## **Order-parameter symmetries in high-temperature superconductors**

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We present two representations of the order-parameter (OP) eigenfunctions for tetragonal cuprate superconductors, and for orthorhombic YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7- $\delta$ </sub> and Bi<sub>2</sub>Sr<sub>2</sub>CaCu<sub>2</sub>O<sub>8+ $\delta$ </sub> (BSCCO). Recent BSCCO *c*-axis twist Josephson junction experiments of Li *et al.* demonstrate that the tunneling is strongly incoherent, and that the OP contains an isotropic *s*-wave component, but not any  $d_{x^2-y^2}$ -wave component. We propose a *c*-axis twist tetracrystal ring experiment.

Several developments have reopened the question of the symmetry of the order parameter (OP) in the high transition temperature  $T_c$  superconductors.<sup>1–5</sup> Although phasesensitive tricrystal experiments were interpreted in terms of an OP in YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7- $\delta$ </sub> (YBCO) dominated at low temperature *T* by a  $d_{x^2-y^2}$ -wave component,<sup>6</sup> other YBCO experiments indicated a substantial *s*-wave component,<sup>7</sup> or even a nodeless OP.<sup>4,5</sup> *c*-axis Pb/Bi<sub>2</sub>Sr<sub>2</sub>CaCu<sub>2</sub>O<sub>8+ $\delta$ </sub> (BSCCO) Josephson demonstrated the existence of an *s*-wave OP component in BSCCO at low *T*.<sup>3</sup>

In BSCCO *c*-axis twist experiments, the critical current densities  $J_c^J$  and  $J_c^S$  of each twist junction and of the constituent single crystals were *the same*, regardless of twist angle  $\phi_0$ .<sup>1</sup> As shown in the following, this implies that BSCCO has strongly incoherent *c*-axis tunneling, and its OP contains an isotropic *s*-wave component for  $T \leq T_c$ , but no  $d_{x^2-y^2}$ -wave component.

The CuO<sub>2</sub> plane bands cross the Fermi energy  $E_F$ .<sup>8,9</sup> The crystal point group of a tetragonal CuO<sub>2</sub> plane is  $C_{4v}$ .<sup>10</sup> The group operations are the identity *E*, mirror reflections  $\sigma_x$ ,  $\sigma_y$  in the *ac*, *bc* planes, respectively, mirror reflections  $\sigma_{d1}$ ,  $\sigma_{d2}$  in the  $d_1c$ ,  $d_2c$  planes containing the diagonals  $d_1$ ,  $d_2$ , respectively, and rotations  $C_4^{\pm 1}$ , and  $C_2 = C_4^2$  by  $\pm 90^\circ$  and 180° about the *c* axis, respectively.<sup>10</sup> The bases of spin singlet functions belonging to the one-dimensional irreducible representations (IR's)  $\Gamma$  of  $C_{4v}$  are denoted  $|s\rangle$ ,  $|d_{x^2-y^2}\rangle$ ,  $|d_{xy}\rangle$ , and  $|g_{xy(x^2-y^2)}\rangle$ . Under a given symmetry operation  $\mathcal{O}_s$ , each function  $|f\rangle$  within a given basis satisfies  $\mathcal{O}_s|f\rangle = \lambda_{sf}|f\rangle$ , where  $\lambda_{sf}$  is the corresponding group character or eigenvalue. Functions within the same basis are *compatible*.

We use two representations of the OP basis sets. For Fermi-surface (FS) restricted pairing (BCS) on a cylindrical FS, the intralayer-paired quasiparticle momenta are described by  $\phi_{\mathbf{k}}$ , where  $\mathbf{k}_F = k_F(\cos \phi_{\mathbf{k}}, \sin \phi_{\mathbf{k}})$ . The OP basis is then described using angular momentum quantum numbers *l*. The respective four eigenfunction bases are represented as a constant,  $\cos(2\phi_{\mathbf{k}})$ ,  $\sin(2\phi_{\mathbf{k}})$ , and  $\sin(4\phi_{\mathbf{k}})$ , each multiplied by some function  $\sum_{l=0}^{\infty} A_l \cos(4l\phi_{\mathbf{k}})$ , behaving as a constant under  $C_{4\nu}$ .

For a tight-binding band structure, we use the real-space representation, in which the paired quasiparticles are located on distinct lattice sites, their separations along the Cu-O bond directions counted by the quantum numbers (n,m). The wave vectors  $\mathbf{k} = (k_x, k_y)$  are within the first Brillouin zone (BZ) given by  $|k_x|, |k_y| \le \pi/a$ . The respective four eigenfunction bases are represented as a constant,  $\cos(k_x a) - \cos(k_y a)$ ,  $\sin(k_x a)\sin(k_y a)$ , and  $\sin(k_x a)\sin(k_y a)[\cos(k_x a) - \cos(k_y a)]$ , each multiplied by some function  $\sum_{n,m=0}^{\infty} A_{nm} \cos(nk_x a)\cos(mk_y a)$  with  $A_{nm} = A_{mn}$ , behaving as a constant under  $C_{4n}$ .

In orthorhombic YBCO, the *b* (or *y*) axis is longer than the *a* (or *x*) axis. The  $C_{2v}^1$  crystal point group of the CuO<sub>2</sub> planes only allows the group operations *E*,  $\sigma_x$ ,  $\sigma_y$ , and  $C_2$ . The two allowable OP eigenfunctions denoted  $|s + d_{x^2-y^2}\rangle$  and  $|d_{xy} + g_{xy(x^2-y^2)}\rangle$  are listed for both representations in Table I. In YBCO, *s*- and  $d_{x^2-y^2}$ -wave functions are compatible.

In BSCCO, the orthorhombic axes are the *diagonals*  $d_1$  (a) and  $d_2$  (b). The bc plane containing the periodic lattice distortion **Q** forms a crystallographic  $\sigma_{d2}$  mirror plane.<sup>2,11</sup> The  $C_{2v}^{13}$  point group of the CuO<sub>2</sub> planes allows the

TABLE I. Spin singlet OP eigenfunctions in the FS restricted (*l*) and real space (n,m) representations, their IR notations, and character table for the orthorhombic point group  $C_{2v}^1$ , appropriate for YBCO.

OP Ti û i	$\sigma_x, \sigma_y$
Eigenfunction	$(\sigma_a, \sigma_b)$
$ s+d_{x^2-y^2}\rangle$	+1
$\Rightarrow a_0 + \sqrt{2} \sum_{l=1}^{\infty} a_l \cos(2l\phi_{\mathbf{k}})$	
1	
$ = \sum_{n,m=0} a_{nm} \cos(n\kappa_x a) \cos(n\kappa_y b) $ $n,m$	
$ d_{xy}+g_{xy(x^2-y^2)}\rangle$	-1
$\Rightarrow \sqrt{2} \sum_{l=1}^{\infty} b_l \sin(2l\phi_{\mathbf{k}})$	
1	
$\Rightarrow \mathcal{L}_{n,m=1}\mathcal{D}_{nm}\operatorname{SIII}(n\kappa_{x}a)\operatorname{SIII}(n\kappa_{y}b)$	
	Eigenfunction $ s + d_{x^{2}-y^{2}}\rangle$ $\Rightarrow a_{0} + \sqrt{2} \sum_{l=1}^{\infty} a_{l} \cos(2l\phi_{\mathbf{k}})$ $\Rightarrow \sum_{n,m=0}^{\infty} a_{nm} \cos(nk_{x}a) \cos(mk_{y}b)$ $ d_{xy} + g_{xy(x^{2}-y^{2})}\rangle$ $\Rightarrow \sqrt{2} \sum_{l=1}^{\infty} b_{l} \sin(2l\phi_{\mathbf{k}})$ $\stackrel{l}{\Rightarrow} \sum_{n,m=1}^{\infty} b_{nm} \sin(nk_{x}a) \sin(mk_{y}b)$

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TABLE II. Spin singlet OP eigenfunctions in the FS restricted (*l*) and real space (n,m) representations, their IR notations, and character table for the orthorhombic point group  $C_{2v}^{13}$ , appropriate for BSCCO, where  $k_{d1}a = \tilde{k}_x + \tilde{k}_y$  and  $k_{d2}b = \tilde{k}_x - \tilde{k}_y$ .  $\tilde{k}_x$  and  $\tilde{k}_y$  are dimensionless coefficients of the appropriate basis vectors of the reciprocal lattice, which point along the Cu-O bond directions.

IR	OP Eigenfunction	$\sigma_{d2} \ (\sigma_b)$
$A_1$	$ s+d_{xy}\rangle$	+1
	$ \Rightarrow a_0 + \sqrt{2} \sum_{l=1}^{\infty} a_l \cos \left[ 2l(\phi_{\mathbf{k}} - \pi/4) \right]  \Rightarrow \sum_{n,m=0}^{\infty} a_{nm} \cos(nk_{d1}a) \cos(mk_{d2}b) $	
<i>A</i> <sub>2</sub>	$ d_{x^{2}-y^{2}}+g_{xy(x^{2}-y^{2})}\rangle$ $\Rightarrow \sqrt{2}\sum_{l=1}^{\infty}b_{l}\sin[2l(\phi_{\mathbf{k}}-\pi/4)]$ $\stackrel{l}{\Rightarrow} \sum_{n,m=0}^{\infty}b_{nm}\sin(nk_{d1}a)\sin(mk_{d2}b)$	-1

symmetry operations *E*,  $\sigma_{d2}$ , and *C*<sub>2</sub>. The OP eigenfunctions  $|s+d_{xy}\rangle$  and  $|d_{x^2-y^2}+g_{xy(x^2-y^2)}\rangle$  are represented both ways in Table II. In BSCCO, *s*- and  $d_{x^2-y^2}$ -wave OP functions are *incompatible*.

The orthorhombic crystal symmetry requires the interaction for spin singlet pairing to have the form

$$\lambda(\mathbf{k},\mathbf{k}') = \sum_{ij} \sum_{\Gamma=A_1}^{A_2} \varphi_{i\Gamma}(\mathbf{k})\lambda_{ij,\Gamma}\varphi_{j\Gamma}(\mathbf{k}'), \qquad (1)$$

where each  $\varphi_{i\Gamma}$  is a basis element from either Table I or II. An OP  $\Delta(\mathbf{k},T) = \sum_{i=1}^{2} \Delta_i(T) \varphi_{iA_i}(\mathbf{k})$  with two incompatible components arises when one element  $\varphi_{iA_i}$  appears from each IR basis  $A_i$ , or  $\lambda_{ij,\Gamma} = \lambda_{ii,A_i} \delta_{ij}$ . The Ginzburg-Landau (GL) free energy  $\mathcal{F}_0$ , from which the *C* numbers  $\Delta_i(T)$  can be determined, is

$$\mathcal{F}_0 = \sum_{i=1}^2 \mathcal{F}_{0i} + \epsilon |\Delta_1 \Delta_2|^2 + \delta \operatorname{Re}(\Delta_1^2 \Delta_2^{*2}), \qquad (2)$$

where  $\mathcal{F}_{0i} = \alpha_i |\Delta_i|^2 + \beta_i |\Delta_i|^4$  and  $\alpha_i(T) = \alpha_{i0}(T - T_{ci})$ . In weak coupling (BCS) theory, the bare transition temperatures  $T_{ci}$  are obtained from the  $\lambda_{ii,A_i}$ , and the constants satisfy  $\beta_i > 0$ ,  $\epsilon > \delta > 0$ . Assuming  $T_{c1} > T_{c2}$ , the S<sub>1</sub> regime,  $T_{c2}^{<} \leq T < T_c = T_{c1}$ , has  $|\Delta_1(T)| \neq |\Delta_2(T)| = 0$ . In regime S<sub>2</sub>,  $T < T_{c2}^{<}$ , both  $|\Delta_1| \neq 0$  and  $|\Delta_2| \neq 0$ , and the OP is the nodeless  $\Delta_1 + i\Delta_2$  state. This *only* occurs below the second phase transition at  $T_{c2}^{<}$ .

An OP having two compatible components arises when both basis elements i, j = 1, 2 belong to the same IR,  $A_{\alpha}$ , where  $\alpha$  is either 1 or 2. Then,  $\lambda(\mathbf{k}, \mathbf{k}') = \sum_{i,j=1}^{2} \varphi_{iA_{\alpha}}(\mathbf{k})\lambda_{ij,A_{\alpha}}\varphi_{jA_{\alpha}}(\mathbf{k}')$ , where  $\lambda_{ij,A_{\alpha}} = \lambda_{ji,A_{\alpha}}$ . Diagonalizing  $\lambda(\mathbf{k}, \mathbf{k}')$ , we obtain  $\tilde{\lambda}(\mathbf{k}, \mathbf{k}') = \sum_{i=1}^{2} \tilde{\varphi}_{iA_{\alpha}}(\mathbf{k})\tilde{\lambda}_{ii,A_{\alpha}}\tilde{\varphi}_{iA_{\alpha}}(\mathbf{k}')$  and the OP eigenfunction,  $\Delta(\mathbf{k}, T) = \sum_{i=1}^{2} \Delta_i(T)\tilde{\varphi}_{iA_{\alpha}}(\mathbf{k})$ . The GL free energy  $\mathcal{F}_1$  then acquires a term additional to  $\mathcal{F}_0$ , Eq. (2),



FIG. 1. Plot of  $I_c(\phi_0)/I_c(0)$  for the case considered in Ref. 14 of a dominant  $d_{x^2-y^2}$ -wave and subdominant  $d_{xy}$ -wave OP, the relative amounts varying with layer index away from the twist junction. The parameters for these curves are  $T_{c2}/T_c=0.2$ ,  $T_{c2}^</T_c=0.1304$ ,  $\epsilon/6\beta=0.5$ ,  $\delta/6\beta=0.1$ , and the curves for  $\eta_d=\eta_d'=0.1$  and  $\eta_d=\eta_d'=0.001$  are indicated. Curves for  $t=T/T_c=0.99,0.9,0.5$  are presented.

$$\mathcal{F}_1 = \mathcal{F}_0 + \operatorname{Re}(\Delta_1 \Delta_2^*) \sum_{i=1}^2 \mu_i |\Delta_i|^2.$$
(3)

The compatible OP components mix in *two* ways. First, the original basis functions  $\varphi_{iA_{\alpha}}(\mathbf{k})$  mix via the linear transformation which diagonalizes  $\lambda(\mathbf{k}, \mathbf{k}')$ . Second, the subdominant OP  $\Delta_2$  is coupled to  $\Delta_1$  just below  $T_c = T_{c1}$  via the  $\mu_1$  term. Depending upon the sign of  $\mu_1$ , the phases of  $\Delta_2$  and  $\Delta_1$  differ by 0 or  $\pi$ , and  $|\Delta_2(T)| \propto |\Delta_1(T)|^3 \propto (T_c - T)^{3/2}$  just below  $T_c$ . This modifies the relative *T* dependences of  $\Delta_1, \Delta_2$  in the " $\Delta_1 + \Delta_2$ " state. Both mixings occur without a second phase transition. Thus, the **k** dependence of  $\Delta(\mathbf{k}, T)$  changes smoothly with T.<sup>13</sup>

We now consider the case of a high-temperature superconducting (HTSC) Josephson junction formed by twisting bicrystal halves an angle  $\phi_0$  about the c axis, as pictured in Fig. 1 of Ref. 14. For weak tunneling between adjacent layers *n* and *n'*,  $J_c^i = |4eT\Sigma_{\omega}\langle f^i(\mathbf{k}-\mathbf{k}')F_n(\mathbf{k}_i)F_{n'}(\mathbf{k}_i')\rangle_{n\cap n'}|$ for i=J,S, where  $\mathbf{k}_{S}=\mathbf{k}$ ,  $\mathbf{k}'_{S}=\mathbf{k}'$ , but  $\mathbf{k}_{I}=\mathbf{k}_{+}$  and  $\mathbf{k}'_{I}=\mathbf{k}'_{-}$ are rotated by  $\pm \phi_0/2$  about the *c*-axis, respectively.  $\langle \cdots \rangle$  $\cdot \rangle_{n \cap n'}$  is an integral over the overlapping first BZs, and  $f^{i}(\mathbf{q})$  is the spatial average of the quasiparticle tunneling matrix element squared.<sup>14</sup> For  $J_c^i$  to  $\mathcal{O}(f^i)$ ,  $F_n = \Delta_n / [\omega^2]$  $+\xi_n^2+|\Delta_n|^2$ , etc., where  $\omega$  is a Matsubara frequency, and the quasiparticle dispersions  $\xi_n$  and OPs  $\Delta_n$  are *independent* of  $f^{i}$ . In this limit, Bloch's theorem and group theory require each component of the  $\Delta_n(\mathbf{k}_i, T)$  and the  $\Delta_{n'}(\mathbf{k}'_i, T)$  to lock onto the local Cu-O bond orientation. Thus,  $\Delta_n(\mathbf{k},T)$  $=\sum_{i=1}^{2} \Delta_{ni}(T) \varphi_{iA}(\mathbf{k})$  for two incompatible OP components on the *n*th layer. Although  $f^{i}(\mathbf{q})$  can contain both coherent (q=0) and incoherent (q arbitrary) parts, for purely incoherent tunneling (AB),  $f^{i}(\mathbf{q}) = f_{0}^{i}$ ,  $\langle F_{n} \rangle = \langle F_{n'} \rangle$ , and each  $J_{c}^{i}$ =0, except for an s-wave OP, projecting out its FS average near  $T_c$ .

If the FS were cylindrical and  $f^{J}(\mathbf{q})$  were coherent, then the twist experiments would infer an isotropic *s*-wave OP. However, this scenario is unlikely, since the FS is tight binding.<sup>8,9,13</sup> The dominant coherent tunneling processes involve quasiparticle states near the FSs on each side of the junction. For both the BCS and most strong coupling models,  $\Delta_n = 0$ , except for a narrow region of the BZ near the FS. Since the intersection of tight-binding FSs twisted  $\pm \phi_0/2$  about the *c* axis is vanishingly small, *c*-axis twisting reduces such coherent processes, causing  $J_c^J(\phi_0)/J_c^J(0)$  to be largest for  $\phi_0 = 0,90^\circ$ , even for an isotropic *s*-wave OP. The twist experiments thus imply strongly incoherent tunneling.<sup>1</sup>

Previously, we investigated whether in the GL regime tunneling to higher order in the  $f^i$  might allow a dominant  $d_{x^2-y^2}$ -wave OP to twist, by locally mixing in a subdominant *s*- and/or  $d_{xy}$ -wave OP component by the proximity effect, thereby compensating for the junction twist.<sup>14</sup> We found that such OP proximity twisting is possible, allowing  $J_c^J(\phi_0) \neq 0$  for all  $\phi_0$ . However, the amplitude of the  $d_{xy}$  OP component is very small for  $T \gg T_{c2}^<$ , and  $T_{c2}^<$  must be so low as to be unobservable. Also, the proximity effect locally suppresses the dominant OP component, reducing  $J_c^J(\phi_0)$  for  $\phi_0 \neq 0$ .

The amplitude of the  $d_{xy}$  OP component is determined somewhat by the ratio  $T/T_{c2}^{<}$ , but mostly by the *d*-wave Josephson coupling strengths  $\eta'_d$  and  $\eta_d$  obtained from the  $\langle f^i(\mathbf{k}-\mathbf{k}')\varphi_{jA_j}(\mathbf{k}_i)\varphi_{j'A_{j'}}(\mathbf{k}'_i)\rangle_{n\cap n'}$  with j,j'=1,2 at  $E_F$  for i=J,S, respectively. The weaker the  $\eta'_d$ , the lower the *T* at which OP twisting can partially compensate for the junction twist. For  $T_{c2}/T_c=0.2$  and strong  $(\eta_d=\eta'_d=1)$  coupling, we found a small but finite  $J_c^J(45^\circ)$  at  $t=T/T_c=0.7$ . However, reducing  $\eta'_d$  to 0.01 with  $\eta_d=1$  dramatically reduced  $J_c^J(45^\circ)$  at t=0.7.<sup>14</sup>

Moreover, since BSCCO is highly anisotropic,<sup>2,3</sup> we have recalculated  $J_c^J(\phi_0)/J_c^J(0)$  for  $\eta_d = \eta_d' = 0.1,0.001$ , and plotted the results for t = 0.5, 0.9, 0.99 in Fig. 1. For  $\eta_d = 0.1$ , these effects are small but clearly visible. For  $\eta_d = 0.001$ , the suppression of the dominant  $d_{x^2-y^2}$ -wave OP component would be noticeable only very close to  $T_c$ . For *T* below the fluctuation regime,  $J_c^J(\phi_0)/J_c^J(0) \approx |\cos(2\phi_0)|$ . Clearly, *d*-wave OP twisting *cannot* explain the twist experiments.<sup>1</sup>

We therefore consider only one OP eigenfunction constructed from *compatible* functions. A full calculation in the real space representation will be presented elsewhere.<sup>16</sup> In the FS restricted representation,  $f^{J}(\mathbf{q})$  contains coherent  $|J|^2 \delta^{(2)}(\mathbf{k} - \mathbf{k}')$  and incoherent  $\sum_{l=0}^{\infty} f_{l0} \cos[l(\phi_{\mathbf{k}} - \phi_{\mathbf{k}'})]$ parts.<sup>14,15</sup> For either YBCO or BSCCO, when the OP eigenfunctions belong to the same IR,  $A_1$  or  $A_2$  in Tables I and II,  $J_c^J(\phi_0)/J_c^J(0)$  is the same. Just below  $T_c$ , the integrals can be performed exactly. Since the single crystal and twist junctions are chemically identical, with identical *c*-axis spacings, we take  $|J|^2$  and the  $f_{l0}$  to be independent of  $\phi_0$ .<sup>1,2,11</sup> We  $\eta_0^{\rm inc} = em^2 f_{00}/(4T_c)$ define and  $\eta_l = f_{l0} / f_{00}$  $+C|J|^{2}/(mf_{00}T_{c})$  for  $l \ge 0$ , where  $C = 7\zeta(3)/(2\pi^{5})$  and m is the planar effective mass. We note that  $\eta_2 \propto \eta'_d$  in Fig. 1. We find

$$J_{c}^{J}(\phi_{0})/\eta_{0}^{\text{inc}} = \left| \alpha_{0} + \sum_{l=1}^{\infty} \alpha_{l} \cos(2l\phi_{0}) \right|, \qquad (4)$$

where for  $l \ge 1$ ,  $\alpha_l = \eta_{2l}a_l^2$ ,  $\eta_{2l}b_l^2$  for the  $A_1, A_2$  states, respectively. The  $a_l$  and  $b_l$  are the OP eigenfunction expansion



FIG. 2. Plots of  $|A + \cos(2\phi_0) + B\cos(4\phi_0)|/|1 + A + B|$ , along with the *A*, *B* values, of the lowest *l* contributions to  $J_c^J(\phi_0)/J_c^J(0)$  in Eq. (4). Upper two curves:  $A_1$  states. Lower three curves:  $A_2$  states.

coefficients in the FS restricted representation in Tables I and II. Most important,  $\alpha_0 = a_0^2(1 + \eta_0)$  can be finite for the  $A_1$  states, but for the  $A_2$  states,  $\alpha_0 = 0$ !

For pure *l*-wave states,  $J_c^J(\phi_0)/J_c^J(0) = |\cos(l\phi_0)|$ , as shown for l=0,1,2 in Fig. 4 of Ref. 1. For an isotropic (l=0) *s*-wave state,  $J_c^J(\phi_0)/J_c^J(0)=1$ , consistent with the data.<sup>1</sup> The other pure *l*-wave states have some  $\phi_0^*$  at which  $J_c^J(\phi_0^*)=0$ . For l=2,  $\phi_0^*=45^\circ$ , and for l=4,  $\phi_0^*=22.5^\circ,67.5^\circ$ . Including higher *l* compatible components will not change these qualitative results. As shown in Fig. 2, an  $A_2$  state always has some  $\phi_0^*$  at which  $J_c^J(\phi_0^*)=0$ . If the isotropic component of an  $A_1$  state were nearly negligible, then  $J_c^J(\phi_0)/J_c^J(0)$  could also be highly anisotropic. Otherwise, an  $A_1$  state gives the least anisotropic  $J_c^J(\phi_0)$  behavior. Thus, the observation  $J_c^J(\phi_0)/J_c^S=1$  at and below  $T_c$  in BSCCO is prima facie evidence that the dominant OP is |s $+ d_{xy}\rangle$ , with an isotropic component.<sup>1</sup>

In addition, if the tunneling across the twist and intrinsic junctions were equal and purely incoherent,  $(f_{l0} \rightarrow 0 \text{ for } l \neq 0)$ , then the  $|d_{x^2-y^2}+g_{xy(x^2-y^2)}\rangle$  state would have  $J_c^J(\phi_0)=J_c^S=0 \forall \phi_0$ , and the  $|s+d_{xy}\rangle$  state would have  $J_c^J(\phi_0)/J_c^S=1$ , as observed, provided that it had an isotropic *s*-wave component.<sup>1</sup> An example of an OP consistent with many experiments is the "extended *s*-wave" OP,  $\{[\cos(k_x a)-\cos(k_y a)]^2+\epsilon^2\}^{1/2}$ , which is expandable as a tetragonal *s*-wave function with near nodes for  $\epsilon \ll 1$ . Although the *c*-axis tunneling in BSCCO must be strongly incoherent, it need not be purely incoherent. If it is not purely incoherent, however, then the OP must be isotropic.

Since the *c*-axis twist and tricrystal experiments are incompatible,<sup>1,6</sup> we propose an experiment to settle the issue. The new geometry is the tetracrystal ring shown in Fig. 3. A single crystal of BSCCO is cleaved twice into three pieces, shown as dark, medium, and light. The medium piece is cut into two equally thick pieces, which are placed across the dark crystal, forming angles  $\phi_{12}$  and  $\phi_{23}$ . The light crystal is then placed atop the medium ones, forming a straight (0° or 180°) angle  $\phi_{34}$  with one of them, and  $\phi_{41} = \pm \phi_{31}$  with the other, the sign depending upon  $\phi_{34}$ . Then, the entire triangular ring is fused together.



FIG. 3. Proposed configuration of a *c*-axis version of the tricrystal ring experiment. Dark crystal: bottom. Light crystal: top. Medium shading: equal thickness crystals. Arrows indicate the direction of a given single crystal axis.

One calculates the  $I_c(\phi_{ij})$  for each OP eigenfunction. Whenever  $I_c(\phi_0^*)=0$ ,  $I_c$  changes sign at  $\phi_0^*$ , and  $I_c<0$  implies a  $\pi$  junction. The phase of a single junction can be chosen to minimize the free energy, thereby changing  $I_c$  to  $|I_c|$  when  $I_c<0$ . But, for the tetracrystal, the relative junction phases are fixed by the loop.

The equations for a ring of *ab*-planar junctions also apply

to a ring of *c*-axis junctions.<sup>6</sup> The self-inductance *L* should satisfy the requirement  $\beta = LI_c^J/\Phi_0 \ge 1$  with large single crystals, where  $I_c^J$  is the minimum junction critical current value.<sup>17</sup> However, for high flux detector sensitivity, the area inside the ring should be sufficiently small. If the *s*-wave OP component dominates, there would be no  $\pi$  junctions in the ring, and integral multiples of the flux quantum  $\Phi_0 = hc/2e$ would be trapped inside. However, if the  $d_{x^2-y^2}$  OP component dominates, then if all three  $\phi_{ij}$  satisfy  $\cos(2\phi_{ij}) < 0$  (as in an equilateral triangle), the ring would contain an odd number of  $\pi$  junctions, trapping half-integral multiples of  $\Phi_0$ . Other OP scenarios can be studied by varying the  $\phi_{ij}$ .

Thus, the *c*-axis twist experiments provide strong evidence that the OP is  $|s+d_{xy}\rangle$ , with a nonvanishing isotropic component, and that the *c*-axis tunneling is strongly incoherent. In addition, either the OP is isotropic or the *c*-axis tunneling is purely incoherent, as in AB.<sup>12</sup> We propose a *c*-axis tetracrystal experiment to settle the conflict with the tricrystal experiment.

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