Large skyrmions in an $Al_{0.13}Ga_{0.87}As$ quantum well

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We report tilted-field magnetotransport measurements of two-dimensional electron systems in a 200-Å-wide $Al_{0.13}Ga_{0.87}As$ quantum well. We extract the energy gap for the quantum Hall state at Landau-level filling $\nu=1$ as a function of the tilt angle. The relatively small effective Landé g factor ($g\simeq 0.043$) of the structure leads to skyrmionic excitations composed of the largest number of spins yet reported ($s\simeq 50$). Although consistent with the skyrmion size observed, Hartree-Fock calculations, even after corrections, significantly overestimate the energy gaps over the entire range of our data.

In two-dimensional electron systems (2DES's), the quantum Hall effect (QHE) at the Landau-level filling factor $\nu=1$ has attracted much theoretical[1–6] and experimental[7–14] attention. At this filling, the Coulomb (exchange) energy is so influential that the QHE excitation gap is more than an order of magnitude larger than the single-particle Zeeman energy, the gap expected if the Coulomb energy were ''turned off.'' In fact, given a small enough Zeeman energy, the interplay between these two energies leads to a lowest-lying charged excitation, called a *skyrmion*, composed of electrons arranged in a canted, nearly parallel spin texture. Properties of this excitation, such as its energy gap and physical extent, are determined by the ratio $\tilde{g}=|g|\mu_B B_{\rm tot}/(e^2/\epsilon l_B)$ of the single-particle Zeeman energy, which limits the number of spin-flips in an excitation, to the Coulomb energy which favors local ferromagnetic ordering. (ϵ is the dielectric constant, $l_B=\sqrt{\hbar c/eB_\perp}$ is the magnetic length, g is the effective Landé g factor, μ_B the Bohr magneton, and $B_{\rm tot}$ and $B_\perp$ are the total applied magnetic field and the component perpendicular to the layer plane, respectively.)

The limit of $\tilde{g}\to 0$ is of particular interest where, in an ideal system, the excitation gap is predicted to exist even in the absence of Zeeman energy and the skyrmion size diverges—$s\to\infty$. (We choose s to denote the total spin of a thermally activated skyrmion-antiskyrmion pair; the effective spin of a single skyrmion or antiskyrmion would therefore be $s/2$ if particle symmetry holds.) Experiments employing diverse techniques such as optically pumped nuclear magnetic resonance,[7] magnetotransport,[8,14] and magneto-optical absorption spectroscopy[9] in GaAs 2DES's, where $|g|\simeq 0.44$, have yielded $s\sim 7$–9. By using hydrostatic pressure to tune g, Refs. 11–13 can access the regime $\tilde{g}\to 0$, where they extract a larger number of spin-flips ($s=36$) from the temperature dependence of their magnetotransport data. Unfortunately, this technique requires a separate cooldown for each g, which leads to a different disorder potential every time. Furthermore, since applying hydrostatic pressure lowers the density of the 2DES, to compensate the sample must be illuminated. Because of these complications, a controllable, systematic study using pressure is nontrivial.

In this paper, we report on an observation of the largest skyrmions yet reported ($s\simeq 50$), to our knowledge by using an alternate approach to access the low $\tilde{g}$ regime. In bulk $Al_xGa_{1-x}As$, g increases monotonically from $g=-0.44$ at $x=0$ (GaAs) to $g\approx+0.5$ at $x=0.35$ vanishing at $x\simeq 0.13$.[15] We fabricated a wafer with a 200-Å $Al_{0.13}Ga_{0.87}As$ quantum well bounded first by a thin 12.6-Å AlAs layer followed by thick $Al_{0.35}Ga_{0.65}As$ barriers on each side. Grown by molecular-beam epitaxy, this symmetric structure is modulation doped with Si. While many experimental techniques measure g,[15–20] determining an extremely low g is difficult and subject to great relative uncertainty. From calculations utilizing the Kane model,[21,22] our best estimate of the g factor for this wafer is $g=0.043\pm 0.010$, an order of magnitude lower than that in bulk GaAs.[23] The slightly positive value of g is attributed to the spillover of the electron wave function into the barrier region where g is positive, as well as the nonparabolicity of the energy bands.[21,24] In the right inset to Fig. 1, we show a calculation of the longitudinal g factor for a symmetrically distributed electron system as a function of Al concentration x in a 200-Å $Al_xGa_{1-x}As$ quantum well with barriers as specified above. We have taken the g factor to be isotropic; we will address this assumption below.

We measure two samples (identified as A and B) from different parts of the wafer with mobility $\mu\approx 5\times 10^4$ cm^2/V s in a Van der Pauw geometry. Samples A and B have total areal densities $n=1.37\times 10^{11}$ and 1.28×10^{11} cm^{-2}, respectively. We collect the low-temperature magnetotransport data in a dilution fridge and a ^{3}He system. In our experiment, we extract the excitation energy gap (Δ_1) for the QHE at filling $\nu=1$ from the temperature dependence of its longitudinal resistance (R_{xx}) minimum. We gather Δ_1 for several $\tilde{g}$ by tilting the sample and thus applying a mag-

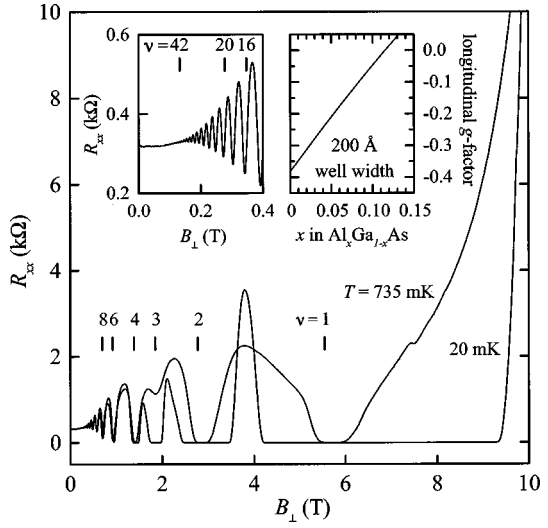

FIG. 1. Magnetoresistance traces for sample A at $T=20$ and 735 mK, which show QHE states for only $\nu=1$ and 3 among odd ν. The left inset shows the onset of Shubnikov–de Haas oscillation at $\nu=42$ ($B_\perp=0.13$ T). The right inset shows a calculation of the longitudinal g factor g_l for a 200-Å $Al_xGa_{1-x}As$ well bounded by $Al_{0.35}Ga_{0.65}As$; $g_l=0.043\pm0.010$ for $x=0.13$.

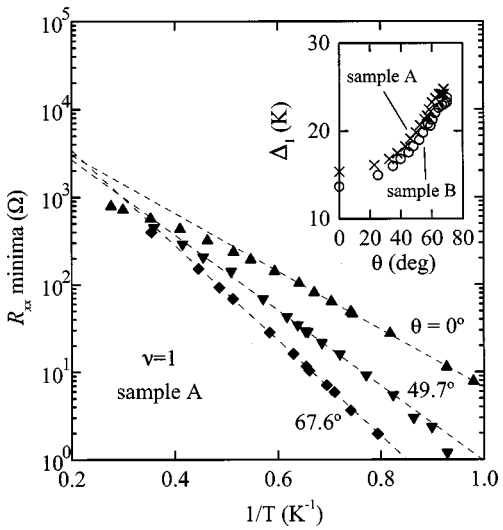

FIG. 2. Arrhenius plot of R_{xx} minima for the $\nu=1$ QHE state at several angles θ for sample A. The activation energy Δ_1 is plotted in the inset vs θ for both samples A and B.

netic field $B_{\rm tot}$ at an angle θ with respect to the normal of the sample plane. This technique allows the Zeeman energy ($\propto B_{\rm tot}$) to be tuned *in situ* while the other parameters in the system are nearly unaffected.[8] Unlike the experiments in Refs. 11–13, the areal density n, the disorder, and the Coulomb energy $e^2/\epsilon l_B$ remain constant for different values of $\tilde{g}$ leading to a relatively straightforward analysis of our data. By finding the energy gap for several angles θ (or equivalently $\tilde{g}$), we can determine the number of spin-flips involved in an excitation since any change in the gap is almost entirely attributable to the change in the Zeeman energy contribution. As in Ref. 8 and Refs. 11–13, we use the formula $s=\partial\tilde{\Delta}_1/\partial\tilde{g}$, where $\tilde{\Delta}_1=\Delta_1/(e^2/\epsilon l_B)$ is Δ_1 normalized by the Coulomb energy, to extract the number of spin-flips in an excitation.

The traces in Fig. 1 attest to the high quality of the sample as well as the small value of the g factor. The longitudinal magnetoresistance of sample A is plotted for temperatures $T=20$ and 735 mK. The 20-mK trace exhibits minima for QHE states with even integer fillings as high as $\nu=42$ at $B_\perp=0.13$ T, marking the lower bound for the Shubnikov–de Haas (SdH) oscillation regime. (See the left inset to Fig. 1.) We believe that impurity scattering, rather than alloy scattering, is the dominant mechanism limiting the mobility in our $Al_xGa_{1-x}As$ quantum wells. A Born approximation treatment[25,26] estimates the alloy scattering mobility limit to be $\approx 4\times10^5$ $cm^2/V\,s$, an order of magnitude larger than our measured mobility. Impurity scattering, however, may explain the low measured mobility. It is known that Al, a relatively reactive element, incorporates impurities in $Al_xGa_{1-x}As$ layers during growth.[27] Such a mechanism is consistent with the relatively low mobility (even after considering the occupation of multiple ellipsoids) observed in AlAs quantum wells[28] where there is no alloy scattering.

Although minima corresponding to QHE states exist for many even integer fillings, there are none for odd integers other than $\nu=1$ and 3. At higher odd fillings, the influence of the Coulomb exchange energy is progressively diminished since the states occur at lower $B_\perp$ and the fraction of electrons affected is $1/\nu$;[29] instead, the excitation gap at higher odd ν is determined primarily by the competition of the single-particle Zeeman energy and disorder broadening of Landau levels. In our system the Coulomb exchange energy appears to be significant enough to overcome disorder broadening only for the odd-integer fillings $\nu=1$ and 3. In fact, calculations which consider the finite width of the 2DES (Ref. 6) predict skyrmionic excitations for our sample at both these fillings. Unfortunately, the measured excitation gap at $\nu=3$ is only $\simeq 3$ K, which is of the order of the Landau-level broadening,[30] making further analysis inconclusive.

We now discuss our data for $\nu=1$. Shown in Fig. 2 are the Arrhenius plots of R_{xx} minima for the $\nu=1$ QHE in sample A for three angles in the temperature range $1<T<4$ K. We extract the activation energy Δ_1 from the slope of a best-fit line (dashed line) to the data using the relation $R_{xx}\sim\exp(-\Delta_1/2T)$. For $\theta=0°$, 49.7°, and 67.6°, we have $\Delta_1=15$, 20, and 25 K, respectively. Our measured Δ_1 is a monotonically increasing function of θ lying in the range $13<\Delta_1<25$ K, as shown in the inset to Fig. 2. Since the data for both samples A and B are qualitatively very similar, we focus on sample A below.

In Fig. 3, we plot $\tilde{\Delta}_1=\Delta_1/(e^2/\epsilon l_B)$ vs $\tilde{g}$ and discuss our results in light of other experiments and theoretical calculations. The data from sample A, which occupies the extreme lower left portion of the figure, are expanded in the inset to Fig. 3. An asymptote (dashed line) fit to the lower range of the data reveals $s=50.2\pm1.0$. Similar analysis on sample B yields $s=49.2\pm2.1$. Compared to our experiment, Ref. 8 explores higher $\tilde{g}$ in GaAs samples represented by various closed symbols in Fig. 3. In the case of the pressure-tuned data (not shown) from Refs. 11–13, $\tilde{g}$ is in the same vicinity as our data, although there is more scatter in the reported $\tilde{\Delta}_1$, presumably because of variations in disorder as discussed

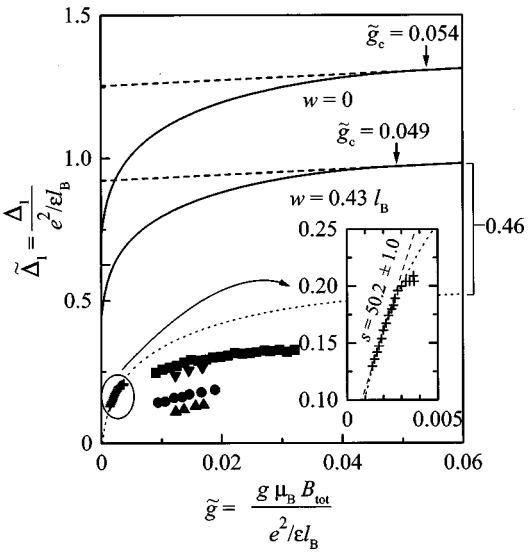

FIG. 3. Normalized activation energy $\tilde{\Delta}_1 = \Delta_1/(e^2/\epsilon l_B)$ vs normalized Zeeman energy $\tilde{g} = g\mu_B B_{\rm tot}/(e^2/\epsilon l_B)$ from experiments and calculations. The experimental data are from sample A (+) and Ref. 8 (closed symbols). In the inset, the asymptote (dashed line) fit to the lower range of sample A data reveals $s = 50.2 \pm 1.0$. The results of Hartree-Fock calculations for a 2DES with zero-layer thickness ($w = 0$) and for $w = 0.43 l_B$ are also shown in the main figure. In the inset, the $w = 0.43 l_B$ skyrmion excitation gap (dotted line) shifted down by $0.46 e^2/\epsilon l_B$ matches the lower range of sample A data. (See text for details.)

before. A noteworthy commonality in these experiments is the range of the measured skyrmionic excitation gap ($\tilde{\Delta}_1 < 0.33$).

We now contrast our experimentally obtained $\tilde{\Delta}_1$ with calculations. The top solid curve in Fig. 3 represents the results of Hartree-Fock calculations[5,6] for the skyrmion excitation energy gap in an ideal, infinitely thin 2DES, i.e., one with an electron probability density width $w = 0$. The calculated skyrmion gap declines steeply as $\tilde{g} \to 0$, reflecting the decreasing cost in Coulomb energy for an excitation with an increasing degree of nearly parallel spins. On the other hand, the (exchange-enhanced) single spin-flip excitation gap (dashed line) expected in the absence of skyrmions has a constant slope corresponding to $s = 1$. At $\tilde{g} = 0$, the skyrmion gap is $\frac{1}{2}\sqrt{\pi/2}\, e^2/\epsilon l_B$, half the single spin-flip excitation gap. The skyrmion remains the favored excitation for $\tilde{g} < \tilde{g}_c = 0.054$ (marked by a vertical arrow).

Note the striking discrepancy between the calculations and experiments. The calculated skyrmion gap for the ideal case is a factor of 4.3–6.7 larger than the experimental data! The ideal case, however, ignores important effects such as finite thickness correction (FTC), Landau-level mixing (LLM), and disorder broadening of Landau levels—all of which reduce the energy gap. Unfortunately, since no calculation currently treats these three corrections simultaneously for skyrmions, we must consider them in cumulative succession. Figure 3 includes Hartree-Fock calculations with FTC (Ref. 6) for a layer thickness $w = 0.43 l_B$, appropriate for our sample. (We determine w by fitting a Gaussian function to the electron probability density from a self-consistent local-density approximation calculation.) By softening short-range interactions, the FTC reduces the predicted gap by $\approx 30\%$. To assess the effect of LLM, we first focus on the exchange-enhanced single spin-flip excitation gap corrected for FTC and LLM. Existing calculations are not in quantitative agreement; based on the trends in Refs. 31 and 32, we estimate the corrected single spin-flip gap to be about $0.58 e^2/\epsilon l_B$ and $0.69 e^2/\epsilon l_B$, respectively for our sample (plus the Zeeman energy $|g|\mu_B B_{\rm tot}$). If the role of disorder is limited to the disorder broadening of Landau levels, we expect the predicted gap to diminish by only $\Gamma \simeq 0.06 e^2/\epsilon l_B$.[30] We then deduce the gap in the large skyrmion limit (small $\tilde{g}$) by shifting the $w = 0.43 l_B$ curve for the skyrmion excitation gap by a constant to match (at $\tilde{g} > \tilde{g}_c$) the corrected single spin-flip gaps estimated. The shifted curve still overestimates our experimental gaps for $\nu = 1$ by factors of 1.3–1.5 and 1.8–2.3 for Refs. 31 and 32, respectively.

Although the *absolute* values of the calculation for the $\nu = 1$ QHE cannot be reconciled with the experimental data, the size of skyrmions (from the slope of the curve) predicted by the calculation agrees with our data. This agreement is evidenced by the $w = 0.43 l_B$ curve shifted down by $0.46 e^2/\epsilon l_B$ (shown by the dotted line in the inset to Fig. 3) which fits the entire lower range of our data remarkably well. In fact, we can use the calculation to check the validity of our value for the g factor. We find that this agreement is valid only in a very narrow range of assumed values for g which includes our estimate of $g \simeq 0.043$.[33] This congruity may be viewed, perhaps, as an independent confirmation of g factor in our sample.

Thus far, we have interpreted our data assuming an isotropic g factor for our sample. In general, however, the g factor can be anisotropic in confined systems, with g_l and g_t denoting the longitudinal and transverse components of the g factor with respect to the growth axis (for a review, see Ref. 22 and references therein). The electron g-factor anisotropy is governed by the low-symmetry electron quantum confinement, and changes strongly with the quantum well width; it can be qualitatively estimated from the energy splitting between the light- and heavy-hole bands. Based on a time-resolved photoluminescence experiment measuring electron spin quantum beats,[34] Le Jeune *et al.* concluded that the g factor is indeed anisotropic for narrow GaAs quantum wells bounded by $Al_{0.30}Ga_{0.70}As$ barriers, as found in Refs. 20 and 35. However, in quantum wells 120 Å and wider, the anisotropy vanishes. We note here that the g factor anisotropy may be reduced as the electron kinetic energy in the 2DES, the thermal energy, or localization energy (because of imperfections or magnetic field) become comparable in value to the quantum confinement energy.

A simple Kane-model calculation for electrons at the bottom of the first subband in our system yields a transverse component of the g factor $g_t = 0.085$. If we reinterpret our data so that $g(\theta) = \sqrt{g_l^2 \cos^2(\theta) + g_t^2 \sin^2(\theta)}$, then s from the asymptote to the lower range of the data is reduced—$s = 19$. However, experimental findings mentioned above support an isotropic g factor for the parameters in our sample. Our system with a quantum well of width 200 Å and a finite 2DES concentration should exhibit even less of a tendency toward g-factor anisotropy. As already noted, the size of skyrmions

from the calculations and our data are no longer consistent if an anisotropic g factor is assumed. Therefore, we believe that the isotropic value of 0.043 is the best estimate for the g factor in our samples.

In summary, we have focused on the thermal excitation energy for $\nu=1$ gathered for several tilt angles from magnetotransport measurements of two-dimensional electron systems in a 200-Å-wide $Al_{0.13}Ga_{0.87}As$ quantum well. In this structure with a small g factor ($g\simeq 0.043$), we observe skyrmions of the largest size yet reported ($s\simeq 50$), to our knowledge. The magnitude of the energy gaps measured are consistent with those from other experiments. While matching the experimentally determined size of skyrmions, Hartree-Fock calculations, even after treatment for corrections, significantly overestimate the energy gaps in our data. Understanding this disparity requires further studies in the large skyrmion regime.

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