Isotope effect in the presence of a pseudogap

T. Dahm*
Max-Planck-Institute for Physics of Complex Systems, Nöthnitzer Straße 38, D-01187 Dresden, Germany
and Institut für Theoretische Physik, Universität Tübingen, Auf der Morgenstelle 14, D-72076 Tübingen, Germany
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We investigate the question of whether the unusual doping dependence of the isotope exponent observed in underdoped high-T_c superconductors might be related to another unusual phenomenon observed in these systems: the pseudogap phenomenon. Within different approximations we study the influence of a phenomenological pseudogap on the isotope exponent and find that it generally strongly increases the isotope exponent, in qualitative agreement with experiments on underdoped high-T_c compounds. This result is stable against strong-coupling self-energy corrections and also holds for recently proposed spin-fluctuation exchange models, if a weak additional electron-phonon coupling is considered.

I. INTRODUCTION

The isotope effect in high-T_c cuprate superconductors is unconventional in different respects. Optimally doped samples show a very small isotope exponent α of the order of 0.05 or even smaller, in contrast to the conventional Bardeen-Cooper-Schrieffer (BCS) value of 0.5, which one expects for a conventional phonon induced pairing interaction.[1] This unusually small value in connection with the high value of T_c lead to early suggestions that the pairing interaction in high-T_c cuprates might be predominantly electronic in origin with a possible small phononic contribution.[2] This scenario, however, is difficult to reconcile with the fact that the isotope coefficient also shows an unusually strong doping dependence, reaching values of 0.5, in some cases even higher, in the underdoped, T_c reduced, compounds.[1,3,4]

Many different models have been advanced in order to try to understand this unusual doping dependence in connection with the small isotope exponent at optimal doping, e.g., influence of van Hove singularities,[5–7] anharmonic phonons,[8,9] electron-phonon coupling in the presence of strong antiferromagnetic correlations,[10,11] pair breaking effects,[12] magnetic impurities, or Jahn-Teller nonadiabaticity,[13,14] but no consensus has been reached so far.[15]

In recent years it became apparent that the physics of underdoped high-T_c superconductors is governed by the pseudogap phenomenon. A behavior which is reminiscent of the presence of a pseudogap, growing upon successive underdoping, has been observed consistently in a large number of different experiments, e.g., nuclear magnetic resonance (NMR) Knight-shift and relaxation rate experiments, specific heat, angular-resolved photoemission spectroscopy (ARPES), tunneling, *c*-axis, and *ab*-plane dynamical conductivity.[16,17] Currently there is no consensus about the origin of this pseudogap and many different proposals exist.[18–25] Williams and co-workers[17] have shown that a phenomenological model for a pseudogap having *d*-wave symmetry can account well for thermodynamic quantities in the underdoped cuprates.

In the present manuscript we want to study whether the influence of the pseudogap might give a, perhaps more natural, explanation for the unusual isotope effect in underdoped high-T_c cuprates. As has been shown by Carbotte and co-workers,[26,27] an energy dependence of the electronic density of states (DOS) varying on the pairing energy scale can modify the isotope effect, and therefore we ask whether there might be a link between the pseudogap phenomenon and the isotope effect. Since there exists no widely accepted theory for the pseudogap at present, here we will follow the idea of Williams and co-workers and treat the pseudogap on a phenomenological basis, introducing it into the single-particle excitation spectrum. Such a procedure is reasonable, if the pseudogap itself does not show an isotope effect, as suggested by recent NMR experiments.[28,29]

In the following we will study the influence of such a pseudogap on the isotope effect within different models. We shall start with the weak-coupling (BCS) approximation where we consider *s*- and *d*-wave symmetry of the superconducting order parameter and of the pseudogap. In order to see whether these results are stable for more realistic cases, we will study two recently proposed models based on a spin-fluctuation exchange pairing interaction, e.g., the nearly antiferromagnetic Fermi-liquid (NAFL) model due to Monthoux and Pines[30] and the self-consistent fluctuation-exchange (FLEX) approximation for the two-dimensional Hubbard model.[31]

II. WEAK-COUPLING APPROXIMATION

The linearized gap equation in the weak-coupling limit for an anisotropic pairing interaction $V(\vec{k},\vec{k}')$ reads

$$\Delta(\vec{k}) = \frac{1}{N}\sum_{k'} V(\vec{k},\vec{k}')\frac{\tanh(\epsilon_{k'}/2T_c)}{2\epsilon_{k'}}\Delta(\vec{k}'). \qquad (1)$$

Here, ϵ_k is the band dispersion and $\Delta(\vec{k})$ the superconducting gap function. We want to assume that the pairing interaction consists of two parts: a phononic part $V_p(\vec{k},\vec{k}')$ and an electronic part $V_e(\vec{k},\vec{k}')$, such that $V(\vec{k},\vec{k}') = V_p(\vec{k},\vec{k}') + V_e(\vec{k},\vec{k}')$. The dominant contribution shall be V_e. Then, in weak-coupling approximation we have

$$V_e(\vec{k},\vec{k}')=\begin{cases} V_{e0}\psi_\eta(\vec{k})\psi_\eta(\vec{k}') & \text{if } |\epsilon_k|,|\epsilon_{k'}|\leqslant\omega_e \\ 0 & \text{else,}\end{cases} \tag{2}$$

where ω_e is the characteristic energy scale of the electronic part and is assumed to be independent of isotopic mass. $\psi_\eta(\vec{k})$ is the basis function for the pairing symmetry considered. For *s*-wave pairing $\psi_s(\vec{k})=1$, for $d_{x^2-y^2}$-wave pairing $\psi_{d_{x^2-y^2}}(\vec{k})=\cos 2\Theta_k/\sqrt{2}$, and for d_{xy}-wave pairing $\psi_{d_{xy}}(\vec{k})=\sin 2\Theta_k/\sqrt{2}$, where $\Theta_k=\arctan(k_y/k_x)$ is the angular direction of the momentum $\vec{k}$.

The phononic part may consist of different contributions having different symmetries. However, since we assumed that the electronic part is dominating with a symmetry specified by ψ_η, only the ψ_η component of V_p, having the same symmetry, will affect T_c. Therefore we can assume without loss of generality

$$V_p(\vec{k},\vec{k}')=\begin{cases} V_{p0}\psi_\eta(\vec{k})\psi_\eta(\vec{k}') & \text{if } |\epsilon_k|,|\epsilon_{k'}|\leqslant\omega_p \\ 0 & \text{else,}\end{cases} \tag{3}$$

where ω_p is the characteristic phonon energy. In harmonic approximation, which we will adopt here, ω_p varies with the isotopic mass M like $1/\sqrt{M}$, while ω_e is assumed to be independent of M.

For such an interaction the gap function can be separated into two parts: $\Delta(\vec{k})=\Delta_e(\vec{k})+\Delta_p(\vec{k})$, with

$$\Delta_{e,p}(\vec{k})=\begin{cases} \Delta_{e0,p0}\psi_\eta(\vec{k}) & \text{if } |\epsilon_k|\leqslant\omega_{e,p} \\ 0 & \text{else.}\end{cases} \tag{4}$$

With this ansatz Eq. (1) becomes a 2×2 matrix equation for the two order-parameter components Δ_{e0} and Δ_{p0}. Assuming a cylindrical Fermi surface with a constant density of states, Eq. (1) can be written in the form

$$\begin{pmatrix}\Delta_{e0}\\ \Delta_{p0}\end{pmatrix}=\begin{pmatrix}V_{e0}L(\omega_e) & V_{e0}L(\omega_e)\\ V_{p0}L(\omega_e) & V_{p0}L(\omega_p)\end{pmatrix}\begin{pmatrix}\Delta_{e0}\\ \Delta_{p0}\end{pmatrix}, \tag{5}$$

where we defined the function $L(\omega)$

$$L(\omega)=N(0)\int_0^\omega d\epsilon\frac{\tanh\left(\frac{\epsilon}{2T}\right)}{\epsilon}\simeq N(0)\ln\left(\frac{1.13\omega}{T}\right). \tag{6}$$

The last expression holds in the weak-coupling limit $\omega\gg T$. $N(0)$ denotes the density of states at the Fermi level. In deriving Eq. (5) we assumed $\omega_e\leqslant\omega_p$, as is usually the case for spin-fluctuation exchange models (see the following sections). However, the final result Eq. (13) does not depend on this choice. Letting $L_p=L(\omega_p)$ and $L_e=L(\omega_e)$ the leading eigenvalue of the matrix in Eq. (5) is

$$\lambda(\omega_e,\omega_p,T)=\frac{V_{e0}L_e+V_{p0}L_p}{2}+\frac{1}{2}\sqrt{(V_{e0}L_e-V_{p0}L_p)^2+4V_{e0}V_{p0}L_e^2} \tag{7}$$

and T_c is determined from the implicit equation

$$\lambda(\omega_e,\omega_p,T_c)=1. \tag{8}$$

From this the isotope exponent α_0 can be calculated:

$$\alpha_0=\frac{1}{2}\frac{d\ln T_c}{d\ln\omega_p}=-\frac{1}{2}\frac{\omega_p}{T_c}\frac{\frac{\partial\lambda}{\partial L_p}\frac{\partial L_p}{\partial\omega_p}}{\frac{\partial\lambda}{\partial L_p}\frac{\partial L_p}{\partial T_c}+\frac{\partial\lambda}{\partial L_e}\frac{\partial L_e}{\partial T_c}}. \tag{9}$$

In the weak-coupling limit $\omega_p,\omega_e\gg T_c$ this gives

$$\alpha_0=\frac{1}{2}\frac{V_{p0}(1-V_{e0}L_e)}{V_{p0}(1+V_{e0}L_e)+V_{e0}(1-V_{p0}L_p)}. \tag{10}$$

Note, that for a purely electronic interaction $V_{p0}=0$ this expression yields $\alpha_0=0$ and for a purely phononic interaction $V_{e0}=0$ it gives $\alpha_0=0.5$, as one should expect. For a mixed interaction α_0 will generally lie between 0 and 0.5. In fact, one can easily show that for given values of ω_p and ω_e one can always choose V_{p0} and V_{e0} in such a way that a given value of T_c and $\alpha_0\in[0,0.5]$ is reached.[32]

Now we wish to consider the influence of a pseudogap. In the presence of a pseudogap we have to modify the single-particle excitation spectrum. Following Williams and coworkers, we replace in Eq. (1)

$$\epsilon_k\Rightarrow\sqrt{\epsilon_k^2+E_g^2(\vec{k})}, \tag{11}$$

where $E_g(\vec{k})$ is the pseudogap and will be chosen to be either $E_{g,s}(\vec{k})=E_{g0}=\text{const}$ for an *s*-wave pseudogap or $E_{g,d}(\vec{k})=E_{g0}\cos 2\Theta_k$ for a *d*-wave-type pseudogap. Note that this symmetry of the pseudogap does not necessarily have to be identical with the pairing symmetry and we will allow them to be independent in this section. However, the study in Ref. 17 suggests that both symmetries are of *d*-wave type in underdoped high-T_c compounds and we will focus on this case in the following sections. With the replacement Eq. (11) the function L becomes

$$L(\omega)=\frac{N(0)}{2\pi}\int_0^{2\pi}d\Theta\,\psi_\eta^2(\Theta)\int_0^\omega d\epsilon\frac{\tanh\left(\frac{\sqrt{\epsilon^2+E_g^2(\Theta)}}{2T}\right)}{\sqrt{\epsilon^2+E_g^2(\Theta)}}. \tag{12}$$

Equations (7) and (9) still remain valid, if one uses this expression for $L(\omega)$. In the weak-coupling limit $\omega_p,\omega_e\gg T_c,E_g$ we then find for the isotope exponent

$$\alpha=\alpha_0\left(\frac{1}{4\pi T_c}\int_0^{2\pi}d\Theta\int_0^\infty d\epsilon\frac{\psi_\eta^2(\Theta)}{\cosh^2\frac{\sqrt{\epsilon^2+E_g^2(\Theta)}}{2T_c}}\right)^{-1}, \tag{13}$$

where α_0 is the isotope exponent Eq. (10) in the absence of a pseudogap. Equation (13) shows that α/α_0 only depends on E_{g0}/T_c, the pairing symmetry $\psi_\eta(\Theta)$ and the symmetry of the pseudogap. Since T_c is a function of E_{g0}, determined from Eq. (8), for a given symmetry of both the pseudogap and the pairing state α/α_0 is a *universal* function of T_c/T_{c0}. Here, $T_{c0}=T_c(E_g=0)$. In Fig. 1 we show α/α_0 as a function of T_c/T_{c0} for different symmetries. The solid line shows

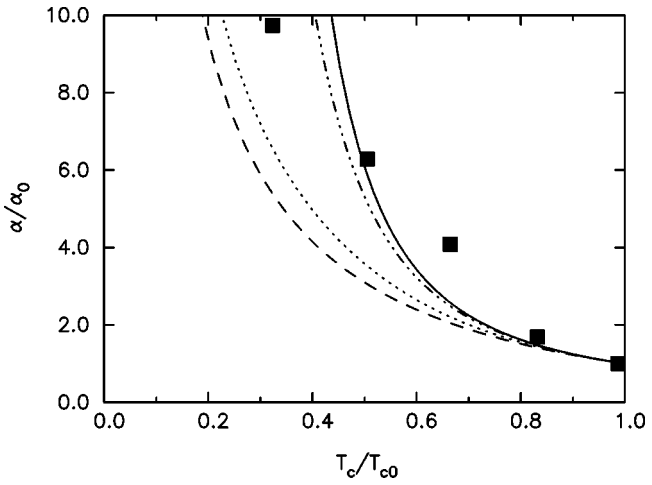

FIG. 1. Weak-coupling result Eq. (13) for the isotope exponent α/α_0 as a function of T_c/T_{c0} in the presence of a pseudogap. α_0 and T_{c0} denote the values in the absence of the pseudogap. The solid line shows the result for an *s*-wave pseudogap. For a $d_{x^2-y^2}$-wave pseudogap the results for an *s*-wave pairing symmetry (dotted line), a $d_{x^2-y^2}$-wave pairing symmetry (dashed-dotted line), and a d_{xy}-wave pairing symmetry (dashed line) are shown. The solid squares are experimental results on Pr-doped YBCO from Ref. 1.

the isotope exponent for an *s*-wave pseudogap. This result is independent of the pairing symmetry, as can be seen by performing the angular integration in Eq. (13). For an anisotropic pseudogap having $d_{x^2-y^2}$-wave symmetry, however, the pairing symmetry does affect the result. The dotted line shows the result for an *s*-wave superconductor with a $d_{x^2-y^2}$-wave pseudogap, while the dashed-dotted line shows the result for a $d_{x^2-y^2}$-wave superconductor with a $d_{x^2-y^2}$-wave pseudogap. The weakest T_c/T_{c0} dependence is found for a d_{xy} superconductor with a $d_{x^2-y^2}$-wave pseudogap (dashed line). In all cases one can see from Eq. (13) that α/α_0 diverges for $T_c \to 0$. Thus in principle arbitrarily high values of α can be reached. As an illustration, experimental results on Pr-doped YBCO are shown in this figure as solid squares.[1] Here, it should be noted that experimental results on different compounds can differ somewhat and also vary with the dopant used (see Ref. 1). Certainly these differences need further explanation (e.g., see the review in Ref. 14) and cannot be understood solely due to the influence of the pseudogap. Here, we only want to focus on the influence of a pseudogap alone and investigate the general tendency and order of magnitude of the effect, which is similar in many compounds.

As an important conclusion we can draw from these weak-coupling results that a pseudogap in general leads to an increase of the isotope exponent α over its value α_0 in the absence of a pseudogap. The quantitative size of this effect depends on the symmetries of the pseudogap and the pairing state. However, the qualitative behavior is very similar in all cases. The size of α_0 can become small, if a strong electronic coupling constant V_{e0} and a small phononic coupling V_{p0} is considered.

III. STRONG-COUPLING EFFECTS: NAFL MODEL

Having seen that a pseudogap can lead to an increase of the isotope exponent in a weak-coupling superconductor, one might wonder whether this effect will survive in more realistic models for superconductivity. In order to see, how strong-coupling effects affect the results, we want to consider a recently proposed spinfluctuation exchange model, the nearly antiferromagnetic Fermi liquid (NAFL) model due to Monthoux and Pines.[30] Within this model the pairing interaction is provided by exchange of antiferromagnetic spinfluctuations and the pairing symmetry is $d_{x^2-y^2}$. The (frequency-dependent) pairing interaction is given by

$$V(\vec{q}, i\nu_m) = g^2 \chi(\vec{q}, i\nu_m), \tag{14}$$

where g is a coupling constant, ν_m the Bose-Matsubara frequencies, and the spin susceptibility χ is given by

$$\chi(\vec{q}, i\nu_m) = \frac{\chi_Q}{1 + \xi^2(\vec{q} - \vec{Q})^2 + \nu_m/\omega_s}. \tag{15}$$

Here, $\vec{Q} = (\pi, \pi)$ is the antiferromagnetic wave vector, ξ is the magnetic correlation length, and ω_s the characteristic spin-fluctuation frequency. Using this interaction the Migdal-Eliashberg equations for strong-coupling superconductors are solved self-consistently. Here one has to solve for the self-energy Σ,

$$\Sigma(\vec{k}, i\omega_n) = \frac{1}{N} \sum_{k',n'} V(\vec{k} - \vec{k}', i\omega_n - i\omega_{n'}) G(\vec{k}, i\omega_n), \tag{16}$$

along with Dyson's equation for the Green's function G,

$$G(\vec{k}, i\omega_n) = \frac{1}{i\omega_n - \epsilon_k - \Sigma(\vec{k}, i\omega_n)} \tag{17}$$

self-consistently. Using this solution, T_c is determined from the linearized gap equation

$$\phi(\vec{k}, i\omega_n) = -\frac{1}{N} \sum_{k',n'} V(\vec{k} - \vec{k}', i\omega_n - i\omega_{n'}) \times |G(\vec{k}, i\omega_n)|^2 \phi(\vec{k}, i\omega_n), \tag{18}$$

where ϕ is the gap function. For the band structure ϵ_k a tight-binding band with next-nearest-neighbor hopping has been used in Ref. 30 and we will adopt that here.

In order to have a small nonzero isotope exponent at optimal doping we consider coupling to an additional phonon mode, the ''buckling'' mode studied in Refs. 10, 33, and 34. This mode provides an attraction in the $d_{x^2-y^2}$-wave channel and its pairing interaction reads

$$V_p(\vec{q}, i\nu_m) = V_{p0} \left(\cos^2 \frac{q_x}{2} + \cos^2 \frac{q_y}{2} \right) \frac{\omega_p^2}{\nu_m^2 + \omega_p^2}. \tag{19}$$

We do not expect the main results to depend strongly on the details of the electron-phonon spectrum, as long as its coupling strength is small compared with the spin-fluctuation interaction. It is important, however, that the electron-phonon interaction has an attractive component in the $d_{x^2-y^2}$-wave channel, as has been discussed above Eq. (3). For the calculations we choose the parameters given by Monthoux and Pines: $\xi = 2.3a$, $\chi_Q = 44$ states/eV, hopping

TABLE I. Coupling constants for the NAFL model with an additional coupling to the buckling mode Eq. (19) for different values of the spin-fluctuation frequency ω_s. $E_{g,\mathrm{supp}}$ denotes the value of the pseudogap E_{g0}, which completely suppresses T_c.

ω_s/t	g/t	λ_{ph}	$E_{g,\mathrm{supp}}/T_{c0}$
0.03	5.1	0.31	12.3
0.06	3.2	0.15	6.5
0.2	2.4	0.10	4.0

matrix element $t=250$ meV, where a is the lattice constant.[30] For the spin-fluctuation frequency ω_s we choose three different values $0.03t$, $0.06t$, and $0.2t$, in order to study the crossover from weak-coupling to strong-coupling behavior. For the characteristic phonon frequency we choose a typical value of $\omega_p=0.2t$. Following Ref. 30, the interaction strengths g and V_{p0} are adjusted such that the transition temperature becomes $T_{c0}=90$ K and the isotope exponent reaches $\alpha_0=0.05$. Results for the coupling constants are shown in Table I. Here, the electron-phonon coupling constant λ_{ph} is defined in the usual way:

$$\lambda_{ph}=2\int_0^\infty \frac{d\Omega}{\Omega}\frac{1}{N}\sum_q \frac{1}{\pi}\,\mathrm{Im}\{V_p(\vec{q},\Omega+i\delta)\}. \qquad (20)$$

From Table I we can see that higher coupling constants g are required for smaller spin-fluctuation frequencies. Comparatively small values of the electron-phonon coupling constant are sufficient to yield an isotope exponent $\alpha_0=0.05$.

In order to study the influence of a pseudogap we introduce a d-wave pseudogap $E_g(\vec{k})=E_{g0}\cos 2\Theta_k$, as suggested by the analysis of Williams and co-workers,[17] into the single-particle excitation spectrum by replacing in the single-particle Green's function Eq. (17),

$$\epsilon_k+\mathrm{Re}\,\Sigma \Rightarrow \pm\sqrt{(\epsilon_k+\mathrm{Re}\,\Sigma)^2+E_g^2(\vec{k})}. \qquad (21)$$

Here it is necessary to take into account the real part of the self-energy Σ, since the pseudogap opens at the Fermi surface, which is renormalized due to the self-energy. Table I also shows the amplitude of the pseudogap, denoted by $E_{g,\mathrm{supp}}$, which completely suppresses T_c to 0. Experimentally, the ratio of $E_{g,\mathrm{supp}}$ to T_{c0} is about 6–15, depending on the material.[16,17] The values found here indeed turn out to be of this order of magnitude. Note, that the renormalization of the pseudogap $E_g(\vec{k})$ due to the self-energy Σ is taken into account in this approximation. In contrast to the weak-coupling approximation in the previous section, the pseudogap as seen in the density of states is washed out now and thus is a real ''pseudo'' [25] gap.

In Fig. 2 we show α as a function of T_c for three different values of the spin-fluctuation frequency ω_s along with the weak-coupling result for a $d_{x^2-y^2}$-wave superconductor with a $d_{x^2-y^2}$-wave pseudogap from Fig. 1. For higher values of the spin-fluctuation frequency α gradually approaches the weak-coupling result as one should expect. For small $\omega_s=0.03t$ there are some deviations from the weak-coupling limit. However, the results are not affected very much.

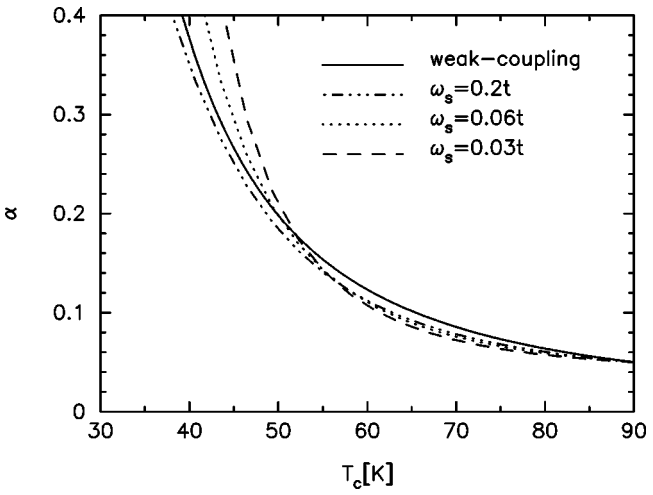

FIG. 2. Isotope coefficient α as a function of T_c for the NAFL model with an additional coupling to the buckling phonon mode. The opening of the pseudogap leads to a suppression of T_c and an increase of α. The solid line shows the corresponding weak-coupling result from Fig. 1. Results are shown for different values of the characteristic spin-fluctuation frequency: $\omega_s=0.03t$ (dashed line), $\omega_s=0.06t$ (dotted line), and $\omega_s=0.2t$ (dashed-dotted line). In each case the coupling constants have been adjusted such that $T_{c0}=90$ K and $\alpha_0=0.05$.

IV. SELF-CONSISTENT FLEX APPROXIMATION

Within the NAFL model the spin-fluctuation pairing interaction is fixed and does not change with the electronic properties. However, it is clear both experimentally and theoretically that the pseudogap does affect the spin susceptibility and thus should affect the spin-fluctuation pairing interaction itself. To study such kind of effects it is necessary to treat the electronic properties and the electronic pairing interaction in a self-consistent way. Such a self-consistent treatment is provided by the so-called fluctuation-exchange (FLEX) approximation[31] for the two-dimensional Hubbard model and also yields a $d_{x^2-y^2}$-wave superconducting state.[35–37] The main difference with the NAFL model is that the spin susceptibility is calculated from the interacting Green's functions within an random-phase approximation-type approximation. Then the pairing interaction reads

$$V(\vec{q},i\nu_m)=\frac{3}{2}U^2\frac{\chi_0(\vec{q},i\nu_m)}{1-U\chi_0(\vec{q},i\nu_m)}, \qquad (22)$$

where the bubble susceptibility χ_0 is calculated from the fully dressed single-particle Green's function G [Eq. (17)] self-consistently:

$$\chi_0(\vec{q},i\nu_m)=-\frac{1}{N}\sum_{k,n} G(\vec{k}+\vec{q},i\omega_n+i\nu_m)G(\vec{k},i\omega_n). \qquad (23)$$

This guarantees that any change in the one particle Green's function is reflected in the pairing interaction and vice versa. In Fig. 3 we show $\alpha(T_c/T_{c0})$ within FLEX approximation with an additional coupling to the buckling mode already considered in the NAFL model above. Results are shown for different values of the on-site Hubbard repul-

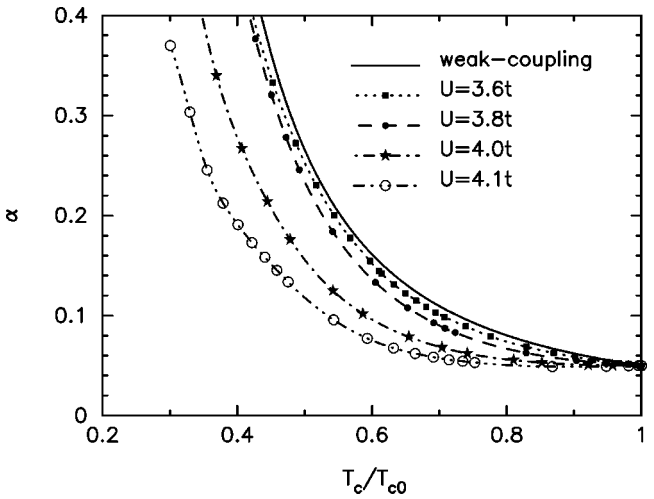

FIG. 3. Isotope coefficient α as a function of T_c/T_{c0} within FLEX approximation with an additional coupling to the buckling phonon mode. Results for different values of the on-site Hubbard repulsion U are shown: $U=3.6t$ (dotted line), $U=3.8t$ (dashed line), $U=4.0t$ (dashed-dotted line), and $U=4.1t$ (dashed-dot-dotted line). The solid line shows the corresponding weak-coupling result from Fig. 1.

sion U along with the weak-coupling result. Here, a band filling of $n=0.84$ for a simple tight-binding band has been assumed. The electron-phonon coupling strength again has been adjusted to give $\alpha_0=0.05$ for $E_{g0}=0$. For $U\leqslant 3.6t$, $\alpha(T_c/T_{c0})$ very much follows the weak-coupling limit. Only if U reaches values of the order of $4t$ or higher, deviations from the weak-coupling limit become apparent. For higher values of U, $\alpha(T_c/T_{c0})$ becomes flatter and starts to rise only at smaller values of T_c. This is a consequence of the influence of the pseudogap on the spin-fluctuation pairing interaction. In contrast to the NAFL model the opening of the pseudogap not only reduces T_c via the reduction of single-particle spectral weight at the Fermi level, but it also suppresses the lowest frequency spin fluctuations. Since these are predominantly pair breaking,[38,39] this increases the effective coupling strength of the spin fluctuations[40] as compared with the phonons and thus leads to a reduction of α.

V. CONCLUSIONS

We studied the influence of a pseudogap on the isotope exponent for different models having an electronic pairing interaction with a subdominant electron-phonon interaction. In the weak-coupling limit we found that the introduction of a pseudogap leads to a strong increase of the isotope exponent above its value in the absence of a pseudogap. For $T_c \rightarrow 0$ the isotope exponent diverges, allowing arbitrarily high values. The symmetries of the order parameter and the pseudogap only lead to quantitative, but not qualitative changes of these results. Strong-coupling effects within the NAFL model do not affect the results very much. The size of the pseudogap compared with T_c turns out to be of the right order of magnitude. Self-consistent treatment of the spin-fluctuation pairing interaction in the presence of the pseudogap can lead to stronger deviations from the weak-coupling limit. The general tendency that the isotope exponent rises upon opening of the pseudogap still remains, however. From these results it does not seem unreasonable that the pseudogap indeed can have an important influence on the isotope effect and might be, at least partially, responsible for the increasing isotope exponent in the underdoped cuprates.

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*Electronic address: thomas.dahm@uni-tuebingen.de

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