

Quantum fluctuations and the c -axis optical conductivity of high- T_c superconductors

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A theory of the frequency dependence of the interplane conductivity of a strongly anisotropic superconductor is presented. The form of the conductivity is shown to be a sensitive probe of the strength of quantum and thermal fluctuations of the phase of the superconducting order parameter. The temperature dependence of the superfluid stiffness and of the form of the absorption at frequencies of the order of twice the superconducting gap is shown to depend on the interplay between superconducting pairing, phase coherence, and the mechanism by which electrons are scattered. Measurements of the c -axis conductivity of high- T_c superconductors are interpreted in terms of the theory.

I. INTRODUCTION

One of the important unresolved questions in high- T_c superconductivity is the strength of quantum fluctuations in the superconducting ground state. In this paper we show that the frequency dependence of the c -axis conductivity is a useful probe of the strength of these fluctuations. Our results apply, with minor modifications which are indicated at appropriate points below, to any sufficiently anisotropic superconductor and therefore may also be useful for interpreting data on layered organic superconductors and on the “spin-ladder” compounds.

Another important issue concerns the nature of the electronic states in the CuO_2 planes of high- T_c materials. We will show that the c -axis conductivity can be used to extract information about these states.

In a previous paper¹ we presented a theory for the c -axis optical spectral weight of layered superconducting systems. We showed, among other things, that the strength of the quantum fluctuations could be inferred from the ratio of the spectral weight in the c -axis superfluid response to the spectral weight lost from the c -axis conductivity as the temperature is decreased from high T to $T=0$. A ratio of 1 indicates mean-field-like superconductivity with negligible quantum phase fluctuations, whereas a ratio greater than 1 shows that quantum fluctuations are important. In this paper we demonstrate that quantum and thermal fluctuations also have important consequences for the form of the c -axis conductivity. In particular, if quantum fluctuations are significant, then under conditions believed to occur in high- T_c materials, $\sigma_c(\omega)$ acquires a peak for frequencies $\omega > 2\Delta$, where Δ is the superconducting gap. Our previous spectral weight analysis provided evidence for strong quantum fluctuations in $T_c = 70$ K $\text{YBa}_2\text{Cu}_3\text{O}_{6.6}$,^{1,2} but not in $\text{YBa}_2\text{Cu}_4\text{O}_8$ or $\text{YBa}_2\text{Cu}_3\text{O}_7$,^{1,2} and a peak in $\sigma_c(\omega)$ is found in the former material but not in the latter two.³

The qualitative idea behind our calculations is as follows. We consider materials, such as the high- T_c superconductors, which consist of weakly coupled layers. The weak interlayer coupling means that the conductivity may be calculated by second-order perturbation theory in the interplane coupling, and is therefore given by a convolution of two in-plane Green’s functions. Thus, as emphasized by Anderson,^{4,5} the

c -axis conductivity is in effect a spectroscopy of the in-plane properties. In this paper we show by explicit calculation in several models what can be learned from this spectroscopy.

Specific details of the high- T_c crystal chemistry imply^{6,7} that in these materials the interlayer coupling is dominated by the states near $(0, \pi)$ points of the two-dimensional Brillouin zone, where the superconducting gap is maximal. The c -axis conductivity thus reflects properties of electronic states near these points. The in-plane Greens function of high- T_c superconductors has been studied by photoemission spectroscopy and in the superconducting state at the momenta relevant for the interplane conductivity is very small for frequencies ω less than the gap energy, Δ and has a peak for $\omega \sim \Delta$ (which is interpreted as a quasiparticle pole), followed by a shallower minimum at an energy $\approx 2\Delta$ followed by a broad continuum, interpreted as the incoherent part of the spectral function. This structure has been referred to as “peak-dip-hump.”

In the usual theory of superconductivity,⁸ which neglects phase fluctuations, the type-II coherence factors associated with conductivity mean that the quasiparticle peak in $\text{Im } G(\omega)$ would not contribute to $\sigma_c(\omega)$ which would therefore vanish at the gap edge, 2Δ , and only begin to rise when the incoherent part of $\text{Im } G(\omega)$ appears. There would be no sharp structure in $\sigma_c(\omega)$ corresponding to the sharp structure in $\text{Im } G(\omega)$. However, we shall show that if phase fluctuations are important, then the effect of the coherence factors is reduced and the quasiparticles contribute, leading to a peak in σ_c for $\omega \approx 2\Delta$.

The c -axis conductivity is a useful spectroscopy of in-plane properties only if the tunnelling matrix element is known. In this paper we assume it has the usual band theory form, namely an interplane hopping which conserves in-plane momentum. A large literature exists in which the anomalous properties of the c -axis conductivity are attributed to a highly nontrivial interplane coupling, in which passage from one plane to another involves a strong scattering from some excitation or defect which resides between planes and does not couple to in-plane electron motion: see, for example, Refs. 9–11 and references therein. As discussed in Sec. VI, we believe there is substantial experimental evidence against this proposal. In any event, the theory of the effect of superconductivity on σ_c in the more straightforward

case of conventional interplane tunnelling and anomalous in-plane properties is given in this paper.

The rest of this paper is organized as follows. Section II presents a model for the in-plane Green's function introduced by Norman and Ding¹² and similar in many respects to a model of Chubukov and co-workers.¹³ Section III uses the model to obtain formulas for $\sigma_c(\omega)$ and shows how quantum and thermal fluctuations affect the results. Section IV evaluates the results in several limits. Section V discusses the f -sum-rule spectral weight, Sec. VI compares our results to data and to alternate theories, and Sec. VII is a conclusion and discussion of open problems. Readers uninterested in the technical details of the calculation are advised to read Sec. II and then proceed to Secs. VI and VII.

II. MODEL

The ‘‘peak-dip-hump’’ structure discussed above is generally agreed^{12–14} to imply that the electrons in high- T_c materials are subject to a strong scattering due ultimately to an electron-electron interaction—strong, because the spectral function at fixed momentum is spread over a wide energy range, and due to electron-electron interaction because the opening of the superconducting gap changes the form of the scattering, and in particular weakens it at low frequencies. Because in optimally doped and underdoped materials the normal-state spectral function is spread over a wide frequency range and has only a weak structure at fixed momentum near $(0, \pi)$, the imaginary part of the normal-state self-energy must be large and only weakly frequency dependent. One theoretical model which leads to such a self-energy involves electrons scattered by some bosonic mode, which is thought of as an electronic collective mode and has spectral weight concentrated near $\omega=0$ in the normal state. If this mode has electronic origin it must acquire a gap or pseudogap in the superconducting state. Following Refs. 12 and 13 we assume that the mode has a gap Ω at $T < T_c$ where the peak-dip-hump structure is observed, but has no gap ($\Omega=0$) at $T \geq T_c$, where the structure is not observed.

Conventional impurity scattering corresponds to $\Omega=0$ in both normal and superconducting states. Electron-electron scattering in an s -wave superconductor at $T=0$ would lead to an Ω of the order of 2Δ ,¹⁴ but possibly reduced by excitonic effects. In a two-dimensional d -wave superconductor (such as high- T_c materials are believed to be), one would expect a small $\sim \omega^2$ contribution at low frequencies from states near the gap nodes. We will ignore this small effect, and interpret Ω as the scale at which the scattering returns to its $T > T_c$ value.

In underdoped cuprates the electron spectral function (and many other properties) exhibits a ‘‘pseudogap’’ in a wide range of temperatures above T_c . The pseudogap is the superconducting energy gap, which seems^{16,17} in underdoped materials to persist in a wide range of temperatures above the resistively defined T_c . In these materials the superconducting transition corresponds to the onset of long-ranged phase coherence.^{18–20} The peak-dip-hump structure seems to be associated with the establishment of phase coherence, and not with the formation of the gap. This behavior is not at present understood.²⁰ In the present paper we simply assume it oc-

curs and examine its consequences for the c -axis conductivity.

For several reasons, including that discussed just above, the properties of the mode required by the models of Refs. 12 and 13 seem somewhat unusual, raising the question of whether a description in terms of scattering of conventional electrons off of a bosonic mode is the physically correct one. The models, however, are reasonably successful in fitting the electron spectral function, and this is all that we require here. The issue of the proper physical picture of the unusual behavior of the electron spectral function is, however, a crucial issue in the physics of high-temperature superconductivity. We return to it in the conclusion. We now turn to the mathematical formalism we need.

We may write in general for the normal (G) and anomalous (F) propagators in Matsubara formalism:

$$G(p, i\omega_n) = \frac{-i\omega_n Z_p(\omega_n) - \epsilon_p}{[\omega_n^2 + \Delta_{\omega_n}(p)^2] Z_p(\omega_n)^2 + \epsilon_p^2} \quad (1)$$

and

$$F(p, i\omega_n) = \frac{\Delta_{\omega_n}(p) Z_p(\omega_n)}{[\omega_n^2 + \Delta_{\omega_n}(p)^2] Z_p(\omega_n)^2 + \epsilon_p^2}. \quad (2)$$

Here $G(p, \omega_n) = \int d\tau e^{i\omega_n \tau} \langle T_\tau c_p^\dagger(0) \rangle$, $F(p, \omega_n) = \int d\tau e^{i\omega_n \tau} \langle T_\tau c_p(0) \rangle$, and $Z_p(\omega)$ is the renormalization function defined by $i\omega - \Sigma(p, i\omega) = i\omega Z_p(i\omega)$ where $\Sigma(i\omega)$ is the self-energy which contains the effects of coupling to the mode which produces the strong normal-state scattering. The approximation of Norman and Ding,¹² which is adequate for our purposes and which we adopt henceforth, consists of neglecting the frequency dependence of Δ and the momentum dependence of Z . The frequency dependence of Σ is given by

$$\begin{aligned} \Sigma(i\omega_n) = & \frac{\Gamma}{2\pi} \frac{\omega_n - i\Omega}{\sqrt{(\omega_n - i\Omega)^2 + \Delta(p)^2}} \\ & \times \ln \left[\frac{\omega_n - i\Omega + \sqrt{(\omega_n - i\Omega)^2 + \Delta(p)^2}}{i\Delta(p)} \right] - \text{c.c.} \end{aligned} \quad (3)$$

This form corresponds to a single-particle scattering rate which tends to $\Gamma/2$ for $\omega \gg \Omega, \Delta$ and reduces to the familiar expressions for a dirty superconductor when $\Omega \rightarrow 0$. The observed¹⁵ frequency independence of σ_1 at $T > T_c$ in the range $\omega < 1/2$ eV leads us to choose $\Gamma \sim 0.8$ eV $\gg \Delta$. At large frequencies we therefore have very strongly scattered electrons, corresponding to an $\text{Im} G(p, \omega)$ which is small $\sim 1/\Gamma$ and essentially independent of p, ω . However, for $\omega < \Delta + \Omega$ Z is real, so G will have a pole at a frequency $\omega_{qp} = \sqrt{\Delta^2 + \epsilon_p^2} / Z(\omega_{qp})^2$ if $\omega_{qp} < \Delta + \Omega$. A large Γ such as we have assumed implies that $Z(\omega_{qp}) \gg 1$ so the quasiparticles have small weight and negligible dispersion. References 12, 13, and 17 argue that the combination of a large Γ and an $\Omega=0$ in the superconducting state but not in the normal state accounts for the peaks observed in photoemission experiments at $T < T_c$ but not for $T > T_c$.^{16,17} The resulting spectral

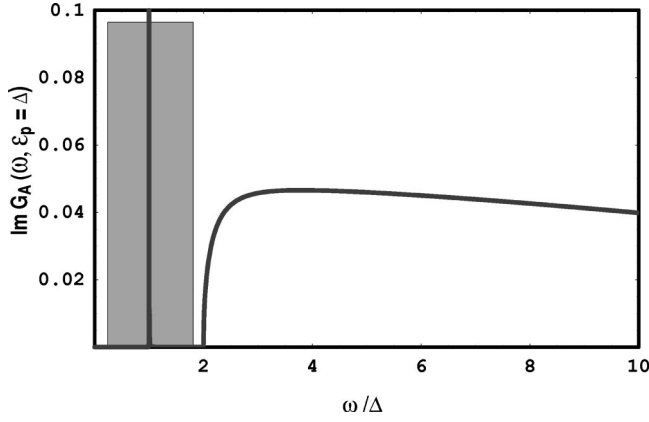


FIG. 1. Imaginary part of the electron Green's function for parameters $\Gamma=40$ and $\Omega=\Delta$. The quasiparticle peak is indicated by the vertical line; its weight is shown as the shaded box.

function is shown in Fig. 1 for $\epsilon_p=\Delta$ and $\Gamma=40\Delta$. Here the quasiparticle pole is shown as a sharp line and its strength corresponds to the area in the shaded box. The onset of scattering at $\omega=\Omega+\Delta$ causes the broad only weakly frequency-dependent continuum.

The spectral function predicted by Eq. (3) was fit to photoemission data by Ref. 12. Differences between the data and the model were attributed by Ref. 12 to an extra elastic-scattering term and to an extrinsic background. The elastic scattering was introduced to broaden the low-frequency peak, which is a delta function in the model; we feel the observed broadness is more likely due to a variation of the gap over the surface of the sample, but in any event this extra broadening (which we do not include) will not affect our results in any significant way.

The extrinsic background requires more discussion. In the data analyzed in Ref. 12 the weight in the quasiparticle peak was somewhat smaller than the “missing” area obtained by multiplying the higher ω value of $\text{Im } G$ by the frequency interval $0 < \omega < \Delta + \Omega$. As can be seen from our Fig. 1, in the strong-coupling limit of the model self-energy the strength of the quasiparticle peak is in fact somewhat larger than this missing area. The authors of Ref. 7 apparently dealt with this discrepancy by introducing an additional extrinsic background, nonzero only for $\omega > \Delta + \Omega$, and adjusted so the weight in the observed quasiparticle peak is roughly equal to the “missing” area calculated from the background-subtracted part of $\text{Im } G$. It seems to us that the discrepancy requires further consideration; however, the issue is not crucial to the present paper, which focusses on the qualitative consequences for the c -axis conductivity of the quasiparticle peak in $\text{Im } G$.

III. INTERPLANE CONDUCTIVITY

We now consider the effect of our chosen form of G and F on σ_c , focussing on the extent to which the quasiparticle poles contribute to the observed conductivity and on the effects of the offset Ω . We emphasize that for these considerations the precise forms of G and F do not matter, as long as they have the general properties outlined above.

We assume the Hamiltonian is

$$H = H_{in-plane} + \sum_{p,\sigma,i} [t_{\perp}(p)c_{p,\sigma,i}^{\dagger}c_{p,\sigma,i+1} + \text{H.c.}] \quad (4)$$

Here $c_{p,\sigma,i}^{\dagger}$ creates an electron of in-plane momentum p and spin σ on plane i . $H_{in-plane}$ contains the (presumably nontrivial) physics of a single Cu-O₂ plane, and in particular leads to the G and F functions discussed in the previous section.

We calculate the interplane conductivity in the usual way, representing the c -direction electric field by a vector potential A and coupling it to H via the Peierls substitution (in units $\hbar=c=1$) $t_{\perp} \rightarrow t_{\perp} e^{ieAd}$ with d the interplane spacing. We expand to second order in t_{\perp} finding^{1,4,5}

$$\sigma_c(\omega) = \frac{1}{i\omega} \left[K - T \sum_{\omega'} \int d\hat{p} t_{\perp}^2(\hat{p}) \Pi \left(\omega' + \frac{\omega}{2}, \omega' - \frac{\omega}{2}, \hat{p} \right) \right] \quad (5)$$

We have omitted dimensional factors of e^2 and lattice constants, which are not relevant to our arguments.

It is convenient to consider separately contributions to the polarizability Π coming from the different parts of the Fermi surface (as labeled by \hat{p}) and separate Π into normal and anomalous parts, as

$$\Pi = \Pi_{GG} - \Pi_{FF} \quad (6)$$

with

$$\Pi_{GG}(\omega'_+, \omega'_-) = \nu \int d\xi G_i(p, \omega'_+) G_{i+1}(p, \omega'_-), \quad (7)$$

$$\Pi_{FF}(\omega'_+, \omega'_-) = \nu \int d\xi \langle F_i(p, \omega'_+) F_{i+1}(p, \omega'_-) \rangle. \quad (8)$$

Here $\xi = v_F(p - p_F)$, $\omega'_{\pm} = \omega' \pm \omega$ and here and in many places subsequently the \hat{p} label on the Π functions has been suppressed.

The diamagnetic term K may be written

$$K = T \sum_{\omega} \int d\hat{p} t_{\perp}^2(\hat{p}) [\Pi_{GG}(\omega, \omega, \hat{p}) + \Pi_{FF}(\omega, \omega, \hat{p})]. \quad (9)$$

In Eq. (8) Π_{FF} is defined as the expectation value of a product of F functions because it depends on the interlayer phase coherence. To study the phase coherence properties in more detail we write the FF correlator in real space and separate out the term involving the phase difference, finding

$$\Pi_{FF}(r, t) = \Pi_{FF}^0(r, t) \langle e^{i\phi_i(r, t) - i\phi_{i+1}(0, 0)} \rangle. \quad (10)$$

Here Π_{FF}^0 is the usual convolution of F functions with phases set to 0 and may be calculated by standard methods.⁸ In the BCS approximation the phases do not fluctuate: $\phi_i(r, t) = \phi_{i+1}(r', t') = \phi_0$ and $\Pi_{FF} = \Pi_{FF}^0$. In the actual materials the phase in each plane fluctuates. In the pseudogap regime of underdoped materials the superconductivity is destroyed by phase fluctuations while the amplitude of the gap remains nonzero.¹⁸⁻²⁰ For $T < T_c$ the phase has a nonzero average but may fluctuate. We write $\phi(r, t) = \phi_0 + \delta\phi(r, t)$. At $T \ll T_c$ thermal fluctuations are negligible. In

the two-dimensional case of present interest, quantum fluctuations are dominated by short length scales and so are uncorrelated from plane to plane, $\langle \delta\phi_i \delta\phi_{i+1} \rangle = 0$. This allows us to evaluate the correlator entirely in terms of in-plane properties, at low T . Unlike quantum fluctuations, two-dimensional thermal fluctuations are dominated by length scales of the order of the thermal coherence length ξ_T which diverges at T_c . Interplane phase correlations are important if the Josephson energy of a correlated region is larger than the temperature, i.e., if $N_0 t_\perp^2 \xi_T^2 \Delta / \Gamma > k_B T$; where N_0 is the in-plane density of states and the standard dirty-limit factor $\Delta / \Gamma \ll 1$ reflects the fact that the weight in the c -axis conductivity is spread over a wide frequency range of order Γ , so only the fraction Δ / Γ is available to contribute to the Josephson coupling. The small values of interplane couplings and Δ / Γ relevant to high- T_c materials mean that this criterion is only satisfied for temperatures very near T_c . Thus except very near to T_c we have

$$\Pi_{FF}(r, t) = \Pi_{FF}^0(r, t) e^{-\langle (\delta\phi_i)^2 \rangle} \equiv \alpha \Pi_{FF}^0(r, t). \quad (11)$$

In other words, in $d=2$ the effect of quantum and thermal phase fluctuations is to renormalize the interplane $F-F$ correlator by a constant Debye-Waller factor α with $0 < \alpha < 1$. In $d=1$ the phase fluctuation integral has a logarithmic divergence cut off by t_\perp or the length and time scale, so the Debye-Waller factor will have scale or t_\perp dependence.

We shall be interested in either a strongly scattered normal state or in $T \ll \Delta$ so we take the $T \rightarrow 0$ limits of the formulas. After integration over momentum and analytical continuation we find for the imaginary parts

$$\text{Im} \Pi_{FF}^0(\omega'_+, \omega'_-) = \frac{1}{2} \text{Re} \left[\frac{\Delta^2}{(\xi_+^R Z_+^R + \xi_-^R Z_-^R)(\xi_+^R \xi_-^R)} - \frac{\Delta^2}{(\xi_+^R Z_+^R + \xi_-^A Z_-^A)(\xi_+^R \xi_-^A)} \right] \quad (12)$$

and

$$\text{Im} \Pi_{GG}(\omega'_+, \omega'_-) = \frac{1}{2} \text{Re} \left[\frac{1}{\xi_+^R Z_+^R + \xi_-^R Z_-^R} \left(1 + \frac{\omega_+ \omega_-}{\xi_+^R \xi_-^R} \right) - \frac{1}{\xi_+^R Z_+^R + \xi_-^A Z_-^A} \left(1 + \frac{\omega_+ \omega_-}{\xi_+^R \xi_-^A} \right) \right], \quad (13)$$

where $\xi_\pm^R = \sqrt{\Delta^2 - (\omega'_\pm + i\delta)^2}$, $\xi_\pm^A = (\xi_\pm^R)^*$ and $Z_\pm^R = 1 - \Sigma^R(\omega'_\pm) / \omega'_\pm$.

Finally, to compute σ_c we must substitute Eqs. (12) and (13) into Eq. (5) and average over the Fermi surface. As mentioned above, in high- T_c materials t_\perp is very strongly peaked about the $(\pi, 0)$ points where the superconducting gap is maximal. Thus we may approximate $\Delta(p)$ by its maximum value Δ and ignore the d -wave gap structure and the integral over angles. We then obtain for the absorptive part of the conductivity

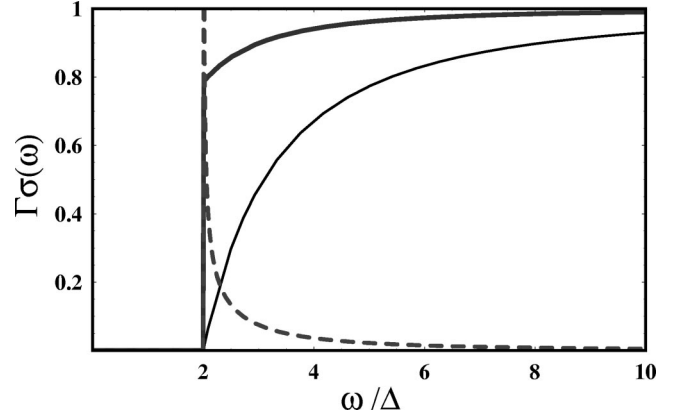


FIG. 2. Optical conductivity multiplied by scattering rate Γ with (thin dark line) and without (thick light line) phase coherence calculated for $\Omega=0$, in the limit $\Gamma \rightarrow \infty$. The normal-state (no gap) conductivity $\Gamma\sigma=1$. Also shown as dashed line is conductivity from unscattered quasiparticle states in absence of phase coherence.

$$\sigma^{(1)}(\omega) = \frac{\sigma_0}{\omega} \int_{-\omega/2}^{\omega/2} d\omega' \text{Im} [\Pi_{GG}(\omega'_+, \omega'_-) - \alpha \Pi_{FF}(\omega'_+, \omega'_-)] \quad (14)$$

with $\sigma_0 = N_0 \int d\hat{p} t_\perp^2(\hat{p})$.

IV. EVALUATION OF CONDUCTIVITY

We begin with the case $\Omega=0$, i.e., with strong scattering unaffected by the onset of phase coherence. Results are shown in Fig. 2. For this choice of Ω , the real part of the normal-state conductivity is $\sigma_c(\omega) = \Gamma / (\omega^2 + \Gamma^2)$. We have chosen a very large Γ so the normal conductivity is a straight line with value $\Gamma\sigma=1$. The light solid line depicts the conductivity of the fully phase coherent ($\alpha=1$) superconducting state. The coherence factor effects, namely $\sigma(\omega=2\Delta)=0$ and the gradual onset of absorption as ω is increased above 2Δ , are evident. Also shown as the heavy solid line in Fig. 2 is the no-phase coherence ($\alpha=0$) conductivity. The coherence factor effects are absent, so the conductivity rises discontinuously from the gap edge (in our approximation, which neglects angular variations of the gap) and is always larger than the phase-coherent σ . No sharp structure is visible in either calculation because everywhere an excitation is allowed, the damping is large.

In the $\Omega=0$ case the conductivity in the presence of the gap is always less than the normal-state ($\Delta=0$) conductivity. In the fully phase coherent ($\alpha=1$) case there is also a delta-function contribution (not shown); the weight in the delta function exactly equals the difference of the fully phase coherent curve from 1. In the $\alpha=0$ case the lack of phase coherence means that there is no superfluid delta function—the conductivity spectral weight is less than in either the no-gap case or the fully phase coherent state. The spectral weight will be discussed in more detail in the next section.

Because it will be useful in our subsequent discussions, we also show as the dashed line in Fig. 2 the conductivity calculated assuming no scattering ($\Gamma=0$) and no phase coherence ($\alpha=0$), but with a nonvanishing superconducting gap. (This conductivity is normalized in a way which does

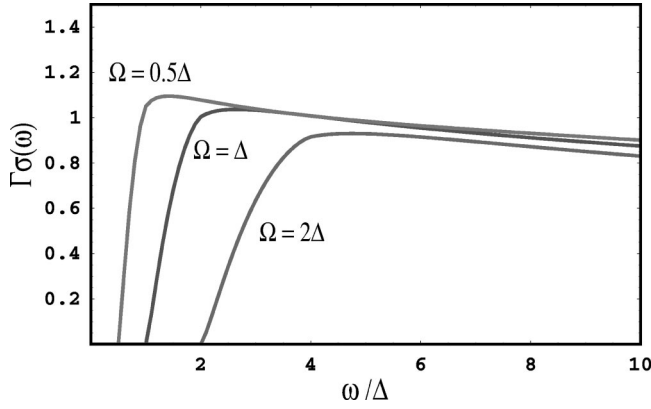


FIG. 3. Optical conductivity in normal state as function of mode offset Ω for $\Gamma=40$.

not involve Γ). In this case quasiparticle absorption above the gap edge is allowed, as in a usual semiconductor, even though there is no scattering, leading to the square-root divergence shown.

We now turn to the effects of the offset. We begin by considering in Fig. 3 the conductivity of a nonpaired ($\Delta=0$) state at $T=0$, for different values of Ω . To facilitate comparison with subsequent figures we measure frequencies and the offset in units of Δ , but we emphasize that these curves pertain to the nonpaired state. At $\Omega=0$ we would have a Lorentzian of half-width $\Gamma=40$ and value 1 at $\omega=0$; this is not shown in the figure. We see from the three displayed curves that $\sigma(\omega < \Omega)=0$ [except for a delta-function contribution of relative strength $\Gamma/Z(\omega=0)=1/(1+\Gamma/\pi\Omega)$ which is not explicitly shown]. The conductivity has a complicated dependence on the combination of scattering rate and offset: if Ω/Γ is sufficiently small (as occurs in the displayed $\Omega=0.5\Delta$ and $\Omega=\Delta$ traces) the conductivity can rise above the no-offset value (basically, $\sigma=1$) for a Γ -dependent range of ω ; for larger offsets (e.g., the $\Omega=2\Delta$) trace, the conductivity is always lower. The total weight (including the delta-function contribution) is conserved as a function of Ω , but as can be seen from the high-frequency behavior of the curves, the differences between σ calculated with different Ω persist up to very high frequencies, so making accurate statements about differences in integrated area requires integration over frequencies of order Γ .

We next show in Fig. 4 the conductivity in the superconducting state, with offset $\Omega=\Delta$ and $\Gamma=40$. In this case in the energy range $\Delta < \omega < \Delta + \Omega$ unscattered quasiparticles exist. In the case of perfect phase coherence ($\alpha=1$) we obtain the lower curve. The onset of the absorption at $\omega=2\Delta + \Omega$ is evident, as is the restoration of the full scattering at $\omega > 2\Delta + 2\Omega$. The freely propagating quasiparticle states at $|\omega| < \Delta + \Omega$ do not contribute to σ because of the type-II conductivity coherence factors. In physical terms, these states are not scattered and one gets no absorption without scattering. Comparison to the appropriate curve in Fig. 3 shows that convergence to the $\Delta=0$ case of the same Ω is very slow. We have verified that the area including the superfluid delta function (not shown) is conserved, but one must integrate to frequencies of order Γ to capture all of the spectral weight.

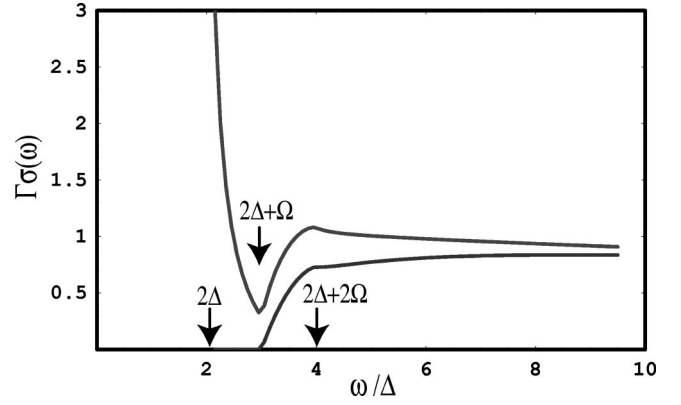


FIG. 4. Optical conductivity in superconducting state with (lower line) and without (upper line) phase coherence, for $\Gamma=40$ and $\Omega=\Delta$.

We now consider the opposite case, namely that the interplane quantum fluctuations are so strong that Π_{FF} is negligible. In this case σ_c is determined from Π_{GG} alone and the resulting conductivity is shown as the upper line in Fig. 4. A quasiparticle contribution is evident in the region $2\Delta + \Omega > \omega > 2\Delta$; this has relative weight $1/Z \ll 1$ and in the simple approximation considered here has the same functional form (and physical origin) as the dashed curve shown in Fig. 3. The threshold of inelastic scattering at $\omega=2\Delta + \Omega$ is evident as is the restoration of the full scattering at $\omega=2\Delta + 2\Omega$. The absorption at $2\Delta < \omega < 2\Delta + \Omega$ is at first sight surprising because it comes from freely propagating quasiparticles which normally cannot lead to finite frequency absorption in a translationally invariant system. Such absorption can be easily understood if the interplane phase coherence is destroyed by the thermal fluctuations. In this case we may think of each plane as having a separate phase, so that translation invariance is effectively broken. In the $T=0$ case of present interest, however, one deals with the ground state of a quantum system which has a translation invariance. The explanation in this case is that $T=0$ interplane fluctuations imply the existence of an interplane charging energy; the presence of this term in the Hamiltonian means that the current carried by the quasiparticles does not commute with the Hamiltonian and this allows the $\omega \sim 2\Delta$ absorption to exist.

V. SPECTRAL WEIGHT

In this section we study in more detail the f -sum-rule spectral weight, i.e., the integrated area under the conductivity. Standard analysis^{1,21-23} shows that this is related to K , Eq. (9), via $K = \sigma_0 \int (2d\omega/\pi) \sigma(\omega)$. The results of Sec. IV imply

$$K(\Omega, \Delta, \alpha) = \sigma_0 \int_0^\infty \frac{2d\omega}{\pi} \int d\xi_p [G(p, \omega)^2 + \alpha |F(p, \omega)|^2]. \quad (15)$$

In the superconducting state, some of the weight is concentrated in a delta function at $\omega=0$. The coefficient of this delta function is conventionally written as $\pi\rho_s$ and is given by the difference between K and $\Pi(\omega \rightarrow 0)$.⁸ We find

$$\rho_s(\Omega, \Delta, \alpha) = 2\alpha\sigma_0 \int_0^\infty \frac{2d\omega}{\pi} \int d\xi_p |F(p, \omega)|^2. \quad (16)$$

These results depend crucially on the assumption that the only important interplane hopping term is that written in Eq. (4) (arbitrary interplane interactions are allowed). Different hopping terms, such as those considered in Refs. 9, 10, and 24 would lead to different results, as noted in Refs. 10 and 24.

It is evident from Eqs. (15) and (16) that if all other parameters remain fixed, then a decrease in α causes related decreases in the total spectral weight and ρ_s . It is natural to assume that if $\alpha=1$ (no phase fluctuations) then spectral weight is conserved as a function of temperature, so that variations in spectral weight as a function of temperature imply variations in α . In a previous paper¹ we studied the consequences of this assumption. We distinguished three cases: (i) no pairing (i.e., $\Delta=0$ and $\sigma \sim \int GG$); (ii) conventional (no fluctuations) superconductivity (i.e., $\Delta \neq 0$ and $\sigma \sim \int GG - FF$); (iii) superconductivity with strong phase fluctuations (i.e., $\Delta \neq 0$ and $\sigma \sim \int GG - \alpha FF$, $\alpha \ll 1$). Cases (i) and (ii) were found to have the same total spectral weight but going from case (ii) to case (iii) by keeping the gap the same but increasing the phase fluctuations (i.e., decreasing α) was shown to decrease the total spectral weight. A particularly interesting case was $\alpha=0$ which has been argued to represent the $T > T_c$ pseudogap regime of underdoped cuprates.^{18,1} Reducing T from room temperature, where no pseudogap is evident, to 100–150 K, where a pseudogap is plainly seen in many measurements in underdoped cuprates, corresponds to going from case (i) to case (iii), i.e., it reduces the total spectral weight. Further reducing T to a $T < T_c$ induces long-ranged phase coherence, implying $\alpha > 0$ and hence an increase in spectral weight. If as $T \rightarrow 0$ phase coherence is fully restored at all scales, i.e., if quantum fluctuations are negligible, we may set $\alpha=1$. In this case the $T=0$ spectral weight equals the high-temperature spectral weight and in particular ρ_s is given by the area lost in $\sigma_c(\omega > 0)$ between high temperature and $T=0$. A $T=0$ value of ρ_s which is less than the “missing area” was therefore argued¹ to imply non-negligible quantum fluctuations.

The results of Ref. 1 relied on the T independence of the spectral weight at $\alpha=1$. This was verified in a BCS-like model¹ but the complicated interplay between superconductivity and scattering indicated in Refs. 12–14 means that additional analysis is required. We show here that the crucial assumption is that the self-energy function $Z_p(\Omega)$ has negligible dependence on band energy ξ_p in the frequency ranges of interest. We consider the difference between $K(\Omega, \Delta, \alpha)$ and the noninteracting spectral weight K_{nonint} , which is given by Eq. (9) with $Z=1$. Because the high frequency, large ξ asymptotics of the two integrands are the same, we may integrate over ξ_p first, using the ξ_p independence of Z and obtaining

$$K(\Omega, \Delta, \alpha) - K_{nonint} = \sigma_0 \int_0^\infty \frac{2d\omega}{\pi} \frac{\pi(\alpha-1)\Delta^2}{2Z(\omega^2 + \Delta^2)^{3/2}}. \quad (17)$$

When $\alpha=1$ the right-hand side of Eq. (17) vanishes: spectral weight is conserved as a function of Ω and Δ . We

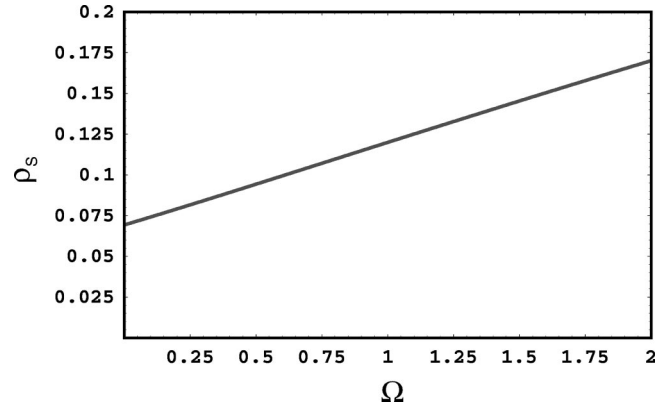


FIG. 5. Dependence of superfluid stiffness on offset Ω for $\Gamma=40$. These numbers must be multiplied by Γ/π in order to be compared directly to areas from scaled conductivities shown in previous figures.

note that because this integral is dominated by $\omega \sim \Delta$ the crucial requirement is that $Z_p(\omega)$ have negligible ξ_p dependence for $\omega \sim \Delta$.

Using the same assumption we find for the superfluid stiffness

$$\rho_s = 2\alpha\sigma_0 \int_0^\infty \frac{2d\omega}{\pi} \frac{\Delta^2}{Z(\omega^2 + \Delta^2)^{3/2}}. \quad (18)$$

The Ω dependence of ρ_s is shown in Fig. 5 for $\Gamma=40$ and $\alpha=1$. One sees that ρ_s increases rapidly with increasing Ω . The physics may be understood most simply from the Ω dependence of ρ_s . For impurity scattering ($\Omega=0$) ρ_s is reduced from the noninteracting value by the usual dirty limit factor Δ/Γ ; this is properly understood as a mass renormalization $\rho_s \rightarrow \rho_s/Z$ coming from a self-energy which (for $\omega < \Delta$) is real but has a strong frequency dependence ($\Sigma \approx \omega Z \approx \omega\Gamma/\Delta$). If we now add a threshold Ω to the scattering mechanism the renormalization decreases: $Z \rightarrow \Gamma/(\Delta + \Omega)$; the decrease in Z leads to an increase in ρ_s .

A similar ω dependence occurs in the spectral weight of the “quasiparticle” peak in the $\alpha=0$ conductivity, again because the quasiparticle mass $\sim 1/Z$. For example, in the case $\Omega=\Delta$ the weight in the $\alpha=0$ quasiparticle peak is approximately equal to the weight in the conductivity at high T (i.e., $\Delta=0$) over the range $(0, 2\Delta)$. Of course, this is less than the total “missing weight” (indeed is about half) because σ is suppressed over the wider range $\omega \sim 2\Delta + \Omega$.

For $\alpha=1$, weight is conserved as Ω is varied, so the increase in ρ_s is compensated by a decrease in the $\omega > 0$ conductivity. We see however from the normal-state calculation shown in Fig. 2 that as Ω is changed the difference in weight is spread out over a wide range of frequencies, of order Γ . If one integrates σ over a range small compared to Γ the total weight (ρ_s plus $\omega > 0$ part) may appear to increase as Ω is increased. By contrast, we find that if Δ is varied at fixed Ω , the change in conductivity is more concentrated at frequencies of the order of Δ and weight is to a reasonable approximation conserved even if one integrates only over a range of the order of a few Δ , especially for $\alpha < 1$.

Weight is not conserved if the Debye-Waller parameter α is varied. Varying α changes the weight in the superfluid component, the weight in the nonzero-frequency component, and also the total weight. In our previous work¹ we found [as can be seen directly from Eqs. (17) and (18)] that increasing α while keeping other parameters fixed increases the weight in the c -axis superfluid response by twice as much as the total spectral weight increase. The physics is that as α is increased, coherence factor effects become more important, leading to a decrease in $\sigma_c(\omega)$ in the region $\omega > 2\Delta$; this decrease reduces the nonzero frequency spectral weight $\int_{\omega=0}^{\infty} \sigma(\omega) d\omega$ by an amount which turns out to be half of the increase in ρ_s . We are most interested in applying this result to underdoped cuprates, in which the superconducting gap is well formed for temperatures of the order of the resistive T_c and the onset of superconductivity leads to an increase in α from 0 to some nonzero value. However, as we have seen, there is evidence that the onset of superconductivity leads also to a change in Ω ; therefore further consideration is necessary.

Further consideration requires further assumptions. We focus on underdoped materials and assume that at temperatures slightly greater than T_c the superconducting gap is well formed and much greater than T , but that thermal phase fluctuations drive α to 0. We further suppose that at $T > T_c$ the offset parameter $\Omega = \Omega_+$. As $T \rightarrow 0$ long-ranged phase coherence is established, so $0 < \alpha \leq 1$ and $\Omega \rightarrow \Omega_-$. The data seem consistent with $\Omega_+ = 0$ and $\Omega_- \approx \Delta$ but we prefer to present a more general treatment. We now use Eqs. (17) and (18) to obtain a relation between ρ_s and the change ΔK , in the $\omega > 0$ spectral weight between $T > T_c$ and $T \rightarrow 0$, finding

$$\frac{\rho_{s,observed}}{\Delta K} = \frac{2\alpha}{1 + \alpha - [\rho_s(\Omega_+, \alpha=1)/\rho_s(\Omega_-, \alpha=1)]}. \quad (19)$$

In this equation, $\rho_{s,observed}$ is the experimental $T \rightarrow 0$ value, $\rho_s(\Omega, \alpha=1)$ is the function shown in Fig. 5, and the equation only applies if the gap Δ is well formed and larger than T for temperatures just above the resistive transition.

If $\Omega_+ < \Omega_-$ (as has been claimed^{12,17} to occur in cuprates), then the ratio is less than 2, if $\alpha < 1$. As shown in Ref. 1 the ratio becomes equal to 2 if $\Omega_+ = \Omega_-$, and could become greater than 2 if the inequality were reversed. The physics of the ratio is most easily understood from the limit $\alpha \rightarrow 0$, $\Omega_+ = 0$, and $\Omega_- \gg \Delta$. In this case turning on an $\Omega \gg \Delta$ strongly suppresses the conductivity in the region $\omega \sim \Delta$ but the small value of α means the compensating increase in ρ_s is negligible. The values $\Omega_+ \approx 0$ and $\Omega_- \approx \Delta$ inferred from photoemission^{12,13} lead to $\rho_s/\Delta K = 2\alpha/(\alpha + 0.4)$, i.e., to a substantial reduction of the ratio from 2 if $\alpha \sim 0.5$ or less.

Care is needed in applying Eq. (19) to data. As is clear from Fig. 3, variations in Ω lead to variations in σ over a wide energy range, of order the basic scattering rate Γ . An integration over a smaller range could miss some contributions to ΔK , leading to a larger apparent value of the ratio.

VI. COMPARISON TO DATA AND TO OTHER THEORIES

We begin by summarizing the results obtained in the previous sections. We used second-order perturbation theory in the interplane coupling to study the interplane conductivity, with particular emphasis on the effect of phase fluctuations in the presence of a pairing gap Δ . We found that the formal consequence of the existence of phase fluctuations is a Debye-Waller factor $\alpha < 1$ which multiplies the anomalous (“ F ”) propagators. The physical consequences include a decrease in the total f -sum-rule oscillator strength, a decrease in the superfluid stiffness ρ_s and a change in the form of the conductivity at frequencies of the order of twice the superconducting gap. Another important parameter is Ω which is defined in Eq. (3) and parametrizes the frequency scale associated with the mechanism by which electrons are scattered.

As already noted in Ref. 1 the most important and model-independent result of this analysis is that the fundamental measure of the strength of quantum fluctuations in the superconducting ground state is the ratio of ρ_s to the area missing in $\sigma(\omega > 0)$ as T is decreased from above the temperature T_{PG} at which the superconducting (or “pseudo”) gap becomes visible to $T=0$. A $T=0$ ρ_s which is smaller than the “missing area” implies non-negligible quantum fluctuations.

We further found that if quantum fluctuations are strong, the value of the c -axis conductivity in the region $\omega \sim 2\Delta$ is increased above the predictions of BCS theory. If the quasiparticles with $\omega \sim \Delta$ are weakly damped (as indicated by photoemission experiments^{16,17}) this increase takes the form of a peak.

Finally, we derived a relation between ρ_s and the area ΔK lost between $T=T_c$ and $T=0$ from the $\omega > 0$ σ . In the usual BCS theory, where the gap closes at T_c , $\rho_s/\Delta K = 1$. If (as is believed to be the case in underdoped cuprates) a pairing gap exists in a wide temperature regime above T_c and the resistive transition signals only the onset of phase coherence, then the ratio of ρ_s to the weight ΔK lost below T_c is different from 1 and as seen from Eq. (19) depends on both the strength of the quantum fluctuations and the variation with temperature of the “offset parameter” Ω . If Ω is T independent, then the ratio is 2 independent of α , whereas if Ω increases as T is decreased below T_c the ratio is less than 2, and if quantum fluctuations are sufficiently strong can be less even than 1. For the values $\Omega(T > T_c) \approx 0$ and $\Omega(T \rightarrow 0) \approx \Delta$ inferred from photoemission data^{12,13,16,17} the ratio would become 1 at $\alpha = 0.4$. The c -axis optics therefore contains information about the T dependence of the electron scattering mechanism. However, to obtain this information one must integrate the conductivity over a wide range because we found changes in Ω led to changes in σ which extended over a range of order the basic scattering rate Γ .

Our results were obtained using second-order perturbation theory in the interplane hopping. This perturbation theory has been used by many workers,⁴ and in the present model may be tested. The theoretical expansion parameter is $N_0 t_{\perp}^2 / \Gamma$, where N_0 is an in-plane density of states, t_{\perp} is an average of the hopping over the Fermi surface, and Γ is the scattering rate. The band structure estimates $t_{\perp}(p) = t_0 [\cos(p_x) - \cos(p_y)]^2$, $t_0 \sim 0.15$ eV (for YBCO; smaller

for others) and $N_0 \sim 2$ states/eV (Ref. 7) lead to $N_0 t_{\perp}^2 \sim 0.02$ eV; the Γ inferred from the high-frequency data on $\text{YBa}_2\text{Cu}_3\text{O}_{6.95}$,¹⁵ then suggests that perturbation theory is very good. One may make a more experimental estimate by writing (with units restored) our theoretical result for the c -axis dc conductivity $\sigma_c^{dc} = (e^2 d/a^2) N_0 t_{\perp}^2 / \Gamma$. Here d is the mean interplane spacing and a is the in-plane lattice constant. The observed $\sigma_c^{dc} \sim 100\text{--}200 \text{ } \Omega^{-1} \text{ cm}^{-1}$ for optimally doped YBCO then implies $N_0 t_{\perp}^2 \sim 1/50$, reasonably consistent with the above estimates and justifying the use of second-order perturbation theory. For other materials the interplane conductivity is even smaller, so the perturbation theory should be even better.

Our results also rely crucially on the assumption that the part of the Hamiltonian involving motion of electrons between planes has the form given in Eq. (4), i.e., is the usual band-theory form which involves hopping of a real electron in a manner which conserves in-plane momentum. Strong scattering the barrier region (i.e., nonmomentum-conserving hopping)^{9,10} or ‘‘occupation modulated hopping’’²⁴ would invalidate our results. We will argue below that the close correspondence between our results and data suggests that the usual band theory form of the hopping is the correct one.

We turn now to the data, beginning with the temperature and doping dependence of the spectral weight. In overdoped materials a small increase in low- ω normal-state spectral weight occurs as T is decreased below room temperature [see, e.g., Fig. 3(a) of Ref. 3]. Our results (cf. Fig. 2) suggest that this may be compensated by a small decrease in conductivity over a wide frequency range. In optimally doped materials spectral weight seems to be conserved as a function of temperature. In optimally doped and overdoped materials the superconducting gap closes at T_c and spectral weight in ρ_s compensates for the area lost below T_c .^{2,3,15,25} Further, in optimally doped and overdoped materials the conductivity in the superconducting state appears to have the usual BCS form, rising smoothly from the gap edge and being always less than the normal state σ .³ This is consistent with our results if in these materials quantum and thermal fluctuations of the order parameter are negligible.

The observed σ_c for optimally doped and overdoped materials provides evidence against the alternative explanation of the c -axis conductivity advanced in Refs. 9 and 10. In these works, the weak frequency dependence of σ_c at $T > T_c$ is attributed to a strongly momentum nonconserving interplane coupling $t_{\perp}(p, p')$, due physically to strong scattering in the interplane barrier layers, rather than to a large value of an in-plane electron scattering rate Γ . This view implies that c -axis conductivity is equivalent to a point contact tunneling; if it were correct then at $T < T_c$ one would expect to observe peaks in σ_c at $\omega = 2\Delta$, corresponding to the sharp peaks observed in photoemission and in c -axis tunneling.^{16,17,26} Further, the momentum mixing caused by this scattering would decrease ρ_s below the BCS ‘‘missing area’’ value, as noted by Kim.¹⁰ We therefore believe the assumption that the tunnelling matrix element always conserves momentum, and that the changes in conductivity with doping are due to changes in the underlying physics of the CuO_2 planes, is correct. The data are also inconsistent with the model of Ref. 24 which for optimally doped materials

would predict a $\rho_s(T=0)$ greater than the area missing below T_c . Another alternative model involves tunnelling through a resonant level residing on a defect between the planes.¹¹ In our view the similarity of the behavior observed in a range of different compounds renders this model implausible; however, if the resonant tunneling is momentum conserving the analysis we have presented may be easily generalized. If it is not momentum conserving, the previous objections apply.

In the present paper we assumed, following Ref. 12, that the frequency dependence of the gap function was not important. In conventional superconductors the frequency dependence of Δ is weak and leads only to minor effects, but because it is possible that in high- T_c materials Δ has a significant frequency dependence for $\omega \sim \Delta$, we present here a brief qualitative discussion of the consequences for σ . The formulas for spectral weight and the Debye-Waller parameter α generalize trivially: one simply includes the proper $\Delta(\omega)$ and the physics we described is not changed. In particular, the conclusion that a reduction of spectral weight between high temperatures and $T=0$ implies quantum phase fluctuations in the superconducting ground state [i.e., $\alpha(T=0) < 1$] still holds. The detailed Ω dependence of $\sigma(\Omega)$ of course depends on the detailed ω dependence of Δ . In any reasonable model the real part of $\Delta(\omega)$ is a decreasing function of frequency. If Δ drops slowly, our conclusions are not significantly altered. If it drops rapidly, all material properties approach their high-frequency (normal state) values more rapidly as a function of ω . In the case of present interest the normal state σ_c is just a constant, so an ω -dependent Δ will tend to fill in the minimum shown in Fig. 4 and reduce the strength of the quasiparticle peak (in the no-phase-coherence case).

We now consider the evolution of the spectral weight as the doping is decreased. Underdoped materials exhibit a normal-state pseudogap, which begins to be visible as the temperature is decreased below a temperature T_{PG} , which increases with decreasing doping. Spectral weight appears to be independent of T at $T > T_{PG}$, with one exception: in $\text{La}_{1-x}\text{Sr}_x\text{CuO}_4$, $\int \sigma_c(\omega) d\omega$ appears to have a strong temperature dependence at all measured frequencies between room temperature and low 100 K. This behavior may be due to the LO-TO structural phase transition, which will change the numerical value of the interplane coupling. In any event, this nonconservation of weight over a wide scale seems to be peculiar to the $\text{La}_{1-x}\text{Sr}_x\text{CuO}_4$ materials and not to be related to superconductivity.

As the temperature is reduced below T_{PG} the onset of the pseudogap causes a decrease in total spectral weight.^{2,3,25} As the temperature is further reduced below T_c the weight [including both ρ_s and $\sigma(\omega > 0)$] increases again, but the $T=0$ weight is never greater than the $T > T_{PG}$ weight. This behavior is consistent with our results if the pseudogap is the superconducting pairing gap, and T_c corresponds to the onset of phase coherence. By contrast, if the pseudogap were caused by a charge or spin-density wave fluctuation, then the methods we have used here would predict that weight would be conserved even in the $T_{PG} > T > T_c$ fluctuation regime: weight lost at low frequencies due to the opening of the gap would be shifted for frequencies just above $2\Delta_{PG}$. Thus we believe the c -axis optical data provides strong support to the

hypothesis that the pseudogap is due to superconducting pairing without phase coherence.¹⁸ We note also that the model of Ref. 24 did not consider the pseudogap regime explicitly, but the results seem to imply a $\rho_s(T=0)$ greater than the weight lost below T_{PG} , again in contradiction to the data.

In underdoped $\text{YBa}_2\text{Cu}_3\text{O}_{6+y}$, ρ_s at $T=0$ is less than the area lost below T_{PG} ,¹⁵ implying strong quantum fluctuations; in other materials there is as yet no evidence for strong quantum fluctuations. In $\text{YBa}_2\text{Cu}_4\text{O}_8$ the evidence suggests ρ_s equals the area lost below T_{PG} .

The ratio of ρ_s to the change in $\omega > 0$ spectral weight between $T > T_c$ and $T \rightarrow 0$ appears qualitatively consistent with the results presented here. In optimally doped and overdoped materials the gap closing coincides with T_c and the expected ratio of 1 is apparently observed.^{3,15,27} As the doping is decreased and the pseudogap becomes more apparent, the measured ratio grows. A number of underdoped materials appear to exhibit a ratio of 2,^{3,15,25} while a very recent measurement²⁷ finds a ratio of ≈ 1.6 in a slightly underdoped BSCCO sample. It is not yet clear whether the effects of the T -dependent offset are visible in the spectral weight. An experimental study of the interrelationship of $\alpha(T=0)$ (as defined by the ratio of the observed ρ_s to the area missing below T_{PG}), the T -dependent offset (from photoemission) and the ratio of ρ_s to the area missing below T_c would be very valuable.

We now turn to the frequency dependence of σ_c . The effects discussed here seem clearly to be visible in the $\sigma_c(\omega, T)$ presented in Fig. 3 of Tajima *et al.*,³ although it should be noted that there are large uncertainties in the experimental determination of the electronic contribution to σ_c because the observed conductivity is dominated by phonon lines which must be subtracted out. Also the YBCO family of materials studied by Tajima *et al.* may have (especially for optimally and overdoped $\text{YBa}_2\text{Cu}_3\text{O}_{6+y}$ and for $\text{YBa}_2\text{Cu}_4\text{O}_8$) large contributions from electronic states involving the CuO_2 chains, which are not included in our theory. Additionally the YBCO materials have a bilayer structure which may allow other excitations, including in particular ‘‘optical’’ Josephson plasmons.^{28,29} Further study of systems without chains and of single-plane systems is needed.

Consider first the optimally doped sample [Fig. 3(b) of Ref. 3]. The $T > T_c$ conductivity is small and has negligible frequency and temperature dependence, consistent with a large, quasistatic T -independent scattering rate Γ . [References 15 shows the conductivity of a similar sample over a wide frequency range—frequency dependence is only apparent for frequencies > 0.5 eV confirming the large Γ .] As T is decreased below T_c , the opening of the superconducting gap leads to a decrease in σ for $\omega < 600$ cm^{-1} . We believe (cf. Fig. 4) that these data are consistent with a maximum gap value $\Delta_{max} \approx 200$ cm^{-1} and an offset $\Omega \sim \Delta$. The presence of absorption at the lowest measured frequencies and the rounded shape of $\sigma_c(\omega)$ are presumably due to a combination of defects and the fact that in real materials σ_c is determined by an average over the Fermi surface of an angle dependent energy gap and an angle dependent c -axis hopping: $\sigma_c \sim \int d\hat{p} t_{\parallel}^2 \Pi(\hat{p})$ (of course, additional contributions

from chain states are also possible). The observed ρ_s is found to compensate for the missing area. Thus in optimally doped $\text{YBa}_2\text{Cu}_3\text{O}_7$ the T dependence of the total spectral weight, of ρ_s , and of the ω and T dependence of the σ are consistent with the behavior expected for a BCS superconductor with negligible quantum fluctuations at low T and a negligible pseudogap above T_c .

As the doping level is decreased, the behavior changes. As seen in Figs. 3(c) and (d) of Ref. 3, a pseudogap (suppression of the absorption at $\omega < 2\Delta$) is visible for a range of temperatures $T > T_c$. The rise of the conductivity at the pseudogap edge seems rather abrupt, as expected from the results shown in Fig. 2. The total spectral weight (superfluid part plus contribution from $\omega > 0$) is less than the room-temperature spectral weight. Next, a peak appears in σ at $\omega \sim 400\text{--}500$ cm^{-1} ; the peak is stronger in the $\text{YBa}_2\text{Cu}_3\text{O}_{6.6}$ sample than in the $\text{YBa}_2\text{Cu}_3\text{O}_{6.7}$ one, but the position does not shift. Finally, there is a hint of the minimum at $\omega \sim 600$ cm^{-1} in the $\text{O}_{6.6}$ sample.

This behavior is qualitatively consistent with the theoretical curves shown in Fig. 4. It is reasonable to assume that as doping is decreased, quantum fluctuations increase; this is shown by the analysis of the optical weight presented in our previous paper and consistent with the analysis of the Hall conductivity presented in Ref. 31. The larger peak in the $\text{O}_{6.6}$ material is consistent with the stronger quantum fluctuations expected there. The peak at $\omega > 400$ cm^{-1} and minimum at 600 cm^{-1} are expected from the $2\Delta, 2\Delta + \Omega$ structure of the self-energy if the threshold scattering frequency Ω is $\approx \Delta \approx 200$ cm^{-1} . Finally, we note that in the $\text{O}_{6.6}$ material the spectral weight in the peak is about 50% of the spectral weight lost below 2Δ in superconducting state as expected if the quantum fluctuations are very strong. The theory predicts an anticorrelation of the weight in the peak and in $\rho_s(T=0)$. To make the comparison quantitative one should measure both in units of the weight lost below 2Δ . The data needed to make this comparison are not available at present.

As noted above, an alternative interpretation of the peak as a bilayer plasmon has been presented.^{28,29} In support of our interpretation we note that the strength of the feature seems correlated with the strength of quantum fluctuations. However, our results imply that the peak should be visible in single-layer materials where to date no peak has been reported. Indeed, in high- T_c materials apart from $\text{YBa}_2\text{Cu}_3\text{O}_{6+y}$ no clear peak has been observed in σ_c below T_c (except, perhaps in $\text{YBa}_2\text{Cu}_4\text{O}_8$ where a slight hint of a peak is visible). The absence of a strong peak in this material is consistent with the observation that the optical spectral weight is the same at room temperature as it is at $T=0$, implying quantum fluctuations are weak in $\text{YBa}_2\text{Cu}_4\text{O}_8$. Further experimental investigation of these issues would be very helpful.

VII. CONCLUSIONS AND OPEN PROBLEMS

In this paper we have shown how quantum and thermal fluctuations of the superconducting order parameter affect the integrated spectral weight and frequency dependence of the interplane conductivity. We showed that the data are consistent with the notion that the normal-state pseudogap is due

to superconducting pairing without long-range phase coherence, and imply that in very underdoped materials, quantum fluctuations of the phase of the superconducting order parameter are strong. No further assumptions are required to account for the data. In particular, while the increase in spectral weight observed below T_c in underdoped materials may be interpreted as a change in c -axis kinetic energy on entering the superconducting state,^{4,25,27} this increase was shown to be a simple consequence of the hypothesis mentioned above, and therefore provides little additional insight into the microscopics of the normal and superconducting states of high- T_c materials.

The crucial microscopic information which can be extracted from σ_c involves the mechanism by which electrons are scattered within a single CuO_2 plane. We have argued above, following Anderson,⁴ that the c -axis conductivity is in effect a spectroscopy of the in-plane Green's function and implies that at least for momenta near the $(\pi, 0)$ points which dominate the c -axis conductivity the in-plane Green's function is characterized by a self-energy which is large, imaginary, and only weakly frequency dependent, i.e., by a large frequency-independent scattering rate. The physical origin of the scattering is not at present understood. One possibility, adopted by many workers^{12,13,30} is that the scattering rate may be thought of in a relatively conventional way, as the scattering of a usual electron off of some fluctuation. An alternative view, propounded by Anderson⁴ is that the c -axis conductivity reflects a fundamentally unconventional ("non-Fermi-liquid") physics of the CuO_2 planes, which have reasonably well defined excitations which however have very small overlap with the conventional electron.

In this paper we studied in detail the consequences of the more conventional picture, because it is well enough defined to allow detailed calculation. Because our results depend mainly on the shape of the electron spectral function near the $(0, \pi)$ points, it seems likely they would follow from a non-Fermi-liquid picture also.

Within the conventional picture, at least, it is natural to assume that if the superconducting order parameter exhibits strong quantum fluctuations then electron-electron interactions are strong generally and therefore make an important contribution to the scattering rate. If this is the case, then it seems reasonable that this scattering will be affected by the onset of superconductivity, and specifically that the low-frequency part of it will be suppressed.¹⁴ The hypothesized suppression of the scattering rate seems to have been confirmed for high- T_c materials by photoemission data. The data are however inconsistent with theoretical expectation in a manner which deserves further discussion and investigation. The argument for the suppression was this:¹⁴ electron-electron scattering involves the creation of a particle-hole pair. In the superconducting state the density of states of these pairs is reduced for energies less than 2Δ ; hence in the range $\Delta < \omega < 3\Delta$ one would have weakly scattered quasiparticles. In fact, photoemission¹⁷ suggests that the range of weak scattering is $\Delta < \omega < \Delta + \Omega$ with $\Omega \sim \Delta$ not 2Δ . Nor-

man and Ding,¹² from a phenomenological point of view, and Chubukov and Morr,¹³ from a calculation of a model of electrons coupled to spin fluctuations, explained this as a strong-coupling effect: electrons do not scatter off a pair, but off of a collective mode of some kind. In the gapped state, what amount to excitonic effects reduce the final-state energy below the naive 2Δ threshold.

In order to produce the observed effects, the coupling of electrons to the mode must be very strong, and in the normal state the spectral weight in the mode must be concentrated at $\omega = 0$. This is already somewhat unusual, but the data exhibit a further anomalous feature. In underdoped materials the gap appears at a high temperature $T_{PG} \sim 200$ K while phase coherence appears at the much lower resistive superconducting transition temperature $T_c \sim 60$ K. *The appearance of the scattering rate offset, Ω , seems to be tied to the onset of phase coherence, and not to the opening of the gap.* This feature is not expected from the arguments given above, and is not understood at present.

We have shown that, within the conventional picture at least, the T -dependent offset leads to two effects in σ_c : the appearance of a peak at $\omega \sim 2\Delta$, if quantum fluctuations are strong, and a decrease in the ratio between ρ_s and the change between T_c and $T=0$ in the $\omega > 0$ spectral weight. The experimental status of these two effects is unclear; further studies would be valuable.

On the theoretical side, two possible avenues of investigation present themselves. One is that the mode which scatters electrons is the phase fluctuations of the superconducting order parameter. This idea was advanced by Geshkenbein, Ioffe, and Larkin³¹ and has been adopted by us and by others.^{30,32,33} Another possibility is that the more or less conventional physical picture of usual electrons strongly scattered by some bosonic mode is simply inadequate, and that the explanation should be sought in the physics of an underlying non-Fermi-liquid state which becomes more conventional when long-ranged phase coherence is established. This idea has been advanced by P. W. Anderson⁴ and receives at least qualitative support from the slave-boson gauge theory approach to the t - J model,^{34,35} where establishment of phase coherence is related to the restoration of more Fermi-liquid-like behavior.

In our opinion, understanding the physics of the very large scattering is one of the key problems in high- T_c superconductivity. In this paper we have shown how measurements of the c -axis conductivity can provide insights into this problem.

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