the cos term by the effective quadratic term as

$$S_{\text{vortex}} = \int d^3x \frac{1}{2} [K\psi_0^2 (\nabla\phi + \vec{c})^2 + 2z_{\text{eff}}\psi_0\phi^2] \quad (24)$$

which gives the mass $m_0 = \sqrt{2z_{\text{eff}}(K\psi_0)}$ of the ϕ field corresponding to the screening. After integrating over ϕ , we obtain the effective action for the gauge fields \vec{c} , \vec{a} and the external electromagnetic field \vec{A} as

$$\begin{aligned}
S &= \frac{1}{2} \sum_q \sum_{\mu\nu} \left\{ \left[\frac{q^2}{\kappa} \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) + K\psi_0^2 \left(\delta_{\mu\nu} \right. \right. \right. \\
& \left. \left. - \frac{q_\mu q_\nu}{q^2 + m_0^2} \right) \right] c_\mu(q) c_\nu(-q) + \frac{1}{g} \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \\
& \times a_\mu(q) a_\nu(-q) - i\delta_{\mu\nu} \{ [a_\mu(q) + A_\mu(q)] c_\nu(-q) \\
& \left. + c_\mu(-q) [a_\nu(q) + A_\nu(q)] \} \right\}. \quad (25)
\end{aligned}$$

After integrating over \vec{c} , we obtain the propagator of the gauge flux $\vec{b} = \nabla \times \vec{a}$ as

$$\begin{aligned}
& \langle b_\mu(q) b_\nu(-q) \rangle \\
&= \frac{1}{g + K\psi_0^2 + q^2/\kappa} \delta_{\mu\nu} \\
& \quad - q_\mu q_\nu \frac{g[m_0^2 + \kappa K\psi_0^2 + q^2]}{[m_0^2(K\psi_0^2 + g)/g + q^2][\kappa(K\psi_0^2 + g) + q^2]}. \quad (26)
\end{aligned}$$

It should be noted that while the pole $1/q^2$ disappeared and the gauge field is massive, there is no Meissner effect because Eq. (26) is not proportional to q^2 as in Eq. (3). Rather, the small q behavior is essentially the same as that in the confining phase in the pure gauge model shown in Eq. (8).¹¹

After integrating over the \vec{b} field, the effective action for the external e.m. field is $\sim (\nabla \times \vec{A})^2$, which means that the system is insulating. This is perhaps not surprising if viewed in the strong coupling limit $g \rightarrow \infty$. Then the gauge field does not have its own dynamics and serves to impose the constraint of integer occupation at each lattice site. The bosons are just frozen into place on each site, resulting in an insulator. This Mott insulator phase appears to extend to include the entire phase diagram, as shown in Fig. 1(b), with the exception of the line $1/g = 0$.

At finite temperature T , the imaginary time axis becomes finite, i.e., $[0, \beta = 1/T]$, in Eq. (23). Therefore Eq. (23) describes the sine-Gordon model in 2D in the long wavelength limit. Therefore we expect the KT transition, i.e., confinement-deconfinement transition, occurs at some critical temperature T_c .

III. BOSONS WITH NONINTEGER DENSITY

The Abelian Higgs model in Eq. (1) corresponds to the case of integer boson filling per each site. The deviation from it is taken care of by introducing the winding number of the phase θ along the time direction.¹⁹ In the dual picture, the boson density is represented by the z component of the gauge flux $\nabla \times \vec{c}$, and the deviation from the integer filling is represented by adding the term $-\mu(\nabla \times \vec{c})_z$ to the action Eq. (22b). Here the chemical potential μ acts as the magnetic field. Therefore the chemical potential and the vortex condensation compete with each other as in the case of the superconductor in a magnetic field. The only new aspect here is the instanton term $z(\psi_V + \psi_V^\dagger)$. When $\mu=0$, this term induces the vortex condensation $\langle \psi_V \rangle$ even if M^2 is positive and large. For $\mu \neq 0$, we may be tempted to consider an Abrikosov vortex state (of the vortex field ψ_V) in analogy with type II superconductors. An ordered array of such vortices correspond to a Wigner crystal of boson.¹⁹ However, the phase of ψ_V changes by 2π around each vortex and we cannot gain the Josephson energy from the term $-z(\psi_V + \psi_V^\dagger)$. We conclude that the Wigner crystal is suppressed by the instantons. A second idea is to consider the analogy of the intermediate state in type I superconductors, where the stable configuration is the laminar structure.²⁰ In type I superconductors the surface energy between the normal and superconducting regions is positive. The surface energy is proportional to the size along the z axis (β in the present context) and the spacing of the laminar structure is macroscopic in size. In the Appendix, we perform a Ginzburg-Landau calculation of the surface energy, and find that in contrast to usual type I superconductors, it is negative in the case $M^2 > 0$, i.e., when the superconductivity is induced by Josephson coupling. This implies that the straight interface is unstable, and the laminar phase will break up. One possibility is that the system breaks up into patches where $\langle \psi_V \rangle \neq 0$, separated by regions where $\langle \psi_V \rangle = 0$. (This can be viewed as the complement of the Abrikosov vortex state.) The order parameter can be real in each patch, gaining an extensive Josephson energy from the term $-z(\psi_V + \psi_V^\dagger)$. The magnetic field $\nabla \times \vec{c}$ can penetrate the normal region and partially penetrate the patches. This state can maximize the surface energy gain for a fixed patch area and the patches will form some ordered structure. In the original boson representation, the absence of long-range order in ψ_V means that the bosons form a superfluid with Meissner effect. The instantons become irrelevant in this case, in contrast to the $\mu = 0$ case. The effect of the instanton is to cause a periodic modulation of the boson density (corresponding to the modulation of the magnetic field $\nabla \times \vec{c}$ by the patches). This modulation is weak for κ large (M^2 positive and large) and grows with decreasing κ . An ordered array of patches lead to a kind of incommensurate order. For very small κ , $M^2 < 0$ and there is a strong tendency for the order parameter ψ_V to form in the dual picture. In this case the interface energy may become positive and we cannot rule out a laminar picture. In the original boson picture this corresponds to stripes of superfluids separated by Mott insulators. The transition between the stripe phase depends on details of the parameters and we have not attempted to work it out quantitatively.

However, since stripes occur only for positive interface energy in our model, the stripe size is expected to be macroscopic, by analogy with the laminar phase of type I superconductors. Microscopic stripes might occur when $M^2 < 0$ and other interactions such as long range Coulomb forces and commensurability energy are introduced. We have not examined this issue in this paper, and this is left for future studies.

IV. OTHER ORDER PARAMETERS

Up to now, we consider the charge e bosons coupled to the gauge field. This model corresponds to the boson sector of the U(1) gauge theory. However other types of order parameter appear in gauge models of high- T_c superconductors, which is the subject of this section.

First we consider the case of $q=2e$, which corresponds to the spinon pairing order parameter coupled to the U(1) gauge field. Here we take the unit where $2e=1$ and the flux quantization is reduced to half. Then the instanton becomes the end point of two vortices, which modifies the z term in Eq. (22b) as

$$S = \int d^3x \left\{ \frac{1}{2\kappa} (\nabla \times \vec{c})^2 + \frac{1}{2g} (\nabla \times \vec{a})^2 - i\vec{c} \cdot \nabla \times \vec{a} + \frac{1}{2} \psi_V^\dagger [-K(\nabla + i\vec{c})^2 + M^2] \psi_V + u(\psi_V^\dagger \psi_V)^2 - z(\psi_V^\dagger \psi_V^\dagger + \psi_V \psi_V) \right\}. \quad (27)$$

It is noted that the z term is the quadratic term and does not necessarily enforce the condensation of ψ_V . Therefore two possibilities arise in this case.

(I) Single vortex condensation, i.e., $\langle \psi_V \rangle \neq 0$. In this case the quantized charge, i.e., the integral of $(\nabla \times \vec{c}_\perp)_z$ is $2e$ and the single charge e cannot appear. Therefore the charge e is confined.

(II) Vortex pair condensation, i.e., $\langle \psi_V \psi_V \rangle \neq 0$ while $\langle \psi_V \rangle = 0$. In this case the quantized charge is reduced to half, i.e., e . Therefore the confinement of the charge e does not occur.

Figure 4 shows the phase diagram for $q=2e$, where the above two possibilities correspond to I and II, respectively. As for the small fluctuation of \vec{a} is concerned, there occurs no Meissner effect in either phase, although \vec{a} is massive and takes the form of Eq. (8). Therefore both phases are better called the confining phase for the charge $2e$. What distinguishes these two phases is the discrete Z_2 symmetry.¹³ It has been discussed that the limit $\kappa = \infty$ is the Ising gauge model, which shows confinement and deconfinement transition of charge e at some critical value of $g = g_c$.¹⁰

Next we study the case of two species of bosons b_1, b_2 coupled to the gauge field with opposite charges e and $-e$, respectively. This situation occurs in the staggered flux state of an SU(2) formulation for underdoped region.²¹ In this case the dual model is given by

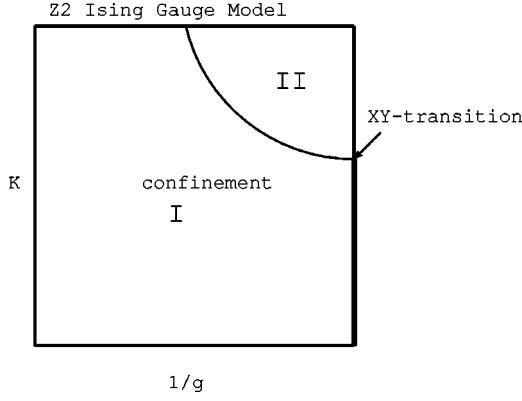


FIG. 4. Phase diagram for $q=2e$. In region I the single vortex condensation occurs and charge e is confined. In region II only vortex pairs condense and charge e is not confined. In the limit $\kappa = \infty$, the model is reduced to Ising gauge model, which shows a phase transition.

$$\begin{aligned}
 S = \int d^3x & \left\{ \frac{1}{2\kappa} [(\nabla \times \vec{c}_1)^2 + (\nabla \times \vec{c}_2)^2] + \frac{1}{2g} (\nabla \times \vec{a})^2 \right. \\
 & - i\vec{c}_1 \cdot \nabla \times (\vec{a} + \vec{A}_1) - i\vec{c}_2 \cdot \nabla \times (-\vec{a} + \vec{A}_2) - i\mu [\nabla \times (\vec{c}_1 \\
 & + \vec{c}_2)]_z + \frac{1}{2} \psi_1^\dagger [-K(\nabla + i\vec{c}_1^2 + M^2)\psi_1 + \psi_2^\dagger [-K(\nabla \\
 & + i\vec{c}_2)^2 + M^2]\psi_2 - z(\psi_1^\dagger \psi_2 + \psi_2^\dagger \psi_1) + u[(\psi_1^\dagger \psi_1)^2 \\
 & \left. + (\psi_2^\dagger \psi_2)^2] + 2w(\psi_1^\dagger \psi_1)(\psi_2^\dagger \psi_2) \right\}, \quad (28)
 \end{aligned}$$

where \vec{c}_i ($i=1,2$) is the gauge field representing the boson current of b_i , ψ_i is the corresponding vortex field and \vec{A}_i is the test field coupled to it. The real electromagnetic field corresponds to $\vec{A}_1 = \vec{A}_2 = \vec{A}$. It is noted here that again the instanton term (z term) is quadratic in ψ 's and cannot induce nonvanishing ψ_1 and ψ_2 when z is not large enough. The Bose condensation of b_1 and b_2 should occur in this case. When g and z is large, which is relevant to the high- T_c problem, the amplitudes of both ψ_1 and ψ_2 are induced and we write $\psi_i = \psi_0 e^{i\varphi_i}$. Here the singular vortex configuration of φ_i is allowed, which corresponds to the original boson. Then the effective action for the phase field φ_i is given by

$$\begin{aligned}
 S_{\text{eff}} = \int d^3x & \left[\frac{1}{2} K \psi_0^2 [(\nabla \varphi_1 + \vec{c}_1)^2 + (\nabla \varphi_2 + \vec{c}_2)^2] \right. \\
 & \left. - 2z \psi_0^2 \cos(\varphi_1 - \varphi_2) \right]. \quad (29)
 \end{aligned}$$

Here we define the symmetric and antisymmetric parts as

$$\begin{aligned}
 \varphi_1 &= \varphi_s + \frac{1}{2} \varphi_a, \\
 \varphi_2 &= \varphi_s - \frac{1}{2} \varphi_a, \quad (30)
 \end{aligned}$$

and \vec{c}_s, \vec{c}_a in a similar way. Then Eq. (29) is written as

$$\begin{aligned}
 S_{\text{eff}} = \int d^3x & \left[\frac{1}{2} K \psi_0^2 \left[2(\nabla \varphi_s + \vec{c}_s)^2 + \frac{1}{2} (\nabla \varphi_a + \vec{c}_a)^2 \right] \right. \\
 & \left. - 2z \psi_0^2 \cos(\varphi_a) \right], \quad (31)
 \end{aligned}$$

where only the antisymmetric part is coupled to the instantons. Therefore the action for the antisymmetric part is the same as that in Eq. (23), i.e., φ_a is fixed by the z term, and the vortex of the φ_a field is forbidden. Calculations similar to Eqs. (23)–(26) show that there is no Meissner effect for the gauge field \vec{a} , although it is massive as in the pure gauge model. This corresponds to the binding or confinement of the two species of bosons b_1 and b_2 because $(\nabla \times \vec{c}_a)_z$ is the difference between the boson densities of b_1 and b_2 . This means that the single Bose condensation is suppressed. The boson pairing condensation, on the other hand, is not disturbed by the z term. Therefore when the field φ_s is disordered, we have the boson pair condensation and finite superfluidity density ρ_s . Then the effective action for the test fields \vec{A}_1, \vec{A}_2 is given after integrating over \vec{c} fields as

$$S_A = \int d^3x \{ \rho_s (\vec{A}_1 + \vec{A}_2)^2 + \chi_a [\nabla \times (\vec{A}_1 - \vec{A}_2)]^2 \}, \quad (32)$$

where χ_a is a diamagnetic susceptibility of the antisymmetric part. Then the system shows the Meissner effect only for the symmetric test field $\vec{A}_1 + \vec{A}_2$. Therefore the system shows the Meissner effect to the external electromagnetic field \vec{A} .

V. CONCLUSIONS

In this paper, we studied the interplay between the confinement and the condensation of the order parameters. At integer-filling of the bosons with charge e , there is only one phase in $(2+1)D$ with the XY transition restricted to the line $g=0$. The nature of this so-called Higgs-confinement phase is the same as the confining phase of the pure gauge model, and no Meissner effect for the gauge field occurs. This is because the instantons act as the ordering field for the vortex condensation. For non-integer filling, Bose condensation is recovered for weak coupling g . However, the Bose condensation and confinement compete with each other, and this competition leads to phase separation for strong coupling.

In this paper we have focused our attention to the problem of bosons coupled to gauge fluctuations. It is only a first step towards addressing the problem which arises out of the gauge theory formulation of the high- T_c problem, which involves both fermions and bosons coupled to gauge fields. Nevertheless, we would like to put the present work in the context of the high- T_c problem and attempt to draw a few inferences. The effect of the fermion is twofold. First, if the gauge field is confining, it allows the possibility of confining fermion-antifermion pairs to form spin excitations and confining fermion and boson to re-constitute the physical electron. The former is believed to happen in the half-filled case where the AF ordering may be described as chiral symmetry breaking and confinement of Dirac fermions.^{22–25} Secondly, the presence of massless Dirac particles changes the dynamics of the gauge field and, in general, it would not take the Maxwellian form assumed in this paper. We can divide the

doping region of the phase diagram into three regimes.

(i) *Doping into the AF* ($x \ll 1$). Here the starting point is a π -flux phase for the fermions with a Dirac spectrum. The gauge propagator is proportional to $\sqrt{q^2}$ instead of q^2 in the Maxwell theory.⁶ The instantons have logarithmic interaction⁶ and undergo a Kosterlitz-Thouless transition in the $2+1$ dimensions as a function of N , the number of fermion flavors. It is believed that the physical case of $N=2$ lies on the disordered side of this transition, so that the instanton gas behaves as free gas (as opposed to instanton anti-instanton bound pairs). Since our consideration is based on assuming the existence of the free instanton gas, this would be the case where our consideration has the best chance of being applicable. Nevertheless, we still have not included the possibility of bosons combining with fermions to form physical holes in an AF background. This would correspond to the formation of small Fermi liquid pockets in a reduced Brillouin zone. Leaving this possibility aside, we can conclude from the results of Sec. III that instantons suppress the formation of a Wigner crystal of doped holes. Furthermore, the possibility of phase separation into microscopic patches is interesting, in that it suggests incommensurate structures which appear experimentally in this part of the phase diagram. However, the superfluid state that appear in our picture does not appear to resemble d -wave pairing, as long as the fermions remain confined in the AF state. It is also interesting that phase separation into larger scale laminar domain is a possibility. Finally in the SU(2) formulation, the result of Sec. IV suggests the possibility of bosons forming a pairing state, leading to a coexistence of superconductivity and AF.

(ii) *Underdoped region*. Here the normal state is the pseudogap state which is described as d -wave pairing of fermions or a staggered flux phase.²¹ Again initially the fermion spectrum is Dirac and the gauge propagation is proportional to $\sqrt{q^2}$ and it is not clear that the present paper is applicable. Nevertheless, we can ask whether the low-temperature phase is a confinement phase where instantons are free and play an important role. There are three possible scenarios for the onset of the low-temperature superconducting phase. The first is a binding of fermions with bosons to form physical quasiparticles. Since the fermions are already paired, a superconducting state appears. This possibility is clearly beyond the scope of the present work. The second and third possibilities are the Bose condensation of single bosons in the U(1) formulation, or the pairing of two kinds of bosons in the SU(2) formulation.²¹ The latter problem is treated in Sec. IV. What we learn from the present study is that instantons tend to suppress Bose condensation when the coupling constant g is larger. Furthermore, instantons favor the binding of the two species of SU(2) bosons to form pairs which then condense, leading to a d -wave superconductor ground state.

(iii) *The overdoped region*. Here the high-temperature phase is the strange metal phase and it has been argued that it is a deconfining phase due to dissipation in the gauge field dynamics.¹² The low temperature Fermi liquid phase is best described as a confinement of fermions and bosons. These are clearly outside of the scope of the present paper.

ACKNOWLEDGMENTS

The authors acknowledge X.G. Wen, C. Mudry, E. Fradkin, R.B. Laughlin, and P.W. Anderson for fruitful discus-

sions. N.N. is supported by Priority Areas Grants and Grant-in-Aid for COE research from the Ministry of Education, Science, Culture and Sports of Japan, P.A.L. acknowledges the support by NSF-MRSFC Grant No. DMR-98-08941.

APPENDIX: SURFACE TENSION

In this appendix, we show the calculation of the surface tension between the normal and superconducting regions of the dual superconductor following Ref. 26. The free energy of the dual superconductor is given by

$$F = \int d^2r \left[\frac{1}{2\kappa'} (\nabla \times \vec{c}_\perp)^2 + \psi_V^\dagger \frac{1}{2} [-K(\nabla + i\vec{c}_\perp)^2 + M^2] \psi_V - z(\psi_V + \psi_V^\dagger) + u(\psi_V^\dagger \psi_V)^2 \right], \quad (\text{A1})$$

where $\kappa'^{-1} = \kappa^{-1} + (dn/d\mu)^{-1}$ ($dn/d\mu$: the charge compressibility). Here the classical (time-independent) configuration is assumed and ∇ is the gradient in 2D space. The Ginzburg-Landau equations are obtained by taking the variation with respect to $\delta\psi_V^\dagger$ and $\delta\vec{c}$:

$$\frac{K}{2} (-i\nabla - \vec{c}_\perp)^2 \psi_V + \frac{M^2}{2} \psi_V + u|\psi_V|^2 \psi_V = z, \quad (\text{A2})$$

$$\nabla \times (\nabla \times \vec{c}_\perp) = \kappa' \vec{j}_V, \quad (\text{A3})$$

where

$$\vec{j}_V = \frac{K}{2i} (\psi_V^\dagger \nabla \psi_V - (\nabla \psi_V^\dagger) \cdot \psi_V) - A|\psi_V|^2 \vec{c}_\perp. \quad (\text{A4})$$

Now we consider the case of instanton driven dual superconductivity. Namely, the z term is the driving force of the vortex condensation and $M^2 > 0$. Then we assume M^2 is large enough and $u(\psi_V^\dagger \psi_V)^2$ term can be neglected. In the absence of the magnetic field \vec{c}_\perp , $\psi_V = \psi_{V0} = 2z/M^2$, and the free energy measured from that in the normal state F_{n0} is given by

$$F - F_{n0} = -V \frac{2z^2}{M^2} \equiv -V \frac{H_c^2}{2\kappa'}, \quad (\text{A5})$$

where H_c is the thermodynamic critical field and V is the volume of the system. Assume that the interface between the superconducting and normal regions is localized near $x=0$, and $x>0$ region is superconducting. Physical quantities depend only on x , and we choose the Coulomb gauge $\nabla \cdot \vec{c}_\perp = 0$. Therefore $\partial_x c_{\perp x} = 0$, and we put $c_{\perp x} = 0$. The boundary condition is

$$\begin{aligned} (\nabla \times \vec{c}_\perp)_z &= \frac{dc_{\perp y}}{dx} = H_c, & \psi_V &= 0, & x &\rightarrow -\infty, \\ (\nabla \times \vec{c}_\perp)_z &= \frac{dc_{\perp y}}{dx} = 0, & \psi_V &= \psi_{V0}, & x &\rightarrow +\infty. \end{aligned} \quad (\text{A6})$$

Here we introduce normalized quantities.

$$x = x/\lambda,$$

$$\begin{aligned}\psi &= \psi_V / \psi_{V0}, \\ c &= c_{\perp y} / (H_c \lambda),\end{aligned}\quad (\text{A7})$$

where λ is the penetration depth and is given by $\lambda = (4K\kappa'Z^2/M^4)^{-1/2}$. The correlation length ξ is given by $\xi = \sqrt{K/|M^2|}$, and we define the ratio $\eta \equiv \lambda/\xi$. Using these normalized quantities, the GL equations become

$$\psi'' = \eta^2 [1 - (c^2 + 1)\psi], \quad c'' = c\psi^2, \quad (\text{A8})$$

where $\psi'' = d^2\psi/dx^2$ etc., and the boundary condition is

$$\begin{aligned}c' &= 1, \quad \psi' = 0, \quad x \rightarrow -\infty, \\ c' &= 0, \quad \psi = 1, \quad x \rightarrow +\infty.\end{aligned}\quad (\text{A9})$$

It can be easily shown from Eq. (A8) that

$$\frac{1}{\eta^2} \psi'^2 + (c^2 + 1)\psi^2 - 2\psi - c'^2 = -1. \quad (\text{A10})$$

The surface tension α_{ns} is given as follows. First define the free energy density \tilde{f} under the magnetic field H as

$$\tilde{f} = f - \frac{HB}{\kappa'}. \quad (\text{A11})$$

Then α_{ns} is given by

$$\begin{aligned}\alpha_{ns} &= \int_{-\infty}^{\infty} dx (\tilde{f} - \tilde{f}_n) \\ &= \int_{-\infty}^{\infty} dx \left[\frac{(\nabla \times \vec{c}_{\perp})^2}{2\kappa'} + \frac{K}{2} (|\psi'_V|^2 \right. \\ &\quad \left. + \vec{c}_{\perp}^2 |\psi_V|^2) + \frac{M^2}{2} |\psi_V|^2 - z(\psi_V + \psi_V^\dagger) \right. \\ &\quad \left. - \frac{H_c(\nabla \times \vec{c}_{\perp})_z}{\kappa'} + \frac{2z^2}{M^2} \right] \\ &= \frac{\lambda H_c^2}{2\kappa'} \int_{-\infty}^{\infty} dx \left[(c' - 1)^2 + \frac{1}{\eta^2} (\psi')^2 \right. \\ &\quad \left. + (c'^2 + 1)\psi^2 - 2\psi \right],\end{aligned}\quad (\text{A12})$$

where \tilde{f}_n is the \tilde{f} in the normal state. Using Eq. (A10),

$$\alpha_{ns} = \frac{\lambda H_c^2}{\kappa'} \int_{-\infty}^{\infty} dx c' (c' - 1). \quad (\text{A13})$$

Because the normalized magnetic flux density c' is $0 < c' < 1$ in the interface region, the integral in Eq. (A13) is negative and $\alpha_{ns} < 0$. Therefore we conclude that the surface tension is negative in the instanton-driven dual superconductor.

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