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Heuristic model for slow relaxation of excess conductance in electron glasses

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We discuss relaxation of excess conductance ΔG observed in recent experiments on electron glasses. Using a simple model based on some salient features of a hopping system with interactions, we show that for relatively short times ΔG scales like $\ln(t)$ and it becomes faster than that as time evolves. The crossover to a faster than $\ln(t)$ dependence occurs when $w_{\min}t \approx 1$, where $w_{\min}$ is the smallest relaxation rate in the system, and t is the observation time.

INTRODUCTION

The existence of a glassy phase in Anderson insulators was theoretically predicted by several authors.[1] Recently there were several reports on the nonergodic transport properties of such systems.[2,3,4] When excited from equilibrium by either exposure to a burst of light or a sudden change of the carrier density, the conductance of such systems increases. This property was shown to be inherent to the hopping system.[2] More surprising is the observation that the excess conductance ΔG persists for long times (in some cases, hours) after the excitation. Recent experiments suggest that such extended relaxation times are associated with interactions.[5]

Another question is the specific form of the relaxation law $\Delta G(t)$. It turns out that for relatively short times, the temporal dependence of ΔG can be well described by a logarithmic law.[6] Computational results based on the Coulomb gap model that predict such a temporal dependence, over similar time scales, has been recently reported.[7] When the measurement is extended over longer times however, a better fit to $\Delta G(t)$ is a power law, namely $\Delta G(t) \propto t^{-\alpha}$ with α of the order of 0.2–0.3.[4] A reasonable fit to $\Delta G(t)$ can be also obtained by a stretched exponent, namely, $\exp[-(t/t')^{\beta}]$ but this entails the price of an additional parameter.

Either form of relaxation, namely $\Delta G(t) \propto \ln(t)$, or $\Delta G(t) \propto t^{-\alpha}$ has no natural time scale. Nevertheless a characteristic time scale τ can be defined empirically by the two-dips experiment (TDE).[4] This experiment employs a MOSFET structure where the Anderson insulator forms the active conductance channel and its conductance is monitored as a function of the gate voltage V_g. In the $G(V_g)$ plots one observes two dips with amplitudes A_1 and A_2. The decaying dip A_1 occurs at the cool-down V_{g1} and the growing dip A_2 at the V_{g2} to which the gate voltage is switched at $t=0$. The amplitude of the dips $A_{1,2}$ are measured as functions of time with a time resolution t_0 (the time it takes to sweep V_g from V_{g1} to V_{g2}, which is typically 30 s). A typical TDE is shown in Fig. 1 illustrating how A_1 diminishes with t while A_2 increases with t. An empirical ''relaxation time'' τ may then be defined by the time at which the two dips have the same magnitude, i.e., by $A_1(t_0+\tau)=A_2(t_0+\tau)$. The TDE is a very useful technique to monitor relaxation over extended periods of time as it is less susceptible to temperature fluctuations and instrumental drifts than the straightforward ''one-shot'' measurement of $\Delta G(t)$. Using this technique it was shown[4] that τ depends on disorder, magnetic fields, and on the carrier-concentration n.[5] The dependence on n turns out to be very dramatic; τ changes by almost three orders of magnitude when n is varied in the range 4×10^{19}–$5\times 10^{20}\,\mathrm{cm}^{-3}$. This nontrivial observation led us to conclude that electron-electron interactions must play a major role in the slow relaxation process.[5]

Figure 2 shows $A_{1,2}(t)$ for two samples with different n and the way τ is determined from such data. This figure illustrates three points. First, the decrease of τ with decreasing n. Second, note that a logarithmic law fits the $A_{1,2}(t)$ data fairly well over three decades. Third, the behavior of $A_1(t)$ and $A_2(t)$ is ''complementary''—one grows at the *same* rate at which the other diminishes. We shall use this interesting symmetry later on.

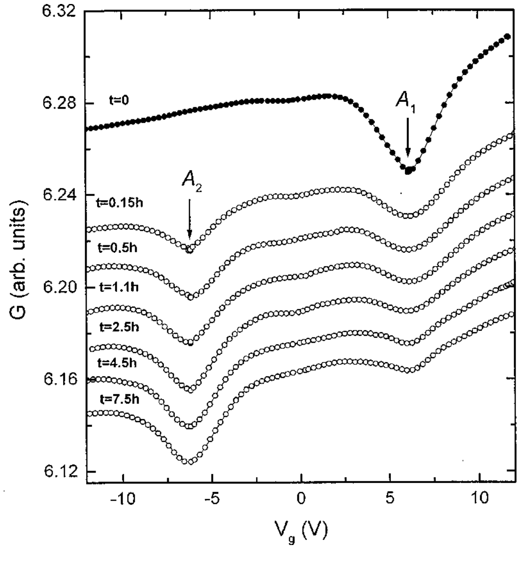

FIG. 1. An illustration of the TDE. The right dip (A_1) occurs at the cool-down value of V_g and decays in time. The left dip (A_2) occurs at the value to which V_g is switched at time $t=0$ and grows with t. $R_\square = 3.8\,\mathrm{M}\Omega$ and $T=4.2\,\mathrm{K}$.

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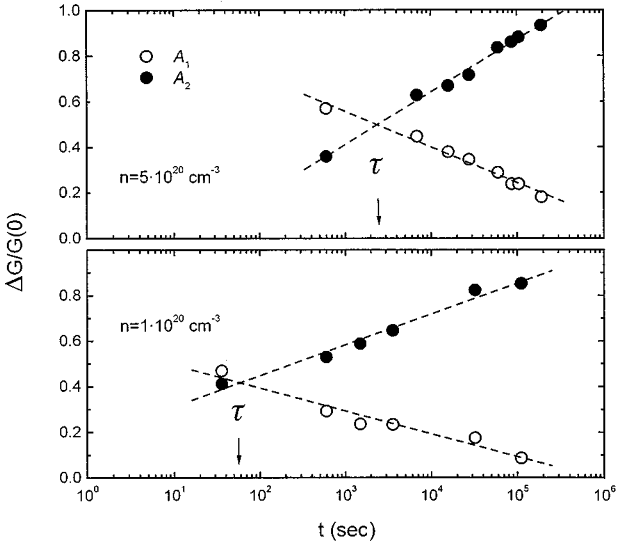

FIG. 2. The time dependence of A_1, A_2 for two different carrier concentrations n. The experimental ''relaxation time'' τ is determined by the t at which $A_1 = A_2$. Notice the logarithmic dependence of A_1, A_2 on t and the symmetry in their slopes.

In this paper we offer a heuristic picture to explain some of these experimental observations. The model we consider assumes the existence of a wide spectrum of transition rates of a variable-range-hopping system. This of course is general and inherent to the hopping system whether interactions are present or not. Depicting the relaxation as a ''dressing'' process pictorially includes the effects due to intercarrier interactions. The model explains the $\Delta G(t) \propto -\ln(t)$ dependence of the relaxation for relatively short times and the faster dependence at later times. It also explains why at a certain limit, the characteristic relaxation time appears to be only weakly dependent on some parameters such as disorder or temperature.

EXPERIMENT

Samples used in this study are thin films (200 Å thick) of amorphous indium-oxide *e*-gun evaporated on 0.5 μm thick $\mathrm{SiO_2}$ layer thermally-grown on a heavily doped Si wafer. The latter served as the gate electrode. In this system we have the advantage of being able to control the carrier-concentration n over a wide range (10^{19}–$3\times10^{22}\,\mathrm{cm}^{-3}$) by varying the oxygen/indium ratio during film deposition. Disorder energy could be varied for each sample by thermal annealing.[8] Combining these two it was possible to produce a series of samples with different n but with similar conductivity at 4.1 K.

Conductivity of the samples was measured using a two terminal ac technique employing a 1211-ITHACO current preamplifier and a PAR-124A lock-in amplifier. Fuller details of sample preparation and measurement techniques are given elsewhere.[4,5]

THE MODEL

We consider the case of an Anderson insulator excited by applying a sudden voltage step at the gate. When a field is applied, the structure of the quasiparticles changes because the field modifies the site potentials or because the concentration of particles changed. To make the transition from one quasiparticle structure to the other, some particles become ''bare'' and via subsequent transitions then rearranged into new quasiparticles. Similarly when the concentration is suddenly changed, bare particles enter the system and they gradually form quasiparticles. The formation of quasiparticles in glasses is a hierarchical process.[9] In such a process the motion of particles becomes gradually correlated with more and more other particles and their mobility decreases rapidly, as the ''dressing'' becomes more complete.

In the simplest possible model, the particles contribute to excess conduction only while they are totally bare. In other words, we neglect the contribution of ''partially dressed'' particles to the conduction. The excess conductance is then proportional to $n(t)$. We farther assume that the generation of bare particles from the ''old'' quasiparticles is as immediate as the excitation, e.g., change of local potential with a field.

To calculate $\Delta G(t)$ we thus need to know the number of bare particles as a function of time $n(t)$, i.e., how many survive at time t as undressed particles. The dressing of particles is controlled by a distribution of transition rates w. As is usual in hopping cases, the rates w depend exponentially on some random variable x (hopping distance and/or energy), $w = w_0 \exp[-x]$. The random variable x is distributed with a function $N(x)$. Then $N(w)dw = N(x)dx$, or

$$N(w) = N(x(w))\frac{dx}{dw} = \frac{d}{dw}\left[-\ln\frac{w}{w_0}\right]N\left[-\ln\frac{w}{w_0}\right]$$
$$= -\frac{w_0}{w}N\left[-\ln\frac{w}{w_0}\right].$$

We now calculate $n(t)$. The number of particles with w, $n(w,t)$ which survive to time t is $n(0)N(w)\exp(-wt)dw$, so

$$\begin{aligned} n(t) &= \int n(0)N(w)\exp(-wt)dw \\ &= \int n(0)\left\{-\frac{w_0}{w}N\left[-\ln\frac{w}{w_0}\right]\right\}\exp(-wt)dw \\ &= \int n(0)\left\{-\frac{w_0}{wt}N\left[-\ln\frac{w}{w_0}\right]\right\}\exp(-wt)d(wt) \\ &= -n(0)w_0\int\left\{-\frac{1}{x}N\left[-\ln\frac{x}{w_0 t}\right]\right\}\exp(-x)dx. \qquad (1) \end{aligned}$$

Ignoring for now the slow t dependence inherent in the logarithmic dependence of N, $n(t)$ appears to be time independent. This of course cannot be exact. The reason for this problem is that we ignored the physical need for the existence of some minimum rate $w_{\min}$. (There is of course also the maximum rate w_0 [see Eq. (1)], but it is clear from the above that this is irrelevant for $t > w_0^{-1} \approx 10^{-12}$ sec.) If we do incorporate the need for $w_{\min}$ as the lower limit of integration over w (in the simplest way by assuming a constant distribution of x between some $x_{\min}$ and $x_{\max}$), the last integral in Eq. (1) becomes $\int_{w_{\min}t}^{\infty}(1/x)e^{-x}dx$. This is a well known[10] function of $w_{\min}t$. The expansion for small $w_{\min}t = x_m$ is

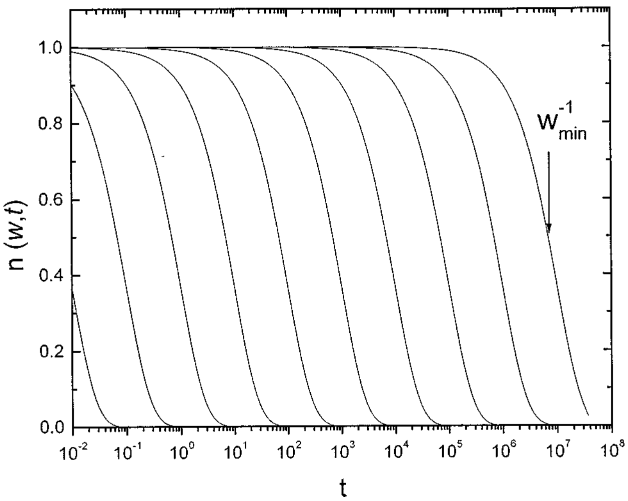

FIG. 3. The decay of ''undressed'' particles (see text) with decay rates w taken to be exponential functions of a random variable x, $w=w_0\exp[-x]$. The figure corresponds to a uniform distribution of x with an upper cutoff at x_m and thus a minimum allowed $w_{\min}=w_0\exp[-x_m]$.

$$\gamma-\ln(x_m)-\sum_{n=1}^{\infty}(-1)^n\frac{x_m^n}{n\cdot n!}, \tag{2}$$

where γ is the Euler constant. The terms of the sum alternate in sign (the first is negative). Since for small x_m the magnitude of the terms in this sum decrease with increasing n, the sum over all terms must be negative so that $-\Sigma_{n=1}^{\infty}(-1)^n(x_m^n/n\cdot n!)>0$, with the absolute value increasing with x_m. It therefore enhances $-\ln(x)$ (also positive for $x_m<1$), and hence renders the relaxation to be gradually faster than $\ln(w_{\max}t)$ as t increases.

It should be stressed that the derivation of Eq. (2) neglected any effects of the distribution $N(w)$ other than to consider the need for a cutoff at some $w_{\min}$. The main justification for such an assumption is that the argument of N changes only logarithmically with t [see Eq. (1)]. Thus, the change in the argument of N may not be significant for a considerable change in t. Over reasonably large intervals of time, the time dependence of Eq. (2) can be approximated by $t^{-\alpha}$ with a small value of α. Such a behavior is indeed observed in our experiments.[4]

The very slow decay of $n(t)$ can be best understood by plotting the integrand of Eq. (1) on a $\log(t)$ scale. On such a scale, the function $\exp(-wt)$ resembles a step function, unity for $t<w^{-1}$ and zero for $t>w^{-1}$. The density $N(w)$ is mainly proportional to $1/w$ which makes the density uniform on a $\log(t)$ scale: $N(w)=dN/dw\propto 1/w$, so $w(dN/dw)=(dN/d\ln w)\propto w^0=\text{const}(w)$. The situation is represented in Fig. 3. The curves represent the functions $n(w,t)$, i.e., the functions $\exp(-wt)$ in accordance with the required distribution $N(w)$, with decreasing w from left to right all the way to $w_{\min}$. At any t, the sum over all the curves is proportional to $n(t)$. The decrease of $n(t)$ within some interval t corresponds to the number of curves which decayed within this interval. Say t progresses for several decades, but such that $w_{\min}t\ll 1$. Then relatively few curves of all those remaining up to $w_{\min}$ have decayed, so the relative decrease of $n(t)$ over its time interval is very small.

We now wish to comment on the assumption that ΔG arises only from the presence of bare particles. We could have considered just as well some later stage of the hierarchy where the particles are already partially dressed. After the free particles undergo the first stage of dressing, i.e., after $1/w_{\min}$, their motion becomes correlated with a gradually larger number of particles and their mobility keeps decreasing. At any stage of the hierarchy, relaxation is governed by a spectrum of rates, which are exponentials of random variables. The decay of the conductivity will thus be described by the same equations as those derived above for the bare particle stage, but with x ranging over successively larger values, with gradually lower mobility, and with gradually smaller $w_{\min}$, during successive stages of the hierarchy. The functional dependence $\Delta G(t)$ will be the same for each stage, but with a scaled down ΔG and a scaled up t. The question of which stage of dressing is observed then merely depends on the range of experimental times. The above discussion suggests that generality is not lost by assuming that the system is at the first stage of the hierarchical evolution.

The above picture would result in a sequence of relaxations of ΔG which may not be smoothly connected. There are two natural ways in which the total relaxation curve is smoothed out. The realistic distribution $N(x)$ is not cut off sharply at x_m, and the various stages of hierarchical dressing will overlap in time, i.e., at any given time different particles will be at different stages of dressing.

We now return to the experimental results. As already noted, Eq. (2) agrees with previously reported measurements of $\Delta G(t)$ inasmuch as at short t (presumably $t<w_{\min}^{-1}$) the relaxation is as $\log(t)$ and at longer t the relaxation becomes somewhat faster and representable by a power law. This behavior was found for a wide variety of parameters, except for samples with low carrier concentration where the relaxation is much faster. In terms of the above model this can be interpreted as a rather large $w_{\min}$, such that $w_{\min}^{-1}$ is no longer much larger than t. All this also ties in well with the experimentally observed behavior of τ. Whenever $\Delta G(t)$ can be fitted to a power law, τ is observed to remain nearly constant. This can be interpreted within the model in the following way. As long as the $t\cdot w_{\min}\ll 1$, even though the rates w for the individual microscopic processes may change drastically, e.g., by a change of the temperature T or by increasing disorder. For demonstration we use the fact that the power law is a scaling function obeying $n(ct)=c^{-\alpha}n(t)$. Then the equation which defines τ gives $A_1(t_0)=2A_1(t_0+\tau)=2A_1(t_0\{[t_0+\tau]/t_0\})$, so $c=[t_0+\tau]/t_0$. The scaling property of A_1 results in $(1/2)A_1(t_0)=c^{\alpha}A_1(t_0)=A_1(ct_0)$, or $1/2=\{[t_0+\tau]/t_0\}^{\alpha}$ which gives

$$\tau=(2^{1/a}-1)t_0. \tag{3}$$

So τ depends only on α and on t_0 and we can expect τ to be the same for all samples as long as α remains the same (and we keep the same t_0).

In experiments reported earlier[5] we performed a systematic study of $\tau(n)$ on a series of samples while the conductance, t_0, and temperature were all kept constant. For n above a certain value we observed a constant value of τ, which was about 1500 s. The experimental behavior of $A_1(t)$ at times $t<\tau$ could be well described by[4] $A_1(ct)\propto(ct^{-0.2})$.

Using Eq. (2) with $\alpha = -0.2$ and $t_0 = 30\,\mathrm{s}$ one obtains $\tau \approx 1000\,\mathrm{s}$, in fair agreement with the experimental value.

The situation changes when the *microscopic* relaxations w become fast enough so $w_{\min}^{-1}$ approaches the observation time t such that x_m in Eq. (2) is no longer small. The contribution from the sum in Eq. (2) then rapidly increases in importance as $w_{\min}^{-1}$ approaches t, and $n(t)$ can no longer be approximated by a power law over a large range of t. As a result the *global* relaxation of $n(t)$ becomes rapidly faster and τ smaller as $w_{\min}^{-1}$ gets closer to t. When $w_{\min}^{-1} < t$, $n(t)$ decays as $\exp(-w_{\min} t)$, i.e., the system becomes ergodic.

A dramatic drop in the value of τ was observed as the concentration n fell below a certain value. Presumably the rapid decrease of $\tau(n)$ signals a transition towards a different, faster mode of decay, perhaps even to ergodic relaxation. It is of interest to see whether this transition might be to a regime where interactions cease to be important.

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