PHYSICAL REVIEW B VOLUME 61, NUMBER 10 1 MARCH 2000-II

Spin-flip transition rate due to electron-magnon scattering in ferromagnetic thin films

Shang Yuan Ren
Department of Physics and Astronomy, Arizona State University, Tempe, Arizona 85287-1540
and Department of Physics, Peking University, Beijing 100871, People's Republic of China

John D. Dow
Department of Physics and Astronomy, Arizona State University, Tempe, Arizona 85287-1540
(Received 22 March 1999; revised manuscript received 13 July 1999)

In a free-electron model of very thin ferromagnetic films, an electron occupies a ''Fermi disk,'' which is reduced from one of the two bulk Fermi spheres, corresponding to spin-up and spin-down electrons. The spin-flip transition rate due to electron-magnon scattering in such ferromagnetic thin films is due to electron scattering between two of these ''Fermi disks.'' We study how electron confinement affects this rate. Normally the spin-flip scattering rate decreases as the film thickness increases. But when the film thickness increases to a point such that electrons of opposite spin start occupying a higher subband, the scattering increases abruptly. This abrupt increase is more prominent at higher temperatures. As the film thickness increases toward infinity, the spin-flip scattering rate in the film decreases to the value for the bulk magnetic material.

I. INTRODUCTION

The recent discovery of giant magnetoresistance (GMR) of magnetic multilayers—a decrease of resistance when an external magnetic field aligns the magnetization vectors in several adjacent layers[1,2]—has renewed interest in developing a new generation of high-speed electronic devices using this phenomenon. Such devices will feature spin-polarized electronic transport,[3] and will involve ferromagnetic materials.

The first model of electronic transport in ferromagnetic materials is a two-fluid model, assuming conduction in parallel by spin-up and spin-down electrons.[4] This model is widely employed because spin-conserving scattering in ferromagnets normally is much stronger than spin-flip scattering, and so the two-fluid model provides a sound basis for exploring a broad range of transport properties in ferromagnetic materials.[5] In this two-fluid model, the total observed resistivity can be written as

$$\rho=\frac{\rho_{\uparrow}\rho_{\downarrow}}{\rho_{\uparrow}+\rho_{\downarrow}}, \tag{1}$$

where $\rho\uparrow$ ($\downarrow$) is the resistivity of the majority (minority) spin electrons. A spin-asymmetry ratio α can be defined

$$\alpha=\frac{\rho_{\downarrow}}{\rho_{\uparrow}}, \tag{2}$$

such that for $\alpha>1(\alpha<1)$, transport by spin-up (spin-down) electrons dominates.

Recently an inverse magnetoresistance effect has been observed in multilayers of the type $(/A/C/B/C/)_N$, where A and B are different ferromagnetic materials which have opposite spin asymmetries (namely, one has $\alpha>1$ and the other has $\alpha<1$) and C is a nonferromagnetic metal such as Cu or Au.[6–8] This inverse magnetoresistance effect has the resistance being higher when the magnetization vectors in the alternate ferromagnetic thin-film layers are parallel to one another, rather than antiparallel.

The traditional two-fluid model can be improved by including the spin-flip scattering in ferromagnetic materials, which has the tendency to equalize the contributions of two spin-currents especially at moderate temperatures. In cases in which the spin-flip scattering cannot be neglected, the total observed resistivity in ferromagnetic materials generally can be written as[9]

$$\rho=\frac{\rho_{\uparrow}\rho_{\downarrow}+\rho_{\uparrow\downarrow}(\rho_{\uparrow}+\rho_{\downarrow})}{\rho_{\uparrow}+\rho_{\downarrow}+4\rho_{\uparrow\downarrow}}. \tag{3}$$

One can write[10]

$$\rho_{\uparrow}=\frac{m^{*}}{ne^{2}\tau_{\uparrow}},\quad \rho_{\downarrow}=\frac{m^{*}}{ne^{2}\tau_{\downarrow}},\quad \rho_{\uparrow\downarrow}=\frac{m^{*}}{ne^{2}\tau_{\uparrow\downarrow}}, \tag{4}$$

where m^* is the effective mass of the electron and $\tau_{\uparrow}$ and $\tau_{\downarrow}$ are the corresponding separate relaxation times, and $\tau_{\uparrow\downarrow}$ is the spin-flip scattering relaxation time.

There are several spin-flip mechanisms in ferromagnetic materials. Direct scattering between the spin-up and spin-down electrons is known to be negligible.[11] Electron-spin-wave scattering, viz., electron-magnon scattering, is the principal mechanism of spin-flip in ferromagnetic materials.[9,10] There are some other ''residual'' spin-flip mechanisms, such as (i) spin-flip scattering by impurities due to spin-orbit coupling and (ii) spin-flip from the combined action of internal magnetic induction and spin-orbit coupling, but these appear to be far less important than electron-magnon scattering for determining the spin-flip relaxation time $\tau_{\uparrow\downarrow}$.[9,10] [Such spin-flip scattering could play an important role in the physics of direct or inverse giant magnetoresistance (GMR) in multilayer ferromagnetic materials.] Although there have been many theoretical investigations of the effects of electron confinement on electron transport in superlattices or heterojunctions of semiconductors[12–15] and in thin films of ordinary metals,[16,17] we are unaware of any corresponding treatment of electron-magnon scattering in confined ferromagnetic thin films or multilayers. Here we present such an investigation.

II. GENERAL ANALYSIS

In a ferromagnetic thin film, both electrons and magnons are confined. In a free-electron model, for electrons confined in the region of $0 \leqslant z \leqslant L$, the states become

$$\frac{\exp(i\mathbf{k}\cdot\mathbf{r})}{(2\pi\hbar)^{3/2}} \Rightarrow \frac{\exp(i\mathbf{k}_{\parallel}\cdot\mathbf{x})}{2\pi\hbar} \times \sqrt{\frac{2}{L}} \frac{\exp(ijz\pi/L)-\exp(-ijz\pi/L)}{2i}, \tag{5}$$

where we use $\mathbf{k}$ to label three-dimensional wave vectors and $\mathbf{k}_{\parallel}$ for two-dimensional wave vectors. The z component of the wave function is a standing wave with wave vector $j\pi/L$. The energies become

$$\frac{\hbar^2 k^2}{2m} \Rightarrow \frac{\hbar^2 k_{\parallel}^2}{2m} + \frac{\hbar^2}{2m}\frac{j^2\pi^2}{L^2}. \tag{6}$$

Here we consider the quantum confinement of the electrons but neglect any confinement of the magnons and simply use a bulklike magnon spectrum. (This kind of approximation has been used successfully for many electron-phonon scattering calculations in semiconductor thin films.[14,15])

In general, the most significant effects of confinement of electron states on the electron-magnon scattering in ferromagnetic thin films are the following: (i) The quantization of electronic states changes the density of states and therefore the final-state distribution. (ii) Confinement affects the envelope functions of electrons, with the major consequence being that crystal momentum in the confinement direction is not strictly conserved. The parameter that quantifies this effect is $G(\mathbf{k}_{\parallel}',j',\mathbf{k}_{\parallel},j,\mathbf{q})$, which is a complicated function of the wave vectors involved.[14,15] (iii) The quantum size effect on the Fermi energy alters the scattering.

The first two of these effects exist in semiconductor thin films and have been analyzed before.[12–15] The Fermi-energy effect only exists in metallic thin films. Here we extend the treatment of the effect from ordinary metallic thin films[18] to ferromagnetic thin films, where the effect is complicated by the existence of two inequivalent electron-spin polarizations.

III. QUANTUM SIZE EFFECT ON THE FERMI ENERGY OF A FERROMAGNETIC THIN FILM

Following Fert,[10] we assume that the electrons have an isotropic effective mass m and can be described by plane waves if not confined. The thin film has the same crystal structure as the bulk but is only confined to a certain region $0 \leqslant z \leqslant L$ in the z direction.

In magnetic materials, we have two Fermi spheres, one for each spin polarization. The Fermi energy of the bulk magnetic material is

$$E_{F,b} = \frac{\hbar^2}{2m} k_{F,b,\uparrow}^2 - NJS = \frac{\hbar^2}{2m} k_{F,b,\downarrow}^2 + NJS, \tag{7}$$

where we use a subscript b to label various bulk material quantities, S is the spin of the magnetic ion, and J is the electron-magnetic-ion interaction parameter. In this work we use the same parameters as in Ref. 10 for the Fermi energy in the bulk $E_{F,b}$, the spin of the magnetic ions S, and the interaction parameter NJ.

The total electronic density in the magnetic bulk material is

$$N_e = \frac{1}{6\pi^2}(k_{F,b,\uparrow}^3 + k_{F,b,\downarrow}^3). \tag{8}$$

In thin magnetic metallic films we have two *sets* of ''Fermi disks'' corresponding to two Fermi spheres in the bulk, one set for the spin-up electrons and the other set for the spin-down electrons. Each Fermi disk corresponds to a specific subband j of electrons with either spin-up or spin-down. Because the total electronic density of a thin film is the same as in the bulk, in a thin film of thickness L we have

$$\frac{1}{4\pi^2}\left(\sum_{j=1}^{j_{F,\uparrow}} \pi k_{F,j,\uparrow}^2 + \sum_{j=1}^{j_{F,\downarrow}} \pi k_{F,j,\downarrow}^2\right) = \frac{L}{6\pi^2}(k_{F,b,\uparrow}^3 + k_{F,b,\downarrow}^3). \tag{9}$$

When the thickness L of the thin magnetic film is very small, all electrons are in the first spin-up subband, so that the condition on L is

$$\frac{\hbar^2}{2m}\left(k_{F,1,\uparrow}^2 + \frac{\pi^2}{L^2}\right) - NJS \leqslant \frac{\hbar^2}{2m}\frac{\pi^2}{L^2} + NJS \tag{10}$$

or

$$L \leqslant \frac{6\pi m}{\hbar^2}\frac{1}{k_{F,b,\uparrow}^3 + k_{F,b,\downarrow}^3} NJS, \tag{11}$$

by combining Eqs. (9) and (10). As the thickness L increases, the electrons start to occupy the first spin-down subband, then the second spin-up subband, then the second spin-down subband, and so on. In general the order in which electrons occupy the jth subband can be determined by the following equations:

$$\frac{1}{4\pi}(j_{F,\uparrow}k_{F,j_F,\uparrow}^2 + j_{F,\downarrow}k_{F,j_F,\downarrow}^2) = \frac{L}{6\pi^2}(k_{F,b,\uparrow}^3 + k_{F,b,\downarrow}^3) - \frac{\pi}{4L^2}[F(j_{F,\uparrow}) + F(j_{F,\downarrow})], \tag{12}$$

where we have

$$F(j) = j^3 - \frac{j(j+1)(2j+1)}{6}. \tag{13}$$

Here $j_{F,\uparrow}(j_{F,\downarrow})$ is the index of the highest occupied spin-up (spin-down) subband and $k_{F,j_F,\uparrow}(k_{F,j_F,\downarrow})$ is the Fermi wavevector of that subband. We have

$$\frac{\hbar^2}{2m}\left(k_{F,j_F,\uparrow}^2 + \frac{j_{F,\uparrow}^2\pi^2}{L^2}\right) - NJS = \frac{\hbar^2}{2m}\left(k_{F,j_F,\downarrow}^2 + \frac{j_{F,\downarrow}^2\pi^2}{L^2}\right) + NJS. \tag{14}$$

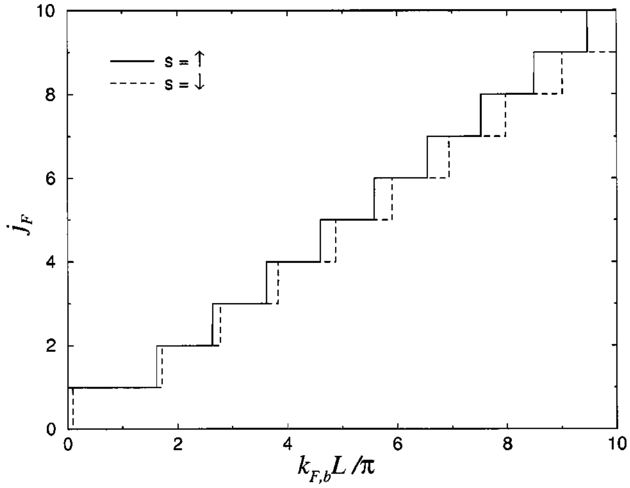

FIG. 1. Highest occupied subband indices j_F, namely, $j_{F,\uparrow}$ and $j_{F,\downarrow}$, for electrons of two different spin directions as functions of reduced thickness of the magnetic thin film $k_{F,b}L/\pi$. Solid lines: $j_{F,\uparrow}$; dashed lines: $j_{F,\downarrow}$.

In Fig. 1 we show j_F for electrons of two different spin directions as function of $k_{F,b}L/\pi$, determined by Eqs. (12) and (14) with parameters $E_{F,b}=7.1\,\text{eV}$, $S=1.06$, and $NJ=0.4\,\text{eV}$. The reduced thickness $k_{F,b}L/\pi$ for the electrons to occupy the first 20 subbands are 0 (1 ↑), 0.090 (1 ↓), 1.621 (2 ↑), 1.713 (2 ↓), 2.637 (3 ↑), 2.783 (3 ↓), 3.620 (4 ↑), 3.830 (4 ↓), 4.600 (5 ↑), 4.870 (5 ↓), 5.576 (6 ↑), 5.906 (6 ↓), 6.551 (7 ↑), 6.940 (7 ↓), 7.524 (8 ↑), 7.794 (8 ↓), 8.497 (9 ↑), 9.006 (9 ↓), 9.470 (10 ↑), and 10.038 (10 ↓). In Fig. 2 we show the Fermi wave vectors in different subbands as functions of the reduced thickness of the thin film. In Fig. 3 we show the Fermi wave vectors of the highest occupied spin-up and spin-down subbands as functions of the reduced thickness. The physical picture in which electrons of two different spins sequentially occupy the ''Fermi disks,'' first of one spin and then of another, is the starting point of this investigation.

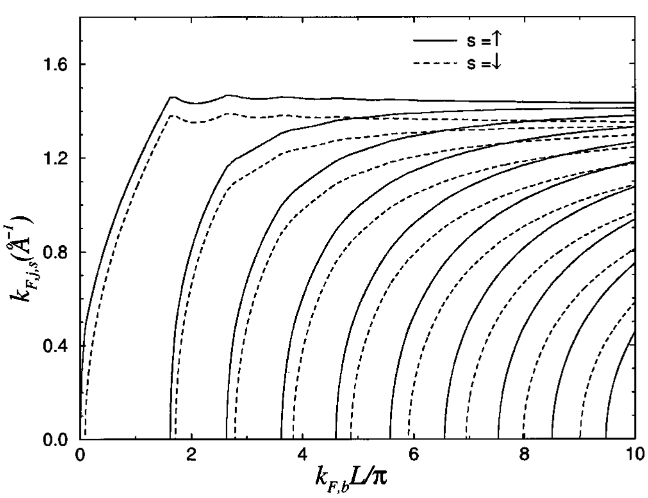

FIG. 2. Fermi wave vectors in different subbands $k_{F,j,s}$, where $s=\uparrow$ or $\downarrow$, namely, $k_{F,j,\uparrow}$, and $k_{F,j,\downarrow}$, as functions of reduced thickness of the magnetic thin film $k_{F,b}L/\pi$. Solid lines: $k_{F,j,\uparrow}$; dashed lines: $k_{F,j,\downarrow}$.

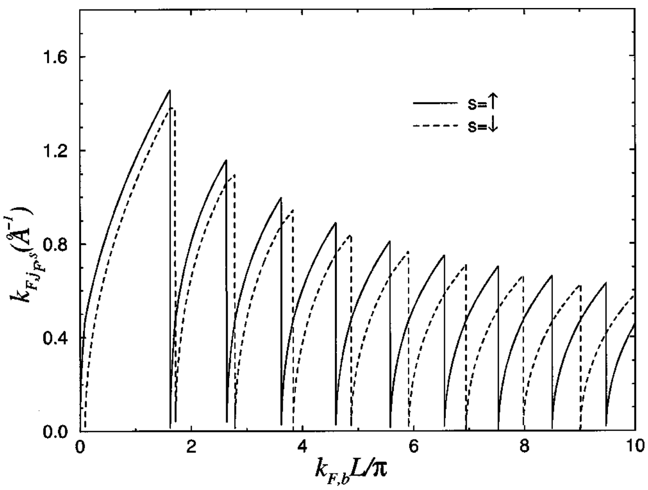

FIG. 3. Fermi wave vectors of the highest occupied spin-up and spin-down subbands as functions of the reduced thickness of the magnetic thin film $k_{F,b}L/\pi$. Solid lines: $k_{F,j_{F,\uparrow}}$; chained lines: $k_{F,j_{F,\downarrow}}$.

IV. THE BOLTZMANN EQUATION AND THE RELAXATION TIME APPROXIMATION

In magnetic thin films, spin-flip scattering occurs between two sets of ''Fermi disks,'' and each set corresponds to a specific spin direction. Following Fert,[10] we begin from the Boltzmann electron transport equation for the distribution function *f*. We consider in-plane electronic transport for electrons in a ferromagnetic thin film:[10]

$$\left.\frac{\partial f_\uparrow(\mathbf{k}_\parallel,j)}{\partial t}\right]_{\text{scatt}}+\left.\frac{\partial f_\uparrow(\mathbf{k}_\parallel,j)}{\partial t}\right]_{\text{field}}=0. \tag{15}$$

In the thin film plane, the field term for the transport is

$$\left.\frac{\partial f_\uparrow(\mathbf{k}_\parallel,j)}{\partial t}\right]_{\text{field}}=-\frac{e\hbar\mathbf{k}_\parallel\cdot\mathbf{E}}{m}\frac{\partial f_\uparrow(\mathbf{k}_\parallel,j)}{\partial E}, \tag{16}$$

and the scattering term is

$$\begin{aligned}\left.\frac{\partial f_\uparrow(\mathbf{k}_\parallel,j)}{\partial t}\right]_{\text{scatt}}=\sum_{s,\mathbf{k}'_\parallel,j'}&\{f_s(\mathbf{k}'_\parallel,j')[1-f_\uparrow(\mathbf{k}_\parallel,j)]\\&\times P(\mathbf{k}'_\parallel,j',s,\mathbf{k}_\parallel,j,\uparrow)\\&-f_\uparrow(\mathbf{k}_\parallel,j)[1-f_s(\mathbf{k}'_\parallel,j')]\\&\times P(\mathbf{k}_\parallel,j,\uparrow,\mathbf{k}'_\parallel,j',s)\}.\end{aligned} \tag{17}$$

Here *s* labels different spin states, either ↑ or ↓, and *P* is a transition rate. The perturbed distribution function is expressed in terms of $\phi_{\mathbf{k}_\parallel,j,s}$:

$$f_s(\mathbf{k}_\parallel,j)=f^0[E_s(\mathbf{k}_\parallel,j)]-\phi_{\mathbf{k}_\parallel,j,s}\frac{\partial f^0[E_s(\mathbf{k}_\parallel,j)]}{\partial E}. \tag{18}$$

Here we can choose the form

$$\phi_{\mathbf{k}_\parallel,j,s}=\alpha_{s,j}\mathbf{k}_\parallel\cdot\mathbf{E}, \tag{19}$$

where $\alpha_{s,j}$ is a proportionality constant.[19]

Treating the Boltzmann equation in the relaxation time approximation, we have for spin up

$$-\frac{e\hbar}{m}\mathbf{k}_{\parallel}\cdot\mathbf{E}\frac{\partial f^{0}[E_{\uparrow}(k_{\parallel},j)]}{\partial E}=\frac{f_{\uparrow}(\mathbf{k}_{\parallel},j)-f^{0}[E_{\uparrow}(\mathbf{k}_{\parallel},j)]}{\tau_{\uparrow,j}}+\frac{f_{\uparrow}(\mathbf{k}_{\parallel},j)-f_{\downarrow}(\mathbf{k}_{\parallel},j)}{\tau_{j,\mathbf{k}_{\parallel},\uparrow\downarrow}}. \quad (20)$$

For spin-down electrons, we have a similar equation to Eq. (20).

In this work we are mainly interested in spin-flip scatterings which change the direction of the electron spin. According to Fert,[10] the transition probability from state $\mathbf{k}\uparrow$ to state $(\mathbf{k}+\mathbf{q})\downarrow$, with absorption of a magnon $\mathbf{q}$ in the bulk, is

$$\begin{aligned}P[\mathbf{k}\uparrow,(\mathbf{k}+\mathbf{q})\downarrow]&=\frac{4\pi SNJ^{2}(\mathbf{q})}{\hbar}\frac{1}{e^{\epsilon_{\mathbf{q}}/k_BT}-1}f^{0}[E_{\uparrow}(\mathbf{k})]\\&\times\{1-f^{0}[E_{\uparrow}(\mathbf{k})+\epsilon_{\mathbf{q}}]\}\delta[E_{\downarrow}(\mathbf{k}+\mathbf{q})-E_{\uparrow}(\mathbf{k})-\epsilon_{\mathbf{q}}].\end{aligned} \quad (21)$$

Similarly, in a ferromagnetic thin film, for the transition from the state $(\mathbf{k}_{\parallel},j,\uparrow)$ to the state $(\mathbf{k}_{\parallel}',j',\downarrow)$ with absorption of one magnon, the equilibrium transition rate is

$$\begin{aligned}P(\mathbf{k}_{\parallel},j,\uparrow,\mathbf{k}_{\parallel}',j',\downarrow)=&\frac{4\pi SNJ^{2}(\mathbf{q})}{\hbar}\frac{1}{e^{\epsilon_{\mathbf{q}}/\mathbf{k}_BT}-1}\\&\times f^{0}[E_{\uparrow}(\mathbf{k}_{\parallel},j)]\{1-f^{0}[E_{\downarrow}(\mathbf{k}_{\parallel}',j')]\}\\&\times\delta[E_{\downarrow}(\mathbf{k}_{\parallel}',j')-E_{\uparrow}(k_{\parallel},j)\\&-\epsilon_{\mathbf{q}}]|I(\mathbf{k}_{\parallel}',j',\mathbf{k}_{\parallel},j,\mathbf{q})|^{2}.\end{aligned} \quad (22)$$

Here $\epsilon_{\mathbf{q}}$ is the energy of the magnon and we have[14,15]

$$I(\mathbf{k}_{\parallel}',j',\mathbf{K}_{\parallel},j,\mathbf{q})=I(\mathbf{k}',\mathbf{k})G(\mathbf{k}_{\parallel}',j',\mathbf{k}_{\parallel},j,\mathbf{q}), \quad (23)$$

where, with $u_{\mathbf{k}}(\mathbf{r})$ the cell-periodic part of the electron wave function, we have

$$I(\mathbf{k}',\mathbf{k})=\int u_{\mathbf{k}'}^{*}(\mathbf{r})u_{\mathbf{k}}(\mathbf{r})d\mathbf{r}, \quad (24)$$

and the integral is over the unit cell. $G(\mathbf{k}',\mathbf{k},\mathbf{q})$ is an overlap integral, $\phi=\exp(i\mathbf{q}\cdot\mathbf{r})$ and ψ is an envelope wave function:

$$G(\mathbf{k}',\mathbf{k},\mathbf{q})=\int\psi^{*}(\mathbf{k}',\mathbf{r})\phi(\mathbf{q},\mathbf{r})\psi(\mathbf{k},\mathbf{r})d\mathbf{r}. \quad (25)$$

The major difference between the scattering of an electron by a magnon in a bulk metal and in a thin film is the function $G(\mathbf{k}',\mathbf{k},\mathbf{q})$. In the bulk metal we have

$$G(\mathbf{k}',\mathbf{k},\mathbf{q})=\delta(\mathbf{k}'-\mathbf{k}-\mathbf{q}), \quad (26)$$

but in the thin film $G(\mathbf{k}',\mathbf{k},\mathbf{q})$ is more complicated.

It is usually a good approximation to take the cell-periodic parts of the electron wave function as unaffected by confinement. Therefore, $I(\mathbf{k}',\mathbf{k})$ can be obtained from the bulk expression, although the dependence on scattering angle could be affected by restrictions on the wave vectors.

V. THE OVERLAP INTEGRAL IN TWO DIMENSIONS

We need to calculate the overlap integral[14]

$$G(\mathbf{k}',\mathbf{k},\mathbf{q})=\int\psi^{*}(\mathbf{k}',\mathbf{r})\phi(\mathbf{q},\mathbf{r})\psi(\mathbf{k},\mathbf{r})d\mathbf{r}. \quad (27)$$

Due to the (assumed) electron confinement in the region $0\leqslant z\leqslant L$, we have

$$\psi(\mathbf{k},\mathbf{r})=\sqrt{\frac{2}{AL}}e^{i\mathbf{k}_{\parallel}\mathbf{x}}\sin\left(\frac{j\pi}{L}z\right). \quad (28)$$

Integration of Eq. (27) over the plane gives

$$G(\mathbf{k}',\mathbf{k},\mathbf{q})=G(j',j,q_z)\,\delta_{\mathbf{k}_{\parallel}',\mathbf{k}_{\parallel}+\mathbf{q}_{\parallel}}, \quad (29)$$

where we have

$$G(j',j,q_z)=\int_0^L\psi_{j'}e^{i\mathbf{q}z}\psi_j(z)dz. \quad (30)$$

The crystal momentum in the plane is conserved, but in the confinement direction we have[15]

$$\begin{aligned}G(j',j,q_z)=\frac{1}{2}\Bigg[&\frac{\sin\{(q_z+k_z'-k_z)L/2\}}{(q_z+k_z'-k_z)L/2}e^{i(q_z+k_z'-k_z)L/2}\\&+\frac{\sin\{(q_z-k_z'+k_z)L/2\}}{(q_z-k_z'+k_z)L/2}e^{i(q_z-k_z'+k_z)L/2}\\&+\frac{\sin\{(q_z+k_z'+k_z)L/2\}}{(q_z+k_z'+k_z)L/2}e^{i(q_z+k_z'+k_z)L/2}\\&+\frac{\sin\{(q_z-k_z'-k_z)L/2\}}{(q_z-k_z'-k_z)L/2}e^{i(q_z-k_z'-k_z)L/2}\Bigg],\end{aligned} \quad (31)$$

where $k_z=j\pi/L$ and $k_z'=j'\pi/L$. Equation (31) shows no restriction on q_z for given initial wave-vector index j and final index j', although maxima in $G(j',j,q_z)$ occur for the four momentum-conserving values

$$q_z=\pm(j'\pm j)\frac{\pi}{L}. \quad (32)$$

Regarding the magnon energy as negligible in the energy conservation allows us to decouple the sum over q_z from energy conservation for a given intra- or intersubband transition. Converting the sum over q_z to an integral, we find[13,14]

$$\sum_{q_z}|G(j',j,q_z)|^2=\int_{-\infty}^{+\infty}|G(j',j,q_z)|^2dq_z\frac{L}{2\pi}=1+\frac{1}{2}\delta_{j',j}. \quad (33)$$

VI. SPIN-FLIP SCATTERING TIME IN A MAGNETIC THIN FILM

For a spin up electron in the $|j,\mathbf{k}_{\parallel}\uparrow\rangle$ state in a thin film of thickness L, the spin-flip scattering rate is

$$\frac{1}{\tau_{j,\mathbf{k}_\parallel,\uparrow\downarrow}}=\frac{2\pi}{\hbar}\int|\langle f_\downarrow|H_{\rm int}|i_\uparrow\rangle|^2\delta(E_{f,\downarrow}-E_{i,\uparrow})dN_f$$
$$=\frac{AL}{8\pi^3}\frac{4\pi SNJ^2}{\hbar}\int dq_\parallel q_\parallel\int d\theta\int dq_z\frac{A}{4\pi^2}$$
$$\times\int d\mathbf{k}'_\parallel\sum_{j'}f^0[E_\uparrow(\mathbf{k}_\parallel,j)]$$
$$\times\{1-f^0[E_\downarrow(\mathbf{k}'_\parallel,j')]\}$$
$$\times\delta(\mathbf{k}'_\parallel-\mathbf{k}_\parallel\mp\mathbf{q}_\parallel)|G(j',j,q_z)|^2$$
$$\times\left[n(\mathbf{q})+\frac{1}{2}\mp\frac{1}{2}\right]\delta(E_{j',\mathbf{k}'_\parallel,\downarrow}-E_{j,\mathbf{k}_\parallel,\uparrow}\mp E_\mathbf{q}) \tag{34}$$

By integrating over $\mathbf{k}'$ and using Eq. (33), we find that

$$\frac{1}{\tau_{j,\mathbf{k}_\parallel,\uparrow\downarrow}}=\frac{A}{4\pi^2}\frac{4\pi SNJ^2}{\hbar}\int dq_\parallel q_\parallel\int d\theta$$
$$\times\sum_{j'}\left(1+\frac{1}{2}\delta_{j,j'}\right)f^0[E_\uparrow(\mathbf{k}_\parallel,j)]$$
$$\times\{1-f^0[E_\downarrow(\mathbf{k}_\parallel\pm\mathbf{q}_\parallel,j')]\}$$
$$\times\left[n(\mathbf{q})+\frac{1}{2}\mp\frac{1}{2}\right]\delta(E_{j',\mathbf{k}_\parallel\pm\mathbf{q}_\parallel,\downarrow}-E_{j,\mathbf{k}_\parallel,\uparrow}\mp E_\mathbf{q}). \tag{35}$$

By neglecting the energy $E_\mathbf{q}$ of the magnon in the energy-conserving δ function, we obtain

$$\frac{1}{\tau_{j,\mathbf{k}_\parallel,\uparrow\downarrow}}=\frac{A}{4\pi^2}\frac{4\pi SNJ^2}{\hbar}\int dq_\parallel q_\parallel\int d\theta$$
$$\times\sum_{j'}\left(1+\frac{1}{2}\delta_{j,j'}\right)f^0[E_\uparrow(\mathbf{k}_\parallel,j)]$$
$$\times\{1-f^0[E_\downarrow(\mathbf{k}_\parallel\pm\mathbf{q}_\parallel,j')]\}\left[n(\mathbf{q})+\frac{1}{2}\mp\frac{1}{2}\right]$$
$$\times\delta\left(2NJS\pm\frac{\hbar^2}{m}k_{F,j,\uparrow}q_\parallel\cos\theta+\frac{\hbar^2}{2m}q^2\right.$$
$$\left.+\frac{\hbar^2}{2m}\frac{(j'^2-j^2)\pi^2}{L^2}\right). \tag{36}$$

By introducing $t=\cos\theta$, we have $\theta=\cos^{-1}t$ and $d\theta=-1/\sqrt{1-t^2}dt$. Therefore,

$$\frac{1}{\tau_{j,\mathbf{k}_\parallel,\uparrow\downarrow}}=\frac{A}{4\pi^2}\frac{4\pi SNJ^2}{\hbar}\int dq_\parallel q_\parallel$$
$$\times\sum_{j'}\left(1+\frac{1}{2}\delta_{j,j'}\right)f^0[E_\uparrow(\mathbf{k}_\parallel,j)]$$
$$\times\{1-f^0[E_\downarrow(\mathbf{k}_\parallel\pm\mathbf{q}_\parallel,j')]\}$$
$$\times\left[n(\mathbf{q})+\frac{1}{2}\mp\frac{1}{2}\right]\int\frac{-1}{\sqrt{1-t^2}}dt$$
$$\times\delta\left(2NJS\pm\frac{\hbar^2}{m}k_{F,j,\uparrow}q_\parallel t+\frac{\hbar^2}{2m}q_\parallel^2\right.$$
$$\left.+\frac{\hbar^2}{2m}\frac{(j'^2-j^2)\pi^2}{L^2}\right). \tag{37}$$

Finishing the integral over t and noticing the role of the factor $f^0[E_\uparrow(\mathbf{k}_\parallel,j)]\{1-f^0[E_\downarrow(\mathbf{k}_\parallel\pm\mathbf{q}_\parallel,j')]\}$, we obtain

$$\frac{1}{\tau_{j,\mathbf{k}_\parallel,\uparrow\downarrow}}=\frac{A}{4\pi^2}\frac{4\pi SNJ^2}{\hbar}\sum_{j'}$$
$$\times\int_{q_1}^{q_2}dq_\parallel\frac{1}{F_{j,j'}(q_\parallel)}$$
$$\times\left(1+\frac{1}{2}\delta_{j,j'}\right)\frac{1}{\frac{\hbar^2k_{F,j,\uparrow}}{m}}\left[n(\mathbf{q})+\frac{1}{2}\mp\frac{1}{2}\right], \tag{38}$$

where we have

$$q_1=|k_{F,j,\uparrow}-k_{F,j',\downarrow}|,$$
$$q_2=|k_{F,j,\uparrow}-k_{F,j',\downarrow}|+2k_{F,j,\uparrow}\quad\text{if}\quad k_{F,j,\uparrow}\leqslant k_{F,j',\downarrow},$$
$$=|k_{F,j,\uparrow}-k_{F,j',\downarrow}|+2k_{F,j,\downarrow}\quad\text{if}\quad k_{F,j,\uparrow}\geqslant k_{F,j',\downarrow},$$

and

$$F_{j,j'}(q)=\sqrt{1-\left(\frac{2NJS+(\hbar^2/2m)[q^2+(\pi^2/L^2)(j^2+j'^2)]}{(\hbar^2/m)k_{F,j,\uparrow}q}\right)^2}, \tag{39}$$

Then we can obtain the averaged spin-flip rate $\tau_{\uparrow\downarrow,L}{}^{-1}$ by taking an average of $\tau_{j,\mathbf{k}_\parallel,\uparrow\downarrow}{}^{-1}$ over $j,\mathbf{k}_\parallel$, weighted by the two-dimensional density of states.

VII. RESULTS AND DISCUSSION

In Fig. 4 we show the major results of our calculations: the ratio of the spin-flip scattering rate $\tau_{\uparrow\downarrow,L}{}^{-1}$ in the ferromagnetic thin film of thickness L to the corresponding spin-flip scattering rate in the bulk ferromagnetic material, $\tau_{\uparrow\downarrow,b}{}^{-1}$, for different temperatures and for different reduced thicknesses L of the magnetic thin film. The parameters we used were those Fert used for Fe:[10] $S=1.06$. $NJ=0.4\,\text{eV}$, $m=m_e$, $\mu=12m_e$, and $E_{F,b}=7.1\,\text{eV}$. From these parameters, the largest value of $k_{F,b}L/\pi$ in Fig. 4 corresponds to a film thickness of about 23 Å.

From this figure, there are several points which are noteworthy: (i) In general, as the thickness increases, the ratio of spin-flip scattering rates decreases. Thus, for very thin films, the scattering is considerably enhanced. This is very similar to the enhancement with the reduced thickness of the phonon scattering rate in semiconductor thin films.[15] (ii) Whenever the film thickness increases to the point that the spin-down electrons can start occupying a higher subband, the spin-flip

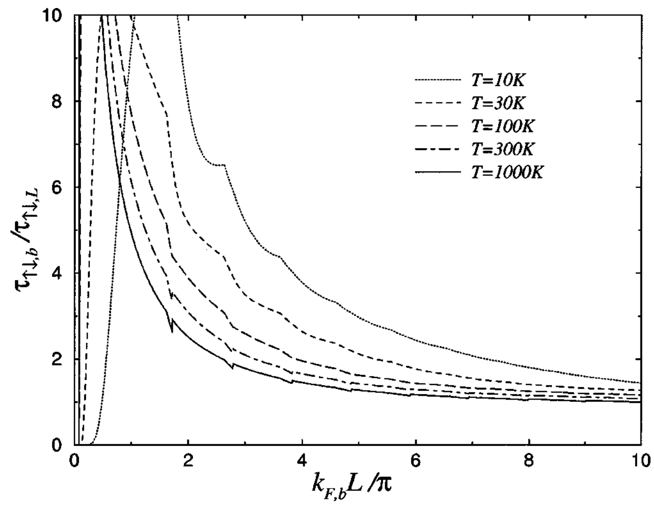

FIG. 4. Ratio of scattering times, $1/\tau_{\uparrow\downarrow,L}/1/\tau_{\uparrow\downarrow,b}=\tau_{\uparrow\downarrow,b}/\tau_{\uparrow\downarrow,L}$, for different temperatures versus the reduced thickness of the magnetic thin film $k_{F,b}L/\pi$, for the case $S=1.06$, $NJ=0.4\,\mathrm{eV}$, $m=m_e$, $\mu=12m_e$, $E_F=7.1\,\mathrm{eV}$. Dotted line, $T=10\,\mathrm{K}$; short dashed line, $T=30\,\mathrm{K}$; long dashed line, $T=100\,\mathrm{K}$; chained line, $T=300\,\mathrm{K}$; solid line, $T=1000\,\mathrm{K}$.

scattering in the thin film (and also the ratio) has an abrupt increase, because a new channel of spin-flip scattering is opened. (iii) This effect is more significant when the temperature is higher, because at low temperature there are not enough magnons to support this scattering to a new Fermi ''disk.'' (iv) For the same film thickness, the lower the temperature is, the larger the ratio is. (v) As the reduced thickness $k_{F,b}L/\pi$ increases, the ratio goes to unity.

The spin-flip scattering effect discussed here is expected to play a role in the physics of spin-polarized electron transport. However, due to the fact that spin-conserving scattering invariably dominates over spin-flip scattering, we do not expect that the spin-flip scattering discussed here will normally have a dominant effect on the conductivity of thin films.

In recently investigated spin-polarized devices, such as those involving giant magnetic resistance or spin-polarized transistors, the interface scattering between the normal-metal films and the ferromagnetic films, and the spin-flip scattering in the normal metal films, will probably play more significant roles than the spin-flip effects discussed here. Nonetheless, especially for very thin films, the spin-flip scattering rate is considerably larger than in bulk material, and this enhancement of the scattering rate should be incorporated into any analysis of the transport of electrons in ferromagnetic thin films.

ACKNOWLEDGMENTS

We are grateful to the Office of Naval Research for their support of this work (Contract No. N00014-98-10137).

[1] M. N. Baibich, J. M. Broto, A. Fert, F. Nguyen Van Dau, F. Petroff, P. Etienne, G. Creuzet, A. Friederich, and J. Chazelas, Phys. Rev. Lett. **61**, 2472 (1988).

[2] G. Binach, P. Grunberg, F. Saurenbach, and W. Zinn, Phys. Rev. B **39**, 4828 (1989).

[3] G. A. Prinz, Phys. Today **48** (4), 58 (1995).

[4] N. C. Mott, Adv. Phys. **13**, 325 (1964).

[5] I. A. Compbell and A. Fert, in *Ferromagnetic Materials*, edited by E. P. Wohlfarth (North-Holland, New York, 1982).

[6] J. M. George, L. G. Pereira, A. Barthélémy, F. Peteroff, L. Steren, J. L. Duvail, A. Fert, R. Loloee, P. Holody, and P. A. Schroeder, Phys. Rev. Lett. **72**, 408 (1994).

[7] J. P. Renard, P. Bruno, R. Mégy, B. Bartenlian, P. Beauvillain, C. Chappert, C. Dupas, E. Kolb, M. Mulloy, P. Verllet, and E. Vélu, Phys. Rev. B **51**, 12 821 (1995).

[8] S. Y. Hsu, A. Barthélémy, P. Holody, R. Loloee, P. A. Schroeder, and A. Fert, Phys. Rev. Lett. **78**, 2652 (1997).

[9] A. Fert and I. A. Compbell, J. Phys. F: Met. Phys. **6**, 849 (1976).

[10] A. Fert, J. Phys. C **2**, 1784 (1969).

[11] A. Bourquart, E. Daniel, and A. Fert, Phys. Lett. **26A**, 260 (1968).

[12] K. Hess, Appl. Phys. Lett. **35**, 495 (1979).

[13] P. J. Price, Ann. Phys. (N.Y.) **133**, 217 (1981).

[14] B. K. Ridley, J. Phys. C **15**, 5899 (1982).

[15] B. K. Ridley, *Electrons and Phonons in Semiconductor Multilayers* (Cambridge University Press, New York, 1997).

[16] V. B. Sandomirskii, Zh. Eksp. Teor. Fiz. **52**, 158 (1967) [Sov. Phys. JETP **25**, 101 (1967)].

[17] For example, N. Trivedi, and N. W. Ashcroft, Phys. Rev. B **38**, 12 298 (1988); L. Sheng, D. Y. Xing, and Z. D. Wang, *ibid.* **51**, 7325 (1995).

[18] S. Y. Ren and J. D. Dow (unpublished).

[19] J. M. Ziman, *Electrons and Phonons* (Oxford University Press, Oxford, 1960), p. 284 ff.