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Bose glass scaling for superconducting vortex arrays revisited

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Lidmar and Wallin have recently pointed out that Bose glass scaling theory predicts a linear cusp in the phase boundary of vortex matter with correlated disorder as a function temperature of temperature and perpendicular magnetic field. Here, we collect a number of consequences of this observation for physical quantities near the Bose glass transition.

In an interesting recent paper, Lidmar and Wallin[1] have pointed out that a consistent theory of the angular scaling associated with Bose-glass superconductors requires careful attention to the distinction between the magnetic *field* ($\mathbf{B}_\perp$) and magnetic *induction* ($\mathbf{H}_\perp$) in the direction perpendicular to the correlated disorder (typically a mosaic of twin boundaries or parallel columnar pins). While their observation does not change the prediction of a sharp cusp near the Bose glass critical point in the phase diagram as a function of T and $H_\perp$, it does change certain details of the scaling predictions derived in Ref. 2. In particular, Lidmar and Wallin find that the predicted cube root cusp in the critical temperature as a function of angle is replaced by a linear one for very small angles, and present numerical evidence in favor of this prediction.[1] A number of experiments which begin to test various predictions of Bose glass scaling theory have now been carried out.[3–6] As an aid to future experiments,[7] we summarize here the changes in the scaling predictions of Ref. 2 necessitated by the observation of Lidmar and Wallin. We also point out related changes in the predictions of Refs. 8 and 9.

The Bose glass transition at temperature $T_{\rm BG}$ in type-II superconductors with parallel columnar defects (or a mosaic of twin boundaries) in an external field $\mathbf{H}$ in the $\mathbf{z}$ direction aligned with the correlated disorder is characterized by a diverging length scale perpendicular to the correlated disorder,

$$l_\perp(T)\sim\frac{1}{|T-T_{\rm BG}|^{\nu_\perp}}. \tag{1}$$

Scaling arguments adapted from treatments of the Bose glass transition in helium films on disordered substrates[10] lead to the prediction[2]

$$l_\parallel(T)=\frac{Tn_0^2}{c_{11}}l_\perp^2(T). \tag{2}$$

where n_0 is the vortex line density (if ϕ_0 is the flux quantum, $n_0=B_z/\phi_0$), c_{11} is the vortex bulk modulus and we have set $k_B=1$.

A simple derivation of this important relation results from the quantum mechanical mapping of Ref. 2. We note that the z axis (i.e., fictitious time) evolution of the coherent states ψ is given by the Schrödinger term $T\psi^*\partial_z\psi\sim T/l_\parallel l_\perp^2$, since the localization radius for a coherent state is $l_\perp[\int d^2r|\psi(\mathbf{r})|^2=\text{const}]$. At the same time the contribution from the vortex repulsion is $v_0|\psi|^4\sim v_0/l_\perp^4$, where v_0 represents the vortex repulsion constant. Balancing these two contributions to the Bose coherent state action and taking into account that for the model with the only core vortex repulsion we have $c_{11}=v_0n_0^2\sim v_0|\psi|^4$, one arrives immediately at Eq. (2).

As a result of Eq. (2), $l_\parallel(T)\sim l_\perp^2(T)$ if, as expected, c_{11} remains finite at the Bose glass transition. The time scale for relaxation of a typical fluctuation with dimensions $l_\perp\times l_\perp\times l_\parallel$ is assumed to diverge with an undetermined exponent z according to

$$\tau(T)\sim l_\perp^z(T), \tag{3}$$

similar to a scaling ansatz proposed earlier for a possible ''vortex glass'' transition mediated by point disorder.[11]

Dynamical predictions for transport experiments with currents $J_\perp$ perpendicular to the disorder follow from a scaling ansatz of the form

$$\mathcal{E}_\perp l_\perp^{1+z}=\mathcal{F}_\pm(l_\perp^3J_\perp,l_\perp^xH_\perp), \tag{4}$$

where $\mathcal{E}_\perp$ is the electric field perpendicular to the columns and different scaling functions $\mathcal{F}_\pm(x,y)$ are required above and below $T_{\rm BG}$. Simple physical arguments[2] fix all powers of the fundamental length $l_\perp(T)$ except the power x which appears in combination with $H_\perp$. Although one might have thought that $H_\perp$ should be multiplied by $l_\parallel(T)l_\perp(T)$ (and hence $x=3$), Lidmar and Wallin show that in fact $x=1$. Their argument can be viewed as a consequence of the form of the diverging tilt modulus derived in Ref. 2. Let us start with the standard renormalization group homogeneity relation[12] with scale factor b for the free energy density $F(t,H_\perp)$ of the three-dimensional vortex array, as a function of the reduced temperature $t=(T-T_{\rm BG})/T_{\rm BG}$ and $H_\perp$,

$$F(t,H_\perp)=b^{-4}F(b^{1/\nu_\perp}t,b^xH_\perp). \tag{5}$$

The scaling exponent x which accompanies $H_\perp$ should be the same as that appearing in Eq. (4). The prefactor b^{-4} arises because the free energy density scales as the inverse characteristic volume which in this case is $l_\perp^2l_\parallel$ (there are two di-

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mensions transverse to the disorder of order $l_\perp^2$, and one is parallel to it of order $l_\parallel \sim l_\perp^2$). After differentiating this expression twice with respect to $H_\perp$ ($c_{44}^{-1} \propto \partial^2 F/\partial H_\perp^2$), we obtain a scaling law for the tilt modulus $c_{44}(t,H_\perp)$, namely,

$$c_{44}(t,H_\perp) = b^{4-2x} c_{44}(b^{1/\nu_\perp} t, b^x H_\perp). \tag{6}$$

After setting $H_\perp = 0$ and choosing the scale factor b so that $b^{1/\nu_\perp} t = 1$, we see that the singular behavior of c_{44} is

$$c_{44}(T) \sim l_\perp^{4-2x}. \tag{7}$$

As discussed in Ref. 2, $c_{44}(T)$ is expected to *diverge* at the Bose glass transition according to

$$c_{44}(T) = T n_0^2 l_\parallel(T) = \frac{(T n_0^2)^2}{c_{11}} l_\perp^2(T) \tag{8}$$

and, upon comparing Eqs. (7) and (8), we see that $x = 1$.

After inserting various factors to make the arguments dimensionless, Eq. (4) becomes

$$\mathcal{E}_\perp l_\perp^{1+z} \sim \mathcal{F}_\pm (l_\parallel l_\perp J_\perp \phi_0 / cT, l_\perp H_\perp \phi_0 / 4\pi T). \tag{9}$$

The first argument is a ratio of the work done by the Lorentz force associated with current $J_\perp$ to move a length $l_\parallel$ of flux line a distance $l_\perp$ to the thermal energy. The second argument follows by starting with the natural dimensionless scaling combination $B_\perp l_\perp l_\parallel / \phi_0$, setting $B_\perp \approx (\partial B_\perp / \partial H_\perp) H_\perp$ and using[2]

$$\left(\frac{\partial B_\perp}{\partial H_\perp} \right) = \frac{\phi_0^2 n_0^2}{4\pi c_{44}} \tag{10}$$

together with Eq. (8).[13] The analogous scaling form for the longitudinal electric field $\mathcal{E}_\parallel$, considered in Ref. 8, is

$$\mathcal{E}_\parallel l_\parallel l_\perp^z \sim G_\pm (l_\perp^2 J_\parallel \phi_0 / cT, l_\perp H_\perp \phi_0 / 4\pi T). \tag{11}$$

Various theoretical predictions now follow from these scaling laws. The results presented for $H_\perp = 0$ in Refs. 2 and 8 are unchanged. However, as pointed out by Lidmar and Wallin, the cusped phase boundary for separating the Bose glass from the entangled flux liquid now takes the form[1]

$$H_\perp^c(T) \sim \pm (T_{BG} - T)^{\nu_\perp}, \tag{12}$$

where it is expected that $\nu_\perp = 1$.[10] The linear resistivity perpendicular to the columns $\rho_\perp$ vanishes for small $H_\perp$ like

$$\rho_\perp(T = T_{BG}, H_\perp) \sim (H_\perp)^{z-2} \tag{13}$$

and obeys more generally the scaling form

$$\begin{aligned} \rho_\perp(t,\theta) &= |t|^{\nu_\perp(z-2)} f_\pm(\theta/|t|^{\nu_\perp}) \\ &\approx \rho_0 |t|^{\nu_\perp(z-2)} [1 + A\theta^2 |t|^{-2\nu_\perp}], \end{aligned} \tag{14}$$

where the last line requires $\theta \ll |t|^{\nu_\perp}$. Here $\theta \approx H_\perp / H_z \ll 1$ is the tilt angle away from the direction of the correlated disorder and the last line applies for $T > T_{BG}$. The corresponding results[8] for the longitudinal resistivity $\rho_\parallel$ (parallel to the columns) are modified to

$$\rho_\parallel(T = T_{BG}, H_\perp) \sim |H_\perp|^z, \tag{15}$$

while the more general scaling form is

$$\rho_\parallel(t,\theta) = |t|^{\nu_\perp z} g_\pm(\theta/|t|^{\nu_\perp}). \tag{16}$$

The Lidmar and Wallin observation also has important implications for a Harris criterion type argument for the stability of the Bose glass phase to *splayed* columnar defects.[9] Random splay acts like a random transverse magnetic field $H_\perp$, whose order of magnitude is given by $\theta_{rms} H_z$, where θ_{rms} is the root-mean-square tipping angle of a set of splayed columnar pins. Assume for simplicity a Gaussian distribution of tipping angles with variance Δ for a set of columnar pins with average spacing d in a plane perpendicular to the mean column direction $\hat{\mathbf{z}}$. In a ellipsoidal correlation volume just below T_{BG}, we have $\theta_{rms} \sim \Delta/\sqrt{N_c}$, where the number of columnar defects piercing that volume is

$$N_c \approx \frac{l_\perp^2}{d^2} + \frac{l_\perp l_\parallel}{d^2} \Delta.$$

The second term dominates near the Bose glass transition, and[9]

$$\theta_{rms} \sim \Delta^{1/2} d/(l_\perp l_\parallel)^{1/2} \sim |t|^{3\nu_\perp/2}.$$

Evidently the root-mean-square tipping angle averaged over a correlation volume vanishes *faster* than the angle $\theta_c(t)$, which defines the limit of the transverse Meissner effect as $T \to T_{BG}$. Indeed, we have from Eq. (12), that

$$\theta_c(t) \sim \frac{H_\perp^c(T)}{H_z} \sim |t|^{\nu_\perp}. \tag{17}$$

Thus, contrary to the conclusion reached in Ref. 9, the Harris criterion argument suggests that the Bose glass phase should be *stable* to a small amount of splay disorder just below T_{BG}.

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ments were carried out on a sample containing a *single* family of twin planes, in contrast to the more isotropic disorder discussed here.

[7] For general experiments, use of the scaling relations discussed in this paper for the description of the experimental data would require taking into account the difference between the applied magnetic field $\mathbf{H}_a$ and the magnetic field in the sample $\mathbf{H}$. However, for the samples of plateletlike shape and fields close to the c axis the transverse applied and internal magnetic fields do not differ. Indeed, general relations for ellipsoids with easy axis along the z direction the general relations read

$$nB_z+(1-n)H_z=H_{za}, \quad \frac{1}{2}(1-n)B_\perp+\frac{1}{2}(1+n)H_\perp=H_{\perp a},$$

where n is the demagnetizing factor. For platelets $n\approx 1$, and for the components entering scaling relations $H_\perp \approx H_{\perp a}$. See, e.g., L. D. Landau and E. M. Lifshitz, *Electro-dynamics of Continuous Media* (Pergamon, New York, 1960), Chap. II.

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