Metal-insulator transition with infinite-range Coulomb coupling: Fractional statistics and quantum critical properties

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We show that the Hubbard model with infinite-range Coulomb coupling is equivalent to an ideal gas of three species of particles obeying fractional exclusion statistics. A full appreciation of this mapping requires an extension of the pertinent formalism. This very simple, but rather peculiar model is exactly solvable in any dimension and exhibits a Mott metal-insulator transition, whose universality class is shown to be that of a free spinless Fermi gas. A modified version of the Luttinger theorem is shown to apply in any dimension. We also characterize the metallic and insulating phases by obtaining the electronic band structure as well as the interacting density of states. The fractional statistics manifests itself on the amplitudes of several thermodynamic quantities and, in particular, the Pauli spin susceptibility is subdominant in all metallic phases, and a Curie-type of response appears.

I. INTRODUCTION

The field of strongly correlated electron systems has, over the years, been the subject of intense activity. Its range of application includes high-T_c superconductivity, heavy fermions, metal-insulator transition (MIT), and fractional quantum Hall effect. A crucial common feature of these systems is the fact that several physical phenomena of interest take place at intermediate or strong Coulomb coupling. In many situations, perturbative approaches are unreliable and controlled nonperturbative calculations become necessary. Unfortunately, non-perturbative schemes have proved rather difficult to implement, and as a consequence many fundamental properties of even the simplest models have been elusive. It is, therefore, important to study non-trivial models or specific limits where exact results are possible, such as one-dimensional lattices with short and long-range interactions,[1] infinite dimensional systems,[2] finite dimensional lattices with infinite-range hopping,[3,4] or infinite-range interaction.[5–7]

Among the subjects of interest concerning strongly correlated electron systems, one of the most important is the Mott-Hubbard MIT.[8,9] Recently, Anderson[10] emphasized that the problem of doping a single Mott-Hubbard band is the starting point to understand the problem of high-T_c superconductivity. A long time ago, Wigner[11] was one of the first to suggest that the electron gas model of a metal is unstable against crystalization at low density due to the long-range Coulomb repulsion. However, a more complete analysis of the MIT induced by electron-electron interactions has been put forward by Mott.[12] Later, Hubbard[13] proposed a microscopic lattice model, with a *short-range* (on site) interaction that was shown[14] to exhibit a MIT at a critical value of the interaction parameter, U. Hubbard's prediction of a second order MIT was based on a kind of coherent phase approximation leading to the absence of a well-defined Fermi surface on the metallic side[15] and no magnetism. Using a variational method due to Gutzwiller,[16] Brinkman and Rice[17] obtained a MIT for the Hubbard model with a proper description of the metallic state, but with an oversimplified picture of the insulating phase, which consisted of noninteracting local moments. We know from the strong-coupling limit of the Hubbard model that the insulating phase has antiferromagnetic order. An account of magnetism in the presence of a MIT can be found in the works by Slater,[18] who suggested a mechanism in which the doubling of the unit cell caused by an underlying long-range antiferromagnetic order splits the Brillouin zone and causes the appearance of a gap in the density of states (DOS). The exact solution of the one-dimensional Hubbard model presented by Lieb and Wu[19] showed no MIT at finite U (the system is an insulator for any $U>0$ at half-filled band). In two dimensions, the MIT is still a controversial issue: results[20] derived using real-space renormalization-group suggest that at half-filled band the ground-state is insulating for any $U>0$ and isotropic hopping ($\alpha=t_y/t_x=1$). For a non-isotropic hopping ($\alpha<1$) the ground-state should be metallic for U/t less than a certain critical value, which depends on α, and insulator otherwise. On the other hand, recent Monte Carlo data[21] suggest that the density-driven MIT in $d=2$ belongs to a special universality class, where the dynamic and correlation length exponents are given respectively by $z=4$ and $\nu=1/4$, instead of $z=2$ and $\nu=1/2$ as in $d=1$.

More recently, considerable progress in our understanding of metal-insulator transitions has been obtained from exactly solvable limits. Gebhard and Ruckenstein[22] introduced a model that consists of a Hubbard chain with long-range hopping and showed that it exhibts a MIT at half filling when U equals the bandwidth Δ. In the limit of infinite dimensions, the pioneering work of Metzner and Vollhardt[23] introduced a nontrivial version of the Hubbard model in which the Brinkman and Rice solution becomes exact. Georges *et al.*[24,2] studied the MIT in the infinite dimensional Hubbard model and demonstrated that the metallic phase is a renormalized Fermi liquid. Nevertheless, several low-dimensional strongly correlated electron system, with close association with the physics of the MIT, have found metallic phases displaying

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non-Fermi liquid (NFL) behavior, specially in the high-T_c and fractional quantum Hall literature. It has been found that the low-energy effective theories of such systems possess very peculiar properties, which cause the breakdown of Fermi-liquid theory.[25] For instance, near a quantum critical point (QCP) the fluctuations of soft modes induce anomalous scattering, which suppress quasiparticles lifetimes. Other examples are systems with enhanced small momentum transfer forward scattering.

An important property of some non-Fermi liquids is the appearence of quasiparticle excitations with fractional statistics intermediate between bosons and fermions. In two dimensions, this can be traced back to the existence of a nontrivial group structure associated with the exchange of identical particles, the so-called *anyons*.[26] Recently, Haldane[27] proposed a generalization of Pauli's exclusion principle, which leads to a new kind of fractional statistics, whose associated particles have been called *exclusons*.[28–32] Several systems have been reported to be described by Haldane's exclusion statistics: fractional quantum Hall effects[28,33–35] $1d$ and higher dimensional solvable models,[29,36–40] low-T properties of one-component Luttinger liquids[41] and Mott insulators.[42] From the above remarks, it is clear that the fractional statistical concept have had substantial importance to the understanding of many interrelated and topical subjects. It seems thus reasonable to expect exclusons to play some role in the mechanism of MIT in low-dimensional systems.

In this paper, we concentrate our effort in the Hubbard model with *infinite-range* Coulomb coupling.[5–7] We show, in Sec. II, that this model is mapped onto an ideal gas of three species of particles obeying fractional exclusion statistics, thus clarifying some disagreements of previous works,[29,36] and allowing the interpretation of several NFL properties of the model in terms of exclusons. The fractional statistics manifests itself on the amplitudes of the response functions, such as specific heat, and by canceling the Pauli spin susceptibility in some metallic regimes. We thus make an attempt in describing a Mott MIT as being caused by the statistical interaction between exclusons. We report, in Sec. III, a thorough analysis of the ground-state properties including a phase diagram, in which the regions separated by critical lines are described in the framework of exclusion statistics. The electronic one-particle dynamical properties of the model, with particular emphasis on the interacting DOS, is also presented. In Sec. IV, we analyze the applicability of the generalized Luttinger's theorem, recently proposed by Haldane. We provide in Sec. V a complete scaling analysis of the interaction-driven MIT, and, by explicit calculation of the exponents, we show that the universality class of its QCP is that of a d-dimensional free spinless Fermi gas. In particular, we show that the mean number of doubly occupied $\mathbf{k}$ states, which is one of the three species of exclusons identified in the model, can be taken as the order parameter of the interaction-driven MIT. In this context, we should mention that Continentino and one of us[43] have shown that the interaction- and the density-driven transitions are in the same universality class. The static spin susceptibility is presented in Sec. VI, where we show that the Pauli response is subdominant in all metallic states due to the strong correlation in the system. Finally, conclusions are presented in Sec. VII.

II. THE MODEL AND EXCLUSON REPRESENTATION

Our model system is a Hubbard-like Hamiltonian with infinite-range Coulomb repulsion[5]

$$H=-t\sum_{\langle i,j\rangle,\sigma}c_{i\sigma}^{\dagger}c_{j\sigma}-(\mu+U/2)\sum_{i\sigma}c_{i\sigma}^{\dagger}c_{i\sigma}+\frac{U}{N}\sum_{j_1j_2j_3j_4}\delta_{j_1+j_3,j_2+j_4}c_{j_1\uparrow}^{\dagger}c_{j_2\uparrow}c_{j_3\downarrow}^{\dagger}c_{j_4\downarrow}. \quad (1)$$

The first term describes the hopping of electrons on a d-dimensional hypercubic lattice of N sites, and $\langle i,j\rangle$ is a notation for nearest neighbors. The second term represents a coupling to a reservoir of particles, and the third term expresses the conservation of the particle mass center in the scattering process, thus selecting the zero momentum transfer forward scattering only, which is an efficient way to generate NFL behavior.[25]

Introducing the lattice Fourier transform

$$c_{j\sigma}^{\dagger}=\frac{1}{\sqrt{N}}\sum_{\mathbf{k}}c_{\mathbf{k}\sigma}^{\dagger}e^{-i\mathbf{k}\cdot\mathbf{r}_j},\quad c_{j\sigma}=\frac{1}{\sqrt{N}}\sum_{\mathbf{k}}c_{\mathbf{k}\sigma}e^{i\mathbf{k}\cdot\mathbf{r}_j}, \quad (2)$$

we get a diagonal Hamiltonian in $\mathbf{k}$ space[5–7,44]

$$H=\sum_{\mathbf{k},\sigma}(\varepsilon_{\mathbf{k}}-\mu-U/2)n_{\mathbf{k}\sigma}+U\sum_{\mathbf{k}}n_{\mathbf{k}\uparrow}n_{\mathbf{k}\downarrow}. \quad (3)$$

Here, $n_{\mathbf{k}\sigma}=c_{\mathbf{k}\sigma}^{\dagger}c_{\mathbf{k}\sigma}$ is the number of electrons with momentum $\mathbf{k}$ and spin σ, and

$$\varepsilon_{\mathbf{k}}=-t\sum_{\langle 0,j\rangle}e^{i\mathbf{k}\cdot\mathbf{r}_j}, \quad (4)$$

is the energy of the electron in this state. We remark that the diagonality of H in $\mathbf{k}$ space does not imply necessarily that the physics of this model is trivial. In fact, a more detailed analysis of the model reveals unexpected features, as will be discussed in the following sections.

We shall now discuss the mapping of the interacting electron system described by Eq. (1) onto an ideal gas of particles obeying fractional statistics. In a recent paper, Wu[28] discussed the thermodynamics of an ideal gas obeying the generalized Pauli exclusion principle introduced by Haldane.[27] In Wu's approach, the total energy of the system is additive

$$E=\sum_{\mathbf{k}\alpha}\varepsilon_{\mathbf{k}\alpha}n_{\mathbf{k}\alpha}, \quad (5)$$

where $\mathbf{k}$ is a good quantum number and α is a species label for the exclusons. The statistical weight W is given by

$$W=\prod_{\mathbf{k}\alpha}\frac{(D_{\mathbf{k}\alpha}+N_{\mathbf{k}\alpha}-1)}{(D_{\mathbf{k}\alpha}-1)!N_{\mathbf{k}\alpha}}, \quad (6)$$

where

$$D_{\mathbf{k}\alpha}=G_{\mathbf{k}}-\sum_{\mathbf{k}'\alpha'}g_{\mathbf{k}\mathbf{k}';\alpha\alpha'}N_{\mathbf{k}'\alpha'}, \quad (7)$$

$G_{\mathbf{k}}$ is the number of available single-particle states when there is no particle in the system (we will assume that $G_{\mathbf{k}}$ is independent of $\mathbf{k}$, i.e., $G_{\mathbf{k}}=G_0$), $N_{\mathbf{k}\alpha}$ is the corresponding number of particles and the matrix $g_{\mathbf{k}\mathbf{k}';\alpha\alpha'}$ defines the mutual statistical interaction. The set of mean occupation numbers, $n_{\mathbf{k}\alpha}=N_{\mathbf{k}\alpha}/G_0$, that minimizes the grand-canonical free energy satisfies[28]

$$(1+w_{\mathbf{k}\alpha})\prod_{\mathbf{k}'\alpha'}\left(\frac{w_{\mathbf{k}'\alpha'}}{1+w_{\mathbf{k}'\alpha'}}\right)^{g_{\mathbf{k}'\mathbf{k};\alpha'\alpha}}=e^{\beta\varepsilon_{\mathbf{k}\alpha}}, \tag{8}$$

where $\beta=1/T$ and $w_{\mathbf{k}\alpha}=D_{\mathbf{k}\alpha}/N_{\mathbf{k}\alpha}$. Equation (8) was also derived[45] using the von Neumann entropy associated with Haldane exclusion statistics. Moreover, the grand-canonical free energy reads

$$\beta\Omega_{frac}=\beta\frac{\Omega}{G_0}=-\sum_{\mathbf{k}\alpha}\ln(1+w_{\mathbf{k}\alpha}^{-1}). \tag{9}$$

The mapping of the interacting electron system described by Eq. (1) onto an ideal gas of particles obeying fractional exclusion statistics has already been proposed in the literature,[36,29] but the results were incomplete and thus some controversy has arisen with respect to the true nature of the excluson particles of the model (see, e.g., the concluding remarks of Ref. 29). In fact, using Eq. (1), the energy is given by

$$E=\langle H\rangle=\sum_{\mathbf{k}\alpha}\varepsilon_{\mathbf{k}\alpha}n_{\mathbf{k}\alpha},\qquad \alpha=1,2,3, \tag{10}$$

where $\varepsilon_{\mathbf{k}1}=\varepsilon_{\mathbf{k}}-\mu-U/2=\varepsilon_{\mathbf{k}2}$, $\varepsilon_{\mathbf{k}3}=U$ and $n_{\mathbf{k}1}=\langle n_{\mathbf{k}\uparrow}\rangle, n_{\mathbf{k}2}=\langle n_{\mathbf{k}\downarrow}\rangle, n_{\mathbf{k}3}=\langle n_{\mathbf{k}\uparrow}n_{\mathbf{k}\downarrow}\rangle$. If we interpret $n_{\mathbf{k}1}$, $n_{\mathbf{k}2}$, and $n_{\mathbf{k}3}$ as excluson species [see Eq. (27) of Ref. 36], one can show that it is impossible to obtain a statistical matrix that gives rise to the correct thermodynamics of the model. To accomplish this we need to introduce a matrix Λ that transforms the variables $N_{\mathbf{k}\alpha}$ of the Hamiltonian (1) into the correct excluson species

$$\tilde{N}_{\mathbf{k}\alpha}=-\sum_{\mathbf{k}'\alpha'}\Lambda_{\mathbf{k}\mathbf{k}';\alpha\alpha'}N_{\mathbf{k}'\alpha'}. \tag{11}$$

The statistical weight has the same form as in Eq. (6)

$$W=\prod_{\mathbf{k}\alpha}\frac{(\tilde{D}_{\mathbf{k}\alpha}+\tilde{N}_{\mathbf{k}\alpha}-1)}{(\tilde{D}_{\mathbf{k}\alpha}-1)\tilde{N}_{\mathbf{k}\alpha}}, \tag{12}$$

where

$$\tilde{D}_{\mathbf{k}\alpha}=\tilde{G}_{\mathbf{k}}-\sum_{\mathbf{k}'\alpha'}\tilde{g}_{\mathbf{k}\mathbf{k}';\alpha\alpha'}\tilde{N}_{\mathbf{k}'\alpha'}. \tag{13}$$

Now we proceed to obtain the set of mean occupation numbers, $\tilde{n}_{\mathbf{k}\alpha}=\tilde{N}_{\mathbf{k}\alpha}/G_0$. Using the statistical weight given by Eq. (12) and assuming $\tilde{N}_{\mathbf{k}\alpha}$, $\tilde{D}_{\mathbf{k}\alpha}$, and $N_{\mathbf{k}\alpha}$ to be large numbers, the free energy can be written as

$$\Omega=\sum_{\mathbf{k}\alpha}\varepsilon_{\mathbf{k}\alpha}N_{\mathbf{k}\alpha}-\frac{1}{\beta}\sum_{\mathbf{k}\alpha}(\tilde{D}_{\mathbf{k}\alpha}+\tilde{N}_{\mathbf{k}\alpha})\ln(\tilde{D}_{\mathbf{k}\alpha}+\tilde{N}_{\mathbf{k}\alpha})+\frac{1}{\beta}\sum_{\mathbf{k}\alpha}\tilde{D}_{\mathbf{k}\alpha}\ln\tilde{D}_{\mathbf{k}\alpha}+\frac{1}{\beta}\sum_{\mathbf{k}\alpha}\tilde{N}_{\mathbf{k}\alpha}\ln\tilde{N}_{\mathbf{k}\alpha}. \tag{14}$$

In equilibrium, the set $\{\tilde{n}_{\mathbf{k}\alpha}\}$ is obtained by the condition $\partial\Omega/\partial N_{\mathbf{k}\alpha}=0$. Thus, we find the following generalized law of mass action

$$\prod_{\mathbf{k}'\alpha'}\frac{\tilde{w}_{\mathbf{k}'\alpha'}^{g_{\mathbf{k}'\mathbf{k};\alpha'\alpha}}}{(1+\tilde{w}_{\mathbf{k}'\alpha'})^{g_{\mathbf{k}'\mathbf{k};\alpha'\alpha}+\Lambda_{\mathbf{k}'\mathbf{k};\alpha'\alpha}}}=e^{\beta\varepsilon_{\mathbf{k}\alpha}}, \tag{15}$$

where

$$g_{\mathbf{k}\mathbf{k}';\alpha\alpha'}=-\sum_{\mathbf{k}''\alpha''}\tilde{g}_{\mathbf{k}\mathbf{k}'';\alpha\alpha''}\Lambda_{\mathbf{k}''\mathbf{k}';\alpha''\alpha'}, \tag{16}$$

and

$$\tilde{w}_{\mathbf{k}'\alpha'}=\tilde{D}_{\mathbf{k}'\alpha'}/\tilde{N}_{\mathbf{k}'\alpha'}. \tag{17}$$

The free energy is obtained by substituting Eq. (15) into Eq. (14)

$$\beta\Omega_{frac}=-\sum_{\mathbf{k}\alpha}\ln(1+\tilde{w}_{\mathbf{k}\alpha}^{-1}), \tag{18}$$

in agreement with Eq. (9). Note that if by some fortunate guess the starting $n_{\mathbf{k}\alpha}$ is the correct set of exclusons, then the matrix Λ reads:

$$\Lambda_{\mathbf{k}\mathbf{k}';\alpha\alpha'}=-\delta_{\mathbf{k}\mathbf{k}'}\delta_{\alpha\alpha'}, \tag{19}$$

in which case, Eq. (15) reduces to Eq. (8). To obtain the correct mean occupation numbers from Eq. (15), we find that the following matrices

$$g_{\mathbf{k}\mathbf{k}';\alpha\alpha'}=\delta_{\mathbf{k}\mathbf{k}'}\begin{pmatrix}1&1&-1\\0&1&0\\0&0&1\end{pmatrix}, \tag{20}$$

$$\Lambda_{\mathbf{k}\mathbf{k}';\alpha\alpha'}=-\delta_{\mathbf{k}\mathbf{k}'}\begin{pmatrix}1&0&-1\\0&1&-1\\0&0&1\end{pmatrix}, \tag{21}$$

satisfy all required conditions. The matrix $g_{\mathbf{k}\mathbf{k}';\alpha\alpha'}$ has been anticipated in Ref. 36 but with an incorrect interpretation. The true statistical matrix of the system is given by

$$\tilde{g}_{\mathbf{k}\mathbf{k}';\alpha\alpha'}=\delta_{\mathbf{k}\mathbf{k}'}\begin{pmatrix}1&1&1\\0&1&1\\0&0&1\end{pmatrix}. \tag{22}$$

Therefore, the energy is written more appropriately in the form

$$E=\sum_{\mathbf{k}\alpha}\tilde{\varepsilon}_{\mathbf{k}\alpha}\tilde{n}_{\mathbf{k}\alpha}, \tag{23}$$

where

$$\tilde{\varepsilon}_{\mathbf{k}\alpha} \equiv -\sum_{\mathbf{k}'\alpha'} \varepsilon_{\mathbf{k}'\alpha'} (\Lambda^{-1})_{\mathbf{k}'\mathbf{k};\alpha'\alpha}. \tag{24}$$

Using the matrix (21), we obtain:

$$\tilde{\varepsilon}_{\mathbf{k}1} = \varepsilon_{\mathbf{k}} - \mu - U/2 = \tilde{\varepsilon}_{\mathbf{k}2},$$
$$\tilde{\varepsilon}_{\mathbf{k}3} = 2(\varepsilon_{\mathbf{k}} - \mu). \tag{25}$$

We may thus interpret $\tilde{n}_{\mathbf{k}1}$ and $\tilde{n}_{\mathbf{k}2}$ as the occupation number of singly occupied $\mathbf{k}$ states, whereas $\tilde{n}_{\mathbf{k}3}$ counts doubly occupied $\mathbf{k}$ states.[36] Moreover, the mean occupation number of exclusons can be written as $\tilde{n}_{\mathbf{k}\alpha} = \langle \psi^{\dagger}_{\mathbf{k}\alpha} \psi_{\mathbf{k}\alpha} \rangle$, where $\psi_{\mathbf{k}\alpha}$ are operators defined by

$$\psi_{\mathbf{k}1} = (1 - c^{\dagger}_{\mathbf{k}\downarrow} c_{\mathbf{k}\downarrow}) c_{\mathbf{k}\uparrow}, \quad \psi_{\mathbf{k}2} = (1 - c^{\dagger}_{\mathbf{k}\uparrow} c_{\mathbf{k}\uparrow}) c_{\mathbf{k}\downarrow},$$
$$\psi_{\mathbf{k}3} = c_{\mathbf{k}\uparrow} c_{\mathbf{k}\downarrow}. \tag{26}$$

We have confirmed this proposal by calculating the retarded *excluson* Green's functions whose poles are located precisely at the energies given in Eq. (25)

$$G^{+}_{\alpha}(\mathbf{k}, \omega) = \frac{\langle \{ \psi_{\mathbf{k},\alpha}, \psi^{\dagger}_{\mathbf{k},\alpha} \} \rangle}{\omega - \tilde{\varepsilon}_{\mathbf{k},\alpha} + i\eta}, \tag{27}$$

$$\langle \{ \psi_{\mathbf{k},\alpha}, \psi^{\dagger}_{\mathbf{k},\alpha} \} \rangle = 1 - \sum_{\gamma \neq \alpha} \tilde{n}_{\mathbf{k},\gamma}, \tag{28}$$

where the angle brackets denote the thermal average and the curly braces define the usual anticommutator. Note that in Eq. (27) the poles suggest independent excluson species, while the numerator, which is given by Eq. (28), shows clearly the mutual statistical interaction.

We stress that the mapping of a model of interacting particles onto a multicomponent system obeying fractional exclusion statistics is of great interest in the literature. Recently, Maskevitch[46–48] has developed an interesting scheme to construct physical realizations of fractional statistics. It consists of incorporating the interaction matrix g into renormalized quantum numbers for the physical particles and has been applied to the problem of anyons in the lowest Landau level. A serious restriction in Maskevitch's approach, however, is the requirement that g be a symmetric matrix. In fact, there is no need for such constraint in Wu's Eq. (8). The map presented in this work is an example of a realization of a multicomponent excluson gas, with a nonsymmetric statistical interaction g, in terms of interacting fermions. Note that we do not impose any renormalization of quantum numbers in constructing the map, since the kinetic and the interaction term in the Hamiltonian commute.

We have thus presented an extension of the fractional-statistics formalism, that has made it possible to map an interacting electron system of two species of particles onto an ideal gas of three species of exclusons. A somewhat surprising and unique feature of this model is the appearance of stable single-particle excitations with nontrivial exclusion statistics in any dimension.

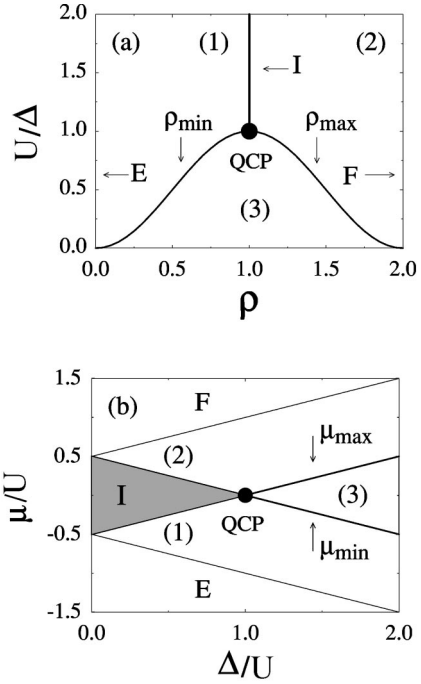

FIG. 1. Ground-state phase diagram in $d=1$, (a), and in any dimension, (b). In region (1) there are only holes and singly occupied $\mathbf{k}$ states; in region (2) we have doubly occupied $\mathbf{k}$ states but no holes; finally, in region (3) we find both singly and doubly occupied particle $\mathbf{k}$ states and holes. The capital letters E and F mean empty and full band, respectively.

III. GROUND-STATE PROPERTIES

Let us now consider the ground-state properties of the interacting electron system. We present in Fig. 1(a) the phase diagram of a one-dimensional lattice, for which a full analytical derivation is possible. The capital letter I denotes a Mott-insulating phase, while E and F represent regions where the band is empty and full, respectively. The lines ρ_{min} and ρ_{max} are defined by

$$\rho_{min} = \frac{1}{2} \int_{-\Delta/2}^{U-\Delta/2} \tilde{\sigma}(x)\, dx = \frac{2}{\pi} \arcsin \sqrt{U/\Delta}, \tag{29}$$

$$\rho_{max} = \left(\int_{-\Delta/2}^{\Delta/2-U} + \frac{1}{2} \int_{\Delta/2-U}^{\Delta/2} \right) \tilde{\sigma}(x)\, dx$$
$$= 2 - \frac{2}{\pi} \arcsin \sqrt{U/\Delta}, \tag{30}$$

where the lattice DOS, $\sigma(x)$, is defined by

$$\sigma(x) \equiv \frac{2}{N} \sum_{\mathbf{k}} \delta(x - \varepsilon_{\mathbf{k}})$$
$$= \tilde{\sigma}(x)\, \theta(x + \Delta/2)\, \theta(\Delta/2 - x), \tag{31}$$

in which $\theta(x)$ is the Heaviside step function. These lines separate two metallic phases, whose properties can be understood using fractional-statistics concepts. Although the actual shape of the lines is dimensional dependent, the point where they meet, $U=\Delta$, is the same for all dimensions as can be seen directly from Eqs. (29) and (30). We also find that the

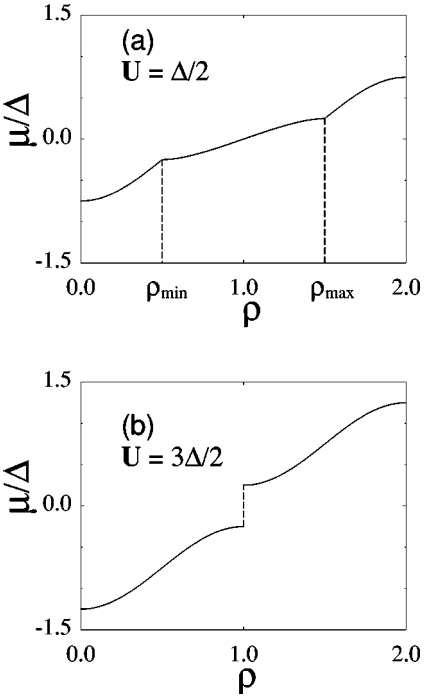

FIG. 2. Chemical potential as a function of band filling in $d=1$ and at $T=0$. In (a) the system displays cusps at ρ_{min} and ρ_{max} but remains always metallic for any filling. In (b) the chemical potential displays a discontinuity at $\rho=1$ and the system exhibits a Mott MIT. (From Ref. 4.)

line ρ_{min} (ρ_{max}) for different DOS cross at $U/\Delta=1/2$, $\rho=1/2(U/\Delta=1/2,\rho=3/2)$. An equivalent phase diagram that is dimensional independent is shown in Fig. 1(b). In region (1) the system is a compressible statistical spin liquid,[6] where double occupancy of particles is absent, i.e., $\tilde{n}_{\mathbf{k}3}=0$. Region (2) represents the mentioned phase with the absence of double occupancy of holes. The insulating phase is an incompressible statistical spin liquid and both the charge compressibility, K, and $\tilde{n}_{\mathbf{k}3}$ vanish. The insulating line I terminates at the QCP [see Figs. 1(a) and 1(b)]. In region (3) the system is a degenerate compressible statistical spin liquid, which is characterized by partial exclusion of doubly occupied states and a finite compressibility.

The chemical potential at $T=0$ is obtained as a function of the band filling using

$$\rho=\int_{-\infty}^{\mu-U/2}\sigma(x)dx+\frac{1}{2}\int_{\mu-U/2}^{\mu+U/2}\sigma(x)dx. \tag{32}$$

In Figs. 2(a) and 2(b) we show a plot of μ as a function of the band filling for a one-dimensional lattice (for similar figures see Ref. 5). In Fig. 2(a) the chemical potential is continuous and displays cusps at $\rho=\rho_{min}$ and $\rho=\rho_{max}$. In Fig. 2(b) we see that the chemical potential has a discontinuity at $\rho=1$ signaling the vanishing of the charge compressibility in the insulating phase. The Fermi energy is obtained from the chemical potential as: $\varepsilon_F=\mu-U/2$. In the ground-state [Fig. 1(a)], the Fermi energy satisfies:

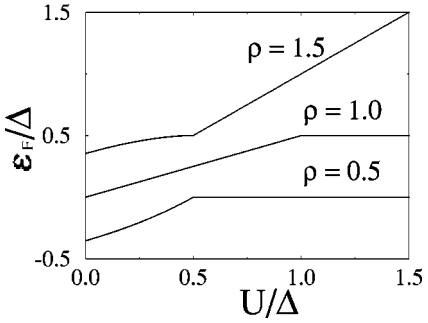

FIG. 3. Fermi energy as a function of the interaction in $d=1$. The curves $\rho=0.5$ and $\rho=1.5$ display cusps as the band filling is such that $\rho=\rho_{min}$ and $\rho=\rho_{max}$, respectively. In the curve $\rho=1$, the cusps appear in the QCP. In this last curve, we have calculated the Fermi energy at $\rho=1^-$ for $U>\Delta$.

$$\varepsilon_F=\begin{cases}-\Delta/2\cos\rho\pi, & \text{region (1)},\\ U+\Delta/2\cos\rho\pi, & \text{region (2)},\\ \dfrac{U}{2}\pm\dfrac{1}{2}\sqrt{\left(\Delta\cos\dfrac{\rho\pi}{2}\right)^2-\left(U\cot\dfrac{\rho\pi}{2}\right)^2}, & \text{region (3)},\end{cases}$$

where the plus (minus) sign is used for $\rho>1(\rho<1)$. In Fig. 3, we plot ε_F as a function of the interaction for three particular values of the band filling. For $\rho=0.5$, ε_F varies as a function of U until the mean number of doubly occupied $\mathbf{k}$ states vanishes ($\rho=\rho_{min}$), where it displays a cusp, after which ε_F is independent of U. For $\rho=1$, we calculated the Fermi energy at $\rho=1^-$: ε_F grows linearly with U up to the QCP, and becomes U independent thereafter. For $\rho=1.5$, ε_F always varies with U and displays a cusp as the mean number of holes in $\mathbf{k}$ space vanishes ($\rho=\rho_{max}$), after which ε_F grows linearly with U. We remark that the Fermi energy does not display a discontinuity at $T=0$ as the interaction is switched on.

The charge compressibility,[5] obtained from Eq. (32), reads

$$K=\frac{1}{2}\{\sigma(\mu-U/2)+\sigma(\mu+U/2)\}. \tag{33}$$

A plot of K as a function of the electron density is shown in Figs. 4(a) and 4(b) for a one-dimensional lattice. Observe in Fig. 4(a) that K exhibits a singular behavior at two special points, ρ_{min} and ρ_{max}. Although Hatsugai and Kohmoto[5] did find these singularities, their physical origin has not been discussed, neither have the lines ρ_{min} and ρ_{max} been identified. It is clear that the divergence of K as the system crosses these critical lines, reveals the existence of two different metallic phases in the ground state for $U<\Delta$. For $U>\Delta$ the compressibility displays the expected one-dimensional singular behavior as one approaches the Mott phase ($\rho=1$) at which $K=0$.

We now proceed to the calculation of the fermionic interacting DOS. Using the Matsubara one-electron Green's function, $\mathcal{G}_\sigma(\mathbf{k},i\omega_n)$,[49,50] where $\omega_n=(2n+1)\pi/\beta,(n=0,\pm1,\pm2,\ldots)$, and its relation with the retarded Green's function

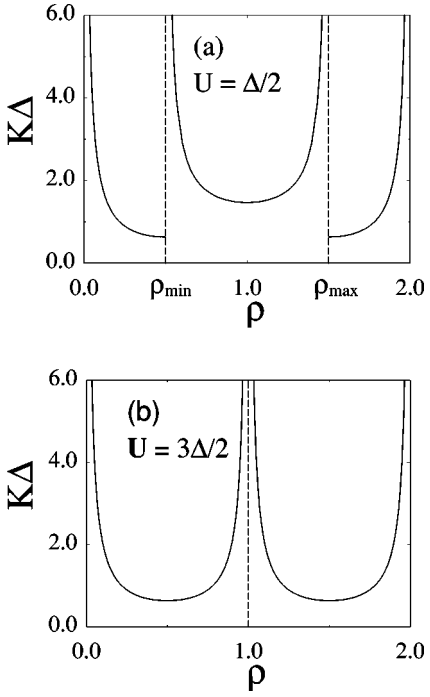

FIG. 4. Charge compressibility as a function of band filling in $d=1$ and at $T=0$. In (a) the charge compressibility displays singular behavior at ρ_{min} and ρ_{max}, revealing the existence of different metallic regimes. In (b) the system is insulator at $\rho=1$, with $K=0$. (From Ref. 4.)

$$G_\sigma^+(\mathbf{k},\omega)=\lim_{i\omega_n\to\omega+i0^+}\mathcal{G}_\sigma(\mathbf{k},i\omega_n), \tag{34}$$

we find for $T\to 0$ the expression

$$G^+(\mathbf{k},\omega)=\frac{1-\langle n_{\mathbf{k}\sigma}\rangle_0}{\omega-\tilde{\varepsilon}_{\mathbf{k}1}+i0^+}+\frac{\langle n_{\mathbf{k}\sigma}\rangle_0}{\omega-(\tilde{\varepsilon}_{\mathbf{k}3}-\tilde{\varepsilon}_{\mathbf{k}1})+i0^+}, \tag{35}$$

where $\langle n_{\mathbf{k}\sigma}\rangle_0$ is defined in Eq. (40), and $G^+\equiv 1/2\Sigma_\sigma G_\sigma^+$. The poles in Eq. (35) determines the fermionic band structure of the model. In particular, for $\rho=1$ we find two branches for ω_k, since the mean occupation number satisfies $\langle n_{k\sigma}\rangle_0=1/2$, when $\arccos(U/\Delta)<k<\arccos(-U/\Delta)$. This special filling shows a clear signature of the fractional statistics by giving rise to a split in the spectral function $A(\mathbf{k},\omega)$, defined in Eq. (36). In fact, the quantitity of interest is the fermionic interacting DOS defined by

$$D(\omega)=\frac{2}{N}\sum_{\mathbf{k}}A(\mathbf{k},\omega)=-\frac{2}{N\pi}\sum_{\mathbf{k}}\mathrm{Im}\{G^+(\mathbf{k},\omega)\}, \tag{36}$$

where $A(\mathbf{k},\omega)$ is the spectral function. Substituting Eq. (35) into Eq. (36), we obtain the interacting DOS at $T=0$:

$$D(\omega)=\frac{1}{2}\sigma(\omega+\mu+U/2)\{\theta(\omega)+\theta(U+\omega)\}+\frac{1}{2}\sigma(\omega+\mu-U/2)\{\theta(-\omega)+\theta(U-\omega)\}. \tag{37}$$

A plot of $D(\omega)$ in Figs. 5(a)–5(f) makes it possible to visualize the Mott MIT in $d=1$. Note that the states with $\omega\leqslant 0$ are filled. The interaction-driven MIT is shown in Figs. 5(a)–5(d), whereas the density-driven MIT can be seen by looking at the figures in the sequence e,d,f. In Fig. 5(b) the system is metallic and the three fractional species are in equilibrium with a Fermi surface consisting of four vectors in k-space (the two positive ones are shown in Fig. 6). As U moves towards Δ, the mean number of the species $\alpha=3$ per site, $N_3=(\Sigma_{\mathbf{k}}\tilde{n}_{\mathbf{k}3})/N$, approaches zero as the QCP [Fig. 5(c)] is reached. Thus, we can take N_3 as the order parameter of the interaction-driven MIT. This result is confirmed by the asymptotical behavior of N_3 close to the QCP and is in agreement with scaling predictions of Sec. V. The Mott phase with a gap $\Delta_{gap}=U-\Delta=\Delta/2$ in the interaction DOS is shown in Figs. 5(d). In the metallic states plotted in Figs.

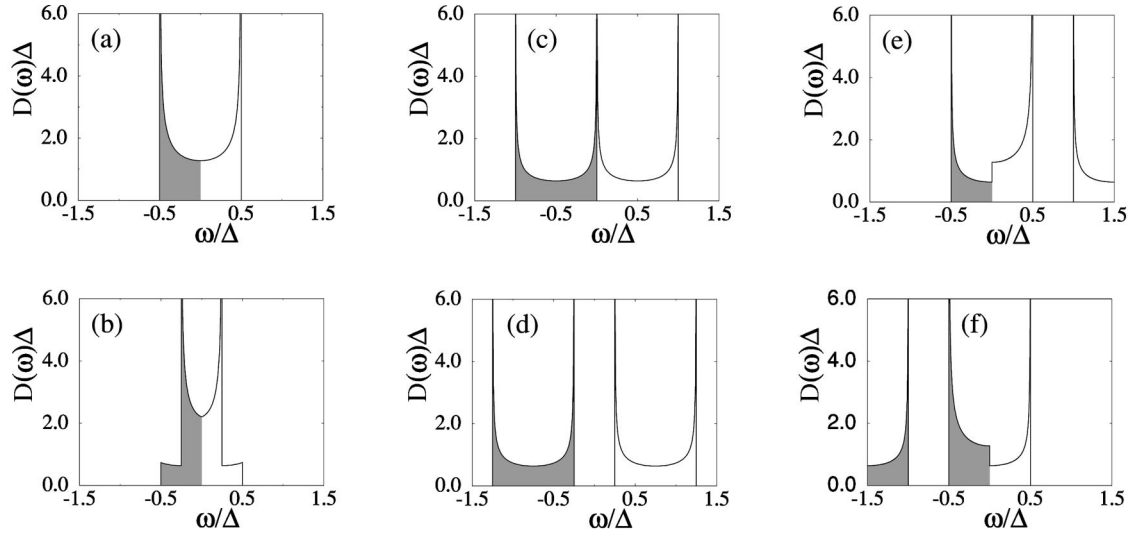

FIG. 5. Interacting DOS in $d=1$ and $T=0$ as the system undergoes an interaction-driven MIT [(a)–(d)] and a density-driven MIT [(e),(d),(f)]. All states with $\omega\leqslant 0$ are filled.

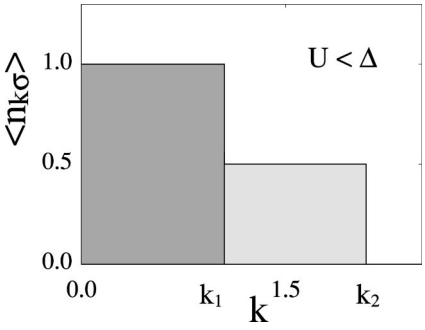

FIG. 6. Mean occupation number of the ground state for a particular band filling in $d=1$ satisfying $\rho_{min}<\rho<\rho_{max}$.

5(e) and 5(f) the Fermi surface is defined by only two k-vectors. The particle-hole symmetry[5] of the model manifests itself in Figs. 5(a)–5(f).

IV. LUTTINGER'S THEOREM

Recently, Haldane[51] has proposed a theory for describing the low-energy excitations of interacting electronic systems in the metallic state. In this approach, the system is assumed to have a ''generalized'' Fermi surface defined by **k**-space vectors at which the mean occupation number $n(\mathbf{k})$ displays some type of singular behavior at $T=0$. In particular, the volume of the Fermi surface, defined by the set of ''singular'' points k_{F_i} is given by

$$V_F=\sum_i \Delta\nu_i \mathbf{k}_{F_i}, \tag{38}$$

where the indices $\Delta\nu_i$ are adiabatic invariants and are related to the particles' statistics. They are determined by the step discontinuity of the distribution function of the noninteracting particles through the formula

$$\Delta\nu_i=\lim_{\delta\to 0}\{n_0(\mathbf{k}_{F_i}-\delta)-n_0(\mathbf{k}_{F_i}+\delta)\}. \tag{39}$$

For fermions, $|\Delta\nu_i|=1$, by virtue of the Pauli principle, but in the fractional quantum Hall effect, $|\Delta\nu_i|$ can assume values given by $1/m$, where m is an integer number. Furthermore, Luttinger's theorem[52] is taken as a general property of the system due to momentum and particle number conservations.

More recently, Byczuk and Spalek[49] showed that in the Hubbard model with infinite range Coulomb coupling the singularities of the Fermi surface satisfy $|\Delta\nu_i|=1/2$ for any $U>0$. The authors however do not use this result to establish any connection with Luttinger's theorem. In this section, we provide such a connection in the context of the model. Using definition (38), the volume of the Fermi surface for the noninteracting case is $V_F^{(0)}=\rho\pi$, where ρ particle density and $|\Delta\nu_i|=1$. As the interaction is switched on, keeping ρ constant, the mean occupation number at $T=0$ satisfies[5]

$$\langle n_{\mathbf{k}\sigma}\rangle_0=\frac{1}{2}[\,\theta(\mu-\varepsilon_{\mathbf{k}}+U/2)+\theta(\mu-\varepsilon_{\mathbf{k}}-U/2)]. \tag{40}$$

In Fig. 6, we plot this function for the one-dimensional lattice and for a particular band filling satifying $\rho_{min}<\rho<\rho_{max}$. Here, the Fermi surface is defined by the set $\{-k_2,-k_1,k_1,k_2\}$ and $|\Delta\nu_i|=1/2$. With this prescription, the volume of the Fermi surface is given by

$$V_F=k_1+k_2, \tag{41}$$

and therefore density of particles reads

$$\rho\pi=k_1+k_2=V_F=V_F^{(0)}. \tag{42}$$

We remark that although Eq. (42) shows that the volume of the Fermi surface is preserved, this cannot be considered as a proof of the validity of the Luttinger's theorem in the generalized form proposed by Haldane, since $|\Delta\nu_i|$ changes as the interaction is switched on. Nevertheless, we stress that as long as $U>0$, $|\Delta\nu_i|$ is indeed an adiabatic invariant.

V. THERMODYNAMICS AND SCALING ANALYSIS

In this section we provide a scaling analysis of the model in the vicinity of the QCP of the interaction-driven Mott MIT, whose hypothesis are confirmed by microscopic calculations. The system described by Eq. (1) exhibits a Mott MIT (Ref. 5) at $T=0$, $\mu=0$, and $U=\Delta$. There are several equivalent formulations of the scaling theory at zero and finite temperature[53–57] and here we choose a scaling form for the singular part of the grand-canonical free energy, $\Omega(T,\mu;g)$, that is valid in the whole vicinity of the QCP ($T=\mu=g=0$)

$$\Omega_s(T,\mu;g)=\xi^{-(d+z)}\Omega_0\left(\frac{T}{g^{\phi}},\frac{\mu}{g^{\beta\delta}}\right). \tag{43}$$

In Eq. (43) ξ is the critical correlation length, Ω_0 is a universal scaling function, the chemical potential, μ, is the field that couples to the order parameter, $g=(\Delta-U)/2$ is the variable that drives the transition at zero temperature, ϕ and $\Delta_{cross}=\beta\delta$ are crossover exponents, d is the Euclidian dimension of the system and z is the dynamic exponent. The presence of z, which governs the scaling properties of the $T=0$ critical dynamics, results in an effective dimension $d_{eff}=d+z$. The main assumption in this scaling theory is that the transition is characterized by a single correlation length ξ, expressed in different equivalent asymptotic forms, corresponding to the approaching path to the QCP

$$\xi_g\sim g^{-\nu_g},\quad T=0,\quad g\to 0^+,$$
$$\xi_T\sim T^{-\nu_T},\quad g=0,\quad T\to 0. \tag{44}$$

According to the values of the ratio T/g^{ϕ} and the sign of g, three different regimes can be identified: a quantum (Q), a quantum critical (QC) and a classical (*thermally activated*) regime. In Fig. 7 we plot a phase diagram as a function of g and T where these regions are separated by crossover lines: $T=g$ and $T=-g$. The hyperscaling relation: $2-\alpha=(d+z)\nu$ is assumed to be valid for any $d\leqslant d_c$, where d_c is the upper critical dimension. Using Toulouse's method[58] to obtain d_c, we will find that the hyperscaling relation is valid for any d.

In the scaling analysis we will use the singular part of the grand-canonical free energy and, consequently, we identify regular and singular terms in corresponding expressions for

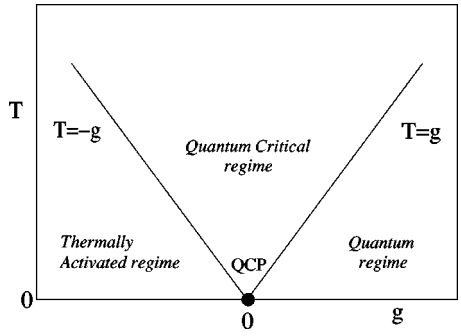

FIG. 7. Phase diagram as a function of g and temperature T. The regions indicated are separated by crossovers lines: $T=g$ and $T=-g$. The point $T=g=\mu=0$ is the QCP of the MIT.

Ω. We are interested in obtaining results at low temperatures ($\beta U \gg 1$) and near the QCP ($T=0, U=\Delta$), and thus we write

$$\Omega(T,\mu;g)=\sum_k\ (\varepsilon_k-\mu-U/2)-NT\ln 2 - T\sum_k\ \ln\left\{1+\frac{1}{2}e^{\beta(\varepsilon_k-\mu-U/2)}\right\}+\mathcal{O}(e^{-\beta U/2}). \tag{45}$$

The first two terms in Eq. (45) being regular functions are useless from the scaling point of view and can be neglected. However, we point out that the second term exhibits the degeneracy of the ground state and gives rise to a residual entropy. The third term is singular for $U\to\Delta^-$ and $T\to 0$, and can be conveniently rewritten using the tight-binding DOS, $\sigma(\varepsilon)$, in the form,

$$\Omega_s(T,0;g)=-T\int_0^{\Delta/2}d\varepsilon\ \sigma(\varepsilon)\ln\left\{1+\frac{1}{2}e^{\beta(\varepsilon-U/2)}\right\}, \tag{46}$$

in which we have set the chemical potential $\mu=0$ and the density of states parity was used. The logaritmic factor of the integrand in Eq. (46) is vanishingly small in the interval $[0,U/2]$ for $\beta\to\infty$, and therefore, as $U\to\Delta^-$, the dominant part of the integral comes from the band edge and $\sigma(\varepsilon)$ can be approximated by[59]

$$\sigma(\varepsilon)=A_d\left(1-\frac{2\varepsilon}{\Delta}\right)^{d/2-1}\left\{1+\mathcal{O}\left(1-\frac{2\varepsilon}{\Delta}\right)\right\},\qquad \frac{2\varepsilon}{\Delta}\to 1^-. \tag{47}$$

In the above, A_d is a dimensional dependent constant:

$$A_d=\frac{4}{\Delta\Gamma(d/2)}\left(\frac{d}{2\pi}\right)^{d/2}, \tag{48}$$

and $\Gamma(s)$ is the Gamma function. After integrating by parts twice, we obtain a series expansion for the free energy per site (see Appendix A),

$$\Omega_s(x,y)/N=-A_d\left(\frac{\Delta}{2}\right)^2\left(\frac{2x}{\Delta}\right)^{(d+2)1/2}\times\{a_0+a_1y+a_2y^2+\cdots\}, \tag{49}$$

where $x=\max\{g,T\}$ and $y=\min\{g,T\}/x$ define the regime of interest. The series' coefficients depend on the system's dimensionality, d, and on f, defined as an index counting the degeneracy of the ground state. We remark that in the exclusion representation $1/f$ equals to $\mathbf{k}$-state mean occupation number for the case of total exclusion at $T=0$ ($n_{\mathbf{k}1}=n_{\mathbf{k}2}=1/f$, $n_{\mathbf{k}3}=0$). In the *quantum* regime ($g>T$) the first three coefficients are listed below:

$$a_0^Q=\frac{4}{d(d+2)}, a_1^Q=-\frac{2\ln f}{d}, a_2^Q=\frac{1}{2}\left[\frac{\pi^2}{3}+(\ln f)^2\right]. \tag{50}$$

In the QC regime ($|g|<T$) we can determine all coefficients via the general relation

$$a_m^{QC}=\frac{1}{fm!}\Gamma\left(\frac{d}{2}\right)\Phi\left(-\frac{1}{f},\frac{d}{2}-m+1,1\right), f=2, \tag{51}$$

where $\Phi(z,v,u)$ is the Lerch function,[60] which replaces the Riemann Zeta function that appears in similar expansions for Fermi liquid systems (see Appendix A for details). Moreover, the presence of the Lerch function in the amplitudes of the specific heat is a signature of the fractional exclusion statistics and identifies the non-Fermi-liquid behavior of the system. Note that a distinct value for a_2^Q was reported in Ref. 6, presumably due to an incorrect treatment of the low-temperature expansion. We should point out that, for $f=1$, the standard coefficients for the spinless Fermi gas ($U=0$) are recovered if one includes the neglected terms $[\mathcal{O}(e^{-\beta U/2})]$ in Eq. (49).

By comparing the singular part in Eqs. (49) and (43) we obtain the critical exponents: $\nu_g=\nu_T=1/2, z=2$ and $\phi_g=\phi_T=1$, and after using the hyperscaling equation, we get $\alpha_g=\alpha_T=1-d/2$. Although in most cases the crossover exponent satisfies the relation $\phi=z\nu$, as we find in our study, there are special situations in which different expressions apply.[57] In the *classical* regime ($T<|g|$) the dominant term of Ω_s is obtained by substituting the series in Eq. (49) by a thermal activation factor: $e^{-|g|/T}$.

For the Mott interaction-driven transition, the chemical potential couples to the order parameter, which we identify as the number of doubly occupied $\mathbf{k}$-states per site: $N_3=(\Sigma_{\mathbf{k}}\tilde{n}_{\mathbf{k}3})/N$. Indeed, by approaching the QCP along the path $T=\mu=0, g\to 0^+$, we find: $N_3(g)\simeq A_d(\Delta/d)(2g/\Delta)^{d/2}$, and $N_3(g)=0$, for $T=\mu=0$ and $g\leqslant 0$. From the expressions above an exponent $\beta=d/2$ results. We can also derive an equation of state: $N_3(\mu)\simeq A_d(\Delta/d)(2\mu/\Delta)^{d/2}, T=g=0, \mu\to 0^+$, from which we identify $\delta=2/d$ and $\Delta=\beta\delta=1$. The vanishing behavior of N_3, as one approaches the QCP along the line $\mu=g=0, T\to 0$ is also determined by the exponent β: $N_3(T)\simeq A_d a_1^{QC}(\Delta/2)(2T/\Delta)^{d/2}$. The above results are consistent with the usual assumption of taking the charge stiffness, D_c, defined as the dc part of the conductivity, $\sigma(\omega)=D_c\delta(\omega)$, as the order parameter for the Mott MIT. In fact, from the scaling prediction,[53] $D_c\sim\xi^{-(d+z-2)}$, and the above-derived results for the order parameter, we find ξ_g

$\sim g^{-1/2}$, for $T=\mu=0$, and $g\to 0^+$, and also $\xi_T\sim T^{-1/2}$, for $\mu=g=0$ and $T\to 0$, with $z=2$ and $\nu=1/2$, as expected.

It is interesting to note that the density-driven MIT in the $d=1$ *short-range* Hubbard model exhibits the same critical exponents as the model we study.[43] In fact, in both cases we have $\alpha=1/2$, $\nu=1/2$, $z=2$, which implies $\xi\sim\delta^{-1}$, $K_c\sim\delta^{-1}$ and $D_c\sim\delta$ where $\delta=|1-n|$. On the other hand, for $d=3$, other similitarities can be found in comparison with the solution of Brinkman and Rice[17] for the interaction-driven Mott MIT, where the critical exponents $\alpha=0$, $\nu=1/2$, $z=1$ describe analogous qualitative behavior for the correlation length, charge compressibility and charge stiffness: $\xi\sim g^{-1/2}, K_c\sim g, D_c\sim g$, but with distinct exponents.

The grand-canonical specific heat can be written as $C=C^{(1)}+C^{(2)}+C^{(3)}$, where

$$C^{(1)}=\beta^2\sum_{\mathbf{k},\sigma}\left(\varepsilon_{\mathbf{k}}-\mu+\frac{U}{2}\right)^2 n_{\mathbf{k}d}(n_{\mathbf{k}\sigma}-n_{\mathbf{k}d}), \tag{52}$$

$$C^{(2)}=\beta^2\sum_{\mathbf{k},\sigma}\left(\varepsilon_{\mathbf{k}}-\mu-\frac{U}{2}\right)^2 \times(1-n_{\mathbf{k}\sigma}-n_{\mathbf{k}\bar{\sigma}}+n_{\mathbf{k}d})(n_{\mathbf{k}\sigma}-n_{\mathbf{k}d}), \tag{53}$$

$$C^{(3)}=\beta^2\sum_{\mathbf{k},\sigma}[2(\varepsilon_{\mathbf{k}}-\mu)]^2(1-n_{\mathbf{k}\sigma}-n_{\mathbf{k}\bar{\sigma}}+n_{\mathbf{k}d})n_{\mathbf{k}d}. \tag{54}$$

We note that in the exclusion representation, the above expressions evidence the mutual statistical interaction between the exclusion species:

$$C^{(1)}=\beta^2\sum_{\mathbf{k}}\sum_{\alpha=1,2}(\tilde{\varepsilon}_{\mathbf{k}3}-\tilde{\varepsilon}_{\mathbf{k}\alpha})^2\tilde{n}_{\mathbf{k}3}\tilde{n}_{\mathbf{k}\alpha}, \tag{55}$$

$$C^{(2)}=\beta^2\sum_{\mathbf{k}}\sum_{\alpha=1,2}\tilde{\varepsilon}^2_{\mathbf{k}\alpha}\tilde{n}_{\mathbf{k}\alpha}(1-\tilde{n}_{\mathbf{k}}), \tag{56}$$

$$C^{(3)}=\beta^2\sum_{\mathbf{k}}\tilde{\varepsilon}^2_{\mathbf{k}3}\tilde{n}_{\mathbf{k}3}(1-\tilde{n}_{\mathbf{k}}), \tag{57}$$

in which we have defined $\tilde{n}_{\mathbf{k}}=\tilde{n}_{\mathbf{k}1}+\tilde{n}_{\mathbf{k}2}+\tilde{n}_{\mathbf{k}3}$ and $\alpha,\gamma=1,2,3$.

At low temperatures and in different regimes we derive expressions for $C^{(1)}$ and $C^{(2)}(C^{(3)}\sim\overline{e^{\beta u/2}}\to 0)$ using the interacting DOS, Eq. (37), at $T=0$. In the *quantum* regime, one finds: $C^Q(T,g)\simeq(3/2)D(\omega=0)a_2^Q T, \mu=0, (T/g)\to 0$, which implies a thermal mass $m_T\propto C^Q(T,g)/T\sim g^{d/2-1}$ and an exponent $\alpha=1-d/2$. Moreover, in the QC regime we obtain: $C^{QC}(T)\simeq[D(\omega=T)+D(\omega=-T)]a_2^{QC}[d(d+2)/4]T, \mu=0, (g/T)\to 0$, whose explicit T dependence reads: $C^{QC}(T)\sim T^{d/z}$, with an exponent d/z (Refs. 53–57) expected for a QCP, and is here a consequence of the dispersion relation and the tight-binding DOS at the band edge [Eq. (47)]. Similar results have also been obtained for the density-driven MIT by allowing the variable interchange: $g\leftrightarrow\mu$. As a peculiar aspect of this model, at $\mu=-g$ ($\rho=\rho_{min}$) or $\mu=g$ ($\rho=\rho_{max}$) the *quantum* and QC contributions compete equally. This peculiar feature will be further considered in a forthcoming publication.

For completeness, we have also calculated the static structure factor, $S(\mathbf{q},T,\mu;g)$, associated with the Fourier transform of the density-density correlation function. In the *quantum* regime the singular part reads: $S_s(\mathbf{q}\to 0;T=\mu=0;g)\sim g^{d/2}, g\to 0$ and thus, from the scaling prediction,[53] $S_s(\mathbf{q}\to 0;T=\mu=0;g)\sim g^{-(2-z-\eta)\nu}$, one obtains $\eta=d$. The exponent η also determines the singular behavior of S in the QC regime: $S_s(\mathbf{q}\to 0;T=\mu=0;g)\sim T^{d/2}, T\to 0$. Moreover, we can show that the charge compressibility satisfies:[5] $K\simeq A_d(2g/\Delta)^{d/2-1}, T=\mu=0, g\to 0^+$, and therefore $\gamma=1-d/2$. From the above anlysis, we conclude that the exponents of the quantum critical fixed point of the MIT are in the same universality class as that of the spinless free Fermi gas[56] at the QCP, $\mu=0$, $T=0$, characterized by the vanishing of the Fermi surface.

VI. THE STATIC MAGNETIC SPIN SUSCEPTIBILITY

In this section we evaluate the zero-field magnetic susceptibility at finite temperature. The grand-canonical free energy Ω in a magnetic field is given by

$$\Omega(T,\mu,H;g)=-T\sum_k \ln\{1+e^{-2\beta(\varepsilon_k-\mu)} +2e^{-\beta(\varepsilon_k-\mu-U/2)}\cosh(\beta\mu_B H)\}, \tag{58}$$

where μ_B is the Bohr magneton. The zero-field magnetic susceptibility is defined as follows

$$\chi_\mu(T;g)\equiv-\frac{1}{N}\left(\frac{\partial^2\Omega}{\partial H^2}\right)_{H=0}, \tag{59}$$

and thus, using Eq. (59) we find

$$\chi_\mu(T;g)=\frac{\mu_B^2}{T}\int_{-\Delta/2}^{\Delta/2}\frac{\sigma(\varepsilon)d\varepsilon}{e^{\beta(\varepsilon-\mu-U/2)}+2+e^{-\beta(\varepsilon-\mu+U/2)}}. \tag{60}$$

At this point, we remark that the zero-field magnetic susceptibility for fixed density, χ_ρ, can be obtained from Eq. (60) by eliminating μ in favor of $\rho\equiv N_e/N$. On the other hand, the mean occupation number of the simple occupied $\mathbf{k}$ states, N_S, is defined as

$$N_S(T,\mu;g)=N_e-N_3=N_1+N_2, \tag{61}$$

with $N_\alpha\equiv\Sigma_{\mathbf{k}}n_{\mathbf{k}\alpha}$, ($\alpha=1,2,3$), the total average number of the excluson species. After some algebra we obtain

$$N_S(T,\mu;g)=N\int_{-\Delta/2}^{\Delta/2}\frac{\sigma(\varepsilon)d\varepsilon}{e^{\beta(\varepsilon-\mu-U/2)}+2+e^{-\beta(\varepsilon-\mu+U/2)}}. \tag{62}$$

Comparing Eq. (60) with Eq. (62), we get

$$\chi_\mu(T;g)=\frac{\mu_B^2}{T}\left(\frac{N_1+N_2}{N}\right). \tag{63}$$

We thus find that, as a consequence of strong interactions in the system, the Pauli response is subdominant at low temperatures, and the susceptibility is essentially Curie-like. On the other hand, for $U\to\infty$, the susceptibility is pure Curie-type since N_1+N_2 is T independent.

We now present an analysis of Eq. (62) in four particular cases:

(a) The simplest one is obtained for $U=0$, when we recover the Pauli magnetic susceptibility in the $T\to 0$ limit.

(b) Another case of interest is the atomic limit, where the ''hopping term,'' t, vanishes and Eq. (62) becomes:

$$\chi_\mu(T)=\frac{\mu_B^2}{T}\frac{2}{e^{\beta(\mu-U/2)}+2+e^{-\beta(\mu+U/2)}}, \tag{64}$$

which is indeed the correct result.[8]

(c) For $U\to\infty$ and fixing $\rho\leqslant 1$, the number of doubly occupied **k** states N_3 vanishes and wet get a Curie-like magnetic susceptibility both in the insulating phase, $\rho=1$, and in the metallic one, $\rho<1$, thus:

$$\chi_\rho(T)=\frac{\mu_B^2}{T}\rho. \tag{65}$$

We remark that Eq. (65) is valid in any dimension and, in particular, for $d=1$ it coincides with the result obtaind in the *short-range* infinite-coupling Hubbard chain.[61] If we fix the chemical potential instead of ρ, we find for the spin susceptibility, obtained by substituting in Eq. (60) $\mu\to\mu-U/2$ and by taking the limit $U\to\infty$ in this sequence, the following result

$$\chi_\mu(T)=\frac{\mu_B^2}{T}\overline{\rho}(T=0)+\frac{\ln 2}{2}\mu_B^2\sigma(\mu)+\cdots, \tag{66}$$

where μ is kept far from the band edge in order to justify the Sommerfeld expansion. This Pauli term can be understood as follows: when the magnetic field is applied, the particles of the system align themselves with H, thus reducing the chemical potential. Therefore, to keep μ constant a macroscopic number of particles, proportional to H, is needed, giving rise to the Pauli term. The factor $\ln 2$ in $\chi_\mu(T)$ is a signature of the fractional statistics obeyed by the particles and the degeneracy of the ground-state.

(d) For $\rho\ll\rho_{min}$, the number of doubly occupied **k** states is exponentially small, and similarly to the case $U\to\infty$, we obtain

$$\chi_\rho(T)=\frac{\mu_B^2}{T}\rho+\cdots, \tag{67}$$

and for χ_μ we find

$$\chi_\mu(T)=\frac{\mu_B^2}{T}\overline{\rho}(T=0)+\frac{\ln 2}{2}\mu_B^2\sigma(\mu-U/2)+\cdots, \tag{68}$$

where we have assumed that $\mu-U/2$ is far from the band edge.

(e) For $\mu_{min}\ll\mu\ll\mu_{max}$ and $U<\Delta$, we find

$$\chi_\mu(T)=\frac{\mu_B^2}{T}\overline{\rho_s}(T=0)+\frac{\ln 2}{2}\mu_B^2\{\sigma(\mu-U/2)+\sigma(\mu+U/2)\}+\cdots, \tag{69}$$

where $\overline{\rho_s}(T=0)$ is the mean particle density with simple occupancy in **k** space. On the other hand, as we fix $\rho=1$ and put $U\ll\Delta$, the chemical potential at low temperature is given by[49]

$$\mu=-T\ln 2+\cdots. \tag{70}$$

Thus, for $\chi_\rho(T)$ we obtain

$$\chi_\rho(T)=\frac{\mu_B^2}{T}\rho_s(T=0)+\mu_B^2\sigma(U/2)\ln 2+\cdots. \tag{71}$$

(f) If one sets $U=U_C=\Delta,\mu=0$ and $T\to 0$, i.e., close to the QCP, we get from Eq. (60)

$$\chi_\mu(T)=\frac{\mu_B^2}{T}\left\{1-\int_0^{\Delta/2}\frac{\sigma(\varepsilon)d\varepsilon}{1+2e^{-\beta(\varepsilon-\Delta/2)}}\right\}+\mathcal{O}(e^{-\beta\Delta/2}). \tag{72}$$

We should notice that the integrand in Eq. (72) is exponentially small as ε moves away from the band edge. By using the approximate form of $\sigma(\varepsilon)$ in the band edge, Eq. (47), we finally obtain

$$\chi_\mu(T)=\frac{\mu_B^2}{T}\{1-N_3+\cdots\},$$

$$\mu=g=0,T\to 0, \tag{73}$$

where $N_3\simeq A_d a_1^{QC}(\Delta_0/2)(2T/\Delta_0)^{d/2}$.

VII. CONCLUSIONS

In this paper, we have studied a Hubbard-type model with infinite-range Coulomb coupling which is mapped onto an ideal gas of particles obeying fractional statistics *à la* Haldane. We have shown that, for doing this, it was necessary to extend the thermodynamic formalism developed by Wu, thus clarifying some disagreements in the literature. We have contructed excluson operators and we have checked its validity by calculating the *excluson* Green's functions and finding poles exactly at the energies associated with the excluson species. We have presented an attempt to describing a Mott MIT as being caused by the statistical interaction between independent exclusons. In particular, we have demonstrated that the mean number of doubly occupied **k** states, which is one of the three species of exclusons identified in the model, can be taken as the order parameter of the interaction-driven MIT. We have described several NFL properties of the ground state caused by the statistical interaction. In particular, we have shown that in the $U\times\rho$ phase diagram there exist two fundamental lines, ρ_{min} and ρ_{max}, separating regions of doubly and singly occupied **k** states, whose interpretation in terms of exclusons is evident. These special fillings in $d=1$ are responsable for cusps at the chemical potential and singularities in the charge compressibility. We have also investigated the Luttinger's theorem in the form presented by Haldane and have shown that although the adiabatic condition is only satisfied for $U>0$, we can extend Haldane's prescription in a way that the volume enclosed by the Fermi surface is preserved for any $U\geqslant 0$. We report a detailed quantum scaling analysis of the interaction-

driven MIT close to its QCP where the scaling hypotheses are confirmed by explicit calculations of the thermodynamic quantities such as the free energy, the specific heat, the order parameter and the static structure factor. We find, by explicit calculation of the critical exponents, that the QCP of the MIT of the Hubbard model with infinite range Coulomb coupling belongs to the same universality class as the QCP ($T=\mu=0$) of a spinless free Fermi gas. Moreover, in $d=1$ we identify the same critical exponents as those of the density-driven MIT of the one dimensional short-range Hubbard model. It is important to note that the coefficients of the grand-canonical free energy expansion and of the specific heat express a clear manifestation of the generalized exclusion principle. We have also obtained the zero field magnetic spin susceptibility in several regimes and we highlight here that the low-temperature behavior is Curie like, as in the infinite coupling short-range $d=1$ Hubbard model, with subdominant, if any, Pauli-like contribution.

We finish by remarking that our findings, albeit derived in an extremal limit of an infinite-range Hubbard-type model, contains several features which may provide insights into more realistic models. We mention in particular the appearence of exclusion statistics in the thermodynamic properties of the model, and the occurrence of a MIT. Both subjects are of much current interest and we emphasize that the mapping presented here, between interacting electrons and exclusons, has proved very helpful for obtaining new results in the context of the excluson formalism, such as the excluson Green's function and the specific heat formula, and thus it represents a concrete step towards the understanding of the physics underlying fractional statistics.

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APPENDIX A: COEFFICIENTS OF GRAND-CANONICAL FREE ENERGY EXPANSION

We sketch the derivation of the coefficients a_m obtained in the grand-canonical free energy expansion. Consider the following changing of variable, $y=\beta(\varepsilon-U/2)$, which transforms Eq. (46) into

$$\Omega_s=-A_d T^2\int_{-\beta U/2}^{\beta g}\left(\frac{2g}{\Delta}-\frac{2T}{\Delta}y\right)^{d/2-1}\ln\left(1+\frac{1}{f}e^y\right)dy. \tag{A1}$$

Integrating by parts twice and neglecting terms of order $\mathcal{O}(e^{-\beta U})$, we obtain

$$\Omega_s=-A_d\left(\frac{\Delta}{2}\right)^2\frac{4}{d(d+2)}\times\int_{-\beta U/2}^{\beta g}\left(\frac{2g}{\Delta}-\frac{2T}{\Delta}y\right)^{d/2-1}\frac{fe^y}{(e^y+f)^2}dy. \tag{A2}$$

Consider first the quantum case ($g>T$). After a binomial expansion in the integral of Eq. (A2), we can extend the limits of the integral to $-\infty$ and ∞. Retaining the first three terms in the expansion and comparing to Eq. (49) one can write

$$a_0^Q=\frac{4}{d(d+2)}\int_{-\infty}^{\infty}\frac{fe^y}{(e^y+f)^2}dy=\frac{4}{d(d+2)}, \tag{A3}$$

$$a_1^Q=\frac{2}{d}\int_{-\infty}^{\infty}\frac{fye^y}{(e^y+f)^2}dy=\frac{2\ln f}{d}, \tag{A4}$$

$$a_2^Q=\int_{-\infty}^{\infty}\frac{fy^2e^y}{(e^y+f)^2}dy=\frac{1}{2}\left[\frac{\pi^2}{3}+(\ln f)^2\right]. \tag{A5}$$

In the QC case ($T>g$), after a similar binomial expansion, the lower limit in the integral of Eq. (A2) can be considered as $-\infty$ while the upper limit may be taken as 0 and thus we get the general term $a_m^{QC}(m=0,1,2,\ldots)$

$$a_m^{QC}=\frac{1}{fm!}\left[\left(\frac{d}{2}-m+1\right)\right]\int_0^{\infty}\frac{y^{d/2-m}}{e^y+1/f}dy. \tag{A6}$$

Using now the integral form of the Lerch functions,[60]

$$\Phi(z,v,u)=\frac{1}{\Gamma(v)}\int_0^{\infty}\frac{t^{v-1}e^{-(u-1)t}}{e^t-z}dt, \tag{A7}$$

$$[\mathrm{Re}v>0,\ \text{or}\ |z|\leqslant 1,\quad z\neq 1,\mathrm{Re}v>0,\ \text{or}\ z=1,\mathrm{Re}v>1],$$

a simple algebra and an integration by parts leads us to

$$a_m^{QC}=\frac{1}{fm!}\Gamma\left(\frac{d}{2}\right)\Phi\left(-\frac{1}{f},\frac{d}{2}-m+1,1\right). \tag{A8}$$

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