Critical currents and the Ambegaokar-Baratoff to Ginsburg-Landau crossover in granular superconductors

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A simple expression for the transport critical current, which describes rather well the Ambegaokar-Baratoff (A-B) to Ginsburg-Landau (G-L) crossover, is obtained from an elementary analysis of a Josephson junction subject to external magnetic fields within the Ginsburg-Landau theory. We show that the order parameter in the intergrain region depends crucially on the interplay of two characteristic lengths: the superconducting coherence length $\xi \propto (1-T/T_c)^{-1/2}$ and the junction-induced superconducting decoherence length $\zeta_q \propto (1-T/T_c)^{-\alpha}$. Data for the transport critical current in granular YBCO samples are presented, and the A-B to G-L crossovers successfully described.

I. INTRODUCTION

Anomalous critical current J_c behaviors have been observed since the early days of high-T_c superconductivity.[1] An interesting phenomenon, apparently peculiar to granular superconductors, is the change of curvature (convex to concave) in the temperature dependence of J_c. This change known as the crossover from the Ambegaokar-Baratoff (A-B) regime, where $J_c \propto (1-T/T_c)^{1/2}$, to the Ginsburg-Landau (G-L) regime where $J_c \propto (1-T/T_c)^{3/2}$ is evident in various experimental reports though rarely noticed.[2–4] To fit the temperature dependence of critical current data, it has been rather common to consider $J_c \propto (1-T/T_c)^{\alpha}$, with different values for the exponent α.[5–10] In some cases, as predicted by de Gennes (dG) for superconductor–normal-metal–superconductor SNS junctions,[11] exponents of the order of 2 have been conveniently used, especially for temperature regions close to T_c.[5,6] In other cases, it was quite favorable to fix α as the ratio m/n, where $m=1$ or 3 when the grains are respectively larger or smaller than the Josephson penetration length and $n=1$ or 2, depending on whether the junctions are SNS- or superconductor-insulator-superconductor (SIS)-like.[8–10] Based on the Ginsburg-Landau equations and assuming, as usual, an exponentially decaying order parameter, with clear junction suppression effects in the intergranular tunneling region, we obtain a critical current formula which fits rather well the temperature and magnetic field behavior of the data. For a fixed magnetic field, the A-B to G-L crossover is described with only one value of the exponent α.

To put it in perspective, we recall that the A-B to G-L crossover effect was originally noticed in granular NbN superconductors which Clem *et al.*[4] interpreted as a current-induced suppression of the gap parameter. The critical current was described in terms of the ratio $\epsilon = E_J/2E_s$, between the Josephson-coupling energy E_J, and the superconducting condensation energy of a grain, E_s. In the dirty limit ($\epsilon \gg 1$) the critical current density reduces to the Ginsburg-Landau depairing current proportional to $(1-T/T_c)^{3/2}$. It was also quite natural to explain the crossover by invoking the interplay of intragrain and intergrain current-induced depairing, grain sizes, proximity effects, flux creep and depinning effects, etc. Recently, Darhmaoui and Jung,[12] observed A-B to G-L crossover for intragrain, intergrain, depairing, and depinning critical currents in thin films and ceramic YBCO samples. From their data, they conclude that the crossover is independent of the type of flux-pinning defect and of the crystallographic direction of the supercurrent's flow. They also find that the A-B to G-L crossover effect is independent of whether the sample is ceramic or a granular film. Xu *et al.*[13] gave a possible explanation for the observed magnitude of the Josephson critical voltage J_cR_n assuming mixed $s+id$ pairing states, but their Josephson critical current expression does not account for the convex-concave crossover. Widder *et al.*[14] present a model where the critical current numerical calculations fit well, in the d-wave pairing for a single grain boundary, and which describe the A-B to G-L crossover by solving self-consistently quasiclassical differential equations.

The experimental work and the theoretical explanations provide clear indications that this effect is a sort of universal property closely related to the granular characteristic of the superconducting samples and strongly induced by external magnetic fields. Although there have been some attempts to obtain expressions for J_c to describe this kind of temperature and magnetic field dependence behavior, we are not aware of any explicit formula which accounts for this behavior in terms of only one critical exponent. It is the aim of the present work to analyze the junction's effect on the superconducting order parameter within the Ginsburg-Landau theory, and to obtain simple expressions for $J_c(B_{\rm ext},T)$ to fit the data and to describe the A-B to G-L crossover for all

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values of the applied magnetic field. It turns out that the order parameter and the critical current sensitivity to thin-tunnel junctions and external magnetic fields is reflected in the interplay of two fundamental characteristic lengths: the superconducting coherence and decoherence length. The very appealing junction-induced decoherence length $\zeta_q = \zeta_0(1-T/T_c)^{-\alpha}$ arises quite naturally from our analysis. This quantity plays an important role not only in the suppression of the order parameter and critical current in granular superconductors, but also in the A-B to G-L crossover.

In the next section we present a simple analysis of the Ginsburg-Landau (G-L) equations[15] and obtain a new expression for $J_c(B_{\rm ext},T)$, which is then successfully used in Sec. III to fit new and old experimental data.

II. SUPPRESSION OF THE ORDER PARAMETER AND THE A-B TO G-L CROSSOVER

The existence of nonzero supercurrents in the polycrystalline cuprates, as a consequence of the coherent interaction of electron pairs across the intergrain regions, has been widely explained in terms of the well-accepted weak-superconductor behavior[16,17] and the well-known quantum extension of the superconducting state into the tunneling region. To describe the A-B to G-L crossover as an effect of intergranular tunneling regions on the critical current, we revisit the well-studied junction's critical current problem within a phenomenological approach where an order parameter, compatible with tunneling and magnetic field penetration effects, is proposed and the temperature behavior of the free parameter is fixed through the Ginsburg-Landau equations. We assume, for the intergrain region (see Fig. 1), the order parameter

$$\Psi(r) = a[g_L(z)e^{i\varphi_L(r)} + g_R(z)e^{i\varphi_R(r)}], \tag{1}$$

where φ_L and φ_R are the superconducting phases in the corresponding grains, and

$$g_{L,R}(z) = \frac{\sqrt{n_{L,R}}}{2}[1-\tanh q(\delta_J + d/2 \pm z)] \tag{2}$$

the attenuation factor that can be written as

$$g_{L,R}(z) = \frac{\sqrt{n_{L,R}}}{2}\exp[-q(\delta_J + d/2 \pm z)]\text{sech}$$
$$\times[q(\delta_J + d/2 \pm z)]. \tag{3}$$

Here d is the grain's separation, n_j (with $j=L,R$) refers to the superconducting carrier concentration, considered equal in both grains, δ_J is the Josephson penetration length, and q is a kind of attenuation parameter, whose temperature dependence will be deduced here. We shall assume the same r dependence for both superconducting phases.

The attenuation factors g_L and g_R, as written in Eq. (3), contain exponentially decaying amplitudes characteristic of quantum tunneling, enhanced by the penetration of the magnetic field, and a hyperbolic function $\text{sech}[q(\delta_J + d/2 \pm z)]$ which approaches 1 as the argument tends to zero, and exponentially to zero in the asymptotic regions. Because of the proximity effect the junction behaves as a microbridge and the intergrain region is taken as a weakened extension of the superconducting domain.

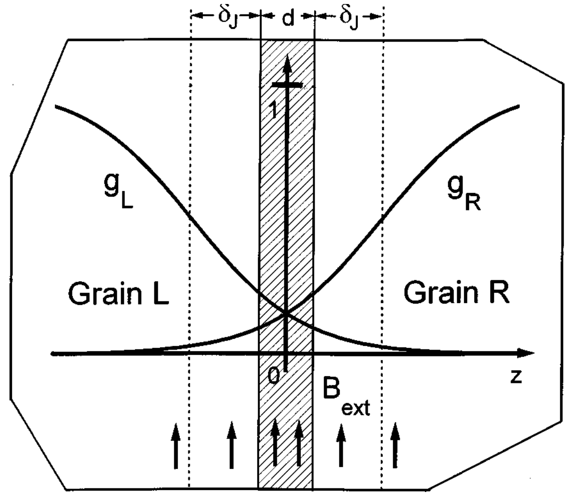


FIG. 1. The intergrain microbridge junction. The order parameter decays because of the intergrain barrier and the magnetic-field penetration.

On the other hand, one of the well-known G-L equations, in the Coulomb gauge $\nabla\cdot\mathbf{A}=0$, is

$$\left\{-\nabla^2 + i\frac{4e}{\hbar}\mathbf{A}\cdot\nabla + \frac{4e^2}{\hbar^2}|\mathbf{A}|^2 - \frac{1}{\xi^2}\left(1-\frac{|\Psi(z)|^2}{|\Psi_o|^2}\right)\right\}\Psi = 0, \tag{4}$$

with $\mathbf{A}$ the magnetic vector potential, Ψ_0 the equilibrium order parameter, and ξ the superconducting coherence length. We assume that the main contribution to the phase shift is caused by the magnetic field in the tunneling region, i.e., $\nabla\varphi = (2e/\hbar)\mathbf{A}$, which leads to the well-known flux quantization for any closed path in the tunneling region.[18,19] Replacing the intergranular order parameter of Eq. (1) in the previous Ginsburg-Landau equation, the real part (at $z=0$) reduces to

$$\left[-q^2\Gamma(q,\delta_J,d) - \frac{1}{\xi^2}\left(1-\frac{|\Psi|^2}{|\Psi_o|^2}\right)\right]|\Psi|^2 = 0, \tag{5}$$

where $\Gamma(q,\delta_J,d) = \tanh[q(\delta_J + d/2)](1+\tanh[q(\delta_J + d/2)])$. It is easy to verify that this equation implies the relation

$$|\Psi|^2 = |\Psi_0|^2(q^2\Gamma\xi^2 + 1). \tag{6}$$

Since $\Psi_0 \propto (1-T/T_c)^{1/2}$ and the coherence length $\xi \propto (1-T/T_c)^{-1/2}$, it is clear that as the critical temperature is reached the vanishing of the order parameter Ψ requires

$$q^2\Gamma\xi^2 \propto (1-T/T_c)^{-1+\beta}, \quad \text{with } \beta>0. \tag{7}$$

Accordingly the attenuation factor q [introduced in Eqs. (1)–(3)] must be a function of the temperature. More precisely, it should behave as

$$q = q_0(1-T/T_c)^{\alpha}. \tag{8}$$

For our purposes, this is an important result that will be used below. This analysis suggests the relevance of another characteristic length, the junction-induced decoherence length ζ_q defined as the inverse of q, closely related to the junction's width and the magnetic-field penetration length. The superconducting decoherence length is then given by

$$\zeta_q = \zeta_o (1 - T/T_c)^{-\alpha},$$

thus

$$|\Psi|^2 = |\Psi_o|^2 (\xi^2 \Gamma / \zeta_q^2 + 1). \quad (9)$$

In this way, the behavior of the order parameter $|\Psi|$ in the intergrain region, extended by the magnetic field penetration, depends basically on the interplay of two characteristic lengths: the coherence length ξ and the coherence-decay length ζ_q. Both quantities depend critically on T and as will be seen now, determine not only the low values of J_c but also its temperature behavior.

If we replace the order parameter of Eq. (1) in the current density formula

$$\mathbf{j} = -\frac{i\hbar e}{2m}(\Psi^* \nabla \Psi - \Psi \nabla \Psi^*) - \frac{2e^2}{m}|\Psi(r)|^2 \mathbf{A}, \quad (10)$$

we obtain the following magnetic field-dependent expression

$$j \propto \frac{1 - \tanh[(\delta_J + d/2)/\zeta_q]}{\zeta_q \cosh^2[(\delta_J + d/2)/\zeta_q]} \sin\left(\varphi_2 - \varphi_1 - \frac{2e}{\hbar}\oint \mathbf{A}\cdot d\mathbf{l}\right). \quad (11)$$

After averaging over the junction's surface and taking into account the temperature dependence of q, we have

$$j_c(B_{\text{ext}}, T) = A\tau^{\alpha+0.5}\left(\frac{1 - \tanh[\tau^\alpha(\delta_J + d/2)/\zeta_0]}{\cosh^2[\tau^\alpha(\delta_J + d/2)/\zeta_0]}\right)\frac{\sin\frac{\pi\Phi}{\Phi_0}}{\frac{\pi\Phi}{\Phi_0}}, \quad (12)$$

where Φ is the magnetic flux (proportional to B_{ext} and the junctions threaded area), Φ_0 is the quantum of magnetic flux, and $A = 2\hbar|a|^2 n_j e/\zeta_0 m_e$. Equation (12) is the function we were looking for. Our analysis was based on the quite plausible assumption of an exponentially decaying order parameter in the tunneling region. The resulting critical current contains a polynomial and a hyperbolic temperature dependence. The hyperbolic factor dominates at low temperatures and causes the suppression of the critical current. Both the junction's width and the magnetic-field penetration length are important quantities, which reflect not only the granular characteristic but determine also the current suppression mechanism. Above the crossover temperature, i.e., in the G-L or in the dG regime, the critical current is dominated by the polynomial factor. We must notice, however, that once the magnetic field is fixed, the critical current of Eq. (12) fits the data all the way from low temperature to T_c *with only one value of the exponent* α. The presence of an external magnetic field increases the exponent in the first factor to $\alpha + 1/2$, and reduces the crossover temperature. As will be shown in the next section, to fit our data α varies from 0.8 at $B = 0$ to $\simeq 2.2$ for larger magnetic fields.

Formally, the main difference between the critical current obtained here and those found in the literature resides in the temperature dependence of the attenuation factor q, or equivalently, on the temperature dependence of its inverse: the decoherence length ζ_q. This gives us not only the possibility to fit data, but also to provide additional insights into the effect of granularity and Josephson couplings on the cuprate's critical currents. Therefore the change of curvature in the transport critical current in polycrystalline superconductors is basically a property of granular origin. The junction-induced order-parameter suppression obviously affects the intergrain and the intragrain currents. The present analysis was done independently of pairing symmetry considerations. It is possible, however, that the attenuation factor could be sensitive to pairing symmetries.

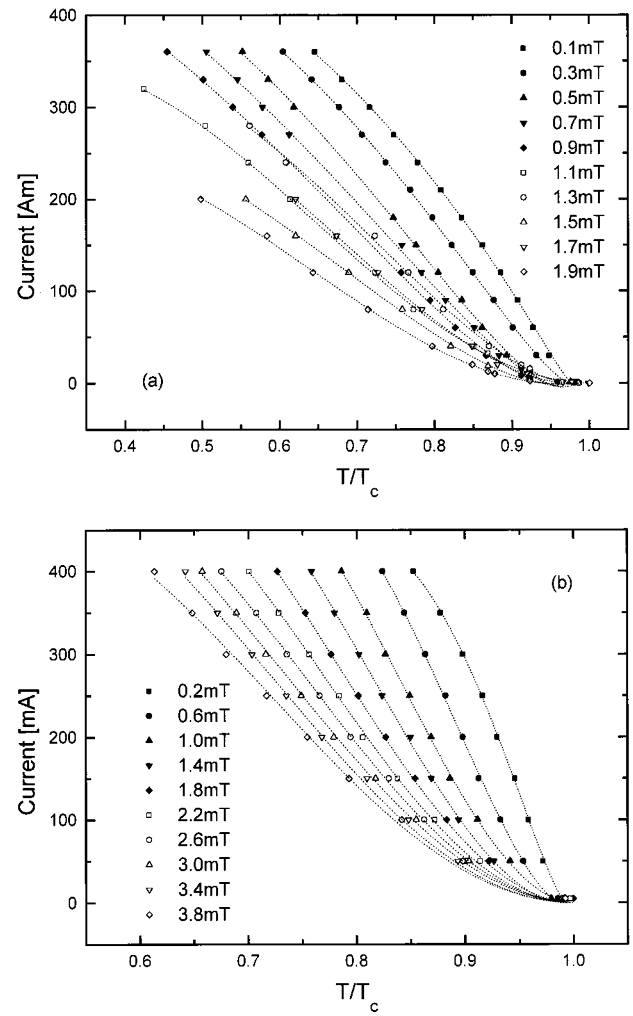
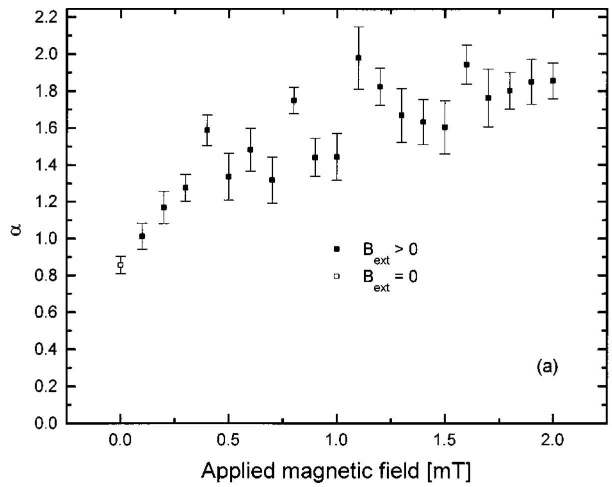


FIG. 2. Transport critical current for samples with different oxygenation periods (5 and 12 h), and for different values of the external magnetic field. For each dotted curve, obtained using Eq. (14), we need only one value of α. The inflection points move to lower temperatures as the magnetic field increases.

III. TRANSPORT CRITICAL CURRENT AND THE A-B TO G-L CROSSOVER IN EXPERIMENTAL DATA

Assuming that the whole current is a linear combination of single-junction critical-current contributions, it is clear that the temperature dependence deduced in the preceding section persists at the macroscopic size. We then propose a total transport critical current described by



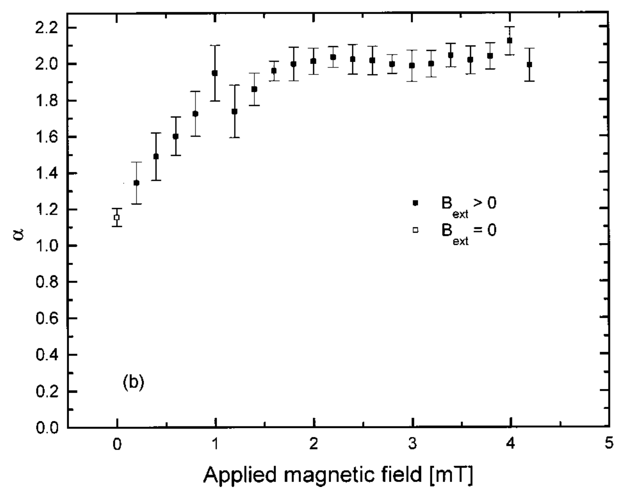


FIG. 3. The exponent α as a function of the external magnetic field. In (a) we have this parameter for sample Y5, and in (b) for Y12.

$$I_c = I_0 \tau^{\alpha+0.5} \left(\frac{1-\tanh \tau^{\alpha}\eta}{1-\tanh \eta_0} \right) \times \left(\frac{\cosh^2 \eta_0}{\cosh^2 \tau^{\alpha} \eta} \right) \frac{\sin\left(\frac{\pi \Phi_{\text{eff}} (1-T/T_c)^{-1/2}}{\Phi_o} \right)}{\sin\left(\frac{\pi \Phi_{\text{eff}}}{\Phi_0} \right)}, \quad (13)$$

where I_0 is the zero temperature critical current, $\eta = (\delta_J + d/2)/\zeta_0$, $\Phi_{\text{eff}} = B_{\text{ext}} a_{\text{eff}}$ is a kind of zero-temperature effective flux through an effective area a_{eff}, which generally depends on the magnetic-field and the zero-temperature Josephson's penetration length δ_0, and η_0 is related with η and defined as follows. In the limit $\delta_J \gg d$, the parameter $\eta \simeq \eta_0 \tau^{-0.5}$, where $\eta_0 \simeq \delta_0/\zeta_0$. These parameters and Φ_{eff} are magnetic-field and sample-dependent quantities, as shown below. On the other hand, in the absence of magnetic field the parameter η reduces to $d/2\zeta_0$. This case will also be discussed below.

In Figs. 2(a) and 2(b) we used the critical current function of Eq. (13) to fit the experimental data. These data have been measured for two polycrystalline Y-Ba-Cu-O samples with different oxygenation periods (5 and 12 h), and critical temperatures of 92 K, which we will denote as Y5 and Y12. To measure the critical current, four-probe indium contacts were made on parallelepiped-shaped samples, which were then cooled to temperatures of 20 K. At this point an external magnetic field B_1 and a transport current I_1 through the sample were fixed, allowing the temperature to slowly go up. At a certain value of the temperature, say T_{c1}, the onset of the transition to the normal state was reached with a precision of 0.1 μV. Once the voltage difference grew to approximately 10 μV, we lowered the transport current to $I_2 < I_1$, and the sample went back to its superconducting state while the temperature continued to slowly rise. With the same magnetic field B_1 and lower transport current I_2, a new transition point at temperature $T_{c2} > T_{c1}$ was reached, and so on. This procedure was repeated increasing the magnetic field in stepwise fashion. An interesting feature of the experimental data is the systematic change in the curvature of J_c as function of T near T_c. The inflection points move to lower temperatures as the magnetic field increases. Each curve corresponds to a fixed value of the external magnetic field and only one value of α is used to fit the data. In other words, for a given sample and fixed magnetic field, the temperature behavior including the A-B to G-L crossover is described with only one value of α. Varying the magnetic field causes α to change as shown in Figs. 3(a) and 3(b). Slightly larger values of α characterize the 12-h oxygenation sample Y12. Even for low magnetic fields the J_c curves of this sample are much more concave than those of the 5-h oxygenation sample Y5. Similar behavior has been observed in Ref. 10, where two exponents α_{far} and α_{close} were used for each curve. The numerical values of α vary there also between 0.8 and 2.2. The conclusions of Ref. 10 concerning the grain sizes, conducting properties of the intergrain material, and the ratio between the weak and the Josephson junctions, remain the same. We also show in Fig. 4 the magnetic field behavior for η_0. From this figure it is clear that the decoherence length $\zeta_0 > \delta_0$ for the Y5 sample while $\zeta_0 < \delta_0$ for the Y12 sample. This is a clear consequence of the different oxygenation times for these samples, as the decoherence length contains both the influence of the intergrain and the intragrain properties.

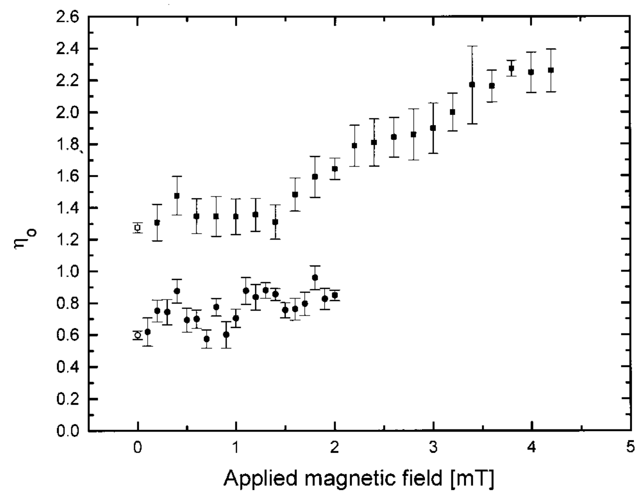


FIG. 4. The parameter η_0 as a function of the external magnetic field. The points lying on the upper part correspond to Y12 and those with lower values to Y5. As explained in the text, the points at $B = 0$ are fitted with the parameter $d/2\zeta_0$.

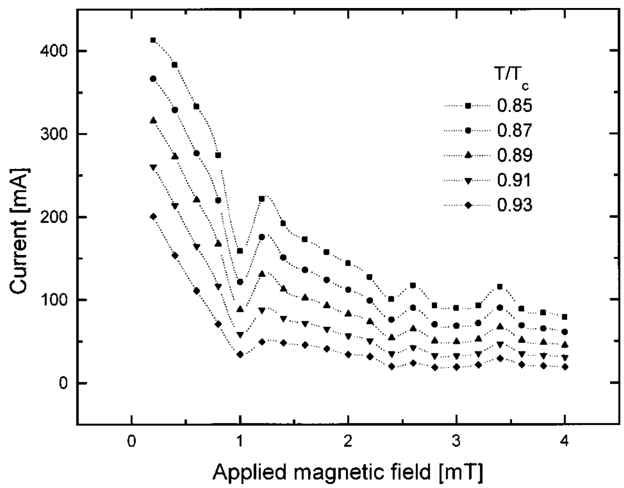


FIG. 5. Transport critical current as a function of the magnetic field. These points were obtained from the data appearing in Fig. 2(b) and using Eq. (13). The dotted curves are guidelines.

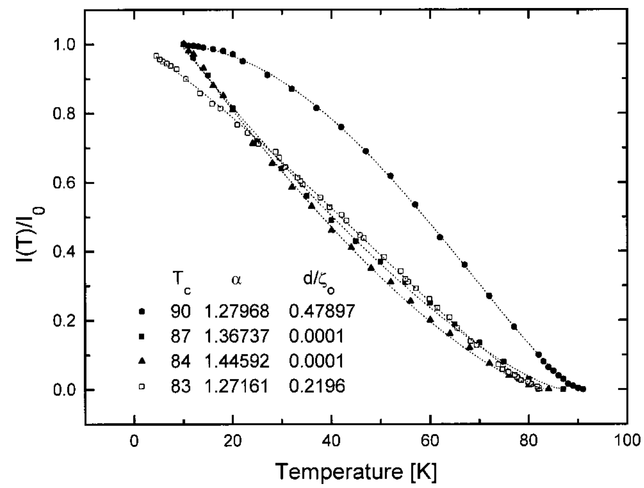


FIG. 6. Experimental data taken from Darhmaoui and Jung (Ref. 12) (filled points) and Widder et al. (Ref. 14) (open points).

From our data for the critical current as a function of temperature and using Eq. (13), we obtain the isothermals shown in Fig. 5. This kind of behavior is predicted for the long junction limit.[20] It is interesting to notice that, as functions of the magnetic field, the critical current decreases while the α exponent reaches a kind of limiting value.

Notice that for $B=0$ and $d/2\zeta_0 \simeq 0$, the critical current of Eq. (13) reduces to

$$I_c(T,0) = I_0(1 - T/T_c)^{\alpha}. \qquad (14)$$

In Fig. 6, we plot data from Refs. 12 and 14 which correspond to $B=0$. Two of these curves have a typical G-L behavior. Using our Eq. (13) we fit these data with $\alpha \simeq 1.4$ and $d/2\zeta_0 \simeq 0$. This means that, for these samples, the grain separation is small or the intragrain depairing mechanisms have strong influence. The other two curves in Fig. 6 exhibit the A-B to G-L crossover. To fit these data we require $d/2\zeta_0 > 0$, while $\alpha \simeq 1.3$. All these tell us that the G-L regime is reached by the confluence of a large decoherence length ζ and α exponents of the order of 3/2, augmented by the magnetic field. On the other hand, it is evident that lower α values correspond to critical currents with the AB-GL crossover temperature closer to T_c.

IV. CONCLUSIONS

In this paper we studied the effect of intergrain regions on the temperature and magnetic field dependences of the critical current. We introduced an exponentially suppressed order parameter for the intergrain region and, in the framework of the Ginsburg-Landau theory, we have deduced the temperature dependence of an attenuation constant which leads us to consider the junction induced superconducting decoherence length ζ_q as a relevant parameter in the description of transport current in tunneling regions. This quantity and the standard current definition is used to deduce a new expression for the critical current, which describes rather well the A-B to G-L crossover observed in the temperature dependence of critical currents measured in thin film and ceramic Y-Ba-Cu-O samples. The current of Eq. (13) allows us also to fit data obtained in the presence of external magnetic fields. The critical exponents take values between 0.8 and 2.2, depending on the magnetic field. As mentioned previously the suppression of the order parameter is mainly due to the granular characteristic of high T_c cuprates. Our result supports the suggested universality of the A-B to G-L and dG crossover for granular superconductors. We believe that a further analysis of the microscopic description of Josephson junctions will lead to different critical exponents of the gap parameter.

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