

## Magnetoresistance fluctuations in short $n$ -type Si/SiGe heterostructure wires

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Magnetoresistance fluctuations in short quasiballistic Si/SiGe wire segments have been investigated as a function of magnetic field and temperature. The segments are measured in a four-probe geometry and the voltage probe distances  $L$  are taken smaller or larger than the phase coherence length  $l_\phi$  ( $\approx 1.5 \mu\text{m}$  at  $T = 0.1 \text{ K}$ ) and the electron mean free path  $l_e$  ( $\approx 0.8 \mu\text{m}$ ). At magnetic fields smaller than  $B = 1 \text{ T}$ , the amplitude  $\delta R$  and the correlation field  $B_c$  of both the symmetric and antisymmetric part of the resistance fluctuations have been studied as a function of probe distance and temperature. It is found that, despite the quasiballistic character of electron transport in our samples, the behavior of the amplitude and correlation field with probe separation is in good qualitative agreement with expressions derived for the diffusive regime. The observed magnitude of  $B_c$ , however, is much larger than expected for the diffusive transport regime. A better agreement for  $B_c$  is obtained using an expression adapted for the quasiballistic regime. The temperature dependence of the correlation field cannot be explained by expressions appropriate for the diffusive or the quasiballistic transport regime.

### I. INTRODUCTION

Studies of electron transport in nanoscale devices during the last 15 years have revealed a number of very fascinating phenomena. One of the first effects observed in this context were device specific, reproducible fluctuations in the magnetoconductance of submicron-sized metal lines.<sup>1</sup> Lee and Stone<sup>2</sup> showed theoretically that at zero temperature the rms amplitude of the fluctuations is  $\sim e^2/h$ , independent of sample size and disorder. The fluctuations are therefore called universal conduction fluctuations (UCF). The original UCF theory contains two important assumptions: first the conductance is calculated for a two-probe geometry and second electron transport is supposed to be diffusive, i.e., the elastic mean free path of the electrons  $l_e$  is much smaller than the length of the sample  $L$  ( $l_e \ll L$ ). It was soon recognized that the presence of additional measuring probes has a significant effect on the fluctuations when the distance between the probes is comparable to the phase coherence length of the electrons ( $l_\phi$ ). This was demonstrated, for instance, by experiments of Skocpol *et al.*,<sup>3</sup> Benoit *et al.*,<sup>4</sup> and Haucke *et al.*<sup>5</sup> on Si metal-oxide-semiconductor field-effect transistors and metal films and by theoretical work of Büttiker,<sup>6</sup> Hershfield, and Ambegaokar,<sup>7</sup> Baranger, Stone, and DiVincenzo,<sup>8</sup> and Chandrasekhar, Santhanam, and Prober.<sup>9</sup> Not only experimental work on various small structures involving the high-mobility two-dimensional electron gas in AlGaAs/GaAs heterostructures (e.g., narrow etched<sup>10,11</sup> or split-gate wires<sup>12,13</sup> and ballistic cavities)<sup>14</sup> but also work on three-dimensional metallic point contacts<sup>15</sup> shows that the condition of strict diffusive transport is not a necessary condition for the observation of conductance fluctuations.

However it is not clear to what extent the theory developed for the pure diffusive transport regime is capable of describing in detail conductance fluctuations in, for instance, the quasiballistic transport regime. It is interesting to note that measurements by Bird *et al.*<sup>10,11</sup> and Thornton *et al.*<sup>12</sup> of conductance fluctuations in *long*, narrow wires defined in a high-mobility  $n$ -type AlGaAs/GaAs heterostructure at low magnetic fields could be straightforwardly interpreted in the framework of the original diffusive UCF theory. At present the number of experiments in the quasiballistic regime on *short* wires ( $L \approx l_\phi$ ) with a four-probe geometry is very limited. As far as we know only the experiments by Timp *et al.*<sup>16</sup> on AlGaAs/GaAs samples provide information on conductance fluctuations in samples with probe distance smaller than  $l_\phi$ .

The work described in this paper concerns a study of conductance fluctuations in short  $n$ -type Si/SiGe heterostructures. An electron mobility of more than  $50 \text{ m}^2/\text{V s}$  has been achieved in this material system.<sup>17</sup> A variety of magnetoconductance phenomena has been observed in structures defined in both  $n$ -type and  $p$ -type Si/SiGe heterostructures. Studies have been made of quantized conductance,<sup>18</sup> the Aharonov-Bohm effect,<sup>19,20</sup> weak localization,<sup>21,22</sup> Shubnikov-de Haas oscillations, and the quantum-Hall effect.<sup>23,24</sup> A comprehensive review of these experiments has been given by Schäffler.<sup>25</sup> To our knowledge so far, no detailed studies have been made of conductance fluctuations in short Si/SiGe heterostructures.

The elastic mean free path  $l_e$  of the electrons in the Si/SiGe material we study in this work is about  $0.8 \mu\text{m}$  for  $T < 4.2 \text{ K}$  (mobility  $\approx 7.9 \text{ m}^2/\text{V s}$ ). We fabricated samples with probe distance varying from  $0.2$  to  $10 \mu\text{m}$ . All length scales important for conductance (or resistance) fluctuations,

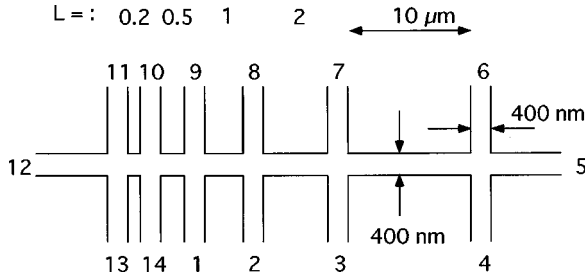


FIG. 1. Overall device layout: 400-nm-wide and 20- $\mu\text{m}$ -long wire structure with probe separations varying between 0.2 and 10  $\mu\text{m}$ .

i.e.,  $l_e$ ,  $l_\phi$ , and  $L$ , are comparable to each other at low temperatures. We will compare the rms amplitude and the correlation field of the fluctuations with expressions developed for the diffusive UCF theory. In this way we obtain a clear indication as to what extent conductance fluctuations measured in this quasiballistic transport regime in a four-probe geometry deviate from diffusive UCF theory. Both the influence of the measuring probes and the role of the elastic mean free path will be discussed. Some results concerning the magnetotransport in 10- $\mu\text{m}$ -long wire segments have been reported in a previous paper.<sup>26</sup> Our results are particularly relevant for future Si-based nanoelectronics. In this field the impact of reduced device dimensions and fabrication technology on electron transport is an important issue.

This paper is organized as follows. In Sec. II we will describe the Si/SiGe heterostructure, the sample layout, and other experimental details. Results of measurements of the magnetic field, probe distance, and temperature dependence of the fluctuations will be presented in Sec. III. The results are discussed in Sec. IV and concluding remarks are made in Sec. V.

## II. EXPERIMENTAL

In Fig. 1 a schematic layout of the devices investigated in this paper is shown. The nominal wire width is 400 nm. The separation between the voltage probes is 0.2, 0.5, 1.0, 2, and 10  $\mu\text{m}$ . This device is made in an *n*-type Si/SiGe heterostructure grown on a Si substrate by molecular beam epitaxy (MBE). The essential part of the heterostructure consists of a 600-nm-thick  $\text{Si}_{0.72}\text{Ge}_{0.28}$  buffer layer on top of which a 25-nm Si layer is grown followed by a 12-nm-thick  $\text{Si}_{0.72}\text{Ge}_{0.28}$  spacer layer. Electrons are provided for by a 15-nm-thick  $3 \times 10^{18} \text{ cm}^{-3}$  Sb-doped supply layer grown on top of the spacer. Details about the Si/SiGe heterostructure material and the fabrication process have been described elsewhere.<sup>26</sup> Resistance measurements are performed in a  $^3\text{He}$ - $^4\text{He}$  dilution refrigerator with magnetic fields, perpendicular to the current, ranging from 0 T to 10 T. The temperature is varied between 0.065 and 4.2 K. The resistance  $R$  of a segment between two adjacent voltage probes is determined by passing an ac current  $I = 1 \text{ nA}$  between probes 5 and 12 and measuring the voltage  $V$  with a standard lock-in technique at 15 Hz.

## III. RESULTS

### A. Transport parameters

To deduce the transport parameters of the material, we investigated the 10- $\mu\text{m}$  segment rather extensively by

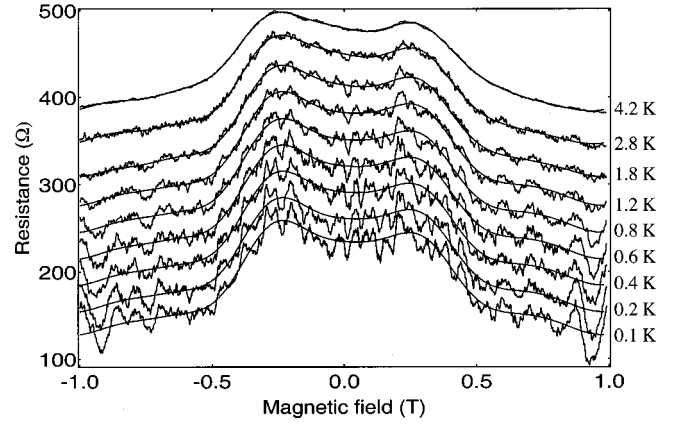


FIG. 2. Resistance curves of the  $L = 200 \text{ nm}$  segment as a function of temperature. For clarity the curves have been given an offset of 25  $\Omega$ . The continuous curves represent the background resistance  $R^{\text{bg}}$ . See text.

Shubnikov–de Haas (SdH), quantum-Hall and weak localization (WL) measurements. These experiments have been described elsewhere and we restrict ourselves here to a summary of the results.<sup>26</sup> We deduced an elastic mean free path  $l_e = 0.8 \pm 0.1 \mu\text{m}$ , a carrier density  $n_s = 7.2 \times 10^{15} \text{ m}^{-2}$ , a diffusion constant  $D = 0.037 \text{ m}^2 \text{ s}^{-1}$ , and a phase coherence length  $l_\phi = 1.5 \pm 0.3 \mu\text{m}$  at  $T = 0.1 \text{ K}$  and an effective wire width  $W_e = 0.225 \pm 0.03 \mu\text{m}$ . The effective wire width  $W_e$  differs by 175 nm from the nominal width  $W = 400 \text{ nm}$ . This difference is caused by a nonconducting depletion region at the boundaries of the wire. Consequently the effective probe separation  $L$  will be larger than lithographically defined. We accounted for this in calculations by adding 175 nm to the nominal probe separation values.

### B. Resistance fluctuations

Figure 2 shows typical magnetoresistance curves of the  $L = 200 \text{ nm}$  segment in the field range between  $-1$  and  $1 \text{ T}$  at temperatures varying from 0.1 to 4.2 K. Clear, aperiodic, and reproducible magnetoresistance fluctuations are observed with an amplitude smoothly decreasing with rising temperature. At fields above 1 T most of the fluctuations are obscured by the Shubnikov–de Haas oscillations. To analyze the fluctuation pattern  $\Delta R(B)$  first a background resistance  $R^{\text{bg}}(B)$  is subtracted from the experimental curves:  $\Delta R(B) = R(B) - R^{\text{bg}}(B)$ . At temperatures  $T \geq 1.8 \text{ K}$  the background resistance is obtained by smoothing the measured data. Below  $T = 1.8 \text{ K}$  we determined  $R^{\text{bg}}(B)$  by adding a magnetic-field-independent offset to the smoothed 1.8 K curve.<sup>27</sup> The solid lines in Fig. 2 represent  $R^{\text{bg}}(B)$  at the various temperatures. As mentioned in the Introduction the voltage probes have an important influence on the behavior of the resistance fluctuations.<sup>1</sup> It has been shown both theoretically and experimentally that the effect of the probes on the resistance fluctuations can be expressed most clearly by separating  $\Delta R(B)$  in a symmetric part  $\Delta R^s(B) = [\Delta R(B) + \Delta R(-B)]/2$  and an antisymmetric part  $\Delta R^a(B) = [\Delta R(B) - \Delta R(-B)]/2$ .

Results for  $\Delta R^s(B)$  and  $\Delta R^a(B)$  at  $T = 0.2 \text{ K}$  are shown in Fig. 3 for  $L = 0.2, 0.5, 1.0,$  and  $2.0 \mu\text{m}$ . The curves have

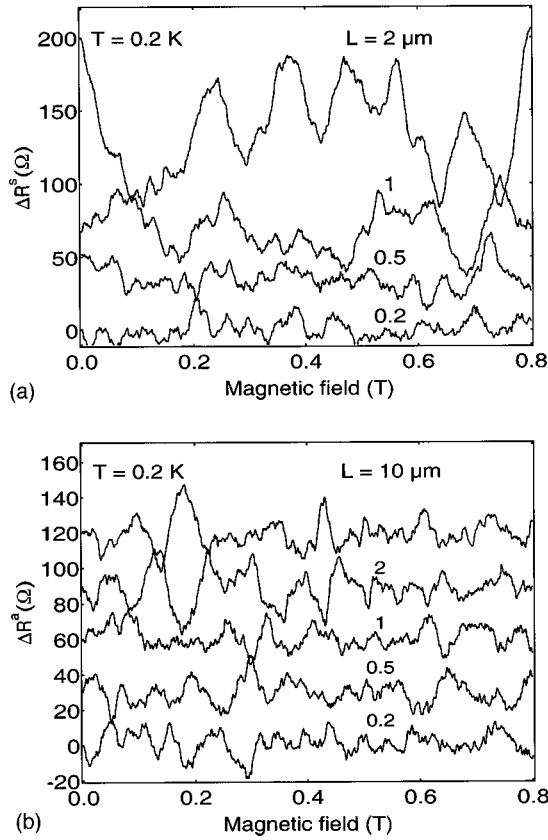


FIG. 3. (a) Symmetric part  $\Delta R^s(B)$  and (b) antisymmetric part  $\Delta R^a(B)$  of the resistance fluctuations vs magnetic field at  $T = 0.2$  K. The curves have been given an offset for clarity.

been given an offset for clarity. In Fig. 3(a) the symmetric part is shown. The amplitude of  $\Delta R^s(B)$  decreases with decreasing probe separation  $L$  in the regime  $L \geq 1 \mu\text{m}$ . At the lowest probe separations,  $L = 0.2 \mu\text{m}$  and  $L = 0.5 \mu\text{m}$ , however, the amplitude becomes almost independent on  $L$ . Figure 3(a) also shows that the typical difference in magnetic field between the maxima of the fluctuations is larger for the 1 and 2  $\mu\text{m}$  segments than for the  $L < 1 \mu\text{m}$  parts. The antisymmetric part of the resistance fluctuations  $\Delta R^a(B)$  is displayed in Fig. 3(b). In contrast to the data of Fig. 3(a) both the amplitude and typical field scale of the fluctuations are almost independent on probe separation  $L$ .

To quantify the observations we determine from the traces of Fig. 3 the rms amplitude of the fluctuations  $\delta R^i$  and the correlation field  $B_c^i$ , where  $i$  refers to symmetric ( $s$ ) or antisymmetric ( $a$ ). This is the field scale over which the fluctuations are correlated. Both  $\delta R^i$  and  $B_c^i$  are calculated from the field-shifted auto correlation function  $F^i(\Delta B)$

$$F^i(\Delta B) = \int \Delta R^i(B + \Delta B) \Delta R^i(B) dB \quad (1)$$

from which  $\delta R^i = \sqrt{F^i(0)}$  and  $B_c^i$  via  $F^i(B_c^i) = \frac{1}{2} F^i(0)$  follow. Because of the presence of the SdH oscillations at fields above  $B = 0.7$  T and the possible influence of the WL effect around  $B = 0$  T we have restricted the range of the analysis to 0.1–0.65 T. This restriction implies that we avoid effects due

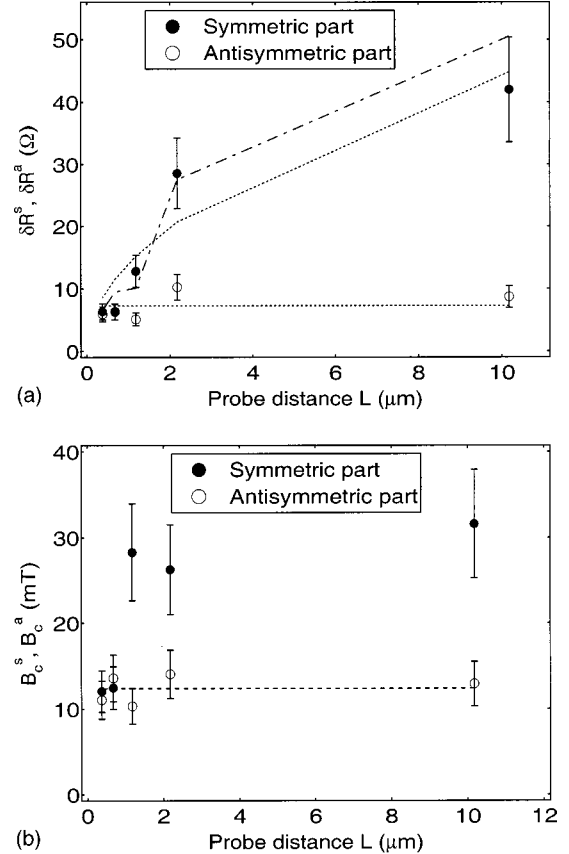


FIG. 4. Characteristics of the symmetric part and antisymmetric part of the resistance fluctuations vs probe separation  $L$ . (a) Amplitudes  $\delta R^s$  and  $\delta R^a$ , with the dashed line representing the theoretical description by Eq. (3) as is explained in the text. (b) Corresponding correlation fields  $B_c^s$  and  $B_c^a$ . The dashed line is a guide to the eye. The measurement temperature  $T = 0.2$  K.

to a magnetic field dependence of the amplitude and correlation field of the fluctuations and do not have to consider electric transport via edge states. Such effects are expected when  $B$  becomes larger than 0.9 T.<sup>11</sup> The resulting dependence of the amplitude and the correlation field on probe separation  $L$  at  $T = 0.2$  K are shown in Fig. 4 (symbols). Figure 4(a) shows  $\delta R^s$  and  $\delta R^a$  vs probe separation  $L$ . It is evident that  $\delta R^s$  increases with  $L$  when  $L \geq 1 \mu\text{m}$  while  $\delta R^a$  is mainly independent of  $L$ . At the lowest probe separations  $L = 0.2 \mu\text{m}$  and  $L = 0.5 \mu\text{m}$ , however, both  $\delta R^s$  and  $\delta R^a$  are independent of  $L$  and, furthermore, have the same magnitude. The various lines in Fig. 4(a) refer to model descriptions to be discussed in Sec. IV.

The correlation fields  $B_c^s$  and  $B_c^a$  as a function of probe separation  $L$  are shown in Fig. 4(b). The difference in behavior between the symmetric part and the antisymmetric part is striking. The correlation field for the antisymmetric part of the oscillations is almost independent of probe separation and has a size  $B_c^a \approx 12 \pm 3$  mT. Remarkably, the correlation field for the symmetric part of the fluctuations seems to increase strongly from 12 to 30 mT as  $L$  becomes larger than 1  $\mu\text{m}$ .

The temperature dependence of the fluctuations is shown in Fig. 5(a) where the amplitude of both the symmetric and the antisymmetric part of the  $L = 0.5 \mu\text{m}$  and  $L = 1 \mu\text{m}$  segments of the fluctuations are displayed on a log-log scale.

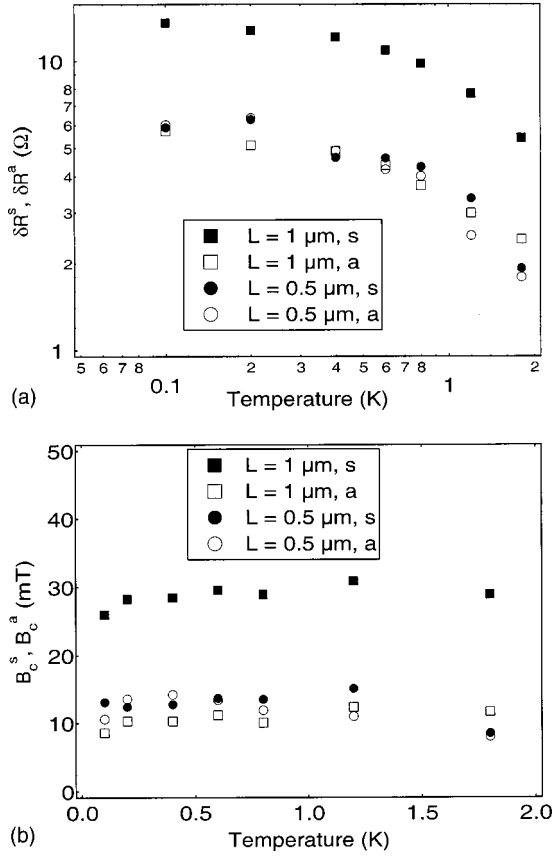


FIG. 5. Characteristics of the symmetric part and antisymmetric part of the resistance fluctuations of the  $L=0.5 \mu\text{m}$  and  $1 \mu\text{m}$  segments vs temperature. (a) rms amplitude, (b) corresponding correlation fields.

Above  $T=0.6 \text{ K}$  the amplitude of the fluctuations strongly decreases with temperature. Below  $T=0.6 \text{ K}$  the fluctuation amplitude has a weaker dependence on temperature and tends to saturate. The same overall temperature dependence of the fluctuations is observed in the other wire segments. The decrease of the fluctuation amplitude at temperatures above  $T=0.6 \text{ K}$  can approximately be expressed as a power law dependence  $\delta R^i (i=s \text{ or } a) \propto T^p$ . The temperature dependence of the different wire segments shows neither a typical dependence of  $p$  on probe separation nor a systematic difference in the temperature dependence of  $\delta R^s$  and  $\delta R^a$ . All measurements can be described by  $p = -0.75 \pm 0.2$ . Therefore the general dependence of  $\delta R^s$  and  $\delta R^a$  on probe separation  $L$  at any temperature  $T$  is similar to the behavior at  $T=0.2 \text{ K}$ , which is displayed in Fig. 4(a).

Figure 5(b) displays the dependence of the correlation field on the temperature for the  $L=0.5 \mu\text{m}$  and  $L=1 \mu\text{m}$  segment. The correlation fields  $B_c^s$  and  $B_c^a$  of the symmetric and antisymmetric part of the fluctuations are both weakly dependent on temperature as can be concluded from Fig. 5(b). We note that the uncertainty in the evaluation of the background resistance  $R^{\text{bg}}$  has its influence on the data of Fig. 5(b). However, after varying  $R^{\text{bg}}$  within the uncertainty interval associated with  $R^{\text{bg}}$ , we still find that the correlation field of the resistance fluctuations is only weakly temperature dependent.

## IV. DISCUSSION

### A. Amplitude of the fluctuations

In this section we discuss the dependence of  $\delta R^s$  and  $\delta R^a$  on probe distance and temperature in relation to expressions derived for four-probe measurements in the diffusive transport regime. We start with the behavior of the amplitude of the fluctuations at  $T=0.2 \text{ K}$ . Theoretical and experimental work on resistance fluctuations in diffusive wires shows that the amplitude of the resistance fluctuations in the wire measured in four-probe geometry can be described as<sup>1,3-9</sup>

$$\delta R^s = a \delta R_\phi \sqrt{L/l_\phi} \quad \text{if } L > l_\phi, \quad (2a)$$

$$\delta R^s = b \delta R_\phi \quad \text{if } L < l_\phi, \quad (2b)$$

and

$$\delta R^a = b \delta R_\phi. \quad (3)$$

In a simple picture the numerical factors  $a, b$  are constants of order 1 and  $\delta R_\phi$  is the fluctuation amplitude of a phase coherent segment of length  $l_\phi$ .

Relations (2) and (3) reflect the different origins of  $\delta R^s$  and  $\delta R^a$ . The symmetric part of the fluctuations is accumulated in the whole wire region of length  $L$  between the voltage probes and up to a length  $l_\phi$  into the voltage probes themselves. In contrast, the antisymmetric part only stems from regions of size  $l_\phi$  around and into the voltage probes<sup>1</sup> and therefore is independent of  $L$ . In short wires ( $L < l_\phi$ )  $\delta R^s$  and  $\delta R^a$  are generated in the same region and consequently they have the same magnitude, see Eqs. (2b) and (3). In long wires ( $L > l_\phi$ ) the contribution of the probe regions is small compared to the region of length  $L$  between the probes. The factor  $(L/l_\phi)^{1/2}$  accounts for statistical averaging over the  $(L/l_\phi)$  phase coherent segments in this area.

The dashed lines in Fig. 4(a) are calculations according to Eqs. (2) and (3). Qualitatively, the symmetric part of the fluctuations follows Eq. (2) quite well. However, the quantitative agreement with Eq. (2) is somewhat less. This can be partly due to the observed variations in the sheet resistance of the various wire segments. To account for this we express  $\delta R^a$  in  $\delta R^s$  using Eqs. (2a) and (3):  $\delta R^s \propto \delta R^a (L/l_\phi)^{1/2}$ . This partly eliminates local variations in material properties since for a certain segment both  $\delta R^s$  and  $\delta R^a$  are affected equally via  $\delta R_\phi$ . The results are displayed in Fig. 4(a) (dash-dotted curve) and show a substantial improvement in agreement. The antisymmetric part is described reasonably well by Eq. (3) as  $\delta R^a$  is almost independent on  $L$ .

The temperature dependence of the amplitude of the fluctuations (Fig. 5) revealed two regions. At temperatures below  $T=0.6 \text{ K}$  the amplitude tended to saturate. At  $T > 0.6 \text{ K}$  the amplitude decreased for all wire segments. The decrease could be described by a power law dependence  $\delta R^i (i=s, a) \propto T^{-p}$  with  $p = 0.75 \pm 0.2$ . According to Eqs. (2) and (3) the temperature dependence of  $\delta R^s$  and  $\delta R^a$  is determined by  $\delta R_\phi$  and  $l_\phi$ .  $\delta R_\phi$  depends on two length scales, the thermal diffusion length  $l_T = \sqrt{\hbar D/k_B T}$  and the phase coherence length  $l_\phi$ , and can be expressed as



$$\delta R_\phi = \frac{e^2}{h} R_\phi^2 \propto l_\phi^2 \quad \text{if } l_\phi \ll l_T, \quad (4a)$$

or

$$\delta R_\phi = \frac{e^2}{h} R_\phi^2 \frac{l_T}{l_\phi} \propto l_\phi l_T \quad \text{if } l_\phi \gg l_T, \quad (4b)$$

where  $R_\phi$  is the resistance of a wire segment of length  $l_\phi$  ( $R_\phi \propto l_\phi$ ) whose conductance fluctuates with a universal amplitude  $e^2/h$ . In the regime  $l_\phi \gg l_T$  the amplitude  $\delta R_\phi$  is reduced by a factor  $l_T/l_\phi$  by thermal averaging.<sup>28</sup> At  $T=0.2$  K we have  $l_T=1.2$   $\mu\text{m}$  and  $l_\phi=1.3$   $\mu\text{m}$ . Therefore our wire segments operate in a regime where  $l_T \approx l_\phi$ .

The analysis of the weak localization peak of the  $L=10$   $\mu\text{m}$  segment, reported in Ref. 26, showed a saturation of  $l_\phi$  below  $T \approx 0.6$  K. The observed saturation of the amplitude of the resistance fluctuations corresponds to the saturation of  $l_\phi$  if we assume that also for  $l_T \approx l_\phi$ ,  $\delta R_\phi$  is correctly described by Eq. (4a). A saturation of the phase coherence length at low temperatures has been observed by many workers in various systems. At present there is a renewed interest in the saturation of  $l_\phi$  since Mohanty *et al.*<sup>29</sup> claim that the saturation of  $l_\phi$  is a fundamental phenomenon caused by zero-point fluctuations of the phase coherent electrons. Theoretical work that supported this claim was heavily criticized by a number of workers. See, for instance Ref. 30 and references therein. We found, using the expressions given by Mohanty *et al.*<sup>29</sup> to calculate the phase coherence length at  $T=0$  K, a value that is a factor of 2 to 3 lower than experimentally observed.

Above  $T=0.5$  K the temperature dependence of  $l_\phi$  in the  $L=10$   $\mu\text{m}$  segment<sup>26</sup> is described by  $l_\phi \propto T^{0.4 \pm 0.1}$ . Given the error margins in  $l_\phi$  both Eqs. (4a) and (4b) predict almost the same temperature dependence for  $\delta R_\phi$ , i.e.,  $\delta R_\phi \propto T^{-0.8 \pm 0.2}$ . This means that in long wires  $L > l_\phi$  [Eq. (2a)],  $\delta R^s$  is expected to vary  $\propto T^{-0.6 \pm 0.15}$ . The temperature dependence of  $\delta R^s$  in short wires  $L < l_\phi$  and  $\delta R^a$  [Eqs. (2b) and (3)] is identical to the temperature dependence of  $\delta R_\phi$ . The calculated temperature dependence corresponds well to the experimental results. So we find that the probe distance and temperature dependence of the amplitude of both the symmetric part and the antisymmetric part of the resistance fluctuations in these short quasiballistic wires are rather well described by expressions derived for the diffusive transport regime.

### B. Correlation field of the fluctuations

In this section we discuss the behavior of the correlation field  $B_c$  with probe separation and temperature [Figs. 4(b) and 5(b)]. First we analyze the dependence of the correlation field with probe separation. Figure 4(b) shows that the critical field of the symmetric part of the fluctuations decreased from  $30 \pm 6$  mT to  $12 \pm 3$  mT as the probe distance became smaller than 1  $\mu\text{m}$ . The correlation field of the antisymmetric part was (also)  $12 \pm 3$  mT and depended within experimental error not on the probe distance.

To interpret these results qualitatively we note that, roughly speaking, the correlation field corresponds to a change of the magnetic flux through an area enclosed by two interfering electron trajectories of length  $l_\phi$  by one flux

quantum.<sup>1</sup> For instance, for wide samples this enclosed area is of the order  $l_\phi^2$  and for long, narrow lines of the order  $l_\phi W_e$ . Geometric considerations show that, for a given  $l_\phi$ , in the neighborhood of the voltage probes of a narrow line, interfering electron trajectories can be envisioned that enclose a larger area than interfering electron trajectories in the narrow regions far away from the probes. Since a larger enclosed area requires a smaller change in magnetic field to change the enclosed flux by one flux quantum, it seems obvious that the correlation field of the asymmetric part of the fluctuations is smaller than the correlation field of the symmetric part of long lines. When the probe distance is decreased the contribution from the probe areas will become more dominant and, by the same token, it is understandable that the observed  $B_c$  will become smaller. The transition will occur when the probe distance becomes of the order  $l_\phi$ . This was confirmed by our experiments: from weak localization experiments on the  $L=10$   $\mu\text{m}$  segment at  $T=0.2$  K we found  $l_\phi=1.3$   $\mu\text{m}$  and we observed the transition near a probe distance of 1  $\mu\text{m}$ . Following the geometric line of reasoning we expect that the transition will be gradual and that the change in  $B_c$  will be relatively small. This is, in fact, confirmed by theoretical work of Chandrasekhar *et al.*<sup>9</sup> who calculated analytically the effect of probes on the fluctuations in narrow lines. These authors present values for  $B_c$  for diffusive transport in short lines with  $l_\phi \ll l_T$ . Their Fig. 8 shows a gradual transition and, for sample parameters not too different from ours, for very short lines ( $L \ll l_\phi$ )  $B_c$  is about 17% smaller than for very long lines ( $L \gg l_\phi$ ). We observed a much larger difference [ $B_c(L > 1 \mu\text{m})/B_c(L < 1 \mu\text{m})=2.5$ ]. In an effort to understand this difference we will focus now on the magnitude of the correlation field of the longer wire segments.

The correlation field of long, narrow lines has been studied theoretically quite extensively [see Beenakker and van Houten<sup>28</sup> (BvH)]. For diffusive transport the correlation field is given by

$$B_c^i = \alpha \frac{h}{e} (W_e l_\phi)^{-1}, \quad (5)$$

where  $\alpha=0.42$  for  $l_\phi \ll l_T$  and  $\alpha=0.95$  for  $l_\phi \gg l_T$ . Evaluation of this expression for our lines at  $T=0.2$  K yielded  $B_c=6.0$  mT and  $B_c=13.5$  mT for  $l_\phi \ll l_T$  and  $l_\phi \gg l_T$ , respectively. Both values are much smaller than the value we observed for the  $L > 1$   $\mu\text{m}$  segments ( $B_c=30 \pm 6$  mT).

BvH (Ref. 28) showed that the correlation field of resistance fluctuations for *quasiballistic* transport will be enhanced (compared to diffusive transport) by the flux cancellation effect: zero flux is enclosed by electron trajectories that return to their initial position by scattering on the wire boundaries only. Using the expressions given by BvH for the correlation field of long, narrow lines in the quasiballistic transport regime we calculated  $B_c$  for different ratios of  $l_\phi$  and  $l_T$  using the parameters for our lines at  $T=0.2$  K. We also applied the interpolation formula given in the same paper<sup>28</sup> to calculate  $B_c$  for  $l_\phi \approx l_T$ . The results and the above given estimates for diffusive transport are listed in the first three rows of Table I. A remarkably good agreement is observed between the quasiballistic estimate for  $l_\phi \approx l_T$  and our experimental value. This result can be regarded as evi-

TABLE I. Theoretical values for the correlation field  $B_c^i$ .

Probe distance ( $i$ )	Regime	$F(0)=[\delta R(l_\phi)]^2$	$B_c^i$ , diffusive (mT)	$B_c^i$ , quasiballistic (mT)	$B_c$ , experiment (mT)
$L > l_\phi(s)$	$l_\phi \ll l_T$	$\propto l_\phi^3$	6.0	22	
$L > l_\phi(s)$	$l_\phi \gg l_T$	$\propto l_\phi$	13.5	81	
$L > l_\phi(s)$	$l_\phi \approx l_T$	BvH <sup>a</sup>	8.2 <sup>a</sup>	35 <sup>a</sup>	
$L \geq 1 \mu\text{m}(s)$					$30 \pm 6$
$L < l_\phi(s)$ or (a) <sup>b</sup>	$l_\phi \ll l_T$	$\propto l_\phi^4$	5.0	17	
$L < l_\phi(s)$ or (a) <sup>b</sup>	$l_\phi \gg l_T$	$\propto l_\phi^2$	7.8	33	
$L < 1 \mu\text{m}(s)$ or (a)					$12 \pm 3$

<sup>a</sup>This is the result of a calculation of  $B_c$  from a correlation function  $F(0)$  obtained in Ref. 28 for the intermediate regime  $l_\phi \approx l_T$ . In this calculation the experimental parameters at  $T=0.2$  K are used:  $l_\phi = 1.33 \mu\text{m}$  and  $l_T = 1.15 \mu\text{m}$ .

<sup>b</sup>The results for the antisymmetric part of the fluctuations are independent of probe distance  $L$ .

dence that the electron transport in the longer segments should be considered as quasiballistic instead of diffusive.

Unfortunately, for quasiballistic transport in very short lines ( $L \ll l_\phi$ ) no expressions to describe  $B_c$  are available in the literature. However, there exists a rather simple procedure to calculate  $B_c$  from the behavior of the amplitude of the fluctuations. This procedure involves a method developed by BvH.<sup>28</sup> These authors show that for quasi-one-dimensional wires the field-shifted autocorrelation function  $F^i(\Delta B)$  can be obtained from  $F^i(0)$  by replacing  $l_\phi \equiv l_\phi(0)$  by a magnetic-field-dependent phase coherence length  $l_\phi(\Delta B)$ .<sup>28,31</sup> After substitution of  $l_\phi(\Delta B)$  in  $F^i(0)$ ,  $B_c$  follows from using the definition  $F^i(B_c^i) = F^i(0)/2$ . In fact, this procedure has also been used by Chandrasekhar *et al.*<sup>9</sup> to calculate  $B_c$  for a narrow wire, four-probe geometry. These authors calculated  $F^i(0)$  analytically and used the expression for  $l_\phi(\Delta B)$  for the diffusive transport listed in Ref. 31. Evaluation of their result for  $L \ll l_\phi$  for our wire segments at  $T=0.2$  K yielded the value  $B_c = 5.0$  mT listed in the fifth row of Table I.

We calculated the remaining three values of  $B_c$  for the short wire block listed in rows 5 and 6 of Table I by using Eqs. (3), (4a), and (4b) to obtain  $F^i(0)$  via  $F^i(0) = \{\delta R^i[l_\phi(0)]\}^2$ . The expressions used for  $l_\phi(\Delta B)$  for both quasiballistic and diffusive transport are listed in Ref. 31. In this way we were able to calculate  $B_c$  for the two limiting situations  $l_\phi \ll l_T$  and  $l_\phi \gg l_T$  (see Table I). Since no expression for  $\delta R^i$  was available for  $l_\phi \approx l_T$ , there was no simple method to calculate  $B_c$  for our experiments' most interesting regime. However, if it is assumed that the trend found for long wires would also hold for short wires, one expects to find a value for  $B_c$  for  $l_\phi \approx l_T$  that is in between the values calculated for the  $l_\phi \ll l_T$  and  $l_\phi \gg l_T$  limits. This expectation is not confirmed by the measurements. Instead, the experimental value for the correlation field  $B_c^s = 12 \pm 3$  mT is in between the calculated diffusive values ( $B_c^s = 5-8$  mT) and quasiballistic values ( $B_c^s = 17-33$  mT). This suggests that in short lines the effect of flux cancellation is apparently less effective and that, consequently,  $B_c$  is less enhanced. Intuitively one might indeed expect that in wire segments, which are shorter than the elastic mean free path, i.e.,  $L < l_e$ , flux cancellation cannot play a major role since in these short segments the electrons hardly scatter in the area between the

voltage probes. So the large difference in  $B_c(L > 1 \mu\text{m})$  and  $B_c(L < 1 \mu\text{m})$  is explained by the assumption that on reduction of the probe distance the electron transport changes from a quasiballistic to a diffusive nature.

Since we used rather plausible arguments so far, we might have concluded that we have a reasonable description of the observed values of the correlation field. However, the description became less satisfactory when we focused on the temperature dependence of  $B_c^s$  and  $B_c^a$ . Figure 5(b) shows that the variation in the correlation field of the fluctuations in the temperature regime 0.1–1.2 K is less than about 30% (above 1.2 K the uncertainty in the experimental data becomes too large). From analyzing the WL effect in the  $L = 10 \mu\text{m}$  segment we found that in this temperature range the phase coherence length is reduced from  $1.5 \mu\text{m}$  to  $0.9 \mu\text{m}$ .<sup>26</sup> We estimated that this reduction in  $l_\phi$  would lead to an increase of the correlation field by a factor of 2 in case of diffusive electron transport and by more than a factor of 3 for quasiballistic electron transport. This is clearly not observed. Another point of concern is the temperature dependence of the correlation field of the  $L = 0.5 \mu\text{m}$  and the  $L = 1 \mu\text{m}$  segments. For one of these segments one expects a crossover from the  $L < l_\phi$  regime to the  $L > l_\phi$  regime as the temperature is increased. This would lead to an additional strong change of the correlation field with temperature. Figure 5(b) provides no evidence for such behavior to take place. Unlike the amplitude of the fluctuations, the correlation field was to a large extent independent of temperature.

A surprisingly weak temperature dependence of  $B_c$  has been reported before in the literature. Studies of conductance fluctuations in ballistic Ag point contacts,<sup>15,32</sup> short epitaxial Bi wires,<sup>33</sup> and ballistic cavities<sup>34</sup> show an almost temperature-independent correlation field. In these cases the devices operated in a regime where  $l_\phi \leq l_e$  (or  $l_\phi$  smaller than the size of a cavity). To explain the behavior of  $B_c$  it was argued or speculated that the length scale of the trajectories that determine  $B_c$  is set by  $l_e$  rather than by  $l_\phi$ . For three-dimensional metallic point contacts this idea was supported by theoretical work of Kozub, Caro, and Holweg.<sup>32</sup> In our case  $l_e \approx 0.8 \mu\text{m}$ , a value that is comparable to the value of  $l_\phi$  and the smallest probe distances. The important point is that  $l_e$  is rather independent of temperature in the range where magnetoconductance fluctuation ex-

periments are done. In case  $B_c$  is determined by  $l_e$  it is obvious that we did not observe in one of the smaller segments a crossover from the  $L < l_\phi$  regime to the  $L > l_\phi$  regime as the temperature is increased. We note that for diffusive transport a naive replacement of  $l_\phi$  by  $l_e$  would lead to higher values of  $B_c$  closer to the experimental value. This could have been a reason not to include quasiballistic transport in the discussion. It is clear that a more fundamental theory describing resistance fluctuations in the regime ( $l_e > W_e$ , and comparable values for  $L$ ,  $l_\phi$ ,  $l_e$ , and  $l_T$ ) is needed to interpret our observations, especially for  $B_c$ , in more detail.

## V. CONCLUSIONS

We studied magnetoresistance fluctuations in short quasiballistic wire segments fabricated in high-mobility  $n$ -type Si/SiGe heterostructures. The length  $L$  of the various wire segments were chosen to be larger or smaller than the phase coherence length  $l_\phi$  and electron mean free path  $l_e$ . Furthermore both  $l_e$  and  $l_\phi$  were comparable in magnitude. This regime has not been described yet theoretically and we compared our data with the existing theoretical model suited to

describe resistance fluctuations in diffusive systems. The dependence of the amplitude of the symmetric and the antisymmetric part of the fluctuations with probe separation is qualitatively well described by this model. The magnitude of the experimental values for  $B_c$  however was significantly larger than predicted by diffusive theory. A comparison of the experimental values for  $B_c$  with an expression derived for the quasiballistic regime shows only a good agreement for the long ( $L > 1 \mu\text{m}$ ) segments. The correlation field was found to be largely temperature independent, where a strong increase of the correlation field with temperature is predicted. This calls for a full description of resistance fluctuations in systems with comparable values of  $W$ ,  $L$ ,  $l_e$ ,  $l_\phi$ , and  $l_T$ .

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- <sup>1</sup>S. Washburn and R. A. Webb, Rep. Prog. Phys. **55**, 1311 (1992), and references therein.
- <sup>2</sup>P. A. Lee and A. D. Stone, Phys. Rev. Lett. **55**, 1622 (1985).
- <sup>3</sup>W. J. Skocpol, P. M. Mankiewich, R. E. Howard, L. D. Jackel, and D. M. Tennent, Phys. Rev. Lett. **58**, 2347 (1987).
- <sup>4</sup>A. Benoit, C. P. Umbach, R. B. Laibowitz, and R. A. Webb, Phys. Rev. Lett. **58**, 2343 (1987).
- <sup>5</sup>H. Haucke, S. Washburn, A. D. Benoit, C. P. Umbach, and R. A. Webb, Phys. Rev. B **41**, 12 454 (1990).
- <sup>6</sup>M. Büttiker, Phys. Rev. Lett. **57**, 1761 (1986).
- <sup>7</sup>S. Hershfield and V. Ambegaokar, Phys. Rev. B **38**, 7909 (1988).
- <sup>8</sup>H. U. Baranger, A. D. Stone, and D. P. DiVincenzo, Phys. Rev. B **37**, 6521 (1988).
- <sup>9</sup>V. Chandrasekhar, P. Santhanam, and D. E. Prober, Phys. Rev. B **44**, 11 203 (1991).
- <sup>10</sup>J. P. Bird, A. D. C. Grassie, M. Lakrimi, P. Meeson, K. M. Hutchings, J. J. Harris, and C. T. Foxon, J. Phys.: Condens. Matter **3**, 2897 (1991).
- <sup>11</sup>J. P. Bird, K. Ishibashi, Y. Ochiai, M. Lakrimi, A. D. C. Grassie, K. M. Hutchings, Y. Aoyagi, and T. Sugano, Phys. Rev. B **52**, 1793 (1995).
- <sup>12</sup>T. J. Thornton, M. Pepper, H. Ahmed, G. J. Davies, and D. Andrews, Phys. Rev. B **36**, 4514 (1987).
- <sup>13</sup>T. Onishi, M. Kawabe, K. Ishibashi, J. P. Bird, V. Aoyagi, and T. Sugano, Phys. Rev. B **48**, 12 353 (1993).
- <sup>14</sup>C. M. Marcus, A. J. Rumberg, R. M. Westervelt, P. F. Hopkins, and A. C. Gossard, Phys. Rev. Lett. **69**, 506 (1992).
- <sup>15</sup>P. A. M. Holweg, J. A. Kokkedee, J. Caro, A. H. Verbruggen, S. Radelaar, A. G. M. Jansen, and P. Wyder, Phys. Rev. Lett. **67**, 2549 (1991).
- <sup>16</sup>G. Timp, A. M. Chang, P. M. Mankiewich, R. Behringer, J. E. Cunningham, T. Y. Chang, and R. E. Howard, Phys. Rev. Lett. **59**, 732 (1987).
- <sup>17</sup>K. Ismail, M. Arafat, K. L. Saenger, J. O. Chu, and B. S. Meyerson, Appl. Phys. Lett. **66**, 1077 (1995).
- <sup>18</sup>D. Többen, D. A. Wharam, G. Abstreiter, J. P. Kotthaus, and F. Schäffler, Semicond. Sci. Technol. **10**, 711 (1995).
- <sup>19</sup>W. X. Gao, K. Ismail, K. Y. Lee, J. O. Chu, and S. Washburn, Appl. Phys. Lett. **65**, 3114 (1994).
- <sup>20</sup>D. Többen, D. A. Wharam, G. Abstreiter, J. P. Kotthaus, and F. Schäffler, Phys. Rev. B **52**, 4704 (1995).
- <sup>21</sup>S. J. Koester, K. Ismail, K. Y. Lee, and J. O. Chu, Phys. Rev. B **54**, 10 604 (1996).
- <sup>22</sup>R. S. Prasad, T. J. Thornton, A. Matsumura, J. M. Fernandez, and D. Williams, Semicond. Sci. Technol. **10**, 1084 (1995).
- <sup>23</sup>D. Többen, F. Schäffler, A. Zrenner, and G. Abstreiter, Phys. Rev. B **46**, 4344 (1992).
- <sup>24</sup>G. Stöger, G. Brunthaler, G. Bauer, K. Ismail, B. S. Meyerson, J. Lutz, and F. Kuchar, Semicond. Sci. Technol. **9**, 765 (1994).
- <sup>25</sup>F. Schäffler, Semicond. Sci. Technol. **12**, 1515 (1997).
- <sup>26</sup>R. G. van Veen, A. H. Verbruggen, E. van der Drift, F. Schäffler, and S. Radelaar, Semicond. Sci. Technol. **14**, 508 (1999).
- <sup>27</sup>This procedure is different than the one described in Ref. 26. Especially for the shorter wire segments of lengths 0.2–2  $\mu\text{m}$  the procedure used in the present paper is found to be more appropriate. We note that a different choice of the background resistance changes the results at the lowest experimental temperatures by about 20%.
- <sup>28</sup>C. W. J. Beenakker and H. van Houten, Phys. Rev. B **37**, 6544 (1988).
- <sup>29</sup>P. Mohanty, E. M. Q. Jariwala, and R. A. Webb, Phys. Rev. Lett. **78**, 3366 (1997).
- <sup>30</sup>Y. Imry, H. Fukuyama, and P. Schwab, Europhys. Lett. **47**, 608 (1999).
- <sup>31</sup>The field-dependent phase coherence length  $l_\phi(\Delta B)$  can be written as (from Ref. 28)  $l_\phi(\Delta B)^{-2} = l_\phi^{-2} + (D\tau_{\Delta B})^{-1}$ , where the magnetic relaxation time  $\tau_{\Delta B}$  is given by  $\tau_{\Delta B} = 12[\hbar/e\Delta B]^2(DW_e^2)^{-1}$  (diffusive regime) and  $\tau_{\Delta B} = 4C_1[\hbar/e\Delta B]^2(v_F W_e^3)^{-1} + 2C_2[\hbar/e\Delta B](l_e/v_F W_e^2)$  (quasiballistic regime),  $C_1 = 9.5$  and  $C_2 = 4.8$  if the boundary scatter-

- ing is specular, and  $C_1=4\pi$  and  $C_2=3$  if the boundary scattering is diffusive.
- <sup>32</sup>V. I. Kozub, J. Caro, and P. A. M. Holweg, *Phys. Rev. B* **50**, 15 126 (1994).
- <sup>33</sup>J. A. van Hulst, Ph.D. thesis, Delft University of Technology, 1995, Chap. 5. Available on request.
- <sup>34</sup>J. P. Bird, K. Ishibashi, D. K. Ferry, Y. Ochiai, Y. Aoyagi, and T. Sugano, *Phys. Rev. B* **52**, 8295 (1995).