Coulomb drag between parallel ballistic quantum wires

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The Coulomb drag between parallel, *ballistic* quantum wires is studied theoretically in the presence of a perpendicular magnetic field B. The transresistance R_D shows peaks as a function of the Fermi level and splitting energy between the one-dimensional subbands of the wires. The sharpest peaks appear when the Fermi level crosses the subband extrema so that the Fermi momenta are small. Two other kinds of peaks appear when either *intra*- or *inter*-subband transitions of electrons have maximum probability; the *intra*-subband transitions correspond to a small splitting energy. R_D depends on the field B in a nonmonotonic fashion: it decreases with B, as a result of the suppression of backscattering, and increases sharply when the Fermi level approaches the subband bottoms and the suppression is outbalanced by the increase of the Coulomb matrix elements and of the density of states.

I. INTRODUCTION

Experimentally[1] and theoretically[2] momentum transfer between spatially separated electron layers or Coulomb drag has been studied mostly between two-dimensional (2D) layers. Theoretically, this drag has also been studied between very long one-dimensional (1D) layers (quantum wires) in which the wire length L is much longer than the mean free path l_i (*diffusive* regime[3]) and recently between 1D layers of length $L \ll l_i$, in which the electron motion along the layer, at low temperatures, is mostly ballistic,[4,5] (*ballistic* regime). Even when most of the electrons pass along the wires without collisions, a few of them experience backscattering due to interaction with the electrons of the other wire and this modifies the time-averaged distribution functions in such a way that the drag effect occurs. In both regimes the drag response is found to be maximal when the subbands in the two wires line up precisely. It is important that in the ballistic regime the transresistance is determined only by the Coulomb interaction between the electrons and such basic properties of the layers as the number of occupied subbands, and does not include the relaxation characteristics of the system such as scattering times. Therefore, the ballistic regime provides the possibility to obtain more direct information about the Coulomb interaction in 1D electron systems.

Motivated by the results of Refs. 4 and 5 we undertook an extended theoretical study of the drag in the ballistic regime, without tunneling between the wires, but in the presence of a perpendicular magnetic field B. In Secs. II and III, we generalize the theory of Ref. 4 to include the effects of intersubband transitions in electron-electron collisions and account for the influence of a magnetic field on the Coulomb drag; a limited account of this influence, valid when only the lowest subbands in the two wires are occupied, appeared in Ref. 5. Concluding remarks follow in Sec. IV.

II. GENERAL FORMALISM

We use a model of a four-terminal double-quantum-wire system, as shown schematically in Fig. 1, similar to the systems investigated in the ''directional coupler'' problem.[6] Two closely spaced quantum wires, numbered 1 and 2 and centered at y_1 and y_2, are contacted independently to four leads at $x=0$ and $x=L$, where L is the length of the wires. The leads have chemical potentials $\mu_{1,2}(0)=\mu_{1,2}^{+}$ and $\mu_{1,2}(L)=\mu_{1,2}^{-}$. Applying a bias $V=(\mu_2^{+}-\mu_2^{-})/e$ to the leads of wire 2 (drive wire) we obtain the current I flowing through it. This current induces a voltage $V_D=(\mu_1^{+}-\mu_1^{-})/e$ in wire 1 (drag wire). This is the typical setup for drag measurements.[1] We assume that the barrier between the wires is high enough to allow the neglect of tunneling.

Below we assume that the electrons in each wire are parabolically confined by the potentials $U_j=\varepsilon_j^0+m^*\Omega_j^2(y-y_j)^2/2$, $j=1,2$. In the presence of a perpendicular magnetic field B, introduced through the vector potential $\mathbf{A}=(-By,0,0)$, the normalized wave functions are $\Psi_{jnk}(x,y)=e^{ikx}\chi_{jnk}(y)$, $\chi_{jnk}(y)=(\pi^{1/2}l_j 2^n n!)^{-1/2}H_n[(y-Y_j)/l_j]\exp[-(y-Y_j)^2/2l_j^2]$, where n is the 1D subband number, k the wave vector of electrons, and $H_n(x)$ the Hermite polynomials. Neglecting spin splitting the corresponding energy spectrum $\varepsilon_{jn}(k)$ reads

$$\varepsilon_{jnk}=\varepsilon_j^0+\hbar\omega_j(n+1/2)+(\hbar^2/2m_j)(k-y_j/l_c^2)^2. \quad (1)$$

Here $\omega_j^2=\omega_c^2+\Omega_j^2$, $\omega_c=eB/m^*$ is the cyclotron frequency, $m_j=m^*\omega_j^2/\Omega_j^2$, $l_c=(\hbar/m^*\omega_c)^{1/2}$ is the magnetic length, $l_j^2=\hbar/m^*\omega_j$, and $Y_j=(\Omega_j^2 y_j+\hbar\omega_c k/m^*)/\omega_j^2$ are the k-dependent centers of the oscillators. The expressions for the kinetic energies $(\hbar^2/2m_j)(k-y_j/l_c^2)^2$ of the electrons can be simplified by a gauge invariant transformation resulting in a shift of k by an arbitrary constant. Since we neglect tunneling, we do not consider electron transitions between the wires and can make such shifts independently for each wire; this does not affect the kinetic equations written below. Explicitly, we will shift the wave vectors in the manner $k-y_1/l_c^2\to k$ for wire 1 and $k-y_2/l_c^2\to k$ for wire 2. Then the

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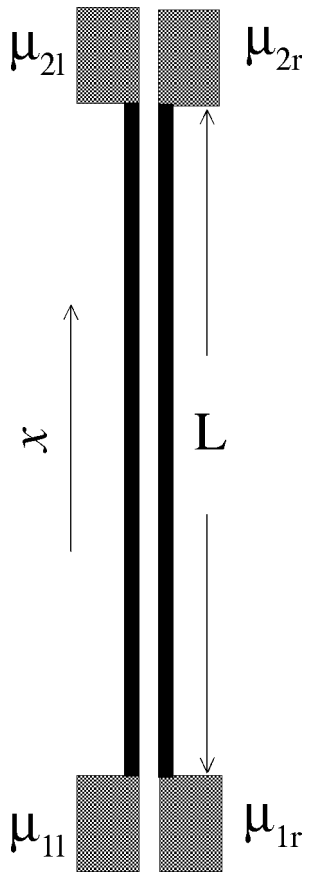

FIG. 1. Schematic diagram of a coupled-quantum-wire device.

kinetic energies in Eq. (1) will read $\hbar^2k^2/2m_j$ and the oscillator centers $Y_j=y_j+(\hbar\omega_c/m^*\omega_j^2)k$.

If the distribution functions $f_{jk}(x)\equiv f_{jk}$ change over distances much longer than both the electronic wavelength π/k and the characteristic radius of the interaction potential, we can write the Boltzmann kinetic equations as

$$\frac{\hbar k}{m_j}\frac{\partial f_{jnk}(x)}{\partial x}=-\frac{4\pi}{\hbar}\sum_{j'k'q}\sum_{nn'n_1n_1'}|M^{jj'j'j}_{n_1n_1'n'n}(k,k',q)|^2\times\delta(\varepsilon_{jnk}+\varepsilon_{j'n'k'}-\varepsilon_{jn_1,k-q}-\varepsilon_{j'n_1',k'+q})\times[f_{jnk}(1-f_{jn_1,k-q})f_{j'n'k'}(1-f_{j'n_1',k'+q})-f_{jn_1,k-q}(1-f_{jnk})f_{j'n_1',k'+q}(1-f_{j'n'k'})], \tag{2}$$

where the collision integral accounts only for electron-electron scattering. The Coulomb matrix elements $M^{jj'j'j}_{n_1n_1'n'n}(k,k',q)$ are given by

$$M^{jj'j'j}_{n_1n_1'n'n}(k,k',q)=\frac{2e^2}{\kappa}\int dy\int dy'K_0(|q||y-y'|)\times\chi_{jnk}(y)\chi_{j'n'k'}(y')\times\chi_{j'n_1'k'+q}(y')\chi_{jn_1k-q}(y), \tag{3}$$

where κ is the dielectric constant and K_0 the modified Bessel function.

It is convenient to write separately the distribution functions for the forward- and backward-moving electrons as $f^+_{j|k|}=f_{jk}|_{k>0}$ and $f^-_{j|k|}=f_{jk}|_{k<0}$, respectively. For these functions the boundary conditions are given in the Landauer-Buttiker approach by $f^+_{jnk}(0)=f(\varepsilon_{jnk}-\delta\mu_j^+)$ and $f^-_{jnk}(L)=f(\varepsilon_{jnk}-\delta\mu_j^-)$, where $\delta\mu_j^\pm=\mu_j^\pm-\mu,\mu$ is the equilibrium chemical potential, $f(\varepsilon)=[e^{(\varepsilon-\mu)/k_BT}+1]^{-1}$, and T is the temperature. For $j=1$ and $j=2$, Eq. (2) gives two coupled kinetic equations whose solution allows us to express the unknown potentials μ_1^- and μ_1^+ through the fixed μ_2^- and μ_2^+ values and thereby calculate the transresistance.

III. RESULTS

If most of the electrons move through the wires ballistically, Eq. (2) can be solved by simple iterations. The zero-order approximation gives $f^+_{jnk}(x)=f(\varepsilon_{jnk}-\delta\mu_j^+)$ and $f^-_{jnk}(x)=f(\varepsilon_{jnk}-\delta\mu_j^-)$. Substitution of these functions in the collision integral gives a nonzero contribution for back-scattering collisions between the electrons of different wires. This is the main contribution which will be considered in detail in the following. If more than a single subband in a wire is occupied, the intersubband transitions within one wire also contribute to the collision integral of Eq. (2) (if only the lowest subband is occupied, the intralayer part of the collision integral completely vanishes because of the relation $q=k-k'$ following from the momentum and energy conservation rules). However, within the iterative approach used here, we can neglect the influence of the intralayer collisions on the distribution functions of the drive layer ($j=2$), since the transport regime is nearly ballistic. Further, the intralayer collisions do not modify considerably the distribution functions of the drag layer ($j=1$) because $\mu_1^+-\mu_1^-$ is assumed to be much smaller than $\mu_2^+-\mu_2^-$, and the main effect on $f^\pm_{1nk}(x)$ results from the interlayer Coulomb interaction. Considering only contributions linear in V, we substitute the equilibrium Fermi-Dirac functions $f^\pm_{1nk}(x)=f(\varepsilon_{1nk})$ in the collision integral and obtain

$$f^+_{1nk}(x)=f(\varepsilon_{1nk}-\delta\mu_1^+)-eV(m_1/\hbar k)\lambda_n(k)x, \tag{4}$$

$$f^-_{1nk}(x)=f(\varepsilon_{1nk}-\delta\mu_1^-)+eV(m_1/\hbar k)\lambda_n(-k)(x-L), \tag{5}$$

where the factor

$$\lambda_n(k)=\frac{4\pi}{\hbar k_BT}\sum_{n_1n'n_1'}\sum_{k'q}\{[|M^{1221}_{n_1n_1'n'n}(k,k',q)|^2\times\delta(\varepsilon_{1nk}+\varepsilon_{2n'k'}-\varepsilon_{1n_1,k-q}-\varepsilon_{2n_1',k'+q})\times f(\varepsilon_{1nk})[1-f(\varepsilon_{1n_1k-q})]\times f(\varepsilon_{2n'k'})[1-f(\varepsilon_{2n_1'k'+q})]]_{k'>0,k'+q<0}-[\ldots]_{k'<0,k'+q>0}\} \tag{6}$$

is determined by the Coulomb matrix elements and the equilibrium distribution functions only. The current flowing in the drag wire is given by

$$I_D=\frac{e}{\pi}\sum_n \int_0^\infty dk(\hbar k/m_1)[f^+_{1nk}(x)-f^-_{1nk}(x)]. \quad (7)$$

I_D does not depend on x due to the property $\Sigma_n\int_{-\infty}^{\infty}\lambda_n(k)dk=0$, which follows from detailed balance. Substituting Eqs. (4) and (5) into Eq. (7), using the requirement $I_D=0$, and defining the transresistance R_D as $R_D=-V_D/I$ through the ballistic current $I=V/R_N$, where $R_N=h/2e^2N$ is the ballistic resistance of wire 2 and N the number of occupied subbands, we finally obtain

$$\begin{aligned}R_D=&\frac{\hbar L}{NN_De^2k_BT}\sum_{n,n_1=0}^{N_D-1}\sum_{n',n_1'=0}^{N-1}\int_{-\infty}^{0}dk'\int_{-k'}^{\infty}dq\int_0^q dk\\&\times|M^{1221}_{n_1n_1'n'n}(k,k',q)|^2\\&\times\delta(\varepsilon_{1nk}+\varepsilon_{2n'k'}-\varepsilon_{1n_1,k-q}-\varepsilon_{2n_1',k'+q})\\&\times f(\varepsilon_{1nk})[1-f(\varepsilon_{1n_1k-q})]f(\varepsilon_{2n'k'})[1-f(\varepsilon_{2n_1'k'+q})].\end{aligned} \quad (8)$$

Here N_D is the number of occupied subbands in the drag wire (wire 1). Note that the introduction of both N and N_D assumes that the theory is valid when the Fermi energy $\mu-\varepsilon_j^0-\hbar\omega_j(n+1/2)$ with respect to the highest occupied level is larger than k_BT. This, of course, implies that the 1D subband separations $\hbar\omega_1$ and $\hbar\omega_2$ are much larger than k_BT and is true at $T\sim 1$ K for electrostatically defined electron channels.

Below we consider the case of identical wires $\Omega_1=\Omega_2=\Omega$, which entails $\omega_1=\omega_2=\omega$, $l_1=l_2=l$, and $m_1=m_2$. To further evaluate expression (8), it is convenient to detach the contribution $R_D^{(1)}$ from R_D that expresses the equality $n+n'=n_1+n_1'$ for which the energy conservation law gives $q=k-k'$. Then we have $R_D=R_D^{(1)}+R_D^{(2)}$ with

$$\begin{aligned}R_D^{(1)}=&\frac{m^{*3}k_BTL\omega^6}{NN_D\hbar^5e^2\Omega^6}\sum_{n,n_1=0}^{N_D-1}\sum_{n',n_1'=0}^{N-1}\frac{\delta_{n+n',n_1+n_1'}}{k_nk_{n'}(k_n+k_{n'})}\\&\times\frac{\Delta^2_{n,n_1'}}{\sinh^2\Delta_{n,n_1'}}|M^{1221}_{n_1n_1'n'n}(k_n,-k_{n'},k_n+k_{n'})|^2.\end{aligned} \quad (9)$$

Here $\Delta_{n,n_1'}=[\Delta+\hbar\omega(n-n_1')]/2k_BT$ and $\Delta=\varepsilon_1^0-\varepsilon_2^0$ is the interwire splitting energy between the lowest subbands. Further, $k_n=(\omega/\Omega)\{2m^*[\mu-\varepsilon_1^0-\hbar\omega(n+1/2)]/\hbar^2\}^{1/2}$ and $k_{n'}=(\omega/\Omega)\{2m^*[\mu-\varepsilon_2^0-\hbar\omega(n'+1/2)]/\hbar^2\}^{1/2}$ are the Fermi wave numbers for the states 1,n and 2,n', respectively. The part $R_D^{(2)}$, corresponding to $n+n'\neq n_1+n_1'$, is obtained as

$$\begin{aligned}R_D^{(2)}=&\frac{m^*L\omega^2}{2\hbar NN_De^2k_BT\Omega^2}\sum_{n,n_1=0}^{N_D-1}\sum_{n',n_1'=0}^{N-1}\int_0^\infty dk\int_0^\infty dk'\\&\times(1-\delta_{n+n',n_1+n_1'})/p(k,k')\\&\times\Theta(kk'+\omega^2(n+n'-n_1-n_1')/\Omega^2l^2)\\&\times|M^{1221}_{n_1n_1'n'n}(k,-k',q)|^2f(\varepsilon_{1nk})f(\varepsilon_{2n'k'})\\&\times[1-f((\varepsilon_{1nk}+\varepsilon_{2n'k'})/2+\Delta(k,k')/2)]\\&\times[1-f((\varepsilon_{1nk}+\varepsilon_{2n'k'})/2-\Delta(k,k')/2)],\end{aligned} \quad (10)$$

where $p(k,k')=[(k+k')^2/4+\omega^2(n+n'-n_1-n_1')/\Omega^2l^2]^{1/2}$, $q=(k+k')/2+p(k,k')$, and $\Delta(k,k')=\Delta+\hbar\omega(n_1-n_1')-(\Omega/\omega)^2\hbar^2p(k,k')(k-k')/m^*$. The statistical factor in Eq. (10) is small unless $|\varepsilon_{1nk}-\mu|,|\varepsilon_{2n'k'}-\mu|$, and $|\Delta(k,k')|$ are small enough and comparable to k_BT. This allows the integrals over k and k' to be carried out in narrow regions around k_n and $k_{n'}$, respectively. We used the same property to reduce the contribution $R_D^{(1)}$ to expression (9). Although the requirement $|\Delta(k,k')|\sim k_BT$ imposes certain restrictions on the values of μ, Δ, and ω, the processes with $n+n'\neq n_1+n_1'$ can give a considerable contribution to R_D, especially for $|\Delta+\hbar\omega(n-n_1')|\gg k_BT$ and $R_D^{(1)}$ small. We stress that the previous theoretical work[4] on the Coulomb drag in the ballistic regime took into account only the processes with $n=n_1$ and $n'=n_1'$, thus neglecting other processes completely from the beginning. The numerical calculations given below demonstrate that this limitation is considerable in many cases.

If only the lowest subbands are occupied in each wire, i.e., for $n=n_1=n'=n_1'=0$, the calculation of the transresistance is considerably simplified. Only $R_D^{(1)}$ contributes to R_D and Eq. (9) can be rewritten as

$$\begin{aligned}R_D=&\frac{2e^2m^{*3}\omega^6Lk_BT}{\pi\hbar^5\kappa^2\Omega^6k_1k_{1'}(k_1+k_{1'})}\frac{(\Delta/2k_BT)^2}{\sinh^2(\Delta/2k_BT)}\\&\times e^{-(\omega_c/\omega)^2l^2(k_1+k_{1'})^2}\\&\times\left(\int_{-\infty}^{\infty}du\,e^{-u^2/2}K_0[(k_1+k_{1'})|d+lu|]\right)^2.\end{aligned} \quad (11)$$

Expression (11) is convenient for assessing the magnetic-field dependence of the transresistance R_D. It directly demonstrates a significant reduction[5] of the drag effect by the magnetic field B, mostly due to the exponential factor. The decrease of R_D starts as $R_D(B)-R_D(0)\sim -B^2$ and becomes exponential with increasing B. The physical reason for this decrease is the suppression of backscattering in electron-electron collisions as the oscillator centers for forward- and backward-moving electrons are pulled apart by the magnetic field. The characteristic field B_0 for this suppression depends on the position of the Fermi level ε_F and is estimated as $B_0\sim(m^*/e)(m^*\Omega^3/\hbar)^{1/2}/(k_1+k_{1'})$. If ε_F is not far from the subband bottoms, B_0 is big and the suppression is weak. If ε_F is well in between the 1D subbands, B_0 is estimated as 1 T for typical wire parameters. However, when ε_F, with the increase of B, approaches the subband bottom, the opposite effect takes place: the transresistance increases because the wave vectors k_1, $k_{1'}$, and q become progressively smaller

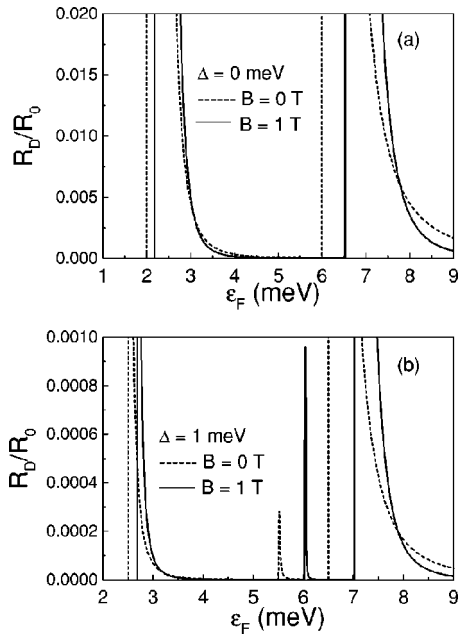

FIG. 2. Dependence of the transresistance R_D on the position of the Fermi level for (a) aligned, $\Delta=0$, and (b) shifted, $\Delta=1$ meV, levels in quantum wires. The dashed and solid curves correspond to $B=0$ and $B=1$ T, respectively. The other parameters are listed in the text.

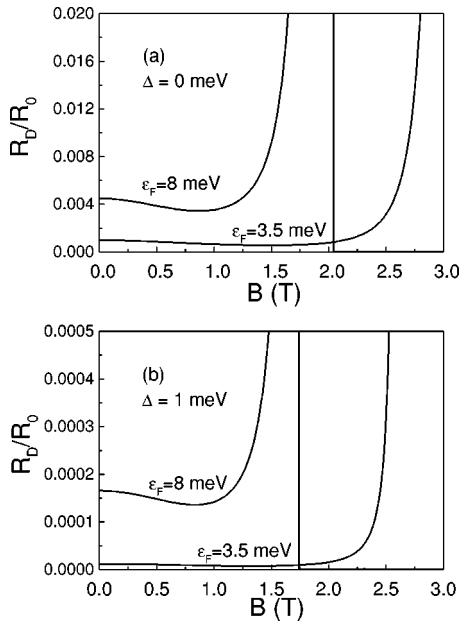

FIG. 3. Dependence of R_D on the magnetic field B, with one ($\varepsilon_F=3.5$ meV) and two ($\varepsilon_F=8$ meV) populated 1D subbands, at $\Delta=0$ (a) and $\Delta=1$ meV (b).

and the suppression of backscattering becomes less important than the increase of the Coulomb matrix element and of the density of states.

Below we present numerical results for the transresistance R_D, expressed in units of the fundamental resistance $R_0=h/2e^2$, at $T=1.3$ K, $L=0.4$ μm, $d=|y_1-y_2|=50$ nm, $\hbar\Omega=4$ meV, $m^*=0.067m_0$, and $\kappa=13$. Figure 2 shows the dependence of R_D on the Fermi energy ε_F, defined as $\varepsilon_F=\mu-(\varepsilon_1^0+\varepsilon_2^0)/2$, calculated at $B=0$ and $B=1$ T. The calculations were done assuming that up to two subbands can be populated in each wire. As seen in part (a), for $\Delta=0$ there are pronounced sharp peaks of R_D when ε_F crosses the bottoms of the first and second subbands. The sharpness of the peaks is explained by a strong enhancement of the Coulomb collision probability when k_n, $k_{n'}$, and q are small, cf. Eqs. (9) and (10). In Fig. 2(b), for $\Delta=1$ meV, one can see three peaks; the middle one appears after ε_F crosses the bottom of the second subband in wire 2. This peak exists due to the processes with $n+n'\neq n_1+n_1'$. The third, most prominent peak in Fig. 2(b), appears after ε_F crosses the bottom of the second subband in wire 1, so that two subbands in both wires are populated. The processes with $n+n'\neq n_1+n_1'$ give the main contribution to this peak as well.

The application of the magnetic field shifts the peaks to higher Fermi energies, due to the increased confinement energy, and sharpens them due to the suppression of backscattering in the regions far from the subband edges. The resulting decrease of the transresistance due to this suppression is illustrated in Fig. 3 for one ($\varepsilon_F=3.5$ meV) and two ($\varepsilon_F=8$ meV) subbands populated. These dependences are nonmonotonic: when, with the increase of B, ε_F approaches the first or second subband bottom, R_D starts to grow sharply. This dependence is essentially the same for both $\Delta=0$ and $\Delta=1$ meV.

Figure 4 shows the dependence of R_D on the level splitting Δ at several constant values of $\mu-\varepsilon_1^0=\varepsilon_F-\Delta/2$. This means that the subband positions of wire 1 remain constant with respect to the Fermi level but those of wire 2 do not; this can be experimentally achieved, for example, by changing the voltage of the gate adjacent to wire 2 while keeping the gate adjacent to wire 1 at a constant voltage. The curves are plotted for one (a) or two (b) subbands populated in wire 1 but for different μ, far from (solid) and close to (dashed)

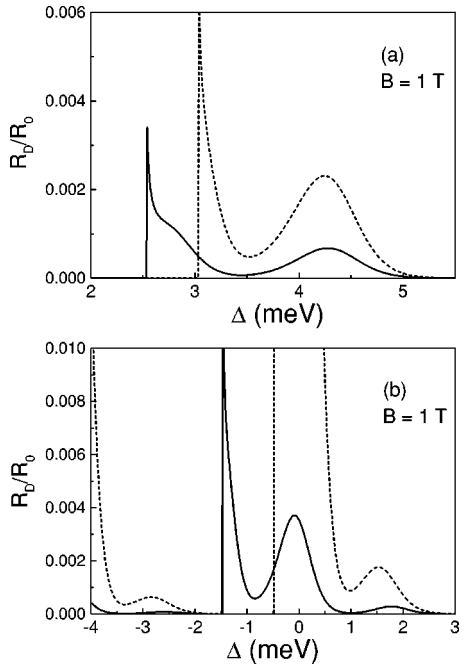

FIG. 4. Dependence of R_D on the level splitting energy Δ at $B=1$ T when one (a) or two (b) populated subbands of wire 1 remain constant with respect to the Fermi level, $\varepsilon_F-\Delta/2=$const. (a) $\varepsilon_F-\Delta/2=3.5$ meV (dashed) and 4 meV (solid). (b) $\varepsilon_F-\Delta/2=7$ meV (dashed) and 8 meV (solid).

the upper populated subband edge. Both curves of Fig. 4(a) show two peaks: the sharp ones appear when the second subband of wire 2 becomes populated while the broad ones appear when the second subband of wire 2 is aligned with the first one of wire 1, at $\Delta=\hbar\omega\simeq 4.36$ meV. A similar behavior is seen in Fig. 4(b). At large negative Δ only one subband is populated in wire 2 while at $\Delta\simeq -1.5$ (solid) and -0.5 meV (dashed) the second subband of wire 2 becomes populated as well. This transition is reflected by strong and sharp peaks in R_D. Other strong peaks appear at $\Delta=0$, when the subbands are aligned; note that on the dashed curve such a peak merges with that at $\Delta\simeq -0.5$ meV and is not resolved. The minor peaks in the regions of negative and positive Δ exist due to the intersubband transitions with $(n,n_1,n',n_1')=(0,1,0,0)$ and $(1,0,0,0)$ and $(n,n_1,n',n_1')=(0,1,1,1)$ and $(1,0,1,1)$, respectively. The maxima of these peaks occur when $\Delta(k,k')\approx\Delta(k_n,k_{n'})$, cf. Eq. (10), goes to zero. Thus, the level-splitting dependence of R_D shows a rich structure of peaks indicating that both *intra-* and *inter-*subband transitions of electrons contribute to R_D.

All calculations described in this section were repeated for different values of the interwire separation d. An increase of d considerably decreases the transresistance: R_D drops by more than one order of magnitude as d varies from 40 to 60 nm, mainly due to the dependence of the Bessel function on its argument. However, all qualitative features presented above are preserved.

IV. REMARKS AND CONCLUSIONS

The treatment of the drag effect in the ballistic transport regime demonstrates the salient properties of electron-electron collisions in double-layer quasi-1D electron systems. The reduced dimensionality dramatically decreases the scattering probabilities at low temperatures due to the restrictions imposed by the momentum and energy conservation laws. As a result, the transresistance shows peaks as a function of either the Fermi level position or the interlayer level-splitting energy. The peaks always appear when the Fermi level crosses the bottom of a subband, so that a new subband n is involved in the scattering prosess; the Fermi wave number k_n for this subband is small, the density of states is high, and this results in a higher scattering probability. When subband n is aligned to another one, the conservation rules allow electron transitions inside the subband n, the corresponding momentum transfer $\hbar q\simeq 2\hbar k_n$ is small, and the Coulomb matrix element is large, thus giving rise to an additional increase of the peak. Next, the peaks appear when two subbands from different layers are aligned; this favors transitions that conserve the sum of the subband numbers, $n+n'=n_1+n_1'$, especially the transitions between the electrons inside the aligned subbands [cf. Eq. (9)]. Finally, peaks occur under special conditions, for $\Delta(k_n,k_{n'})\simeq 0$ [cf. Eq. (10)]; this implies a maximum probability for intersubband transitions with $n+n'\neq n_1+n_1'$. Although the peaks associated with these transitions are usually weaker than those under subband alignment, they give a considerable contribution which cannot be neglected. The described rich structure of the peaks is best seen in the level-splitting dependence of the transresistance shown in Fig. 4.

A magnetic field B applied perpendicular to the wire plane reduces the overlap between the wave functions for forward- and backward-moving electrons and thereby tends to suppress electron-electron scattering. This results in a decrease of the transresistance. In addition, the application of B modifies the quantization energies and leads to a shift of the subbands with respect to the Fermi level. Since the scattering probability increases when a subband edge comes close to the Fermi level, the transresistance R_D may increase with the increase of B. Therefore, the dependence of R_D on B is basically nonmonotonic as shown in Fig. 3.

The results obtained here are valid when the 1D electron gas in either wire is described as a normal Fermi liquid. We used this model because the wires are short, the transport is nearly ballistic, and the properties of the 1D electrons are determined by those of the 2D reservoirs they are injected from. The case of the Coulomb drag between 1D electron systems described as Luttinger liquids has been studied in Ref. 7.

Concerning experimental results we are aware only of those of Ref. 8 where the transresistance R_D was measured as a function of side gate voltages controlling the confining potentials of the parallel, submicron-long quantum wires, thus allowing change in the positions of the 1D subbands with respect to the Fermi level, the interlayer subband splitting Δ, the wire widths W_j, and the interwire distance d. It was found that R_D shows sharp peaks when the Fermi level crosses the bottom of a 1D subband. When the gate adjacent to the drag wire was kept at a constant voltage, correponding to one populated subband in it, the transresistance, as a function of the voltage of the gate adjacent to the drive wire, showed two peaks. The shape and position of these peaks permit us to identify them with those of Fig. 4(a), since the situation described by Fig. 4(a) corresponds roughly to this type of measurement. These experimental results provide qualitative support for our theoretical predictions. However, as no formal connection is made in our model between the gate voltages and the parameters ε_F, Δ, d, and W_j, we cannot attempt a more detailed comparison. Such a connection requires a detailed knowledge of the gate-induced modification of the double-wire confining potential which could be obtained only by a self-consistent solution of the electrostatic problem for the three-gate structure investigated in Ref. 8. We expect though that further experimental and theoretical work will test sufficiently the drag in the ballistic regime and our results.

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