Coherence effects in conventional layered superconductors

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(Received 1 October 1999)

We present and apply a theory for the coherence effects in spatially inhomogeneous layered superconductors. The theory is based upon a comparison of de Gennes's self-consistent equation for the pair potential to that of alternative proximity-effect models, from which the generalized, space-dependent BCS probability amplitudes u's and v's are obtained. Recent microwave surface impedance data of Pambianchi *et al.* on Nb/Cu bilayers are successfully reproduced.

Evidence for coherence effects in spatially inhomogeneous layered superconductors has recently been reported.[1–3] In the microwave surface impedance measurements on Nb/Cu bilayers,[1] Pambianchi, Chen, and Anlage found that the temperature dependences of the real part of the effective conductivity σ_1 show well-characterized peaks. In contrast to the usual BCS coherence peak observed in a Nb film, the σ_1 peak for the Nb/Cu samples is shallower and broader, and it shifts towards lower temperatures as the Cu layer thickness increases.

For the BCS homogeneous superconductors, the concept of coherence effects is well known and it contributes to the successful explanation of the remarkable difference found in ultrasonic experiments (case I) and nuclear relaxation or electromagnetic absorption experiments (case II).[4,5] In this paper we present an approach which extends the theory to the spatially inhomogeneous layered superconductors (bilayers and multilayers). Our approach, which is based upon a comparison of de Gennes's self-consistent equation for the pair potential[6,7] to that of alternative proximity-effect models, is surprisingly simple, and it applies generally to the case-I and case-II experiments. We shall discuss the microwave data on Nb/Cu bilayers,[1] and show that they can be satisfactorily reproduced.

Basic formalism. Coherence effects arise from the cross terms which enter in squaring the matrix element in the computation of transition probabilities under an external perturbation. In homogeneous superconductors, the coherence factors, which describe the effects, can be expressed in terms of the BCS probability amplitudes u's and v's. Namely, for quasiparticle scattering and creation or annihilation of two quasiparticles, we have, respectively,

$$F_{sc}=(uu'\mp vv')^2=\tfrac{1}{2}(1\mp\eta\eta'), \tag{1a}$$

$$F_{ac}=(vu'\pm uv')^2=\tfrac{1}{2}(1\pm\eta\eta'). \tag{1b}$$

Here symbols with or without a prime correspond to the quasiparticle states after or before a transition. η is defined in this work by

$$\eta=2uv. \tag{2}$$

In Eq. (1), the upper and lower signs apply to case I and case II, respectively, and $\eta\eta'/2$ is the cross term. The square terms amount to $\frac{1}{2}$ (Refs. 4 and 5) since after they are expressed as explicit functions of energy, parts of them cancel directly and parts of them vanish on the subsequent energy integration.[8] Considering that $u^2+v^2=1$, we have from Eq. (2)

$$u^2=\tfrac{1}{2}(1+\sqrt{1-\eta^2}), \tag{3a}$$

$$v^2=\tfrac{1}{2}(1-\sqrt{1-\eta^2}). \tag{3b}$$

With the coherence factors, the transition rates for the scattering processes and annihilation or creation processes, when normalized to the normal-state value α_n, can be written as

$$\frac{\alpha_{sc}}{\alpha_n}=\frac{4}{\hbar\omega}\int_{\Omega}^{\infty}F_{sc}[f(E)-f(E+\hbar\omega)]n(E)n(E+\hbar\omega)dE, \tag{4a}$$

$$\frac{\alpha_{ac}}{\alpha_n}=\frac{2}{\hbar\omega}\int_{\Omega}^{\hbar\omega-\Omega}F_{ac}[1-f(E)-f(\hbar\omega-E)]\times n(E)n(\hbar\omega-E)dE, \tag{4b}$$

in which Ω is the energy gap, $\hbar\omega$ is the energy quantum of the external perturbation, and $n(E)$ is the density of states normalized to $N(0)$, the densities of states at the Fermi surface with one spin orientation.

The u's and v's and the related quantities in Eqs. (1)–(4) are all space-independent in the BCS homogeneous case. To extend the formalism to the spatially inhomogeneous case, we recall de Gennes's discussions and derivations on the Bogoliubov equations which govern the space-dependent u's and v's.[7] We note that they appear as the coefficients in a unitary transformation between the free-electron operators and Bogoliubov operators, which is the exact analog of the one in the BCS treatment.[5] Furthermore, in the layered system we are considering, the spatial variations of the superconducting properties are one dimensional. In this case, it is convenient and sufficient to take the u's and v's to be real quantities (see below). Following Tinkham,[4] one easily verifies that Eqs. (1)–(4) also apply to the inhomogeneous case with the u- and v-related quantities being space dependent. The problem therefore reduces to the finding of the space-dependent u's and v's, or identically, from Eqs. (1) and (3), to that of η. Let us replace the summation over **k** with an energy integral and write down de Gennes's self-consistent equation for the pair potential[6,7] in the form

$$\Delta(z)=N(0)V\int_0^{\omega_D}\eta(E,z)[1-2f(E)]n(E,z)dE, \quad (5)$$

where ω_D and $N(0)V$ are the Debye temperature and BCS coupling constant, respectively, and

$$\Delta(z)=V\langle\psi_\uparrow(z)\psi_\downarrow(z)\rangle \quad (6)$$

is the pair potential in the usual notation. Here we have chosen the z axis to be perpendicular to the planes of the layered system. $\eta=2uv$ is related to the condensation amplitude of the Cooper pairs.[7] In the BCS theory, we have

$$\eta_{\mathrm{BCS}}=\Delta/E. \quad (7)$$

Substituting η_{BCS} into Eq. (5), we find that it reduces to the BCS gap equation, as expected. We now show how η can be obtained in the inhomogeneous situations with the following two practical proximity-effect models.

McMillan's proximity-effect model (Ref. 9). In this model, the simplest situation for an SS' bilayer (we shall always have $T_{c,S}>T_{c,S'}\geqslant 0$) is considered in which the film thicknesses $d_{S,S'}$ are thin compared to their coherence lengths $\xi_{S,S'}$ so that the superconducting properties of each film are uniform across its thickness. The coupling between S and S' is described by a tunneling Hamiltonian. The pair potential defined by Eq. (6), which McMillan refers to as the BCS potential,[10] is constant in both S and S' and is given by[9]

$$\Delta_{S,S'}^{ph}=[N(0)V]_{S,S'}\int_0^{\omega_D^{S,S'}}\mathrm{Re}\left[\frac{\Delta_{S,S'}(E)}{\sqrt{E^2-\Delta_{S,S'}^2(E)}}\right]\times[1-2f(E)]dE, \quad (8)$$

in which Re stands for taking the real part. Comparing Eq. (8) with Eq. (5) leads to

$$\eta_M=\frac{\mathrm{Re}[\Delta(E)/\sqrt{E^2-\Delta^2(E)}]}{n(E)}, \quad (9)$$

where we have neglected the subscripts for clarity. $n(E)$ is given by $\mathrm{Re}[E/\sqrt{E^2-\Delta^2(E)}]$ in the model. For an SS' bilayer with vanishing coupling strength, the model has the solutions $\Delta_{S,S'}(E)\to\Delta_{S,S'}^{ph}$, and we have $\eta_M\to\eta_{\mathrm{BCS}}$ in both S and S' films with $\Delta_{S,S'}^{ph}$ being the BCS energy gaps for the films.

The proximity-effect model of Golubov et al. In Golubov's model,[11] which is based upon Usadel's dirty-limit quasiclassical equations,[12] the nonuniformity in a thick S film is considered. Recently, this model has been extended to include the case where the superconducting properties are constant in S but may vary in S'.[13] In these treatments, applicable to bilayers as well as multilayers,[13,14] the pair potential defined by Eq. (6) is known as the order parameter, and can be expressed in the form[13]

$$\Delta_{S,S'}(z)=[N(0)V]_{S,S'}\frac{2T}{T_c}\sum_{\omega_n>0}^{\omega_D^{S,S'}}F_{S,S'}(\omega_n,z), \quad (10)$$

where ω_n is the Matsubara frequency, and T_c the transition temperature of the layered system. F is Gorkov's "anomalous" Green function integrated over energy and averaged over the Fermi surface.[12] We can replace ω_n with $-iE$ in Eq. (10) and using the standard procedure,[5] we find

$$\Delta_{S,S'}(z)=[N(0)V]_{S,S'}\int_0^{\omega_D^{S,S'}}\mathrm{Im}[F_{S,S'}(E,z)]\times[1-2f(E)]dE. \quad (10')$$

Here Im$[F]$ refers to the imaginary part of F. From Eqs. (10′) and (5), we obtain

$$\eta_G=\frac{\mathrm{Im}[F(E,z)]}{n(E,z)}. \quad (11)$$

The results given by Eqs. (10′) and (11) are similar in form to those in Eqs. (8) and (9) considering that in the homogeneous case, the Usadel equations have the analytical solution $F=i\Delta/\sqrt{E^2-\Delta^2}$.

Discussions. Below we shall present our numerical results and compare them with the σ_1 data on Nb/Cu bilayers in Ref. 1. σ_1 corresponds to the transition probability $\alpha_s=\alpha_{sc}+\alpha_{ac}$, given by Eq. (4), with the case-II coherence factors. We shall use the following parameters. From Ref. 1, we have $d_{\mathrm{Nb}}=300$ nm, $d_{\mathrm{Cu}}=9-76$ nm, and the resistivities $\rho_{\mathrm{Nb}}\sim 1\ \mu\Omega$ cm, $\rho_{\mathrm{Cu}}\sim 0.2\ \mu\Omega$ cm. The microwave frequency is 11.7 GHz, which corresponds to $\hbar\omega=4.84\times10^{-2}$ meV. In addition, we have $\omega_D^{\mathrm{Nb}}=275$ K, $\omega_D^{\mathrm{Cu}}=343$ K, and $T_{c,\mathrm{Nb}}=9.2$ K, $T_{c,\mathrm{Cu}}=1.5\times10^{-2}$ K.[15] From $kT_c=1.13\hbar\omega_D e^{-1/N(0)V}$, the BCS coupling constants are evaluated to be $[N(0)V]_{\mathrm{Nb}}=0.2841$ and $[N(0)V]_{\mathrm{Cu}}=0.098\,43$. The coefficients of electronic specific heat are $\gamma_{\mathrm{Nb}}=7.30\times10^{-4}\ \mathrm{J\,cm^{-3}\,K^{-2}}$ and $\gamma_{\mathrm{Cu}}=9.77\times10^{-5}\ \mathrm{J\,cm^{-3}\,K^{-2}}$ for Nb and Cu, respectively.

In McMillan's model, the parameters describing the proximity effect in SS' bilayers are defined by

$$\Gamma_S=T^2Ad_{S'}N_{S'}(0), \quad \Gamma_{S'}=T^2Ad_SN_S(0), \quad (12)$$

where T^2 is the transmission probability between S and S' and A is their area. In Fig. 1, we show the general behavior of α_s under the variation of Γ_{Cu}. The data are calculated from Eq. (4a) since $\hbar\omega\ll\Omega$ in almost the entire temperature range below T_c, and the annihilation or creation process is absent. Γ_{Nb} is determined by $\Gamma_{\mathrm{Nb}}/\Gamma_{\mathrm{Cu}}=d_{\mathrm{Cu}}N_{\mathrm{Cu}}(0)/d_{\mathrm{Nb}}N_{\mathrm{Nb}}(0)=d_{\mathrm{Cu}}\gamma_{\mathrm{Cu}}/d_{\mathrm{Nb}}\gamma_{\mathrm{Nb}}=0.053\,53$, where we have taken $d_{\mathrm{Cu}}/d_{\mathrm{Nb}}=0.4$ considering an effective d_{Nb} in the order of $\xi_{0,\mathrm{Nb}}=43$ nm.[16] From Fig. 1, we see that the shape of $\alpha_s(T)$ in Nb is close to that of the BCS case-II results and does not change much for different Γ_{Cu} values. The shape of $\alpha_s(T)$ in Cu, on the other hand, differs considerably. For larger Γ_{Cu}, its peak is relatively close to T_c. With the decrease of Γ_{Cu}, the peak shifts towards lower temperatures and gradually flattens out. These results are very similar to those observed in Nb/Cu systems[1] except that there the change is due to the increase of d_{Cu}.

From Eq. (12), simultaneous decrease of Γ_{Cu} and Γ_{Nb} means a decrease of T^2 (or simultaneous decrease of d_{Cu} and d_{Nb}). Hence the change from curve $a\to c$ in Cu represents a typical trend of $\alpha_s(T)$ for McMillan's bilayers when the cou-

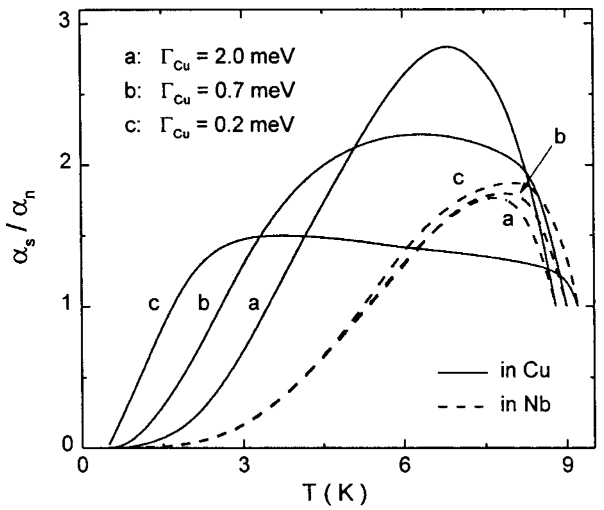

FIG. 1. Transition rates for Nb/Cu bilayers in a 11.7-GHz microwave field, computed from McMillan's proximity-effect model with three Γ_{Cu} values. See text for other sample and material parameters.

pling between S and S' loosens. However, such a trend is less clear if we change $\Gamma_{Nb} \propto d_{Cu}$ alone with other parameters fixed. With increasing d_{Cu}, we usually have a reduced T_c and the shift of the peak in Cu is not obvious. In this sense, McMillan's model does not reproduce the experimental $\alpha_s(T) \sim d_{Cu}$ features observed in the Nb/Cu bilayers.[1]

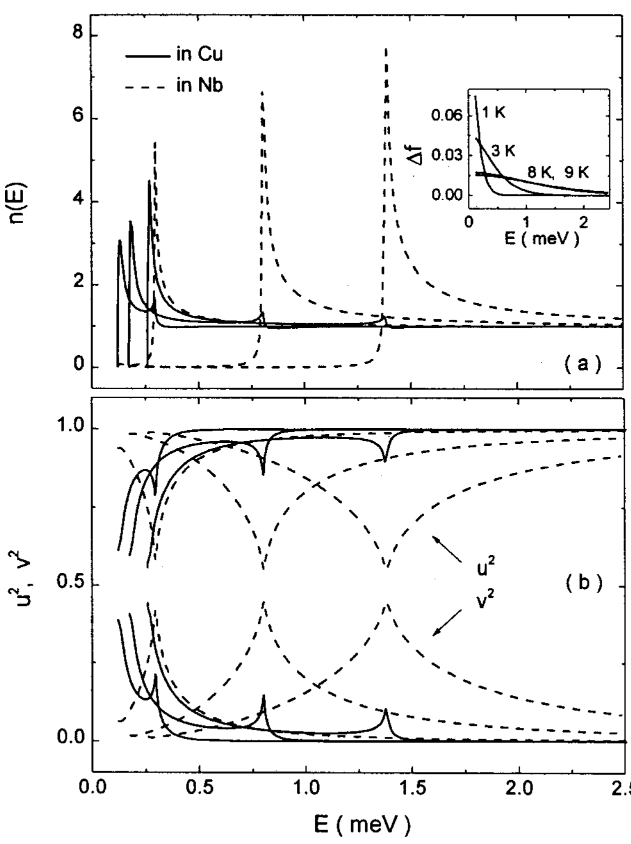

FIG. 2. (a) Densities of states $n(E)$, and (b) u^2 and v^2 at three temperatures of 1, 8, and 9 K for the case of $\Gamma_{Cu} = 0.2$ meV in Fig. 1. In the inset of (a), $\Delta f = f(E) - f(E + \hbar\omega)$ with $\hbar\omega$ being the energy quantum of the microwave field is shown in the same energy range. One more curve at 3 K is added in the inset.

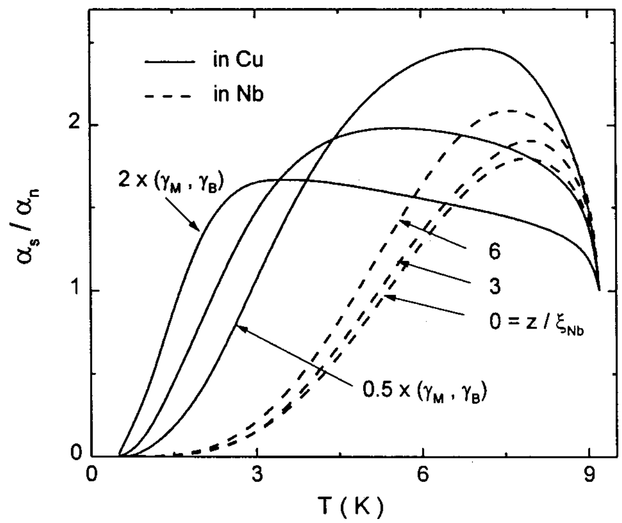

FIG. 3. Transition rates for Nb/Cu bilayers in a 11.7-GHz microwave field, computed from the proximity-effect model of Golubov *et al.* with $\gamma_M = 0.19$ and $\gamma_B = 5$. z is measured from the interface into the Nb layer. Additional curves in Cu with half and twice the values of $\gamma_M, \gamma_B \propto d_{Cu}$ are also shown. See text for other sample and material parameters.

In Fig. 2, $n(E)$, $u^2(E)$, and $v^2(E)$ with $\Gamma_{Cu} = 0.2$ meV are shown for three temperatures $T = 1$, 8, and 9 K. We see that at the energy where the peak in $n(E)$ in Nb locates, there exists a small peak in Cu. Below this energy, $n(E)$ in Nb decreases rapidly and it extends to the system's energy gap Ω near which the main peak in Cu appears. v^2 $(= 1 - u^2)$, which represents the BCS occupation fraction of the Bloch states,[4,5] is seen to have the corresponding peaks in both Cu and Nb. The overall shapes of v^2 and u^2 are similar to those of BCS above the energies where the main peaks in $n(E)$ locate. From Eq. (1), similar situations for the coherence factors are expected. We may conclude from Eq. (4a) that to have an appreciable α_s in Cu at low temperatures there should be a reduced Ω so that a common energy range exists within which both $n(E)$ and $\Delta f = f(E) - f(E + \hbar\omega)$ [see inset to Fig. 2(a)] are not negligibly small.

The fact that McMillan's model does not lead to the experimental $\alpha_s(T) \sim d_{Cu}$ dependences may lie in its inapplicability to the particular Nb/Cu system. We find that if the spatial variations in the thick Nb films are taken into account, as can be described in Golubov's model, the experimental results can be well explained. In Golubov's model, the proximity parameters are[11]

$$\gamma_M = \frac{\rho_S \xi_S}{\rho_{S'} \xi_{S'}} \frac{d_{S'}}{\xi_{S'}}, \quad \gamma_B = \frac{R_B}{\rho_{S'} \xi_{S'}} \frac{d_{S'}}{\xi_{S'}}, \tag{13}$$

in which R_B is the resistance multiplied by area at the SS' interface and $\xi_{S,S'} = \sqrt{\pi \hbar k_B / 6 e^2 \rho_{S,S'} \gamma_{S,S'} T_c}$.[13] With the above quoted data, we find $\xi_{Nb} = 21$ nm $\ll d_{Nb} = 300$ nm and $\xi_{Cu} = 129$ nm $> d_{Cu}$, which confirm uniform and nonuniform situations in Cu and Nb, respectively. In Fig. 3, we show the results for $\gamma_M = 0.19$ taking $d_{Cu} = 30$ nm, and $\gamma_B = 5$ considering a small nonzero R_B.[17] The data at three locations in Nb are shown as dashed lines, which are again similar to the BCS case-II results. From Eq. (13), we see that both γ_M and γ_B are proportional to d_{Cu} for fixed Nb parameters and R_B, and the proportionality is independent of the possible variations of ρ_{Cu} due to $\xi_{\mathrm{Cu}} \propto \sqrt{1/\rho_{\mathrm{Cu}}}$. With increasing $\gamma_M, \gamma_B \sim d_{\mathrm{Cu}}$, the model predicts a decreasing Ω at a given temperature while T_c is not reduced in the thick S layer approximation. Thus the shifting of the α_s peak in Cu to lower temperatures is anticipated. The data with half and twice the values of γ_M and γ_B are shown in the figure, from which the development of the shape of $\alpha_s(T)$ with varying d_{Cu} can be seen. These results, computed with only one parameter R_B which is not determined from experiment, are consistent with the experimental observations.[1]

To summarize, we have developed a theory of coherence effects in spatially inhomogeneous layered superconductors. Recent microwave surface impedance data on the Nb/Cu bilayers have been reproduced. An important result of this work is that the self-consistent equations for the pair potential in different proximity-effect models can be written in very similar forms, and the space-dependent u's and v's can be obtained in a simpler way without solving the Bogoliubov–de Gennes equations directly. This will be useful in the studies of other processes in the layered systems, such as those involving Andreev reflections.

We wish to acknowledge useful discussions with R. F. Wang.

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