

## Compressible anisotropic states around the half-filled Landau levels

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(Received 20 August 1999; revised manuscript received 4 October 1999)

Using the von Neumann lattice formalism, we study compressible anisotropic states around the half-filled Landau levels in the quantum Hall system. In these states the unidirectional charge-density wave (UCDW) state seems to be the most plausible state. The charge-density profile and Hartree-Fock energy of the UCDW are calculated self-consistently. The wavelength dependence of the energy for the UCDW is also obtained numerically. We show that the UCDW is regarded as a collection of the one-dimensional lattice Fermi-gas systems which extend to the uniform direction. The kinetic energy of the gas system is generated dynamically from the Coulomb interaction.

### I. INTRODUCTION

Two-dimensional electron systems in a strong magnetic field have been providing various fascinating phenomena for the last two decades of this twentieth century. The integer quantum Hall effect<sup>1</sup> (IQHE) and fractional quantum Hall effect<sup>2</sup> (FQHE) are observed around the integral filling factor and rational filling factor with an odd denominator,<sup>3</sup> respectively. These effects are caused by the formation of the incompressible liquid state with a finite energy gap. The IQHE state has a Landau-level energy gap or Zeeman energy gap and the FQHE state has an energy gap due to the Coulomb interaction.<sup>4</sup> Remarkable progress of the composite fermion (CF) theory<sup>5,6</sup> for the FQHE shed light on the Fermi-liquid-like state at the half-filled lowest Landau level. Evidence for the Fermi-liquid-like state was obtained in many experiments.<sup>7</sup> It is believed that the state is a compressible isotropic state and has a circular Fermi surface.

Recent experiments at the half-filled Landau levels have revealed the anisotropic nature of the compressible states. At the half-filled third and higher Landau levels, a highly anisotropic behavior was observed in the magnetoresistance.<sup>8,9</sup> At the half-filled lowest and second Landau levels, a transition to the anisotropic state was also observed in the presence of the periodic potential<sup>10</sup> and in-plane magnetic field,<sup>11,12</sup> respectively. Although the origin of the anisotropy is still unknown, the unidirectional charge-density wave (UCDW) state is a candidate for the anisotropic state. Theoretical works in higher Landau levels showed the possibility of the UCDW.<sup>13,14</sup> Recent theoretical works in lower Landau levels support the UCDW or liquid-crystal state.<sup>15-17</sup> In this paper we investigate the compressible charge-density-wave (CCDW) states, which include the UCDW state and compressible Wigner crystal (CWC) state, in the several lower Landau levels. As a result, the UCDW states are found to be the lowest-energy states in the CCDW states at the half-filled Landau levels.

The CCDW state is a gapless state with an anisotropic Fermi surface and has a periodically modulated charge density. Using the von Neumann lattice<sup>18,19</sup> representation, we construct the CCDW state and calculate the charge-density profile and Hartree-Fock energy self-consistently. The von Neumann lattice representation has a quite useful property in

studying the quantum Hall system with the periodic potential.<sup>20,21</sup> The lattice structure of the von Neumann lattice can be adjusted to the periodic potential by varying the modular parameter of the unit cell. In the present case the periodic potential is caused by the charge density modulation through the Coulomb interaction.<sup>22-24</sup> If the translational invariance on the lattice is unbroken, a Fermi surface is formed.

In the Hartree-Fock approximation, we show that the self-consistency equation for the CCDW has two types of solution at half-filling. One has a belt-shaped Fermi sea corresponding to the UCDW and the other has a diamond-shaped Fermi sea corresponding to the CWC. For the belt-shaped Fermi sea, one direction of the momentum space is filled and the other direction is partially filled. Therefore the UCDW is regarded as a collection of the one-dimensional lattice Fermi-gas systems, which was called the quantum Hall gas (QHG) in Ref. 24. In the UCDW state, there are two length scales, the wavelength of the UCDW,  $\lambda_{\text{CDW}}$ , and the Fermi wavelength of the lattice fermions,  $\lambda_{\text{F}}$ . These two parameters obey a duality relation. We obtain the wavelength dependence of the energy. Moreover, we calculate the kinetic energy of the gas system.

The paper is organized as follows. In Sec. II, the Hartree-Fock energy for the CCDW states is calculated. The density profile, wavelength dependence of the energy, and the kinetic energy of the UCDW are obtained in Sec. III. The summary and discussion are given in Sec. IV.

### II. HARTREE-FOCK ENERGY FOR THE CCDW STATE: UCDW VERSUS CWC

In this section we construct the CCDW state in the Hartree-Fock approximation using the formalism developed in Ref. 24. Let us consider the two-dimensional electron system in a perpendicular magnetic field  $B$  in the absence of impurities. The electrons interact with each other through the Coulomb potential  $V(\mathbf{r})=q^2/r$ . In this paper we ignore the spin degree of freedom and use the natural unit of  $\hbar=c=1$ . In the  $l$ th Landau-level space, the free kinetic energy is quenched as  $\omega_c(l+1/2)$ ,  $l=0,1,2,\dots$ , where  $\omega_c=eB/m$ .

In the von Neumann lattice formalism,<sup>19-21</sup> the electron field is expanded as

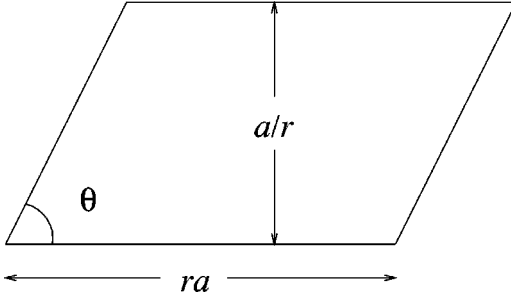


FIG. 1. The unit cell of the von Neumann lattice spanned by the vectors  $\mathbf{e}_1$  and  $\mathbf{e}_2$ .

$$\psi(\mathbf{r}) = \sum_{l, \mathbf{X}} b_l(\mathbf{X}) W_{l, \mathbf{X}}(\mathbf{r}), \quad (1)$$

where  $b$  is an anticommuting annihilation operator and  $\mathbf{X}$  is an integer-valued two-dimensional coordinate. The Wannier base functions  $W_{l, \mathbf{X}}(\mathbf{r})$  are orthonormal complete basis in the  $l$ th Landau level.<sup>21</sup> Expectation values of the position of  $W_{l, \mathbf{X}}(\mathbf{r})$  are located at two-dimensional lattice sites  $m\mathbf{e}_1 + n\mathbf{e}_2$  for  $\mathbf{X} = (m, n)$ , where  $\mathbf{e}_1 = (ra, 0)$ ,  $\mathbf{e}_2 = (a/r \tan \theta, a/r)$ , and  $a = \sqrt{2\pi/eB}$ . The area of the unit cell is  $\mathbf{e}_1 \times \mathbf{e}_2 = a^2$ , which means that a unit flux penetrates the unit cell of the von Neumann lattice. The unit cell is illustrated in Fig. 1. For simplicity we set  $a = 1$  in the following calculation.

The Bloch wave basis, which is given by  $u_{l, \mathbf{p}}(\mathbf{r}) = \sum_{\mathbf{X}} W_{l, \mathbf{X}}(\mathbf{r}) e^{i\mathbf{p} \cdot \mathbf{X}}$ , is another useful basis on the von Neumann lattice.<sup>21</sup> The lattice momentum  $\mathbf{p}$  is defined in the Brillouin zone (BZ),  $|p_i| \leq \pi$ . The wave function  $u_{l, \mathbf{p}}(\mathbf{r})$  extends over the entire plane and its probability density has the same periodicity as the von Neumann lattice.

In the momentum space, the system has a translational invariance,<sup>24</sup> which is referred to as the  $K$  invariance in the CF model.<sup>26</sup> In the following, we show that symmetry breaking of the  $K$  invariance generates a kinetic energy and leads to an anisotropy in the charge density.

We consider a mean-field state of filling factor  $\nu = l + \bar{\nu}$  where  $l$  is an integer and  $0 < \bar{\nu} < 1$ . Let us consider a mean field for the CCDW that has translational invariance on the von Neumann lattice as

$$U_l(\mathbf{X} - \mathbf{X}') = \langle b_l^\dagger(\mathbf{X}') b_l(\mathbf{X}) \rangle \quad (2)$$

and  $U_l(0) = \bar{\nu}$ . Ignoring the Landau-level energy and the inter-Landau-level effect, the Hartree-Fock Hamiltonian within the  $l$ th Landau level  $H_{\text{HF}}^{(l)}$  reads

$$H_{\text{HF}}^{(l)} = \sum_{\mathbf{X}, \mathbf{X}'} U_l(\mathbf{X} - \mathbf{X}') \{ \tilde{v}_l[2\pi(\hat{\mathbf{X}} - \hat{\mathbf{X}}')] - v_l(\hat{\mathbf{X}} - \hat{\mathbf{X}}') \} \\ \times \{ b_l^\dagger(\mathbf{X}) b_l(\mathbf{X}') - \frac{1}{2} U_l(\mathbf{X}' - \mathbf{X}) \}, \quad (3)$$

where

$$\tilde{v}_l(\mathbf{k}) = \left\{ L_l \left( \frac{k^2}{4\pi} \right) \right\}^2 e^{-k^2/4\pi} \tilde{V}(\mathbf{k}), \quad (4)$$

$$v_l(\mathbf{X}) = \int \frac{d^2k}{(2\pi)^2} \tilde{v}_l(\mathbf{k}) e^{i\mathbf{k} \cdot \mathbf{X}}. \quad (5)$$

Here  $\tilde{V}(\mathbf{k}) = 2\pi q^2/k$  for  $\mathbf{k} \neq 0$  and  $\tilde{V}(0) = 0$  due to the charge neutrality condition.  $\hat{\mathbf{X}}$  is a position of the Wannier basis in the real space, that is,  $\hat{\mathbf{X}} = (rm + n/r \tan \theta, n/r)$  for  $\mathbf{X} = (m, n)$ .

By Fourier transforming Eq. (3), we obtain the self-consistency equations for the kinetic energy  $\varepsilon_l$  as

$$\varepsilon_l(\mathbf{p}, \bar{\nu}) = \int_{\text{BZ}} \frac{d^2p'}{(2\pi)^2} \tilde{v}_l^{\text{HF}}(\mathbf{p}' - \mathbf{p}) \theta(\mu_l - \varepsilon_l(\mathbf{p}', \bar{\nu})), \quad (6)$$

$$\bar{\nu} = \int_{\text{BZ}} \frac{d^2p}{(2\pi)^2} \theta(\mu_l - \varepsilon_l(\mathbf{p}, \bar{\nu})), \quad (7)$$

where  $\mu_l$  is the chemical potential and  $\tilde{v}_l^{\text{HF}}$  is defined by

$$\tilde{v}_l^{\text{HF}}(\mathbf{p}) = \sum_{\mathbf{X}} \{ \tilde{v}_l(2\pi(\hat{\mathbf{X}})) - v_l(\hat{\mathbf{X}}) \} e^{-i\mathbf{p} \cdot \mathbf{X}}. \quad (8)$$

Equations (6) and (7) determine a self-consistent Fermi surface. Existence of a Fermi surface breaks the  $K$  invariance inevitably. The mean value of the kinetic energy  $\langle \varepsilon_l \rangle = \int_{\text{BZ}} \varepsilon_l(\mathbf{p}) d^2p / (2\pi)^2$  is equal to  $-v_l(0)\bar{\nu}$ , which is independent of  $r$  and  $\theta$ . The energy per particle in the  $l$ th Landau level is calculated as

$$E^{(l)} = \frac{1}{2\bar{\nu}} \sum_{\mathbf{X}} |U_l(\mathbf{X})|^2 \{ \tilde{v}_l(2\pi\hat{\mathbf{X}}) - v_l(\hat{\mathbf{X}}) \}. \quad (9)$$

$E^{(l)}$  is a function of  $\bar{\nu}$ ,  $r$ , and  $\theta$ . The parameters  $r$  and  $\theta$  are determined so as to minimize the energy  $E^{(l)}$  at a fixed  $\bar{\nu}$ .

There are two types of the Fermi sea that satisfy Eqs. (6) and (7). The one is a belt-shaped Fermi sea illustrated in Fig. 2(a), that is,  $|p_y| \leq p_F$ . This solution corresponds to the UCDW state. At  $\nu = l + \bar{\nu}$ , the Fermi wave number  $p_F$  is equal to  $\pi\bar{\nu}$  and the mean field of the UCDW becomes

$$U_l(\mathbf{X}) = \delta_{m,0} \frac{\sin(p_F n)}{\pi n}, \quad (10)$$

for  $\mathbf{X} = (m, n)$ . We take  $\theta = \pi/2$  for the UCDW without losing generality because the system has the rotational invariance. Then the charge density of the UCDW is uniform in the  $y$  direction and oscillates in the  $x$  direction with a wavelength  $\lambda_{\text{CDW}} = ra$ . In the  $y$  direction,  $p_F$  corresponds to the Fermi wave number  $k_F = p_F/a = \pi\bar{\nu}r/a$  in the real space. The duality relation between  $\lambda_F = 2\pi/k_F$  and  $\lambda_{\text{CDW}} = ra$  exists. Namely  $\lambda_F \lambda_{\text{CDW}} = 2a^2/\bar{\nu}$ . In the CF model, the composite fermion has a wave number  $k_F = \sqrt{2\pi}/a$  at  $\nu = 1/2$ . In the UCDW state for  $r = 1.636$ , which minimizes the energy at  $\nu = 1/2$ ,<sup>24</sup>  $\pi\bar{\nu}r$  equals 2.57, which is very close to the value  $\sqrt{2\pi} = 2.51$ . This implies that there exists unknown connection between the CF state and the UCDW state.

The other Fermi sea that satisfies Eqs. (6) and (7) at  $\bar{\nu} = 1/2$  is a diamond-shaped sea illustrated in Fig. 2(b). This solution corresponds to the CWC state whose density is modulated with the same periodicity as the von Neumann lattice. The mean field of the CWC becomes

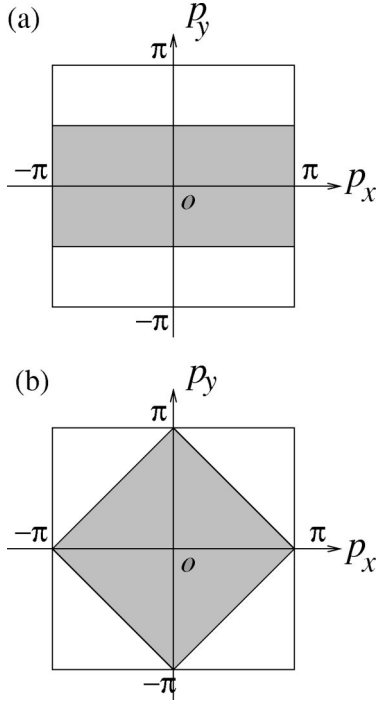


FIG. 2. (a) The belt-shaped Fermi sea (shaded region) for the UCDW at half-filling. (b) The diamond-shaped Fermi sea (shaded region) for the CWC at half-filling.

$$U_l(\mathbf{X}) = \frac{2}{(\pi)^2} \frac{\sin[(\pi/2)(m+n)]\sin[(\pi/2)(m-n)]}{m^2 - n^2} \quad (11)$$

for  $\mathbf{X}=(m,n)$ . Substituting Eqs. (10) and (11) into Eq. (9), we calculate the energy for various CCDW states at  $\bar{\nu}=1/2$ . By varying the parameters  $r$  and  $\theta$ , we obtained the lowest energy numerically.

The results are summarized in Table I. The unit of the energy is  $q^2/l_B$  and  $l_B=\sqrt{1/eB}$ . As seen in the table, the UCDW state is the lowest-energy state in all cases. Therefore the UCDW state is the most plausible state in the CCDW states. For the CWC state,  $\theta=\pi/2$  corresponds to a rectangular lattice and  $\theta=\pi/3$  to a triangular lattice. ( $\theta=\pi/2$  and  $r=1$  means a square lattice and  $\theta=\pi/3$  and  $r=1.075$  means a regular triangular lattice.) For the UCDW state, the wave-

TABLE I. Minimum energy and corresponding parameter  $r$  for the CCDW states at  $\nu=l+1/2$ . For the CWC states, the energies at  $\theta=\pi/2$  (rectangular lattice) and  $\pi/3$  (triangular lattice) are shown. The units of the energy and  $r$  are  $q^2/l_B$  and  $a$ , respectively.

$l$	$E_{\text{UCDW}}$	$E_{\text{CWC}}(\pi/2)$	$E_{\text{CWC}}(\pi/3)$
0	-0.4331	-0.3939	-0.3891
1	-0.3490	-0.3122	-0.3110
2	-0.3074	-0.2715	-0.2703
3	-0.2800	-0.2448	-0.2436
$l$	$r_{\text{UCDW}}$	$r_{\text{CWC}}(\pi/2)$	$r_{\text{CWC}}(\pi/3)$
0	1.636	1.000	1.295
1	2.021	1.000	1.075
2	2.474	1.000	1.075
3	2.875	1.205	1.335

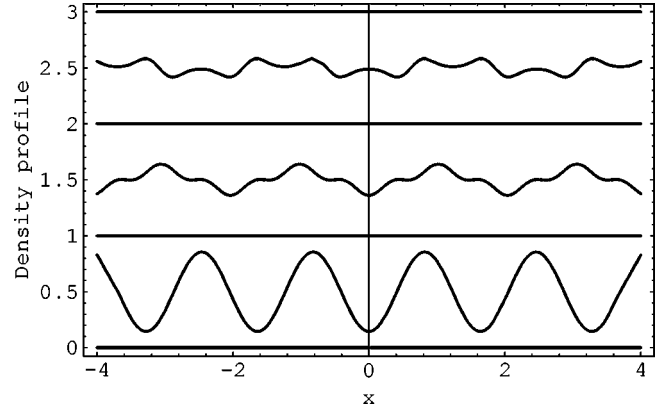


FIG. 3. The density profile function  $\rho(x)$  for  $\nu=0, 0.5, 1, 1.5, 2, 2.5,$  and  $3$ . The units of  $\rho$  and  $x$  are  $a^{-2}$  and  $a$ , respectively.

length  $\lambda_{\text{CDW}}=ra$  increases with the increasing  $l$ . This behavior is consistent with the numerical calculation in finite systems.<sup>16</sup> The Hartree-Fock calculation in the higher Landau level<sup>13,14</sup> predicts  $\lambda_{\text{CDW}}=2.7\sim 2.9\sqrt{2l+1}l_B$ . Our results agree with this at  $l=1,2,3$ . At  $l=0$ , however, our result is much smaller than this.

At  $\nu=1/2$ , the energy of the UCDW state is slightly higher than the value of the charge-density wave (CDW) with gap calculation.<sup>27</sup> In the gapful CDW state, the higher-order correction is small because of the energy gap. In the UCDW state, however, the correction might be large compared with the CDW state. Therefore it is necessary for a more definite conclusion to include fluctuation effects around the mean field. Although this task goes beyond the scope of this paper, the Hamiltonian on the von Neumann lattice could be useful to study fluctuation effects.

### III. PROPERTY OF THE UCDW STATES

In this section, we calculate the density profile of the UCDW and wavelength dependence of the energy. The density of the electron for  $\nu=l+1/2$  reads

$$\rho(\mathbf{r})=l+\int_{-\pi}^{\pi}\frac{dp_x}{2\pi}\int_{-\pi/2}^{\pi/2}\frac{dp_y}{2\pi}|u_{l,\mathbf{p}}(\mathbf{r})|^2. \quad (12)$$

Since  $|u_{l,\mathbf{p}}(\mathbf{r})|^2$  is a periodic function of  $\mathbf{p}$  in the BZ and depends only on the combination of  $\mathbf{r}+(p_y/2\pi)\mathbf{e}_1-(p_x/2\pi)\mathbf{e}_2$ ,  $\rho$  of Eq. (12) is uniform in the  $y$  direction. Here we take  $\mathbf{e}_1=(r,0)$  and  $\mathbf{e}_2=(0,1/r)$  for  $\theta=\pi/2$ . The translation in the momentum space is equivalent to the translation of the charge density of the CCDW. Therefore the symmetry breaking of the  $K$  invariance is same as the symmetry breaking of the translational invariance in the real space.

The density profiles for the UCDW at the half-filled  $l$ th Landau level of  $l=0,1,2$  are plotted in Fig. 3. The unit of the density is  $a^{-2}$  and the wavelength  $\lambda_{\text{CDW}}=ra$  in Table I is used in Fig. 3. As seen in this figure, the amplitude of the wave decreases with the increasing  $l$ .

To minimize the energy, we calculated the  $r$  dependence of the energy, which is plotted in Fig. 4. As seen in Fig. 4, a quasistable state appears for  $l\geq 2$  near  $r=1$  and the  $r$  dependence of the energy becomes flattened as  $l$  grows. This

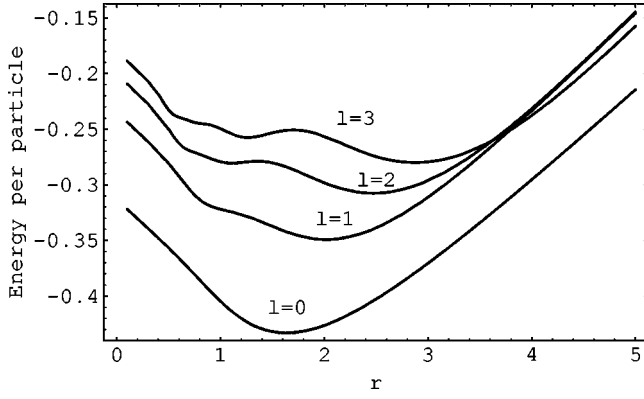


FIG. 4. The energy per particle  $E^{(l)}$  as a function of  $r$  for  $\nu = l + 1/2$ . The units of energy are  $q^2/l_B$ .

means that the UCDW state for  $l \geq 2$  has flexibility against disorder effect. This observation agrees with the absence of the spontaneous formation of the anisotropic state for  $l \leq 1$ .

The kinetic energy  $\varepsilon_l(\mathbf{p}, \bar{\nu})$  for the UCDW is written as

$$\varepsilon_l(\mathbf{p}, \bar{\nu}) = \int_{-\pi}^{\pi} \frac{dp'_x}{2\pi} \int_{-\pi\bar{\nu}}^{\pi\bar{\nu}} \frac{dp'_y}{2\pi} \tilde{V}_l^{\text{HF}}(\mathbf{p}' - \mathbf{p}). \quad (13)$$

This is independent of  $p_x$  after integration and we denote  $\varepsilon_l(\mathbf{p}, \bar{\nu}) = \varepsilon_l(p_y, \bar{\nu})$ .  $\varepsilon_l(p_y, 1/2)$  for  $l=0, 1$ , and  $2$  are shown in Fig. 5. As seen in Fig. 5, the bandwidth  $\Gamma_l$  decreases with the increasing  $l$ , that is,  $\Gamma_0 = 0.7363$ ,  $\Gamma_1 = 0.5682$ , and  $\Gamma_2 = 0.5042$  in the unit of  $q^2/l_B$ . Using the mean field of Eq. (10), the kinetic term in  $H_{\text{HF}}^{(l)}$  is written as

$$K_{\text{HF}}^{(l)} = \sum_m \int \frac{dp_y}{2\pi} a_{l,m}^\dagger(p_y) \varepsilon_l(p_y, \bar{\nu}) a_{l,m}(p_y), \quad (14)$$

where  $a_{l,m}(p_y) = \sum_n b_l(\mathbf{X}) e^{-ip_y n}$  for  $\mathbf{X} = (m, n)$ . Therefore the UCDW state is regarded as a collection of the one-dimensional lattice Fermi-gas systems which extend to the  $y$  direction. In the Buttiker-Landauer formula,<sup>25</sup> the conductance of a one-dimensional channel is equal to  $e^2/2\pi$  in the absence of the backscattering effect. Thus the conductance of the UCDW have an anisotropic value as

$$\sigma_{xx} = 0, \quad (15)$$

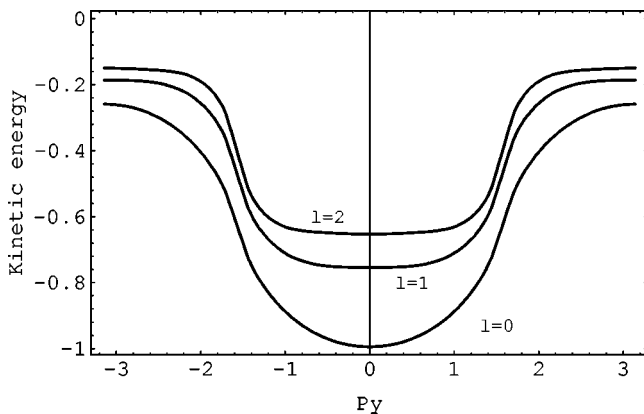


FIG. 5. The kinetic energy  $\varepsilon_l(p_y, 1/2)$  for  $l=0, 1$ , and  $2$ . The units of energy are  $q^2/l_B$ .

$$\sigma_{yy} = n_x \frac{e^2}{2\pi}, \quad (16)$$

where  $n_x$  is a number of the one-dimensional channels that extend from one edge to the other edge. If we take  $\sigma_{xy} = \nu e^2/2\pi$ , the resistance becomes

$$\rho_{xx} = \frac{n_x}{\nu^2} \frac{2\pi}{e^2}, \quad (17)$$

$$\rho_{yy} = 0. \quad (18)$$

Thus the formation of the UCDW leads the anisotropy in the magnetoresistance. For  $\nu = 9/2$ ,  $2\pi/e^2 \nu^2 = 1.3 \times 10^3 \Omega$ , which is of the same order as the experimental value  $\sim 1.0 \times 10^3 \Omega$ . Disorder effects decrease  $n_x$  by destroying the UCDW ordering. Furthermore, the backscattering effect due to impurities reduces  $\sigma_{yy}$  and  $\rho_{xx}$ . In the case for edge modes in the quantum Hall regime, there is no backscattering because of the chirality. The left-mover lives on one edge far from the other edge where the right-mover lives. For the UCDW state, on the other hand, each one-dimensional system has the width of  $ra$  at most. Therefore the backscattering effect strongly affects on the conductance in the UCDW.

To conclude this section, we point out a subtle problem concerned with the  $K$  invariance and the sliding mode. In an ordinary one-dimensional system, the difference of the chemical potentials between the left and right edge of the Fermi sea yields the net electric current. In the one-dimensional system of the UCDW, however, the difference of the chemical potentials can be canceled out by sliding the Fermi sea in Eqs. (6) and (7), thanks to the  $K$  invariance. Then there is no net electric current. This contradicts the above assertion apparently. As mentioned before, the translation in the momentum space is equal to the translation of the charge density in the real space. In other words, to slide the Fermi sea is the same as to slide the CCDW in the real space. The sliding mode is expected to be pinned by impurities. Therefore the violation of the  $K$  invariance due to pinning of the CCDW can remedy the contradiction.

#### IV. SUMMARY AND DISCUSSION

In this paper we have studied the CCDW state whose periodicity of the charge density coincides with that of the von Neumann lattice. The CCDW state is gapless and has an anisotropic Fermi surface. We obtained two types of the CCDW state, the UCDW state and CWC state. By calculating the Hartree-Fock energy, the UCDW is found to have a lower energy at the half-filled Landau levels. Furthermore, the wavelength dependence of the energy, the density profile, and the kinetic energy of the UCDW are calculated numerically. As a result, it is found that the amplitude of the UCDW and bandwidth of the Landau level decrease with the increasing  $l$  and a quasistable state appears for  $l \geq 2$ .

The UCDW state has a belt-shaped Fermi sea. Consequently the system consists of many one-dimensional lattice Fermi-gas systems that extend to the uniform direction. Formation of this structure could be the origin of the anisotropy observed in experiments. To confirm this speculation, experimental works for detecting the wavelength of the UCDW and theoretical works to include fluctuations around the

mean-field solution are necessary. Since there is no energy gap in the CCDW state, the fluctuation effect might be large compared with the CDW state with gaps. Actually Fradkin and Kivelson<sup>15</sup> proposed a rich phase diagram by considering fluctuations around the stripe-ordered state. We believe that the von Neumann lattice formalism presents an appropriate scheme to study the fluctuation effects around the mean field.

Recently a new insulating state was discovered around the quarter-filled third Landau level.<sup>28</sup> This state seems to have gaps and an integral quantized Hall conductance. These facts suggest that the state is a CDW state with gaps that are different from the CCDW. The CDW state whose periodicity is  $q/p$  times that of the von Neumann lattice has gaps and has  $q$  bands with  $p$ -fold degeneracy.<sup>27,21</sup> In the presence of a magnetic field and periodic potential, the Hall conductance of the free-electron system in the gap region is equal to the Chern number.<sup>29,21</sup> Thus the observed quantized Hall con-

ductance is probably the Chern number of the periodic potential problem. The transition between this CDW with gaps and the CCDW studied in this paper is an interesting future problem.

#### ACKNOWLEDGMENTS

I would like to thank K. Ishikawa, T. Ochiai, and J. Goryo for useful discussions. I also thank Y. S. Wu and G. H. Chen for helpful discussions. This work was partially supported by the special Grant-in-Aid for Promotion of Education and Science in Hokkaido University provided by the Ministry of Education, Science, Sports, and Culture, the Grant-in-Aid for Scientific Research on Priority area (Physics of CP violation) (Grant No. 11127201), the Grant-in-Aid for Basic Research (Grant No. 10044043), and the special Grant for Basic Research (Hierarchical matter analyzing system) from the Ministry of Education, Science, Sports, and Culture, Japan.

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