Bose-Einstein condensation of excitons in a single quantum well

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We study the problem of Bose-Einstein condensation of excitons in a single quantum well with infinitely high potential barriers. A BCS-like theory is used to describe the modification of the one-particle Green's functions due to the presence of Bose-condensed excitons. By introducing those single-particle properties into the two-particle Bethe-Salpeter equation, we derive a system of two couple equations for the exciton wave functions, which is solved in the low-density limit in appropriate approximations. We obtain that in a single quantum-well structure, the ground-state quadratic exciton dispersion near $\mathbf{Q}=\mathbf{0}$ is modified and starts linear with momentum in the presence of a Bose condensate. We have calculated the chemical potential of excitons in the quantum well in the low-density limit by the variational method. The first-order density correction to the chemical potential is calculated for different thicknesses. We obtain that in the low-density limit, when the electron-hole excitonic bound states can be considered as composite bosons, the critical temperature of the Bose-Einstein condensation scales linearly with two-dimensional (2D) density of excitons. In a strictly two-dimensional case a system of 2D excitons may undergo a phase transition to a superfluid state. We have calculated the critical temperature of the Kosterlitz-Thouless phase transition to exciton superfluidity as a function of the exciton density.

I. INTRODUCTION

The excitons are the energetically lowest electronic excitations of ideal semiconductors. Depending on the exciton density and temperature they can be observed as a weakly interacting exciton gas, molecular gas (biexcitons), metallic liquid, or electron-hole plasma. Due to the large zero-point energy of the particles, the excitons in a simple direct-gap nondegenerate semiconductor are expected to remain in a gas form down to zero temperature. Such a condition is favorable for the formation of the so-called Bose-Einstein condensate of excitons at a finite temperature. The Bose-Einstein condensation (BEC) of excitons means a macroscopic occupation of the zero-momentum ground state and the collection of particles in the zero-momentum ground state is called the excitonic condensate. It is worth noting that the excitons may exist in the condensate and outside of the condensate (in the excited states and/or with finite center-of-mass momentum). Several early works predicting Bose condensation of excitons are due to Moskalenko,[1] Blatt, Boer, and Brandt,[2] and Casella.[3] In many papers[4–9] the BEC of excitons has been carefully examined by using a formal treatment similar to BCS theory of superconductivity, taking into account the composite nature of excitons, made up with two fermions.

In view of the great advances made in our abilities to design and manufacture low-dimensional semiconductor structure, the search for a low-dimensional condensed phase of excitons has greatly expanded in recent years. Most of the works studied the exciton condensed phase in low-dimensional structures in which the electrons and holes are in two different infinitesimally thin layers separated by a wide barrier material.[10–14] Such structures are favorable for both theoretical and experimental investigation of BEC of excitons because: (i) the barrier increases the exciton lifetime and avoids formation of biexcitons; (ii) from theoretical point of view the electron and hole motions in the infinitesimally thin layers can be regarded as pure two-dimensional (2D) motions without any quantization along the direction perpendicular to the layers. But, in this structure the barrier reduces the electron-hole Coulomb interaction, and so decreases the critical temperature for condensation. From that point of view a single quantum well structure in which the lifetime of excitons is long enough should be more suitable for manifestation of the appearance of exciton condensed phase.

In this theoretical study we also consider the special case of an infinitesimally thin well, when the motion of excitons can be regarded as pure two-dimensional motion. It is known from literature that in a strictly two-dimensional system the phase fluctuations destroy the off-diagonal long-range order, and therefore the BEC cannot exist at finite temperatures. Kosterlitz and Thouless[15] have shown the possibility of a phase transition to superfluidity in a 2D system. This phase transition is a topological phase transition, at which despairing of the vortex pairs takes place. Below the Kosterlitz-Thouless (KT) critical temperature T_c those vortices are bound in pairs with other vortices of opposite circulation and that leads to a dissipationless flow. At temperatures above T_c there exist unbound vortices, which give rise to a dissipation of the flow. The KT phase transition is a topological phase transition and does not violate the Hohenberg theorem[16] that no off-diagonal long-range order can exist in 2D. Lozovik and co-workers[17–20] and Shevchenko[21,22] have shown that in low-dimensional structures in which the electrons and holes are in two different infinitesimally thin layers separated by a wide barrier material with a thickness d, the exciton condensate can become superfluid. Fukuzawa, Mendez, and Hong[23] have reported photoluminescence experiments in coupled quantum wells under the electric field which have been interpreted in terms of a KT phase transition.

In what follows we will investigate the Bose condensation of excitons in a single quantum well with infinitely high

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potential barriers. The framework of Green's function is used to describe the effect of the condensate on the single-particle properties. Our basic assumption is that the presence of Bose-condensed excitons modifies the single-particle Green's functions, and therefore one has to consider the so-called ''anomalous'' one-particle electron and hole Green's functions in a manner formally analogous to the BCS theory of superconductivity. The treatment by ''anomalous'' Green's function is also known as a ''coherent'' approach (the other one, the so-called ''incoherent'' approximation, is known from the literature).[24] The''anomalous'' Green's functions vanish above a certain critical temperature, indicating that there is no longer a condensate of excitons in the system. We will investigate the Bose condensation of excitons in a single quantum well with infinitely high potential barriers. We derive a system of two coupled Bethe-Salpeter (BS) equations for the exciton wave functions. The similar systems have been already found in the 3D case by Jerome, Rice, and Kohn[6] at high density and by Cote and Griffin[9] at low density. It is worth noting that whereas in the normal phase (when the ''anomalous'' single-particle Green's functions equal zero) the excitons are well defined, the presence of condensate leads to the serious difficulties in any attempt to solved exactly the above-mentioned BS equations because of the coupling between different excitonic modes. For this reason we have solved the BS equations in the low-density limit, when: (i) the coupling between the different excitonic modes due to the condensate is ignored; (ii) the original exciton modes will not be affected too much by the presence of condensate. Those approximations enable us to obtain the result, that not only in the 3D case, but in the case of a single quantum well structure as well, the ground-state quadratic exciton dispersion near $\mathbf{Q}=\mathbf{0}$ is modified and starts linear with momentum in the presence of a Bose condensate. This type of dispersion is known in literature as the spectrum of Bogolubov type. We also calculate the critical temperature T_c of KT topological phase transition in an exactly two-dimensional structure. We obtain that the critical temperature of the KT phase transition to exciton superfluidity scales linearly with the exciton density.

An outline of the paper is as follows. In Sec. II we present a theory of BEC of excitons in a single quantum well. In Sec. III we investigate KT phase transition in the 2D case.

II. BOSE-EINSTEIN CONDENSATION OF EXCITONS IN QUANTUM WELL STRUCTURES

A. Single-particle spectrum in the low-density limit

In the present section, we address the problem of BEC of excitons in a single semiconductor quantum well, taking into account the fact that the electron and hole motions along the *z* direction (throughout this paper, we take *x*-*y* plane to be the plane of confinement of the two-dimensional electron-hole system) are quantized into discrete levels due to the presence of a confinement potential along this direction. In what follows we are interested in the case of quantum wells made from direct-gap semiconductors with nondegenerate and isotropic bands when the electron-hole pair is confined between two parallel, infinitely high potential barriers. With the perfect confinement approximation the dispersion laws for electrons $E_c(\mathbf{k},\lambda)$ and holes $E_\nu(\mathbf{k},\xi)$ are as follows (we set $\hbar=1$ throughout this paper):

$$E_c(\mathbf{k},\lambda)=E_g+\frac{\mathbf{k}^2}{2m_e}+\frac{\pi^2\lambda^2}{2m_cL^2};$$

$$E_\nu(\mathbf{k},\xi)=-\frac{\mathbf{k}^2}{2m_\nu}-\frac{\pi^2\xi^2}{2m_\nu L^2},$$

where m_c and m_ν are the electron and hole effective masses, E_g is the energy gap, $\mathbf{k}$ is a 2D wave vector, and the quantum well has a thickness L. $\lambda,\xi=1,2,\ldots$ denote the quantum number of the states in the infinitely deep wells.

We will assume that the excitons in a quantum well behave almost like weakly interacting Bose particles, and therefore one might expect that the BEC of excitons is possible. From the theoretical point of view the presence of Bose-condensed excitons modifies the single-particle Green's functions, and therefore one has to consider the so-called ''anomalous'' one-particle electron and hole Green's functions. In our case the Fourier transforms of the inverse ''normal'' and ''anomalous'' one-particle Green's functions are

$$G_{cc}^{-1}(\mathbf{k},\lambda,\lambda',i\omega_m)=\delta_{\lambda\lambda'}\{i\omega_m-[E_c(\mathbf{k},\lambda)-\mu_c]\}, \quad (1a)$$

$$G_{\nu\nu}^{-1}(\mathbf{k},\xi,\xi',i\omega_m)=\delta_{\xi\xi'}\{i\omega_m-[E_\nu(\mathbf{k},\xi)-\mu_\nu]\}, \quad (1b)$$

$$G_{c\nu}^{-1}(\mathbf{k},\lambda,\xi,i\omega_m)=-\Sigma_{c\nu}(\mathbf{k},\lambda,\xi,i\omega_m), \quad (1c)$$

$$G_{\nu c}^{-1}(\mathbf{k},\xi,\lambda,i\omega_m)=-\Sigma_{\nu c}(\mathbf{k},\xi,\lambda,i\omega_m). \quad (1d)$$

Here G_{cc}^{-1}, $G_{\nu\nu}^{-1}$ and $G_{c\nu}^{-1}$, $G_{\nu c}^{-1}$ are the inverse ''normal'' and ''anomalous'' Green's functions, respectively, μ_e and μ_ν are the chemical potentials of the electrons and holes, and $\Sigma_{ij}\,(i,j=c,\nu)$ denote the corresponding mass operators (the mass operators Σ_{cc} and $\Sigma_{\nu\nu}$ have been included in the effective masses in the corresponding dispersion relations). The symbol ω_m denotes $\omega_m=(2\pi/\beta)(m+\frac{1}{2})$, $\beta=(k_BT)^{-1}$, k_B is the Boltzmann constant, T is the temperature, and $m=0,\pm1,\pm2,\ldots$. The mass operators in the Dyson Eqs. (1c) and (1d) can be written as a sum of a Hartree part and a screened Fock part. The Hartree term vanishes because of the global neutrality of the electron-hole system. In what follows we will extract from the mass operators only the screened static Fock terms $\Sigma_{c\nu}(\mathbf{k},\lambda,\xi)=\Delta_{c\nu}(\mathbf{k},\lambda,\xi)$ and $\Sigma_{\nu c}(\mathbf{k},\xi,\lambda)=\Delta_{\nu c}(\mathbf{k},\xi,\lambda)$. In this approximation Eqs. (1c) and (1d) assume the forms

$$G_{c\nu}^{-1}(\mathbf{k},\lambda,\xi,i\omega_m)=-\Delta_{c\nu}(\mathbf{k},\lambda,\xi), \quad (1e)$$

$$G_{\nu c}^{-1}(\mathbf{k},\xi,\lambda,i\omega_m)=-\Delta_{\nu c}(\mathbf{k},\xi,\lambda). \quad (1f)$$

The ''normal'' $G_{cc}(\mathbf{k},\lambda,\lambda',i\omega_m)$, $G_{\nu\nu}(\mathbf{k},\xi,\xi',i\omega_m)$ and ''anomalous'' $F_{c\nu}(\mathbf{k},\lambda,\xi,i\omega_m)$, $F_{\nu c}(\mathbf{k},\xi,\lambda,i\omega_m)$ one-particle Green's functions can be obtained by solving the following set of equations:

$$\delta_{\lambda\lambda'}=\sum_{\lambda''} G_{cc}(\mathbf{k},\lambda,\lambda'',i\omega_m)G_{cc}^{-1}(\mathbf{k},\lambda'',\lambda',i\omega_m) - \sum_{\xi} F_{c\nu}(\mathbf{k},\lambda,\xi,i\omega_m)\Delta_{\nu c}(\mathbf{k},\xi,\lambda'), \tag{2a}$$

$$0=-\sum_{\lambda'} G_{cc}(\mathbf{k},\lambda,\lambda',i\omega_m)\Delta_{c\nu}(\mathbf{k},\lambda',\xi) + \sum_{\xi'} F_{c\nu}(\mathbf{k},\lambda,\xi',i\omega_m)G_{\nu\nu}^{-1}(\mathbf{k},\xi',\xi,i\omega_m), \tag{2b}$$

$$0=\sum_{\lambda} F_{\nu c}(\mathbf{k},\xi,\lambda,i\omega_m)G_{cc}^{-1}(\mathbf{k},\lambda,\xi',i\omega_m) - \sum_{\xi'} G_{\nu\nu}(\mathbf{k},\xi,\xi',i\omega_m)\Delta_{\nu c}(\mathbf{k},\xi',\lambda), \tag{2c}$$

$$\delta_{\xi\xi'}=-\sum_{\lambda} F_{\nu c}(\mathbf{k},\xi,\lambda,i\omega_m)\Delta_{c\nu}(\mathbf{k},\lambda,\xi') + \sum_{\xi''} G_{\nu\nu}(\mathbf{k},\xi,\xi'',i\omega_m)G_{\nu\nu}^{-1}(\mathbf{k},\xi'',\xi',i\omega_m). \tag{2d}$$

The ''normal'' phase of the system under consideration can be described by setting the nondiagonal parts of mass operator $\Delta_{c\nu}$ and $\Delta_{\nu c}$ equal to zero. Thus $\Delta_{c\nu}$ and $\Delta_{\nu c}$ are the order parameters for the condensed phase. Using the static Fock terms, one can write the order parameters in the following forms:

$$\Delta_{c\nu}(\mathbf{k},\lambda,\xi)=-\sum_{\mathbf{q}}\sum_{\lambda',\xi'}\sum_{\omega_m}\frac{2\pi e^2}{\varepsilon_\infty|\mathbf{q}-\mathbf{k}|}f_{\lambda\xi'\xi\lambda'} \times(L|\mathbf{q}-\mathbf{k}|)F_{c\nu}(\mathbf{q},\lambda',\xi',i\omega_m), \tag{3a}$$

$$\Delta_{\nu c}(\mathbf{k},\lambda,\xi)=-\sum_{\mathbf{q}}\sum_{\lambda',\xi'}\sum_{\omega_m}\frac{2\pi e^2}{\varepsilon_\infty|\mathbf{q}-\mathbf{k}|}f_{\lambda\xi'\xi\lambda'} \times(L|\mathbf{q}-\mathbf{k}|)F_{\nu c}(\mathbf{q},\xi',\lambda',i\omega_m), \tag{3b}$$

The function $f_{\lambda\xi'\xi\lambda'}$ is defined as follows:

$$F_{\lambda\xi'\xi\lambda'}(L|\mathbf{p}|)=\int_0^L dz_1\int_0^L dz_2\exp(-|\mathbf{p}|\cdot|z_1-z_2|) \times\chi_\lambda(z_1)\chi_{\xi'}(z_2)\chi_\xi(z_2)\chi_{\lambda'}(z_1), \tag{3c}$$

where

$$\chi_\lambda(z)=\left(\frac{2}{L}\right)^{1/2}\sin\left(\frac{\lambda\pi z}{L}\right)$$

In the set of Eqs. (2) for the ''normal'' and ''anomalous'' one-particle Green's functions one can observe the difference between the condensate we are studying and this occurring in the case when the electrons and holes are in two different infinitesimally thin layers separated by a wide barrier material. In the case of a single quantum well structure one has to take into consideration the composite nature of excitons, made up with two fermions, each one with different quantized motion along the direction perpendicular to the *x-y* plane. For this reason a complicated mixing of excitations will take place unlike the case of two different infinitesimally thin layers. If one takes into account $\lambda=1,2,\ldots,n$ quantized electron states and $\xi=1,2,\ldots,n_2$ quantized hole states, then n_1+n_2 poles for each of propagators G_{cc}, $G_{\nu\nu}$, $F_{c\nu}$, and $F_{\nu c}$ have to be considered.

It is impossible to solve exactly the set of Eqs. (2) for arbitrary n_1 and n_2, and so we must introduce some approximation. The simplest approximation is to take into account only the first electron and hole confined levels. In this approximation $n_1=n_2=1$ and the solutions of the set of Eqs. (2) are

$$G_{cc}(\mathbf{k},1,1,i\omega_m)=G_{cc}(\mathbf{k},i\omega_m)=\frac{u_k^2}{i\omega_m-\omega_+(\mathbf{k})}+\frac{\nu_k^2}{i\omega_m-\omega_-(\mathbf{k})}, \tag{4a}$$

$$G_{\nu\nu}(\mathbf{k},1,1,i\omega_m)=G_{\nu\nu}(\mathbf{k},i\omega_m)=\frac{\nu_k^2}{i\omega_m-\omega_+(\mathbf{k})}+\frac{u_k^2}{i\omega_m-\omega_-(\mathbf{k})}, \tag{4b}$$

$$F_{c\nu}(\mathbf{k},1,1,i\omega_m)=F_{\nu c}(\mathbf{k},1,1,i\omega_m) =G_{c\nu}(\mathbf{k},i\omega_m) =G_{\nu c}(\mathbf{k},i\omega_m)=\frac{\Delta(\mathbf{k})}{\omega_+(\mathbf{k})-\omega_-(\mathbf{k})} \times\left[\frac{1}{i\omega_m-\omega_+(\mathbf{k})}-\frac{1}{i\omega_m-\omega_-(\mathbf{k})}\right], \tag{4c}$$

where the following notations have been used:

$$u_q^2=\tfrac{1}{2}\left[1+\frac{\eta(\mathbf{q})}{\varepsilon(\mathbf{q})}\right];\quad \nu_q^2=\tfrac{1}{2}\left[1-\frac{\eta(\mathbf{q})}{\varepsilon(\mathbf{q})}\right] \tag{5a}$$

$$\omega_\pm(\mathbf{k})=\xi(\mathbf{k})\pm\varepsilon(\mathbf{k});$$

$$\xi(\mathbf{k})=\tfrac{1}{2}[E_c(\mathbf{k},1)+E_\nu(\mathbf{k},1)-\mu_c-\mu_\nu];$$

$$\Delta_{c\nu}(\mathbf{k},1,1)=\Delta_{\nu c}(\mathbf{k},1,1)=\Delta(\mathbf{k}); \tag{5b}$$

$$\varepsilon(\mathbf{k})=\sqrt{\eta^2(\mathbf{k})+\Delta^2(\mathbf{k})};$$

$$\eta(\mathbf{k})=\tfrac{1}{2}[E_c(\mathbf{k},1)-E_\nu(\mathbf{k},1)-(\mu_c-\mu_\nu)].$$

The dispersion law of the single-particle excitations $\omega_\pm(\mathbf{k})$, given by the poles of the Green's functions (4), depends on the excitonic density. At the temperature $T=0$ K and in the low-density limit $na_0^2\ll 1$ (n is 2D density of free electron-hole pairs, $a_0=\varepsilon_\infty\mu^{-1}e^{-2}$ is the 3D exciton Bohr radius, and $\mu^{-1}=m_c^{-1}+m_\nu^{-1}$ is the exciton reduced mass) the function $\Delta(\mathbf{k})$ and the chemical potential of excitons $\mu_{\rm exc}=\mu_e-\mu_\nu$ can be calculated self-consistently. Defining $\psi_0(\mathbf{q})=\Delta(\mathbf{q})/2\varepsilon(\mathbf{q})$, one can write to the lowest order in the density n

$$\psi_0(\mathbf{q})=(n/2)^{1/2}\varphi_0(\mathbf{q}). \tag{5c}$$

The function $\varphi_0(\mathbf{q})$ is a normalized function

$$\int d^2\mathbf{q}/(2\pi)^2\varphi_0^2(\mathbf{q})=1,$$

which is a solution of the following equation:

$$\left[E_g+\frac{\pi^2}{2\mu L^2}-\mu_{\rm exc}^{(0)}+\frac{\mathbf{k}^2}{2\mu}\right]\varphi_0(\mathbf{k}) - \int\frac{d^2\mathbf{q}}{(2\pi)^2}\frac{2\pi e^2}{\varepsilon_\infty|\mathbf{k}-\mathbf{q}|}f_{1111}(L|\mathbf{k}-\mathbf{q}|)\varphi_0(\mathbf{q})=0, \quad (6)$$

where $\mu_{\rm exc}^{(0)}=\mu_c^{(0)}-\mu_\nu^{(0)}$ is the chemical potential of excitons (the upper symbol ''0'' means that in this approximation the corresponding chemical potentials does not depend on the exciton density n).

Equation (6) is the familiar Wannier equation for an exciton with zero center-of-mass momentum in the infinitely deep well. One can solve Eq. (6) by using a variational method with a function

$$R_{1,0}(\mathbf{k})=\sqrt{\frac{\pi}{2}}\left(\frac{4\beta}{a_0}\right)^2\left[\mathbf{k}^2+\frac{4\beta^2}{a_0^2}\right]^{-3/2}. \quad (7)$$

Here $R_{n,m}(\mathbf{k})=R_{n,m}(k,\vartheta)$ denote the radial functions of a 2D hydrogen atom system[25] $n=0,1,2,\ldots$ is the principal quantum number, and for a given n, the angular momentum quantum number $m=0,\pm1,\pm2,\ldots,\pm n$.

The parameter β can be determined by maximizing the chemical potential $\mu_{\rm exc}^{(0)}$ with respect to β. Thus we obtain the following equation for the chemical potential $\mu_{\rm exc}^{(0)}$:

$$\frac{E_g+[\pi^2/(L/a_0)^2]E_0-\mu_{\rm exc}^{(0)}(\beta)}{E_0} = -4\beta^2+128\beta^3\int_0^{+\infty}dx\,\frac{f(xL/a_0)}{(x^2+16\beta^2)^{3/2}}, \quad (8)$$

where $E_0=(2\mu a_0^2)^{-1}$ is the effective exciton Rydberg and the function $f(x)$ is defined as follows:

$$f_{1111}(x)=f(x)=\frac{3x^2+8\pi^2}{x(x^2+4\pi^2)}-\frac{32\pi^4[1-\exp(-x)]}{x^2(x^2+4\pi^2)^2}. \quad (9)$$

We have numerically maximized the expression (8) for the different values of the well thickness L/a_0. Figure 1 shows the dimensionless chemical potential $\mu_{\rm exc}^{(0)}/E_0$, measured from the total ground-state energy $E_g+[\pi^2/(L/a_0)^2]E_0$ of the electron-hole pair in the well as a function of the dimensionless well thickness L/a_0. In a strictly 2D case $\beta=1$ and $f(x)=1$, so the chemical potential goes to $4E_0$ when L goes to zero. As can be seen, the two-dimensional behavior of the chemical potential disappears very quickly: for $L/a_0=1$, $\mu_{\rm exc}^{(0)}\approx 2E_0$. For $L>4a_0$ the trial function $R_{1,0}$ leads to the chemical potential already smaller than E_0, and so our trial wave function is well suited for narrow well structures, but is not so good for large L limit.

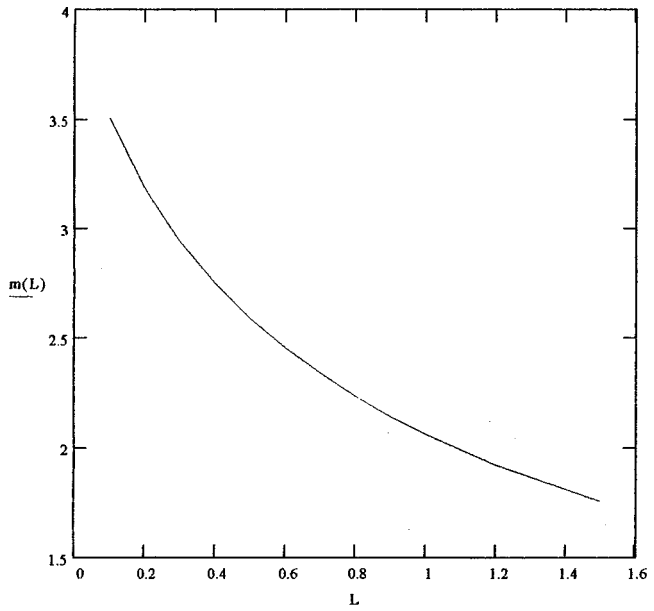

FIG. 1. The calculated dimensionless chemical potential of excitons $m(L)=\mu_{\rm exc}^0/E_0$ (E_0 is the 3D exciton rydberg), measured from the total ground-state energy of the electron-hole pair in the well as a function of the dimensionless well thickness L/a_0 (a_0 is the 3D exciton Born radius).

B. Density correction to the chemical potential

The chemical potential $\mu_{\rm exc}^{(0)}$ does not depend on the exciton density because in that approximation the excitons are completely independent. The density only enters through the magnitude of $\Delta(\mathbf{k})$. The next correction to the chemical potential arises from the exchange repulsion between the electrons and holes that are involved in two excitons. In the 3D case such short-range repulsion due to the overlap of internal wave function is known from the literature.

In the low-density limit the dispersion laws of the single-particle excitations in a quantum well are

$$\omega_\pm(\mathbf{k})=\xi^{(0)}(\mathbf{k})\pm\eta^{(0)}(\mathbf{k}) \pm\frac{n}{2}\varphi_0(\mathbf{k})\int\frac{d^2\mathbf{q}}{(2\pi)^2}\left(\frac{2\pi e^2}{\varepsilon_\infty|\mathbf{k}-\mathbf{q}|}\right)f(L|\mathbf{k}-\mathbf{q}|)\varphi_0(\mathbf{q}), \quad (10)$$

where

$$\xi^{(0)}(\mathbf{k})=\tfrac{1}{2}[E_c(\mathbf{k},1)+E_\nu(\mathbf{k},1)-\mu_c^{(0)}-\mu_\nu^{(0)}];$$

$$\eta^{(0)}(\mathbf{k})=\tfrac{1}{2}[E_c(\mathbf{k},1)-E_\nu(\mathbf{k},1)-\mu_{\rm exc}^{(0)}].$$

In the next order in density (order n) Eq. (6) assumes the form

$$\left[E_g+\frac{\pi^2}{2\mu L^2}-\mu_{\rm exc}^{(1)}+\frac{\mathbf{k}^2}{2\mu}+nS(\mathbf{k},\mathbf{Q}=\mathbf{0})\right]\varphi_0(\mathbf{k}) - \int\frac{d^2\mathbf{q}}{(2\pi)^2}\frac{2\pi e^2}{\varepsilon_\infty|\mathbf{k}-\mathbf{q}|}f(L|\mathbf{k}-\mathbf{q}|)\varphi_0(\mathbf{q})=0, \quad (11)$$

where $S(\mathbf{p},\mathbf{Q})$ denotes

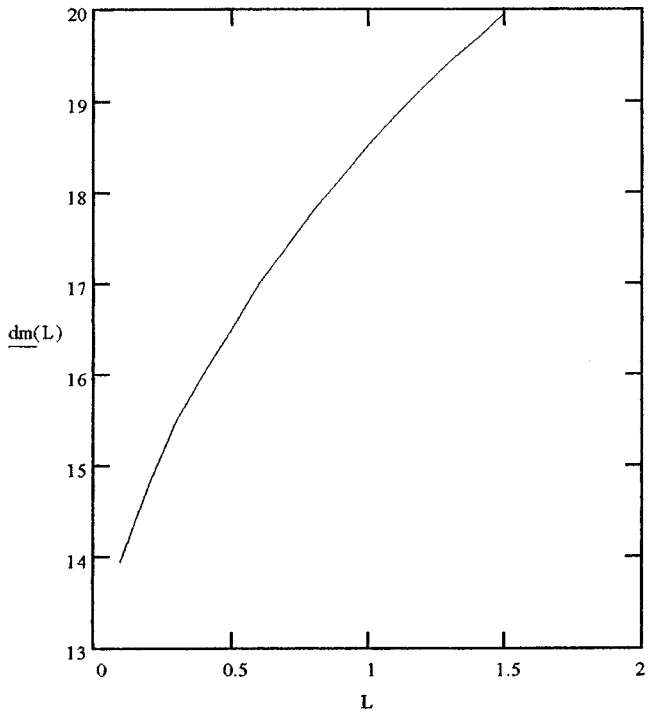

FIG. 2. The calculated correction to the chemical potential $dm(L)=(\mu_{\rm exc}^{(1)}-\mu_{\rm exc}^{(0)})/(na_0^2)E_0$ in the low-density limit $na_0^2\ll 1$ (E_0 is the 3D exciton rydberg, n is 2D density of free electron-hole pairs, a_0 is the 3D exciton Bohr radius) as a function of the dimensionless well thickness L/a_0.

$$S(\mathbf{p},\mathbf{Q})=\tfrac{1}{2}[\varphi_0(\mathbf{p})+\varphi_0(\mathbf{p}-\mathbf{Q})]\times\left(\frac{2\pi e^2}{\varepsilon_\infty}\right)\int\frac{d^2\mathbf{q}}{(2\pi)^2}\,\frac{\varphi_0(\mathbf{q})f(L|\mathbf{p}-\mathbf{q}|)}{|\mathbf{p}-\mathbf{q}|}. \quad (12)$$

From Eq. (11) one can obtain the correction to the chemical potential:

$$\Delta\mu_{\rm exc}^{(1)}=\mu_{\rm exc}^{(1)}-\mu_{\rm exc}^{(0)}=n\int\frac{d^2\mathbf{p}}{(2\pi)^2}\,\varphi_0^2(\mathbf{p})S(\mathbf{p},\mathbf{Q}=\mathbf{0})=(na_0^2)E_0\frac{4\pi}{5}\left[-3+128\beta\times\int_0^{+\infty}\frac{f(xL/a_0)dx}{[x^2+16\beta^2]^{3/2}}\right]. \quad (13)$$

By setting $L\to 0$ one can obtain the density correction $\Delta\mu_{\rm exc}^{(1)}$ in the 2D case. In the limit $L\to 0$, $\beta=1$ and $f(x)=1$. Thus we find that the density correction to the chemical potential in the 2D case is given by

$$\Delta\mu_{\rm exc}^{(1)}=\mu_{\rm exc}^{(1)}-\mu_{\rm exc}^{(0)}=4\pi(na_0^2)E_0. \quad (14)$$

Figure 2 shows $\Delta\mu_{\rm exc}^{(1)}/[(na_0^2)E_0]$ as a function of the dimensionless well thickness L/a_0 calculated by using the 2D hydrogen function $\varphi_0(\mathbf{q})=R_{1,0}(\mathbf{q})$, where the parameter β has been already determined by maximizing the chemical potential $\mu_{\rm exc}^{(0)}$ with respect to β.

C. Two-particle excitation modes in the low-density limit at $T=0$

Our next aim is to obtain the poles of the two-particle Green's function of mechanical excitons K_M^E which takes into account Elliott exchange interaction. In what follows we will assume that the exciton is made up with two fermions each one with a quantum number of the states in the well $\lambda=\xi=1$. We denote by $\omega_l(\mathbf{Q})$ (l is the band index and $\mathbf{Q}$ is the 2D wave vector of the excitons) the poles of the Green's function K_M^E. If we restrict the range of frequencies ω to a neighborhood of the position $\omega_l(\mathbf{Q})$ we may write

$$K_M^E\left(\begin{matrix}\mathbf{r}_1\sigma_1 & \mathbf{r}_3\sigma_3\\ \mathbf{r}_2\sigma_2 & \mathbf{r}_4\sigma_4\end{matrix}\middle|\,\omega\right)\approx\frac{F^{l\mathbf{Q}}(\mathbf{r}_2\sigma_2;\mathbf{r}_1\sigma_1)F^{l\mathbf{Q}*}(\mathbf{r}_4\sigma_4;\mathbf{r}_3\sigma_3)}{\omega-\omega_l(\mathbf{Q})+i0^+}. \quad (15)$$

Here $\mathbf{r}$ is 2D radius vector, σ is the spin index. The exciton wave functions $F^{l\mathbf{Q}}(\mathbf{r}_2\sigma_2;\mathbf{r}_1\sigma_1)$ satisfy the BS equation

$$F^{l\mathbf{Q}}(\mathbf{r}_2\sigma_2;\mathbf{r}_1\sigma_1)=K^{(0)}\left(\begin{matrix}\mathbf{r}_1\sigma_1 & \mathbf{r}_3\sigma_3\\ \mathbf{r}_2\sigma_2 & \mathbf{r}_4\sigma_4\end{matrix}\middle|\,\omega_l(\mathbf{Q})\right)\times\left[I_C\left(\begin{matrix}\mathbf{r}_3\sigma_3 & \mathbf{r}_5\sigma_5\\ \mathbf{r}_4\sigma_4 & \mathbf{r}_6\sigma_6\end{matrix}\right)+I_E\left(\begin{matrix}\mathbf{r}_3\sigma_3 & \mathbf{r}_5\sigma_5\\ \mathbf{r}_4\sigma_4 & \mathbf{r}_6\sigma_6\end{matrix}\right)\right]F^{l\mathbf{Q}}(\mathbf{r}_6\sigma_6;\mathbf{r}_5\sigma_5). \quad (16)$$

Here I_C and I_E are the Coulomb and the Elliott exchange electron-hole interactions, respectively. It is more convenient to write the BS equation in $\mathbf{k}$ representation, taking into account the one-particle band structure of the crystal. In $\mathbf{k}$ representation the BS Eq. (16) assumes the form

$$F_{ji}^l(\mathbf{q},\mathbf{Q})=\sum_{s,t,s',t'}\int\frac{d^2\mathbf{p}}{(2\pi)^2}\int\frac{d^2\mathbf{q}}{(2\pi)^2}K^{(0)}\times\left(\begin{matrix}i,\mathbf{q} & s,\mathbf{p}\\ j,\mathbf{q}-\mathbf{Q} & t,\mathbf{p}-\mathbf{Q}\end{matrix}\middle|\,i\omega_p\to\omega_l+i0^+\right)\times\left[I_C\left(\begin{matrix}s,\mathbf{p} & s',\mathbf{k}\\ t,\mathbf{p}-\mathbf{Q} & t',\mathbf{k}-\mathbf{Q}\end{matrix}\right)+I_E\left(\begin{matrix}s,\mathbf{p} & s',\mathbf{k}\\ t,\mathbf{p}-\mathbf{Q} & t',\mathbf{k}-\mathbf{Q}\end{matrix}\right)\right]F''_{t's'}(\mathbf{k},\mathbf{Q}), \quad (17)$$

where $\{i,j,s,t,s',t'\}=\{c,\nu\}$ and $F_{ji}^l(\mathbf{q},\mathbf{Q})$ is the Fourier transform of the exciton wave function $F^{l\mathbf{Q}}(\mathbf{r}_2\sigma_2;\mathbf{r}_1\sigma_1)$. In Eq. (16) I_C denotes the Coulomb interaction screened by the high-frequency dielectric constant:

$$I_C\left(\begin{matrix}i,\mathbf{p} & s,\mathbf{k}\\ j,\mathbf{p}-\mathbf{Q} & t,\mathbf{k}-\mathbf{Q}\end{matrix}\right)=\left(-\frac{2\pi e^2}{\varepsilon_\infty}\right)\frac{1}{|\mathbf{p}-\mathbf{k}|}(1-\delta_{ij})\,\delta_{is}\delta_{jt}f(L|\mathbf{p}-\mathbf{k}|). \quad (18a)$$

The free two-particle propagator $K^{(0)}$ is defined as

$$K^{(0)}\left(\begin{matrix} i,\mathbf{q} & s,\mathbf{p} \\ j,\mathbf{q}-\mathbf{Q} & t,\mathbf{p}-\mathbf{Q}\end{matrix}\middle| i\omega_p\right) = \delta_{\mathbf{pq}}\sum_{\omega_m} G_{is}(\mathbf{q},i\omega_m+i\omega_p)G_{ij}(\mathbf{q}-\mathbf{Q},i\omega_m). \tag{18b}$$

In the case of a quantum well structure we will neglect the Elliott exchange electron-hole interaction I_E that will simplify the task.

BS Eqs. (17) are equations for the exciton wave functions $F^l_{\nu c}(\mathbf{q},\mathbf{Q})$ and $F^l_{c\nu}(\mathbf{q},\mathbf{Q})$. It is more convenient to introduce another two functions, $F^l_+(\mathbf{q},\mathbf{Q})$ and $F^l_-(\mathbf{q},\mathbf{Q})$, defined as follows:

$$F^l_{\nu c}(\mathbf{q},\mathbf{Q}) = \tfrac{1}{2}[u_{\mathbf{q}}u_{\mathbf{q}-\mathbf{Q}}F^l_+(\mathbf{q},\mathbf{Q}) - \nu_{\mathbf{q}}\nu_{\mathbf{q}-\mathbf{Q}}F^l_-(\mathbf{q},\mathbf{Q})], \tag{19a}$$

$$F^l_{c\nu}(\mathbf{q},\mathbf{Q}) = \tfrac{1}{2}[u_{\mathbf{q}}u_{\mathbf{q}-\mathbf{Q}}F^l_-(\mathbf{q},\mathbf{Q}) - \nu_{\mathbf{q}}\nu_{\mathbf{q}-\mathbf{Q}}F^l_+(\mathbf{q},\mathbf{Q})]. \tag{19b}$$

At the temperature $T=0$ K by means of the BS Eq. (17) we obtain the following coupled equations for the functions $F^l_+(\mathbf{q},\mathbf{Q})$ and $F^l_-(\mathbf{q},\mathbf{Q})$:

$$[\omega_l-\Omega_+(\mathbf{p},\mathbf{Q})]F^l_+(\mathbf{p},\mathbf{Q}) = -\int \frac{d^2\mathbf{q}}{(2\pi)^2}\left(\frac{2\pi e^2}{\varepsilon_\infty}\right)\frac{f(L|\mathbf{p}-\mathbf{q}|)}{|\mathbf{p}-\mathbf{q}|}[u_{\mathbf{p}}u_{\mathbf{p}-\mathbf{Q}}u_{\mathbf{q}}u_{\mathbf{q}-\mathbf{Q}}+\nu_{\mathbf{p}}\nu_{\mathbf{p}-\mathbf{Q}}\nu_{\mathbf{q}}\nu_{\mathbf{q}-\mathbf{Q}}]F^l_+(\mathbf{q},\mathbf{Q}) + \int \frac{d^2\mathbf{q}}{(2\pi)^2}\left(\frac{2\pi e^2}{\varepsilon_\infty}\right)\frac{f(L|\mathbf{p}-\mathbf{q}|)}{|\mathbf{p}-\mathbf{q}|}[u_{\mathbf{p}}u_{\mathbf{p}-\mathbf{Q}}\nu_{\mathbf{q}}\nu_{\mathbf{q}-\mathbf{Q}}+u_{\mathbf{p}}u_{\mathbf{p}-\mathbf{Q}}\nu_{\mathbf{q}}\nu_{\mathbf{q}-\mathbf{Q}}]F^l_-(\mathbf{q},\mathbf{Q}), \tag{20a}$$

$$[\omega_l-\Omega_-(\mathbf{p},\mathbf{Q})]F^l_-(\mathbf{p},\mathbf{Q}) = \int \frac{d^2\mathbf{q}}{(2\pi)^2}\left(\frac{2\pi e^2}{\varepsilon_\infty}\right)\frac{f(L|\mathbf{p}-\mathbf{q}|)}{|\mathbf{p}-\mathbf{q}|}[u_{\mathbf{p}}u_{\mathbf{p}-\mathbf{Q}}u_{\mathbf{q}}u_{\mathbf{q}-\mathbf{Q}}+\nu_{\mathbf{p}}\nu_{\mathbf{p}-\mathbf{Q}}\nu_{\mathbf{q}}\nu_{\mathbf{q}-\mathbf{Q}}]F^l_-(\mathbf{q},\mathbf{Q}) - \int \frac{d^2\mathbf{q}}{(2\pi)^2}\left(\frac{2\pi e^2}{\varepsilon_\infty}\right)\frac{f(L|\mathbf{p}-\mathbf{q}|)}{|\mathbf{p}-\mathbf{q}|}[u_{\mathbf{p}}u_{\mathbf{p}-\mathbf{Q}}\nu_{\mathbf{q}}\nu_{\mathbf{q}-\mathbf{Q}}+u_{\mathbf{p}}u_{\mathbf{p}-\mathbf{Q}}\nu_{\mathbf{q}}\nu_{\mathbf{q}-\mathbf{Q}}]F^l_+(\mathbf{q},\mathbf{Q}), \tag{20b}$$

where $\Omega_+(\mathbf{p},\mathbf{Q})=\omega_+(\mathbf{p})-\omega_-(\mathbf{p}-\mathbf{Q})$ and $\Omega_-(\mathbf{p},\mathbf{Q})=\omega_+(\mathbf{p}-\mathbf{Q})-\omega_-(\mathbf{p})$.

In the low-density limit $na_0^2 \ll 1$ the coupled Eqs. (20) to the first order of density assume the form

$$\left[\omega_l-E_g-\frac{\pi^2}{2\mu L^2}+\mu_{\rm exc}-\frac{1}{2\mu}\left(\mathbf{p}-\frac{m_c}{m_c+m_\nu}\mathbf{Q}\right)^2-\frac{\mathbf{Q}^2}{2(m_c+m_\nu)}-nS(\mathbf{p},\mathbf{Q})\right]F^l_+(\mathbf{p},\mathbf{Q}) = -\left(\frac{2\pi e^2}{\varepsilon_\infty}\right)\int\frac{d^2\mathbf{q}}{(2\pi)^2}\frac{f(L|\mathbf{p}-\mathbf{q}|)}{|\mathbf{p}-\mathbf{q}|}\left[1-\frac{n}{4}(\varphi_0^2(\mathbf{p})+\varphi_0^2(\mathbf{p}-\mathbf{Q})+\varphi_0^2(\mathbf{q})+\varphi_0^2(\mathbf{q}-\mathbf{Q}))\right]F^l_+(\mathbf{q},\mathbf{Q}) + \frac{n}{2}\left(\frac{2\pi e^2}{\varepsilon_\infty}\right)\int\frac{d^2\mathbf{q}}{(2\pi)^2}\frac{f(L|\mathbf{p}-\mathbf{q}|)}{|\mathbf{p}-\mathbf{q}|}[\varphi_0(\mathbf{p})\varphi_0(\mathbf{p}-\mathbf{Q})+\varphi_0(\mathbf{q})\varphi_0(\mathbf{q}-\mathbf{Q})]F^l_-(\mathbf{q},\mathbf{Q}) \tag{21a}$$

$$\left[\omega_l+E_g+\frac{\pi^2}{2\mu L^2}-\mu_{\rm exc}+\frac{1}{2\mu}\left(\mathbf{p}-\frac{m_\nu}{m_c+m_\nu}\mathbf{Q}\right)^2-\frac{\mathbf{Q}^2}{2(m_c+m_\nu)}+nS(\mathbf{p},\mathbf{Q})\right]F^l_-(\mathbf{p},\mathbf{Q}) = +\left(\frac{2\pi e^2}{\varepsilon_\infty}\right)\int\frac{d^2\mathbf{q}}{(2\pi)^2}\frac{f(L|\mathbf{p}-\mathbf{q}|)}{|\mathbf{p}-\mathbf{q}|}\left(1-\frac{n}{4}[\varphi_0^2(\mathbf{p})+\varphi_0^2(\mathbf{p}-\mathbf{Q})+\varphi_0^2(\mathbf{q})+\varphi_0^2(\mathbf{q}-\mathbf{Q})]\right)F^l_-(\mathbf{q},\mathbf{Q}) - \frac{n}{2}\left(\frac{2\pi e^2}{\varepsilon_\infty}\right)\int\frac{d^2\mathbf{q}}{(2\pi)^2}\frac{f(L|\mathbf{p}-\mathbf{q}|)}{|\mathbf{p}-\mathbf{q}|}[\varphi_0(\mathbf{p})\varphi_0(\mathbf{p}-\mathbf{Q})+\varphi_0(\mathbf{q})\varphi_0(\mathbf{q}-\mathbf{Q})]F^l_+(\mathbf{q},\mathbf{Q}). \tag{21b}$$

We look for the solutions $F^l_+(\mathbf{q},\mathbf{Q})$ and $F^l_-(\mathbf{q},\mathbf{Q})$ of the above equations in the form

$$F^l_\pm(\mathbf{q},\mathbf{Q}) = \sum_{n,m} C^\pm_{l,nm}(\mathbf{Q})R_{n,m}(\mathbf{q}). \tag{22}$$

By putting Eq. (22) into the coupled Eqs. (21) we obtain we obtain a set of linear homogeneous equations for $C^\pm_{l,nm}$. The last system cannot be solved exactly. For this reason we (i) ignore the coupling between the different excitonic modes due to the condensate; (ii) suppose that the original exciton modes will not be affected too much by the presence of condensate. This is equivalent to keep in the summation over the hydrogen quantum numbers in Eq. (22) only those terms with quantum numbers n,m, which have energies $E_g+(\pi^2/2\mu L^2)-E_n \approx \omega_l+\mu_{\rm exc}$. In this approximation the chemical potential is $\mu_{\rm exc}=\mu^{(1)}_{\rm exc}$ and the non-zero-momentum ground-state energy $\omega_{l=0}(\mathbf{Q})=\omega_0(\mathbf{Q})$ is obtained to be

$$\omega_0(\mathbf{Q})=\sqrt{V^2Q^2+\left(\frac{Q^2}{2M}\right)^2};\quad V=\sqrt{\frac{\Delta\mu_{\rm exc}^{(1)}}{M}},\tag{23}$$

where $M=m_c+m_\nu$. As can be seen the exciton ground state in a single quantum well structure at finite but small wave vectors shows a phononlike dispersion relation with the speed of sound $V=\sqrt{(n/(M)(d\mu_{\rm exc}^{(1)}/dn)}$, where $\mu_{\rm exc}^{(1)}$ is the chemical potential, defined by Eq. (13).

In the 2D case the velocity is given by

$$V=\sqrt{4\pi(na_0^2)E_0M^{-1}}.\tag{24}$$

D. Bose-Einstein condensation temperature

In this section we concern the problem at which temperatures the condensate should be readily observed. If we want to calculate the BEC temperature $T_{\rm BE}$ by solving the equation $\Sigma_{c\nu}(T_{\rm BE},n)=0$, we must include the effects of the two-particle modes on the single-particle excitations. In our analysis we neglect the fact that both the mass operator and the one-particle ''anomalous'' Green's functions depend on the two-particle Green's function.[9,26,27] For this reason we will estimate the temperature $T_{\rm BE}$ using the fact that in the low-density limit the two-fermion electron-hole bound states can be considered as bosons with a spectrum (23). The density of particles populating the state $\omega_0(\mathbf{Q})$ is given by summing over the corresponding Bose-Einstein distributions

$$n(T)=\frac{1}{V_{2D}}\sum_{\mathbf{Q}}\frac{1}{\exp\{\beta[\omega_0(\mathbf{Q})-\mu_{\rm exc}]\}-1}=\frac{M}{2\pi}\int_0^{+\infty}\frac{EdE}{\sqrt{E^2+M^2V^4}(\exp\{\beta[E+E_g+\pi^2(L/a_0)^{-2}-\mu_{\rm exc}]\}-1)},\tag{25}$$

where V_{2D} is a 2D volume. Thus we define the temperature $T_{\rm BE}$ as a temperature at which the actual density equals the density of particles populating the state $\omega_0(\mathbf{Q})$. For $T<T_{\rm BE}$ the actual density exceeds this density and the extra amount particles will occupy the zero-momentum ground state and Bose-condensed excitons should be observed. In order to determine the critical temperature in terms of the density n, we assume $\mu_{\rm exc}\approx\mu_{\rm exc}^{(0)}$. As a result, we obtain a linear relationship between the critical temperature and the exciton density

$$T_{\rm BE}(L)=\frac{1}{z_0(L)}E_0(na_0^2)k_B^{-1}.\tag{26}$$

Here z_0 denotes the solution of the following equation:

$$z-\frac{1}{4\pi}\frac{(m_c+m_\nu)^2}{m_cm_\nu}\int_0^{+\infty}\frac{xdx}{\sqrt{x^2+b^2z^2}\{\exp(x)-1\}}=0,\tag{27}$$

where

$$b=\frac{4\pi}{5}\left[-3+128\beta\int_0^{+\infty}\frac{f(xL/a_0)dx}{[x^2+16\beta^2]^{3/2}}\right]\tag{28}$$

and the parameter β has been already determined by maximizing the chemical potential $\mu_{\rm exc}^{(0)}$ with respect to β. A plot of $T_{\rm BE}$ in Kelvin versus the dimensionless parameter $x=na_0^2$ for different GaAs structures is shown in Fig. 3.

III. EXCITONS IN TWO DIMENSIONS: PHASE TRANSITION TO SUPERFLUIDITY

In this section we will calculate the critical temperature T_c of KT topological phase transition in an exactly two-dimensional structure made from direct-gap semiconductors with nondegenerate and isotropic bands. In the low-density limit the excitons, which are made of an even number of fermions, can be considered as bosons, and we would thus expect that no BEC of excitons is possible, but KT phase transition to a superfluid state takes place. It is worth noting that the excitons may exist in the superfluid phase and out-

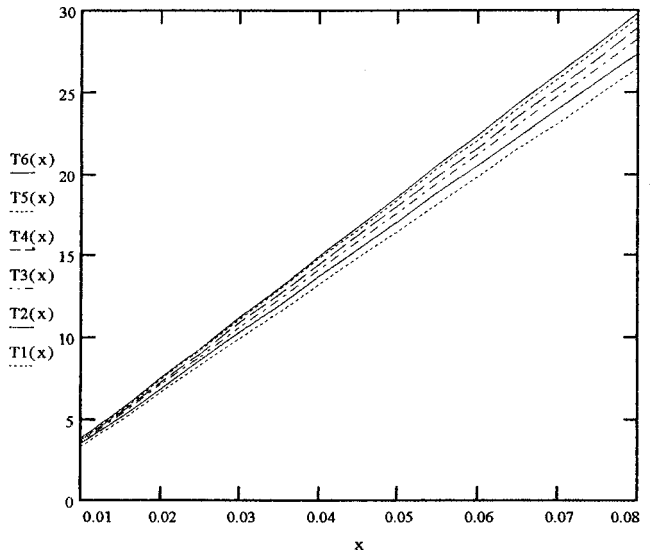

FIG. 3. The Bose-Einstein condensation temperature $T_{\rm BE}$ in kelvin versus the dimensionless parameter $x=na_0^2$ for GaAs structures for different quantum well thicknesses. $T1(x)-T6(x)$ represent the temperature $T_{\rm BE}$ as a function of parameter $x=na_0^2$ for $L/a_0=0.25$, 0.5, 0.75, 1, 1.25, and 1.5, respectively. The exciton is made of an electron with effective mass $m_c=0.067m_0$ and a heavy hole with effective mass $m_v=0.197m_0$. The values of a dielectric constant, 3D exciton Bohr radius and 3D exciton Rydberg, used in calculations are as follows: $\varepsilon_\infty=10.9$, $a_0=115$ Å, and $E_0=6$ meV.

side of the superfluid state, i.e., in the normal phase. Thus one can write for the total exciton concentration n at the temperature T:

$$n = n_n(T) + n_s(T), \tag{29}$$

where n_n and n_s are 2D densities of excitons in a normal exciton phase and in a superfluid phase, respectively. According to the work by Kosterlitz and Thouless[15] the critical temperature T_c of KT topological phase transition is given by

$$T_c = \frac{\pi n_s(T_c)}{2k_B M}, \tag{30a}$$

where $n_s(T_c)$ is the value of the density of the superfluid component at the KT transition temperature. The density of the normal phase is given by the well-known Landau expression

$$n_n(T) = -\frac{1}{2M}\int \frac{d^2\mathbf{Q}}{(2\pi)^2} Q^2 \frac{\partial n[E(\mathbf{Q})]}{\partial E(\mathbf{Q})}, \tag{30b}$$

where $n(E)$ is the Bose-Einstein distribution function and $E(\mathbf{Q})$ is the exciton energy.

The total exciton density at the critical temperature is given by the expression

$$n = \frac{2Mk_BT_c}{\pi} + n_n(T_c). \tag{30c}$$

The last equation can be solved and the critical temperature T_c of a KT phase transition to a superfluid state can be obtained as a function of the total density of excitons n. If we assume the quadratic dispersion $E(\mathbf{Q}) = (2M)^{-1}Q^2$ for the excitons, than the integral in Eq. (30b) diverges logarithmically, which means the absence of a KT phase transition to a superfluidity. In the previous section we have obtained that the exciton ground state in the 2D case at finite but small wave vectors shows a phononlike dispersion relation

$$\omega_0(\mathbf{Q}) = \sqrt{V^2Q^2 + \left(\frac{Q^2}{2M}\right)^2} \tag{31a}$$

with the speed of sound

$$V = \sqrt{(n/M)(d\mu_{\mathrm{exc}}^{(1)}/dn)} = [4\pi E_0(na_0^2)M^{-1}]^{1/2}. \tag{32b}$$

It is worth noting that the above speed of sound is different from that, obtained in Ref. 18. The speed obtained by Lozovik and co-workers has the form

$$V = \sqrt{(n/M)(d\mu_{\mathrm{exc}}^{(1)}/dn)} = [4.71E_0(na_0^2)M^{-1}]^{1/2}. \tag{32c}$$

In order to take into account the contribution of the collective modes (31a) to the concentration of the normal phase we put the dispersion of the collective modes (31a) into (30b). Thus by using for the chemical potential in a strictly 2D case $\mu_{\mathrm{exc}} \approx \mu_{\mathrm{exc}}^0 = E_g - 4E_0$ one can obtain for the density of excitons in a normal exciton phase in the low-density limit

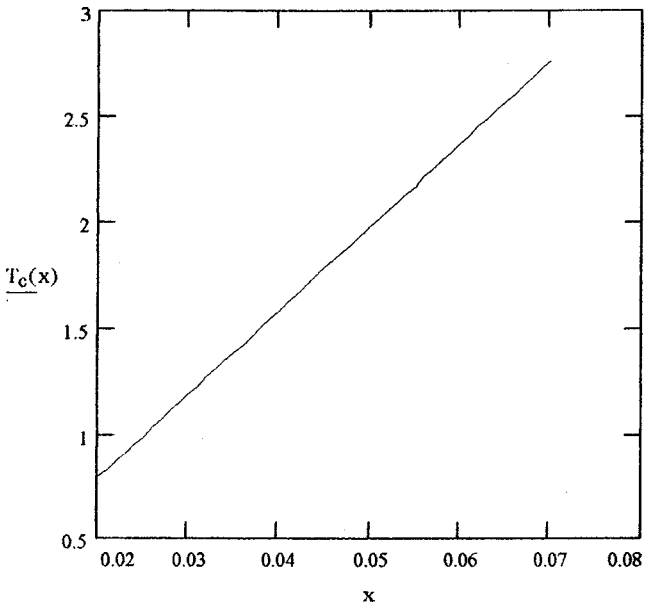

FIG. 4. The critical temperature T_c of the Kosterlitz-Thouless phase transition in kelvin versus the dimensionless parameter $x = na_0^2$ for 2D GaAs structures. The exciton is made of an electron with effective mass $m_c = 0.067m_0$ and a heavy hole with effective mass $m_v = 0.197m_0$. The values of a dielectric constant, 3D exciton Bohr radius and 3D exciton rydberg, used in calculations are as follows: $\varepsilon_\infty = 10.9$, $a_0 = 115$ Å, and $E_0 = 6$ meV.

$$n_n(T) = \frac{Mk_BT}{2\pi}\int_0^{+\infty} dx \frac{\left[x^2 + \left(\frac{MV^2}{k_BT}\right)^2\right]^{3/2} - \left(\frac{MV^2}{k_BT}\right)^3}{\left[x^2 + \left(\frac{MV^2}{k_BT}\right)^2\right]^{3/2}[\exp(x) - 1]}. \tag{33a}$$

As can be seen from Eq. (33a) at $T \to 0$ $n_n(T \to 0) = 0$ and all excitons are in a superfluid state. It is worth noting that in Ref. 18 the contribution of the collective excitation to the density of excitons of the normal phase is estimated to be

$$n_n(T) = \frac{\pi^2(k_BT)^3}{30MV^4} + \frac{\pi^2 k_BT}{6M}. \tag{33b}$$

Obviously, the above two expressions are quite different.

Substituting Eq. (33a) into Eq. (30c) we obtain a linear relationship between the KT critical temperature and the exciton density

$$T_c = \frac{1}{z_c} E_0(na_0^2)k_B^{-1}. \tag{34}$$

Here z_c denotes the solution of the following equation:

$$z - \frac{1}{\pi}\frac{M}{\mu}\left[1 + \frac{1}{4}\int_0^{+\infty} \frac{dx\{(x^2 + a^2z^2)^{3/2} - a^3z^3\}}{[x^2 + a^2z^2]^{3/2}\{\exp(x) - 1\}}\right] = 0, \tag{35}$$

where $a = 4\pi$. A plot of T_c in Kelvin versus the dimensionless parameter $x = na_0^2$ for 2D GaAs structures (for $m_c = 0.067m_0$ and $m_v = 0.197m_0$, z_c is 1.684) is shown in Fig. 4.

The superfluidity in a two-dimensional system is due to the presence of vortices. The superfluid density $n_s(T)$ at the

KT temperature (30a) undergoes a discontinuing from some finite value to zero. In contrast to this global superfluid density, the difference $n-n_n(T)$ is a smooth function of temperature and does not equal to zero even above T_c. One can obtain from Eq. (33a) the temperature $T_0 \approx 10.7T_c$ at which $n_n(T_0)=n$. At the temperature T_c the superfluid density $n_s(T)$ disappears with a jump, but at somewhat higher temperatures $T_0>T>T_c$ we have a nonzero local superfluid density $n-n_n(T)$, which can be manifested in completely accessible to measurement effect—an observation of a third sound,[22] i.e., the propagation of density oscillations of a superfluid component in a two-dimensional system.

IV. CONCLUDING REMARKS

In this paper we discuss the one- and two-particle excitations of the electron-hole gas in a single quantum well using the Green's function method. The combination of Green's function technique for dealing with one- and two-particle properties enables us to obtain some results that can be applied in the long search for the Bose condensed phase of excitons in semiconductors. The following conclusions summarize our results:

(i) We have obtained that not only in 3D case, but in the case of a single quantum well structure as well, the ground-state quadratic exciton dispersion near $\mathbf{Q}=\mathbf{0}$ is modified and starts linear with momentum in the presence of a Bose condensate.

(ii) The critical temperature of the Bose-Einstein condensation scales linearly with 2D density of excitons.

(iii) We have obtained the critical temperature of the Kosterlitz-Thouless phase transition to exciton superfluidity as a function of the exciton density.

Our BCS-like treatment can be applied also to the quantum well structures in which the electrons and holes are in two different quantum wells with thicknesses L_c and L_ν, separated by a wide barrier material with a thickness d. Naveh and Lakhtman[11] have considered the case $L_{c,\nu} \gg d$, and Zhu *et al.*[10]—the case $L_c=L_\nu=0$. In order to apply our treatment to those structures, one has to use instead of Eq. (9) the following function:

$$f(x,\xi_c,\xi_\nu,d) = \frac{16\pi^4[1-\exp(-\xi_c x)][1-\exp(-\xi_\nu x)]\exp(-dx)}{\xi_c\xi_\nu(4\pi^2+\xi_c x^2)(4\pi^2+\xi_\nu x^2)},$$

where $\xi_{c,\nu}=L_{c,\nu}/a_0$.

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