ARTICLES

Weak-localization effect on thermomagnetic phenomena

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The quantum transport equation (QTE) is extended to study weak localization (WL) effects on galvanomagnetic and thermomagnetic phenomena. QTE has many advantages over the linear response method (LRM): (i) particle-hole asymmetry, which is necessary for the Hall effect is taken into account by the nonequilibrium distribution function, while the LRM requires expansion near the Fermi surface, (ii) when calculating response to the temperature gradient, the problem of WL correction to the heat current operator is avoided, (iii) the magnetic field is directly introduced to the QTE, while the LRM deals with the vector potential and special attention should be paid to maintain gauge invariance, e.g., when calculating the Nernst-Ettingshausen effect the heat current operator should be modified to include the external magnetic field. We reproduce in a very compact form known results for the conductivity, the Hall and the thermoelectric effects and then we study our main problem, WL correction to the Nernst-Ettingshausen coefficient (transverse thermopower). We show that in a quasi-two-dimensional film the Nernst-Ettingshausen coefficient has a large logarithmic factor similar to that of the conductivity and the Hall conductivity, while the thermoelectric coefficient does not have such a factor.

I. INTRODUCTION

In many complex cases the quantum transport equation (QTE) turns out to be physically clear and more convenient than the linear response method (LRM). Calculating manybody corrections to the electrical conductivity choice of the QTE or the LRM is a matter of taste, the LRM requires many diagrams to be considered, while the QTE deals only with the electron self-energy diagrams but includes specific terms like Poisson brackets corrections. The QTE shows special advantages when thermoelectric and galvanomagnetic effects are considered.

1. Calculating the thermoelectric coefficient by the QTE as a response of the electron system to the temperature gradient, one avoids difficulty associated with corrections to the heat current operator due to the electron-electron and electron-phonon interactions.^{1,2}

2. Calculating the Hall coefficient by the QTE the electron-hole asymmetry is accounted automatically by the nonequilibrium distribution function of noninteracting electrons, while the LRM requires all electron parameters to be expanded near the Fermi surface.

3. The QTE incorporates real electric and magnetic fields in the gradient terms and also through the terms in the form of the Poisson brackets. Contrary to that, the LRM deals with two vector potentials for the electric and magnetic fields and special attention is required to obtain the Hall component of the electric current. When calculating the Nernst-Ettingshausen coefficient the gauge invariance requires the vector potential corresponding to the magnetic field to be included in the heat current operator (we will discuss this subject in detail in the Appendix).

The QTE was initially developed for the interaction effects on the conductivity in the diffusion regime³ and was later extended to the weakly disordered regime for the conductivity,⁴ thermoelectric power,^{1,2} and the Hall effect.⁵ In the present paper we first derive the QTE, which incorporates electric and magnetic fields and the temperature gradient on equal footing. Then we obtain the WL corrections to the conductivity, the Hall conductivity, which has been calculated earlier by the LRM in Refs. 6 and 7 (see also Refs. 8 and 9). Then we consider the thermoelectric coefficient, which has also been considered earlier by LRM.¹⁰ Finally we find a WL correction to the Nernst-Ettingshausen coefficient, which to our knowledge has not been studied before, and for the reasons mentioned above, it is a very difficult problem to study by the LRM.

II. QUANTUM TRANSPORT EQUATION

The quantum transport equation method is based on the Keldysh diagrammatic technique, where transport phenomena are described by the 2×2 matrix electron Green function \hat{G} , as well as the matrix electron self-energy $\hat{\Sigma}$,

$$\hat{G} = \begin{pmatrix} 0 & G^A \\ G^R & G^C \end{pmatrix}, \quad \hat{\Sigma} = \begin{pmatrix} \Sigma^C & \Sigma^R \\ \Sigma^A & 0 \end{pmatrix}, \quad (1)$$

where A and R stand for advanced and retarded components of the matrix function and C corresponds to the kinetic component.

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The electric current is expressed by the kinetic Green's function G^{C} .

$$\mathbf{J} = \int \frac{d^4 P}{(2\pi)^4} e \mathbf{v} \operatorname{Im} \Delta G^C(P), \qquad (2)$$

where $P = (\mathbf{p}, \boldsymbol{\epsilon})$ and ΔG^C is the nonequilibrium correction to G^{C} , which will be calculated in each particular case of external disturbance.

Assuming that weak localization corrections are small, we will calculate the kinetic electron Green function by iteration. Without weak localization effects, the retarded (advanced) Green function is

$$G^{R}(P) = [G^{A}(P)]^{*} = \frac{1}{\epsilon - \xi_{p} + i/2\tau}, \quad \xi_{p} = \frac{p^{2} - p_{F}^{2}}{2m}, \quad (3)$$

where τ is the elastic scattering time due to electron-impurity scattering.

The normalization condition for the matrix Green function in the coordinate representation

$$\int dY \hat{G}(X_2, Y) \hat{G}(Y, X_1) = \hat{1}$$
 (4)

[X is a four-dimensional coordinate (\mathbf{r}, t)], results in the following form of the Keldysh component

$$G^{C}(X_{2}, X_{1}) = \int dY [S(X_{2}, Y)G^{A}(Y, X_{1}) - G^{R}(X_{2}, Y)S(Y, X_{1})],$$
(5)

where the function S plays the role of the electron distribution function.

In equilibrium $S = S_0 = -\tanh(\epsilon/2T)$. In the presence of the electric and weak magnetic fields (the quantization of the electron levels is neglected) S is determined from the following transport equation:

$$e(\mathbf{v} \cdot \mathbf{E}) \frac{\partial S}{\partial \epsilon} - (\mathbf{v} \cdot \nabla T) \frac{\epsilon}{T} \frac{\partial S}{\partial \epsilon} + \frac{e}{c} (\mathbf{v} \times \mathbf{H}) \frac{\partial S}{\partial \mathbf{p}} = I_{e-imp}, \quad (6)$$

where I_{e-imp} is the collision integrals corresponding to the electron-impurity interaction in the lowest order (without WL corrections) and is chosen in the simplest form

$$I_{e-imp} = \frac{2}{\pi \nu \tau} \int \frac{d\mathbf{k}}{(2\pi)^3} [S(\mathbf{k}, \boldsymbol{\epsilon}) - S(\mathbf{p}, \boldsymbol{\epsilon})] \text{Im} G_0^A(\mathbf{k}, \boldsymbol{\epsilon})$$
$$= \frac{S_0(\boldsymbol{\epsilon}) - S(\boldsymbol{\epsilon})}{\tau}. \tag{7}$$

Performing the Fourier transformation of Eq. (5) from the coordinate representation to the momentum-energy representation we get

$$G^{C}(P) = S(P)[G^{A}(P) - G^{R}(P)] + \delta G^{C}(P), \qquad (8)$$

where δG^{C} is the correction in the form of the Poisson bracket

$$\delta G^{C} = \frac{i}{2} \{ S(P), G^{A}(P) + G^{R}(P) \}.$$
(9)



FIG. 1. Diagrams of the kinetic electron self-energy Σ^{C} contributing to the WL corrections to conductivity.

Poisson bracket corrections arise when integrals in the coordinate representation [like Eq. (5)] transform to the momentum-energy representation,

$$\int dX \ A(X_1, X)B(X, X_2) \Rightarrow A(P)B(P) + \frac{i}{2} \{A(P), B(P)\},$$
(10)

where

$$\{A,B\} = \left(\frac{\partial' A}{\partial \epsilon} \frac{\partial' B}{\partial t} - \frac{\partial' B}{\partial \epsilon} \frac{\partial' A}{\partial t}\right) - \left(\frac{\partial' A}{\partial \mathbf{p}} \frac{\partial' B}{\partial \mathbf{r}} - \frac{\partial' B}{\partial \mathbf{p}} \frac{\partial' A}{\partial \mathbf{r}}\right).$$
(11)

The potentials **A** and Φ enter as

$$\frac{\partial'}{\partial t} = \frac{\partial}{\partial t} - \frac{e}{c} \frac{\partial \mathbf{A}}{\partial t} \frac{\partial}{\partial \mathbf{p}} - e \frac{\partial \Phi}{\partial t} \frac{\partial}{\partial \epsilon},$$
$$\frac{\partial'}{\partial \mathbf{r}} = \frac{\partial}{\partial \mathbf{r}} - \frac{e}{c} \frac{\partial \mathbf{A}}{\partial r_i} \frac{\partial}{\partial p_i} - e \frac{\partial \Phi}{\partial \mathbf{r}} \frac{\partial}{\partial \epsilon}.$$
(12)

Therefore Poisson brackets due to the electric and magnetic fields, and also due to the temperature gradient, are

$$\{A,B\}_E = e \mathbf{E} \left(\frac{\partial A}{\partial \epsilon} \frac{\partial B}{\partial \mathbf{p}} - \frac{\partial B}{\partial \epsilon} \frac{\partial A}{\partial \mathbf{p}} \right), \tag{13}$$

$$\{A,B\}_{H} = \frac{e}{c} \mathbf{H} \cdot \left(\frac{\partial A}{\partial \mathbf{p}} \times \frac{\partial B}{\partial \mathbf{p}}\right), \tag{14}$$

$$\{A,B\}_T = \nabla T \left(\frac{\partial A}{\partial T} \frac{\partial B}{\partial \mathbf{p}} - \frac{\partial B}{\partial T} \frac{\partial A}{\partial \mathbf{p}} \right).$$
(15)

Our goal is to find the WL correction ΔG^C and calculate the electric current [Eq. (2)]. The kinetic Green function G^{C} may be found by two methods. First, G^C may be expressed through the nonequilibrium electron distribution function [see Eq. (5)], which in turn is determined from the transport equation.¹⁻³ Second, the transport equation may be written directly for G^{C} . Both methods are equivalent, we chose the second method as more convenient for the current problem.

Note, that a retarded (advanced) component of the electron self-energy consists of integrals of retarded (advanced) Green functions, therefore, in the main approximation it is equal to zero. Thus, the WL correction to G^C is determined only by the kinetic component Σ^{C} . In the coordinate representation the transport equation for G^C is

$$\Delta G^{C}(X,X') = \int dY dZ G^{R}(X,Y) \Sigma^{C}(Y,Z) G^{A}(Z,X').$$
(16)

For the problem of WL the self-energy diagrams Σ are shown in Fig. 1. To solve Eq. (16) we use the iteration pro-

cedure. As in the case of the QTE for the nonequilibrium electron distribution function,^{1–3} field terms and scattering terms in Eq. (16) cannot be separated. In the present formalism, field terms accounting for the electric (magnetic) field and temperature gradient appear due to zero order (without WL corrections), nonequilibrium distribution functions obtained from Eq. (6), and due to Poisson brackets [Eqs. (13)-(15)] after transformation of Eq. (16) to the energymomentum representation. Further for each particular external perturbation the procedure described above will be presented in detail.

III. CONDUCTIVITY

For the conductivity we retain in Eq. (6) only the term corresponding to the electric field. The solution of Eq. (6) with the electron-impurity collision integral, Eq. (7), in the energy-momentum representation is given by

$$S = S_0 + \phi_E, \quad \phi_E(P) = -e \tau(\mathbf{v} \cdot \mathbf{E}) \frac{\partial S_0(\boldsymbol{\epsilon})}{\partial \boldsymbol{\epsilon}}. \tag{17}$$

The Poisson bracket term for G^C [Eq. (9)] is

$$\delta_E G^C = \frac{i}{2} \{ S_0(\boldsymbol{\epsilon}), G^A(\boldsymbol{P}) + G^R(\boldsymbol{P}) \}_E.$$
(18)

Now we take into account WL corrections for G^C , using Eq. (16). This equation in the momentum representation with all terms in the form of the Poisson brackets is

$$\Delta G^{C} = G^{R} \Sigma^{C}(\phi_{E}) G^{A} + G^{R} \Sigma^{C}(\delta_{E} G^{C}) G^{A} + G^{R} \delta_{E} \Sigma^{C}(S_{0}) G^{A} + \frac{i}{2} \{ G^{R}, \Sigma^{C}(S_{0}) \}_{E} G^{A} + \frac{i}{2} G^{R} \{ \Sigma^{C}(S_{0}), G^{A} \}_{E}.$$
(19)

The first three terms take into account the nonequilibrium corrections due to ϕ_E , δG^C , and internal Poisson brackets in the electron self-energy Σ^{C} . The last two terms are the Poisson brackets between the self-energy and external Green functions. Keeping the linear in the electric field terms, one should calculate the Poisson bracket terms with the equilibrium electron self-energy. But as we have already mentioned, $\Sigma^{A}=0$, and therefore $\Sigma^{C}(S_{0})=2iS_{0}(\epsilon)\operatorname{Im}\Sigma^{A}=0$. So the last two terms are equal to zero, and we need to take into account only nonequilibrium corrections to the electron self-energy.

Let us consider diagrams shown on Fig. 1. We denote corresponding self-energies as Σ_1 , Σ_2 , and Σ_3 . The nonequilibrium corrections to the first diagram are

$$\Sigma_{1}^{C}(\phi_{E}) = \phi_{E}(Q-P)C(Q)[G^{A}(Q-P) - G^{R}(Q-P)],$$

$$\Sigma_{1}^{C}(\delta_{E}G^{C}) = C(Q)\delta_{E}G^{C}(Q-P), \qquad (20)$$

where for brevity we dropped the integral $\int dQ/(2\pi)^4$, Q $=(\mathbf{q},\omega)$, the Cooperon C(Q) is

$$C(Q) = \frac{1}{\pi\nu\tau^2} \frac{1}{-i\omega + 1/\tau_{\phi} + Dq^2},$$
 (21)

where τ_{ϕ} is the phase relaxation time. The first correction in Eq. (19), $\Delta_1 G^C$ due to $\Sigma_1(\phi_E)$, is given by

$$\Delta_1 G^C(P) = G^R(P) C(Q) \phi_E(P) [G^A(Q-P) - G^R(Q-P)] G^A(P).$$
(22)

Substituting $\Delta_1 G^C(P)$ into Eq. (2) we find corresponding correction to the conductivity. In the quasi-two-dimensional case $d \ll L_{\phi}$, where d is the thickness of the film and L_{ϕ} $=(D\tau_{\phi})^{1/2^{\tau}}$ is the localization length, and we have

$$\Delta_1 \sigma_{xx} = -\frac{2e^2 D}{\pi d} \int \frac{d^2 q}{(2\pi)^2} \frac{1}{1/\tau_{\phi} + Dq^2} = -\frac{e^2}{\pi^2 d} \ln\left(\frac{L_{\phi}}{l}\right),\tag{23}$$

where $l = v_F \tau$ is the mean free path.

The second correction in Eq. (19), $\Delta_2 G^C$ due to $\Sigma_1(\delta G^C)$ [Eq. (20)], is

$$\Delta_2 G^C(P) = G^R(P) C(Q)(i/2) \{ S_0(\epsilon), G^A(Q-P) + G^R(Q-P) \}_E G^A(P),$$
(24)

which results in the correction to the conductivity $\Delta_2 \sigma_{xx}$ $=(1/2)\Delta_1\sigma_{xx}$.

Now we consider diagrams Σ_2 and Σ_3 . These diagrams contain G^{C} inside the electron-impurity ladder. For this reason, nonequilibrium corrections due to the distribution function ϕ_E and due to the Poisson bracket $\delta_E G^C$ give zero after the angular integration over the electron momentum.

The next correction $\Delta_3 G^C$ comes from the Poisson brackets inside the self-energy diagrams. The Poisson bracket should include the electron distribution function S_0 in G^C . As the total contribution of all three equilibrium diagrams is zero, and in all diagrams G^{C} has the same position with respect to the Cooperon, the Poisson brackets between $S_0(\epsilon)$ and Green's functions inside the Cooperon are equal to zero. The only nonzero term in $\Delta_3 G^C$ appears when the Poisson brackets are taken between $S_0(\epsilon)$ and the electron Green functions $G^{A,R}(Q-P)$ (external with respect to the Cooperon) in the second and third diagrams,

$$\Delta_{3}G^{C} = G^{R}(P) \left[-\frac{i}{2}C(Q) \{ S_{0}(\epsilon), G^{A}(Q-P) + G^{R}(Q-P) \}_{E} \right] G^{A}(P), \qquad (25)$$

which results in $\Delta_3 \sigma_{xx} = -(1/2)\Delta_1 \sigma_{xx}$. Thus, the second and third corrections to the conductivity originating from the Poisson brackets mutually cancel out and $\Delta_1 \sigma_{xx}$ describes the full WL correction to the conductivity that coincides with Ref. 9. However, we will show that the Poisson brackets corrections are very important for the Hall and Nernst-Ettingshausen effects.

IV. HALL EFFECT

Calculating the Hall conductivity we assume that the magnetic field is directed along the z axis and the electric field is directed along the x axis. The Hall current J_{y} is proportional to $\mathbf{E} \times \mathbf{H}$.

Without WL effects the electron distribution function has the form

$$S = S_0 + \phi_E + \phi_H, \qquad (26)$$

where the nonequilibrium distribution function ϕ_E is given by Eq. (17), and ϕ_H is

$$\phi_{H}(\mathbf{p},\boldsymbol{\epsilon}) = -\tau \frac{e}{c} (\mathbf{v} \times \mathbf{H}) \frac{\partial (S_{0} + \phi_{E})}{\partial \mathbf{p}} = -\frac{e^{2} \tau^{2}}{cm} \mathbf{v} \cdot (\mathbf{E} \times \mathbf{H}) \frac{\partial S_{0}}{\partial \boldsymbol{\epsilon}}.$$
(27)

The Hall conductivity of noninteracting electrons is given by

$$\sigma_{xy} = \frac{J_y}{E} = 2e \int \frac{d^4P}{(2\pi)^4} v_y \phi_H \operatorname{Im} G^A(P) = \Omega \tau \sigma_{xx}, \quad (28)$$

where $\Omega = eH/mc$ is the cyclotron frequency and σ_{xx} is the Drude conductivity.

It is noteworthy here that the Hall effect is proportional to the particle-hole asymmetry and is taken into account in the nonequilibrium distribution function in Eq. (27) by the factor $\partial^2 \xi_p / \partial^2 p = 1/m$, which is a measure of the average curvature of the Fermi surface.

For electrons in the magnetic field the Poisson bracket corrections to G^C ,

$$\delta_H G^C = \frac{i}{2} \{ \phi_E(P), G^A(P) + G^R(P) \}_H, \qquad (29)$$

are also proportional to the same particle-hole asymmetry factor as $\phi_H(\mathbf{p}, \boldsymbol{\epsilon})$.

To get the WL correction to the Hall conductivity we keep in the momentum representation of Eq. (16) only terms proportional to $\mathbf{E} \times \mathbf{H}$,

$$\Delta G^{C} = G^{R} \Sigma^{C}(\phi_{H}) G^{A} + G^{R} \Sigma^{C}(\delta_{H}G^{C}) G^{A} + G^{R} \delta_{H} \Sigma^{C}(\phi_{E}) G^{A} + G^{R} [\delta_{H} \delta_{E} \Sigma^{C}(S_{0})] G^{A} + \frac{i}{2} \{ G^{R}, \Sigma^{C}(\phi_{E}) \}_{H} G^{A} + \frac{i}{2} G^{R} \{ \Sigma^{C}(\phi_{E}), G^{A} \}_{H},$$

$$(30)$$

where $\Sigma^{C}(\phi_{H})$ stands for the self-energy with nonequilibrium distribution function $\phi_{H}, \Sigma^{C}(\delta_{H}G^{C})$ stands for the selfenergy with the magnetic Poisson bracket correction to G^{C} from Eq. (29), $\delta_{H}\Sigma^{C}(\phi_{E})$ denotes the magnetic Poisson bracket in the self-energy with the nonequilibrium function ϕ^{E} , $\delta_{H}\delta_{E}\Sigma^{C}$ is the correction due to internal (inside Σ^{C}) double Poisson brackets. In the last two terms we calculate the magnetic Poisson bracket between the nonequilibrium self-energy and $G^{R,A}$ functions. As we discussed in the previous section, in equilibrium $\Sigma^{C}=0$, thus external electric Poisson brackets between $\Sigma^{C}(S_{0})$ and G^{R} and G^{A} are equal to zero.

Let us start with the first diagram Σ_1 . The correction $\Delta_1 G^C$ corresponding to the first term in Eq. (30) is

$$\Delta_1 G^C(P) = G^R(P) \Sigma^C(\phi_H) G^A(P)$$

= $G^R(P) C(Q) \phi_H(P)$
× $[G^A(Q-P) - G^R(Q-P)] G^R(P).$ (31)

Calculating the electric current, we find the correction to the Hall conductivity $\Delta_1 \sigma_{xy} = \Omega \tau \delta \sigma_{xx}$.

From the second term in Eq. (30) we have

$$\Delta_2 G^C(P) = G^R(P) \Sigma^C [\delta_H G^C(Q-P)] G^A(P)$$

= $-\frac{i}{2\tau} G^R(P) \phi_H(P) \{ [G^A(Q-P)]^2 + [G^R(Q-P)]^2 \} G^A(P).$ (32)

The corresponding correction to the Hall conductivity is $\Delta_2 \sigma_{xy} = (1/2) \Omega \tau \delta \sigma_{xx}$. Note that, as for the conductivity, the corrections to the Hall conductivity from ϕ_H and $\delta_H G^C$ in Σ_2 and Σ_3 give zero after the angular integration.

Now we calculate the internal Poisson brackets [the third term in Eq. (30)]. First let us consider the magnetic Poisson brackets between ϕ_E and external with respect to the Cooperon Green functions $G^{A,R}(Q-P)$ in the second and the third diagrams. For the Hall conductivity these contributions cancel each other, because the electron momentum in ϕ_E changes its sign in these diagrams. Terms with the magnetic Poisson bracket between ϕ_E and the Cooperon for all three diagrams Σ_1 , Σ_2 , and Σ_3 are proportional to

$$[G^{A}(Q-P)-G^{R}(Q-P)]+G^{A}(Q-P)$$

$$\times \langle G^{A}(P')-G^{R}(P'),G^{A}(Q-P')\rangle + G^{R}(Q-P)$$

$$\times \langle G^{R}(P'),G^{A}(Q-P')-G^{R}(Q-P')\rangle, \qquad (33)$$

where $\langle G_1, G_2 \rangle$ stands for $(\pi \nu \tau)^{-1} \int d\mathbf{p} G_1 G_2$. Performing calculations, we find that these terms cancel each other. In a similar way one can prove that the double Poisson brackets corrections $\delta_H \delta_E \Sigma^C$ also mutually cancel out.

Now we calculate contributions of the fifth and sixth terms in Eq. (30),

$$\Delta_5 G^C + \Delta_6 G^C = \frac{e}{c} \mathbf{H} \cdot \mathbf{v} \times \frac{\partial \Sigma^C}{\partial \mathbf{p}} [(G^A)^2 G^R - G^A (G^R)^2].$$
(34)

Calculations show that only Σ_2 and Σ_3 give nonzero contribution, thus

$$\Delta_{5}G^{C} + \Delta_{6}G^{C}$$

$$= \frac{e}{c} \mathbf{H} \cdot \mathbf{v} \times \frac{\partial}{\partial \mathbf{p}} [\Sigma_{2}^{C}(\phi_{E}) + \Sigma_{3}^{C}(\phi_{E})]$$

$$\times [(G^{A})^{2}G^{R} - G^{A}(G^{R})^{2}]$$

$$= \frac{e}{c} \mathbf{H} \cdot \mathbf{v} \times \mathbf{E}C(Q) \frac{e\tau}{m} \frac{\partial S_{0}}{\partial \epsilon} [G^{A}(Q-P) + G^{R}(Q-P)]$$

$$\times [(G^{A})^{2}G^{R} - G^{A}(G^{R})^{2}]. \qquad (35)$$

The corresponding correction to the Hall conductivity is $\Delta_{5,6}\sigma_{xy} = \sigma_{xy}(1/2)\Omega \tau \delta \sigma_{xx}$.

Finally, the total contribution to the Hall conductivity is

$$\frac{\Delta \sigma_{xy}}{\sigma_{xy}} = 2 \frac{\Delta \sigma_{xx}}{\sigma_{xx}},$$
(36)

which coincides with Refs. 6 and 7.

V. THERMOELECTRIC AND NERNST-ETTINGSHAUSEN EFFECTS

Now we consider the thermoelectric and Nernst-Ettingshausen coefficients. Without the WL correction, the nonequilibrium distribution function in the crossed temperature gradient and the magnetic field is

$$S = S_0 + \phi_T + \phi_N, \qquad (37)$$

$$\phi_T(P) = \tau(\epsilon) (\mathbf{v} \cdot \nabla T) \frac{\epsilon}{T} \frac{\partial S_0(\epsilon)}{\partial \epsilon}, \qquad (38)$$

$$\phi_{N}(\mathbf{p}, \boldsymbol{\epsilon}) = -\tau \frac{e}{c} (\mathbf{v} \times \mathbf{H}) \frac{\partial (S_{0} + \phi_{T})}{\partial \mathbf{p}}$$
$$= \frac{e \tau^{2}(\boldsymbol{\epsilon})}{cm} \mathbf{v} \cdot (\nabla T \times \mathbf{H}) \frac{\boldsymbol{\epsilon}}{T} \frac{\partial S_{0}}{\partial \boldsymbol{\epsilon}}.$$
(39)

The Poisson bracket corrections to G^c are

$$\delta_T G^C = \frac{i}{2} \{ S_0(\boldsymbol{\epsilon}), G^A(\boldsymbol{P}) + G^R(\boldsymbol{P}) \}_T, \qquad (40)$$

$$\delta_N G^C = \frac{i}{2} \{ \phi_T(P), G^A(P) + G^R(P) \}_H.$$
(41)

The thermoelectric coefficient is defined from the equation $J = \eta \nabla T$, therefore substituting ϕ_T from Eq. (38) into Eq. (2) we have the following equation for the thermoelectric coefficient without weak localization effect:

$$\eta_0 = \frac{2e}{3T} \int \frac{d^4P}{(2\pi)^4} v^2 \tau(\epsilon) \epsilon \frac{\partial S_0(\epsilon)}{\partial \epsilon} \operatorname{Im} G^A(P).$$
(42)

To get a nonzero result from Eq. (42) requires the electronhole asymmetry to be taken into account. Therefore we expand all electron quantities near the Fermi surface,

$$v(\epsilon) = v_F \left(1 + \frac{\epsilon}{2\epsilon_F} \right), \quad v(\epsilon) = v_0 \left(1 + \frac{\epsilon}{2\epsilon_F} \right),$$

$$\tau(\epsilon) = \tau_0 \left(1 - \frac{\epsilon}{2\epsilon_F} \right), \quad \frac{1}{\tau_0} = \pi v_0 n_i U^2, \quad v_0 = \frac{mp_F}{\pi^2}, \quad (43)$$

where n_i is the impurity concentration and U is the electronimpurity potential. Finally we have

$$\eta_0 = -\frac{2}{9} e \,\tau_0 p_F T. \tag{44}$$

The weak localization correction to the thermoelectric coefficient is obtained by substituting ϕ_T for ϕ_E and $\{A,B\}_T$ for $\{A,B\}_E$ into the equations of Sec. III. As a result we get

$$\Delta \eta = -\frac{e}{\pi T} \int \frac{d^3 q}{(2\pi)^3} \int d\epsilon \epsilon \frac{\partial S_0(\epsilon)}{\partial \epsilon} \frac{1}{q^2 + L_{\phi}^{-2}(\epsilon)}, \quad (45)$$

where $L_{\phi}^{2}(\epsilon) = D(\epsilon)\tau_{\phi}(\epsilon)$. For a film, which is twodimensional with respect to the WL $(d \ll L_{\phi})$, the momentum integral in Eq. (45) is logarithmically divergent on the upper limit. Taking the cutoff to be $1/l(\epsilon)$, $l(\epsilon) = v(\epsilon)\tau(\epsilon)$, and making the expansion near the Fermi surface, we get

$$\Delta \eta = -\frac{e}{2\pi^2 T d} \int d\epsilon \epsilon \frac{\partial S_0(\epsilon)}{\partial \epsilon} \ln\left(\frac{L_{\phi}(\epsilon)}{l(\epsilon)}\right) = \frac{eT}{3d} \left[\frac{\partial}{\partial \epsilon} \ln\left(\frac{L_{\phi}(\epsilon)}{l(\epsilon)}\right)\right]_{\epsilon=0},$$
(46)

which is analogous to the result of Ref. 10. Equation (46) shows that the correction to the thermoelectric coefficient does not have a large logarithmic factor compared to the correction to the conductivity, Eq. (23). This result means that for the experimentally measured Seebek coefficient

$$B = B_0 \left(1 - \frac{\Delta \sigma_{xx}}{\sigma_{xx}} + \frac{\Delta \eta}{\eta_0} \right), \tag{47}$$

where $B_0 = -\eta_0 / \sigma_{xx}$, only the term corresponding to the correction to the conductivity is important.

The Nernst-Ettingshausen coefficient *N* is defined by the equation $\mathbf{J} = N(\nabla T \times \mathbf{H})$. Substituting ϕ_N for ϕ_H in Eq. (2) we get for the Nernst-Ettingshausen coefficient without weak localization effects

$$N = -\frac{\pi^2}{9} \frac{e^2 T}{cm} \frac{\partial}{\partial \epsilon} [v^2(\epsilon) \tau^2(\epsilon) \nu(\epsilon)]|_{\epsilon=0}$$
$$= -\frac{\pi^2}{6} \frac{T}{\epsilon_F} (\Omega \tau_0) \frac{\sigma_{xx}}{H} = -\frac{\pi^2}{6} \frac{T}{\epsilon_F} \sigma_{xy} = \frac{\Omega \tau_0}{2H} \eta. \quad (48)$$

Equation (48) shows that the Nernst-Ettingshausen coefficient is proportional to the product of the electron-hole asymmetry factors, one from the Hall conductivity σ_{xy} and another factor T/ϵ_F arises after expansion of all electron parameters near the Fermi surface similar to the thermoelectric coefficient. Note that this important fact is clear when calculating the Nernst-Ettingshausen coefficient by the transport equation. When the LRM is applied special attention is required to find this double-electron-hole asymmetry, as discussed in detail in the Appendix.

The WL correction to the Nernst-Ettingshausen coefficient may be obtained from the above presented calculation of the Hall conductivity by substituting ϕ_T for ϕ_E and also $\{A,B\}_T$ for $\{A,B\}_E$ in all equations of Sec. III. As a result we have

 $\Delta N =$

$$-\frac{2e^2}{\pi cm} \int \frac{d^3q}{(2\pi)^3} \int d\epsilon \frac{\epsilon}{T} \frac{\partial S_0(\epsilon)}{\partial \epsilon} \frac{\tau(\epsilon)}{q^2 + [D(\epsilon)\tau_{\phi}(\epsilon)]^{-1}}.$$
(49)

According to Eq. (49), in the quasi-two-dimensional case $(d < L_{\phi})$ the main contribution to the Nernst-Ettingshausen coefficient arises from the expansion of $\tau(\epsilon)$ [see Eq. (43)], as a result

$$\frac{\Delta N}{N} = -2\frac{\Delta\sigma_{xx}}{\sigma_{xx}},\tag{50}$$

which is different by sign compared with the corresponding correction to the Hall conductivity [Eq. (36)]. Equation (50) shows that in the quasi-two-dimensional case the WL correction to the Nernst-Ettingshausen coefficient has a large logarithmic factor. Note, that for exactly two-dimensional (2D) electrons, the electron momentum relaxation time does not depend on the energy, and the large logarithmic factor in the WL correction to the Nernst-Ettingshausen effect disappears.

VI. CONCLUSIONS

In the present paper we apply the QTE for the WL effect on different transport coefficients. We reproduce the WL corrections to the conductivity, the Hall conductivity^{6,7} and the thermoelectric effect⁹ obtained earlier by the LRM. Then we calculate WL corrections to the Nernst-Ettingshausen coefficient. To our knowledge, this problem has not been considered before.

The QTE gives a convenient and universal description of all transport phenomena. A specific feature of an external disturbance is taken into account by the nonequilibrium distribution functions of noninteracting electrons [Eqs. (17), (27), (38), and (39)] and by quantum corrections to the quasiclassical transport equation in the form of the Poisson brackets [Eqs. (13), (14), and (15)].

The quasiclassical transport equation has been applied to the WL effect on the electrical conductivity in Ref. 11, where all terms in the form of the Poisson brackets have been ignored. As we demonstrated above, these terms mutually cancelled for the correction to the conductivity but they are very important for the Hall conductivity and the Nernst-Ettingshausen coefficient.

QTE also allows us to avoid lengthy calculations of the particle-hole asymmetry, which is important for the Hall and Nernst effects. In the Hall effect the particle-hole asymmetry appears as a measure of the average curvature of the Fermi surface, and is expressed through the electron mass (some effective electron mass in the general case). Using the QTE we get Hall's particle-hole asymmetry in the distribution function of noninteracting electrons, Eq. (27), and in the magnetic Poisson bracket, Eq. (14). To obtain the same result by the LRM one needs to expand all electron parameters in all Green's functions near the Fermi surface.

The Nernst-Ettingshausen coefficient is proportional to the double particle-hole asymmetry. It means that in the linear response method one should keep second-order terms in expansion of electron parameters near the Fermi surface. With many-body corrections, it is a complicated problem. Ignoring at least one of the diagrams gives a nonzero Nernst-Ettingshausen coefficient in zero order in the particle-hole asymmetry (without any expansion). We discuss this problem in detail in the Appendix for noninteracting electrons. For interacting electrons the problem is even more complicated. As an example of the problem for which wrong results have been obtained, we mention the effect of superconducting fluctuations on the Nernst-Ettingshausen coefficient.^{12,13}

We also found that in the quasi-two-dimensional case the WL correction to the Nernst-Ettingshausen coefficient is proportional to a large logarithmic factor similar to WL corrections to the conductivity and the Hall conductivity, while for the WL correction to the thermoelectric coefficient such a correction is absent.

It is interesting to note, that, though it is not obvious in advance, the WL correction to any transport coefficients is given by the corresponding "classical" formula, where the relaxation time is changed for its Cooperon correction $\Delta \tau = -2\tau \int d^3 q/(2\pi)^3 C(q)$.

Thermoelectric phenomena in low-dimensional systems attracted considerable attention recently.^{14–17} Our experimental study of the Nernst-Ettingshausen coefficient improves our understanding of low-dimensional electron transport.

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APPENDIX

In this appendix we calculate the Nernst-Ettingshausen coefficient of noninteracting electrons using the LRM. We demonstrate that the heat current correction due to the magnetic field should be taken into account. The diagram with this additional heat current vertex cancels the basic diagrams in zero order in the electron-hole asymmetry. The nonzero Nernst-Ettingshausen coefficient arises only in the second order in the asymmetry. These conclusions are also relevant to many-body corrections to the Nernst coefficient.

In the linear response method the external magnetic field is introduced through the corresponding vector potential $\mathbf{H} = i(\mathbf{k} \times \mathbf{A})$. In the presence of the external magnetic field the electronic kinetic energy is given by

$$K = \frac{1}{2m} \left(\mathbf{p} + \frac{e}{c} \mathbf{A} \right)^2 = \frac{p^2}{2m} + \frac{e}{c} \left(\mathbf{v} \cdot \mathbf{A} \right) + \frac{e^2}{2mc^2} A^2.$$
(A1)

We choose the gauge: $\mathbf{k} \cdot \mathbf{A} = 0$, and consider a response of the electron system to the temperature gradient, which is perpendicular to the magnetic field.

Then the Nernst-Ettingshausen current is proportional to $\nabla T \times \mathbf{H} = -i\mathbf{A}(\mathbf{k} \cdot \nabla T).$

In the LRM the Nernst coefficient is given by

$$N = \frac{1}{\Omega T H} \operatorname{Im} Q^{R}(\Omega, \mathbf{A}).$$
 (A2)

Here $Q^{R}(\Omega, \mathbf{A})$ is the Fourier representation of the retarded correlation function of the heat and charge current operators $\hat{\mathbf{J}}_{h}$ and $\hat{\mathbf{J}}_{e}$ in the presence of the vector potential \mathbf{A} ,

$$Q^{R}(\mathbf{x}-\mathbf{x}',t-t',\mathbf{A}) = -\Theta(t-t') \times \langle [\mathbf{\hat{J}}_{h}(\mathbf{x},t,\mathbf{A})\cdot\mathbf{n},\mathbf{\hat{J}}_{e}(\mathbf{x}',t',\mathbf{A})\cdot\mathbf{n}_{1}] \rangle,$$
(A3)

where **n** is a unit vector directed along the temperature gradient, \mathbf{n}_1 is a unit vector perpendicular to **n** and the external magnetic field **H**.

The external magnetic field is taken into account by inserting an additional magnetic vertex $(e/c)(\mathbf{v}\cdot\mathbf{A})$ [the second term on the right-hand side of Eq. (A1)] into the electronic Green's function of the corresponding diagrams of the linear response correlators. For example, diagrams for the Hall coefficient are obtained by inserting this vertex into the conductivity diagrams as it was done in Refs. 8 and 9.



FIG. 2. Diagrams of the linear response method for the Nernst-Ettingshausen coefficient of noninteracting electrons.

For the Nernst-Ettingshausen coefficient, however, it is not enough to insert the magnetic vertex in the diagrams of the off-diagonal thermoelectric coefficient. An additional diagram appears due to modification of the heat current vertex. According to Ref. 2 the heat current vertex for noninteracting electrons is $\gamma_h = \xi_p \mathbf{v}$, which corresponds to the energy current measured with respect to the electron chemical potential. For electrons in the magnetic field the vertex of the heat current is modified according to Eq. (A1)

$$\gamma_h = \xi_p \mathbf{v} + (e/c) (\mathbf{v} \cdot \mathbf{A}) \mathbf{v}. \tag{A4}$$

Diagrams of the LRM for the Nernst-Ettingshausen coefficient are presented in Fig. 2. In the first and second diagrams the vertex $(e/c)(\mathbf{v} \cdot \mathbf{A})$ describes the effect of the magnetic field on the electron states. In the third diagram the same vertex is a part of the heat current operator, Eq. (A3). To get the term $\mathbf{A}(\mathbf{k} \cdot \nabla T)$, one should expand the Green function $G(\mathbf{p}+\mathbf{k})$ in powers of $(\mathbf{k} \cdot \mathbf{v})$. Then the contribution of the first and second diagrams is given by

$$N_{1}+N_{2} = \frac{e}{TH} \int \frac{d^{4}P}{(2\pi)^{4}} \frac{\partial S_{0}(\epsilon)}{\partial \epsilon} \xi_{p} \mathbf{v}_{c}^{e}(\mathbf{v} \cdot \mathbf{A})$$
$$\times (\mathbf{v} \cdot \nabla T)(\mathbf{v} \cdot \mathbf{k})(I_{1}+I_{2}), \qquad (A5)$$

where the combination of the Green functions is

$$I_1 + I_2 = 2iG^A(P)G^R(P)\text{Im}[G^A(P)]^2 = 2i\tau^2\text{Im}[G^A(P)]^2.$$
(A6)

The contribution of the third diagram is

$$N_{3} = \frac{e}{TH} \int \frac{d^{4}P}{(2\pi)^{4}} \frac{\partial S_{0}(\boldsymbol{\epsilon})}{\partial \boldsymbol{\epsilon}} \frac{e}{c} \mathbf{v}(\mathbf{v} \cdot \mathbf{A}) (\mathbf{v} \cdot \nabla T) (\mathbf{v} \cdot \mathbf{k}) I_{3},$$
(A7)

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where the combination of the Green functions I_3 is

$$I_3 = 2iG^A(P) \operatorname{Im}[G^A(P)]^2 = 2i\tau^2 \operatorname{Im} G^A(P).$$
(A8)

The angular integral in Eqs. (A4) and (A6) gives the term proportional to $\mathbf{A}(\mathbf{k} \cdot \nabla T)$,

$$\int \frac{d\Omega_{\mathbf{p}}}{4\pi} \mathbf{v}(\mathbf{v} \cdot \mathbf{A})(\mathbf{v} \cdot \mathbf{k})(\mathbf{v} \cdot \nabla T) = \frac{v^4}{15} \mathbf{A}(\mathbf{k} \cdot \nabla T). \quad (A9)$$

Therefore, the total contribution of the three diagrams is

$$N = \frac{ie^2}{15cTH} \int \frac{d\epsilon}{2\pi} d\xi_p \frac{\partial S_0(\epsilon)}{\partial \epsilon} v^4 \tau^2 \nu \{\xi_p \operatorname{Im}[G^A(P)]^2 + \operatorname{Im} G^A(P)\} \mathbf{A}(\mathbf{k} \cdot \nabla T).$$
(A10)

Without taking into account the particle-hole asymmetry the total contribution of the three diagrams goes to zero after integration over ξ_p ,

$$\int d\xi_p \{\xi_p \text{Im}[G^A(P)]^2 + \text{Im}[G^A(P)]\} = 0.$$
(A11)

Nonzero contribution arises from terms proportional to ϵ^2 , thus we should expand all electron parameters near the Fermi surface:

$$[v(\xi_p)]^4 \nu(\xi_p) = v_0^4 \nu_0 \bigg[1 + \frac{5}{2} \frac{\xi_p}{\epsilon_F} + \frac{15}{8} \bigg(\frac{\xi_p}{\epsilon_F} \bigg)^2 + \cdots \bigg],$$
(A12)

$$\left[\tau(\boldsymbol{\epsilon})\right]^2 = \tau_0^2 \left[1 - \frac{\boldsymbol{\epsilon}}{\boldsymbol{\epsilon}_F} + \cdots\right]. \tag{A13}$$

Taking into account terms proportional to the square of the particle-hole asymmetry, e.g., ξ^2/ϵ_F^2 or $\xi\epsilon/\epsilon_F^2$, we get

$$\int d\xi_p v^4 \nu \tau^2 \{\xi_p \operatorname{Im}[G^A(P)^2 + \operatorname{Im} G^A(P)]\}$$

= $-\pi v_0^4 \nu_0 \tau_0^2 \frac{5}{4} \frac{\epsilon^2}{\epsilon_F^2} = -\pi \frac{5}{2} \frac{v_0^2 \tau_0^2 \nu_0}{m} \frac{\epsilon^2}{\epsilon_F}.$ (A14)

Substituting this result into Eq. (A9), and performing integration over ϵ , we get the Nernst-Ettingshausen coefficient that coincides with Eq. (48).

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