

Spin waves in a two-dimensional p -wave superconductor: Sr_2RuO_4

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We study spin excitations in a two-dimensional p -wave superconductor with $\vec{\Delta} = \hat{d}(\hat{k}_1 \pm i\hat{k}_2)$ symmetry in the context of the newly discovered superconducting Sr_2RuO_4 . The polarization and spectrum of spin-wave excitations are identified and their experimental consequences are discussed.

I. INTRODUCTION

The recent discovery of superconducting Sr_2RuO_4 generates much efforts to determine the pairing symmetry of a possible unconventional superconducting order parameter in this system. Being a $4d$ -orbital analog¹ of the high- T_c cuprate superconductors, Sr_2RuO_4 has the same layered perovskite structure as La_2CuO_4 and becomes superconducting below $T_c \approx 1.5$ K.²⁻⁴ Despite the structural similarity, it behaves very differently from the copper oxides. The normal state below 50 K can be well described by a quasi-two-dimensional Landau-Fermi liquid: The resistivity shows T^2 behavior in the a - b plane and c direction with a large anisotropy ratio.⁴ Quantum oscillations⁵ revealed three cylindrical Fermi-surface sheets in accordance with the band-structure calculations.⁶ There are three $4d$ orbitals (d_{xy}, d_{yz}, d_{zx}) of the Ru^{4+} ions which form three bands crossing the Fermi level. There are two electronlike and one holelike Fermi surfaces.⁵ On the other hand, the mass enhancement (≈ 4) in this material is not small indicating the existence of strong correlation.⁵ The fact that it has a large mass enhancement and a related material, SrRuO_3 , shows ferromagnetism leads to the proposal that the superconducting state of Sr_2RuO_4 is formed by odd-parity pairing (spin-triplet pairing) presumably in the p -wave channel.⁷⁻⁹

There have been a number of experiments indicating unconventional superconductivity in Sr_2RuO_4 . The transition temperature was found to be very sensitive to nonmagnetic impurities¹⁰ and nuclear quadrupole resonance (NQR) found no Hebel-Slichter peak.¹¹ Even though these experiments¹⁰⁻¹² suggested that the pairing symmetry is non- s wave, they could not determine the pairing symmetry itself. More recently, ^{17}O -Knight shift in NMR was measured and it is consistent with the spin-triplet superconductivity¹³ with \hat{d} parallel to the c axis as the case of superfluid³He- A .¹⁴ Here \hat{d} is the unit vector of the triplet order parameter.¹⁴ On the other hand, the specific-heat data are consistent with usual s -wave superconductors if we subtract a persistent T -linear term.^{15,16} In the earlier experiment the ratio of this coefficient of the T -linear term to the one in the normal state

γ_0/γ_N was larger than 0.5. This initiated the proposal of possible nonunitary state in Sr_2RuO_4 .¹⁷ However, more recent data¹⁶ show that $\gamma_0/\gamma_N < 0.25$. This means the nonunitary state is untenable. The specific-heat data further imply that the energy gap should be almost independent of \mathbf{k} like in an s -wave superconductor and also there should be a normal-state-like component. These features are most naturally described by the three orbital band model,¹⁸ where it is assumed that the superconductivity resides mainly in the γ band, while the α and β bands may be considered in the normal state. Of course we cannot exclude the possible small superconducting order parameter associated with the α and β band.¹⁹ In this perspective, the small magnetization seen by the muon spin resonance²⁰ is rather puzzling. In addition, rather strong flux pinning in the vortex state is observed in Sr_2RuO_4 comparable to the one in B phase of UPt_3 and $\text{U}_{0.97}\text{Th}_{0.03}\text{Be}_{13}$.²¹ The latter two systems are considered to be in the nonunitary state.²² Here it is worthwhile to mention that, although all these are consistent with the triplet pairing, both the magnetization and the strong vortex pinning should arise from some topological defects or dislocation in the samples. A recent small-angle neutron-scattering experiment in a magnetic field parallel to the c axis showed that the square vortex lattice is almost everywhere in the B - T phase diagram.²³ This square vortex lattice is also very consistent with p -wave superconductivity we consider here.²⁴⁻²⁶

In the following we take the order parameter

$$\vec{\Delta}(\mathbf{k}) = \Delta \hat{d}(\hat{k}_1 \pm i\hat{k}_2), \quad (1)$$

where \hat{d} is assumed to be parallel to the c axis and \mathbf{k} is the quasiparticle wave vector within the a - b plane. For simplicity, we also assume that the superconductivity resides only in the γ band. This means the α and β bands provide nonsuperconducting background. Recently starting from Eq. (1), microscopic studies of the vortex state,^{25,26} the effect of impurities,²⁷ and the quasiparticle spectrum around a single vortex²⁸ have been discussed.

Here we study spin dynamics in the p -wave superconductors with the order parameter given by Eq. (1). It is important to realize that there are four distinct situations for the mea-

surement of the dynamical spin susceptibility in two-dimensional p -wave superconductors. This is because the direction of the vectorial order parameter $\mathbf{d}(\mathbf{k})$ is fixed along a crystallographic axis which is perpendicular to the basal plane. One can apply a magnetic field along the directions perpendicular and parallel to the RuO₂ plane. In each case, one can measure longitudinal and transverse susceptibilities. Therefore there exist four different susceptibilities. This is a unique property of the two-dimensional p -wave superconductor because, in the three-dimensional case, the direction of the order parameter is not fixed and is always perpendicular to the applied magnetic field.

We also assume that the pinning of \hat{d} vector parallel to the c axis involves a finite pinning energy $-\frac{1}{2}\chi_N\Omega_d^2(d_z)^2$, most likely due to the spin-orbit coupling. Here we do not evaluate the exact value of Ω_d , but we expect that it will be $\Omega_d(T) < \Delta(T)$. Recently the pinning frequency was estimated by Tewordt.²⁹

We believe that the experimental determination of $\Omega_d(T)$ will provide an insight in the pinning mechanism of \hat{d} . In order to avoid future misunderstanding, we shall first explain the role of \hat{d} here. \hat{d} is called the spin vector which has been used in studying ³He. It is perpendicular to the direction of the spin associated with the condensed pair.¹⁴ Under these assumptions, we found the following results for the spin excitations.

Case A: When magnetic field is parallel to the a - b plane (i.e., $\mathbf{H} \perp \hat{d}$). We have the spin waves with the dispersion relations

$$\omega_{\parallel}^2 = (1 - I f_d) \Omega_d^2 + \frac{1}{2} f_d (v_F q)^2 \quad (2)$$

and

$$\omega_{\perp}^2 = \omega_{\parallel}^2 + \omega_L^2 \quad (3)$$

for the longitudinal (ω_{\parallel}) and the transverse (ω_{\perp}) resonance, respectively. The ‘‘pinning’’ frequency Ω_d is associated with the restoring tendency of the order parameter to the initial direction \hat{d} against external perturbations. Here $\omega_L = \mu_B H$ is the Larmor frequency. $I (= -\frac{1}{4} Z_0)$ is the dimensionless on-site Hubbard potential and f_d is the dynamical superfluid density ($\omega \gg v_F q$) given by

$$f_d = \int_{\Delta}^{\infty} dE \frac{\Delta^2 \tanh(E/2T)}{E^2 \sqrt{E^2 - \Delta^2}}, \quad (4)$$

where Δ is the magnitude of the full gap. These can be readily generalized in the presence of Fermi-liquid corrections.¹⁴

Case B: When magnetic field is perpendicular to the a - b plane (i.e., $\mathbf{H} \parallel \hat{d}$). The static susceptibility in this case is given by

$$\chi = \chi_0 \frac{Y}{1 - IY}, \quad (5)$$

where $Y = 1 - f_s$ is the Yoshida function.¹⁴ There is no longitudinal collective mode, but a damped mode with the following dispersion exists in the longitudinal response:

$$\omega = i \frac{1}{\sqrt{2}} v_F |q| \sqrt{(1 - f_d)I}. \quad (6)$$

For the transverse response, we have the spin wave with the same dispersion relation as Eq. (3).

The remainder of this paper is organized as follows. In Sec. II, we show the calculation of the dynamical spin susceptibilities for four different cases mentioned above. We conclude in Sec. III. Some of the technical details are relegated to the appendixes.

II. DYNAMICAL SPIN SUSCEPTIBILITY AND SPIN WAVES

Here we use Green’s-function method of Ref. 30. The single-particle Green’s function in the Nambu space is given by

$$G^{-1}(i\omega_n, \mathbf{k}) = i\omega_n - \left[\xi_{\mathbf{k}} + \frac{\Omega_L}{2} (\vec{\sigma} \cdot \hat{h}) \right] \rho_3 - \Delta (\hat{k} \cdot \vec{\rho}) \sigma_1, \quad (7)$$

where ρ_i and σ_i are Pauli matrices acting on the particle-hole and spin spaces, respectively, \hat{h} is the unit vector parallel to the static magnetic field. $\omega_n = (2n + 1)\pi T$ is the fermionic Matsubara frequency, $\Omega_L = \omega_L / (1 - I)$, $\xi_{\mathbf{k}} = \mathbf{k}^2 / 2m - \mu$, and Δ is the magnitude of the superconducting order parameter. Then the spin susceptibility is expressed as the autocorrelation function of spin operators. For example, the irreducible spin-spin correlation function $\chi_{ij}^{00} \sim \langle [S_i, S_j] \rangle_0$ can be computed from

$$\chi_{ij}^{00}(i\omega_\nu, \mathbf{q}) = T \sum_n \sum_{\mathbf{p}} \text{Tr}[\alpha_i G(\mathbf{p}, \omega_n) \times \alpha_j G(\mathbf{p} - \mathbf{q}, i\omega_n - i\omega_\nu)], \quad (8)$$

where $\alpha_1 = \rho_3 \sigma_1$, $\alpha_2 = \sigma_2$, $\alpha_3 = \rho_3 \sigma_3$ are the spin vertices and come from the expression of spin density: $S_i \sim \text{Tr}[\Psi^\dagger \alpha_i \Psi]$. $\omega_\nu = 2\nu\pi T$ is the bosonic Matsubara frequency.

In the fully renormalized spin susceptibilities, it is crucial to include the fluctuation of the superconducting order parameter related to the rotation of the \hat{d} vector. Now let us look at the specific cases.

A. Magnetic field is perpendicular to the order parameter:

$\mathbf{H} \parallel \hat{\mathbf{x}}$

1. Longitudinal susceptibility

Since the direction of the magnetic field is in $\hat{\mathbf{x}}$ direction, the longitudinal susceptibility is given by $\chi_{xx} \sim \langle [S_x, S_x] \rangle$ and can be obtained from

$$\chi_{xx} = \frac{\chi_{xx}^0}{1 - I\chi_{xx}^0}, \quad (9)$$

where I is the exchange interaction and χ_{xx}^0 is the susceptibility irreducible with respect to I . χ_{xx}^0 includes the coupling between the spin and the order parameter, and can be written as

$$\chi_{xx}^0 = \chi_{xx}^{00} + \frac{V_{xx}g\bar{V}_{xx}}{1 - g\Pi_{xx}}, \quad (10)$$

where g is the strength of the interaction which is responsible for the superconductivity. Here $\chi_{xx}^{00} \sim \langle [S_x, S_x] \rangle_0$ is the bare susceptibility, $V_{xx} \sim \langle [S_x, \delta\Delta_x] \rangle_0$, $\bar{V}_{xx} \sim \langle [\delta\Delta_x, S_x] \rangle_0$ represent the coupling between the spin density and the order-parameter fluctuation, and $\Pi_{xx} \sim \langle [\delta\Delta_x, \delta\Delta_x] \rangle_0$ the correlation between the order-parameter fluctuations. In Nambu's notation, $\delta\Delta_x \sim \Delta \text{Tr}[\Psi^\dagger \alpha_1 \rho_1 \sigma_1 \Psi]$. Notice that χ_{xx}^0 consists of two parts: the quasiparticle contribution and the contribution from the excitation of the condensate. In the case of the longitudinal response, the Ω_L term in Eq. (7) should be dropped out from the final expression. The details of the

computation of each correlation function can be found in Appendix A. We obtain ($\zeta = \mathbf{v}_F \cdot \mathbf{q}$):

$$\chi_{xx}^{00}(\omega, \mathbf{q}) = N(0) \left\langle \frac{\zeta^2 - \omega^2 f}{\zeta^2 - \omega^2} \right\rangle,$$

$$V_{xx}(\omega, \mathbf{q}) = N(0) \left\langle \frac{\omega}{2\Delta} f \right\rangle = \bar{V}_{xx}(\omega, \mathbf{q}),$$

$$\Pi_{xx}(\omega, \mathbf{q}) = g^{-1} - N(0) \left\langle \frac{\zeta^2 - \omega^2}{4\Delta^2} f \right\rangle, \quad (11)$$

where

$$f(\omega, \zeta) = 4\Delta^2(\zeta^2 - \omega^2) \int_{\Delta}^{\infty} dE \frac{\tanh(E/2T)}{\sqrt{E^2 - \Delta^2}} \frac{(\zeta^2 - \omega^2)^2 - 4E^2(\omega^2 + \zeta^2) + 4\zeta^2\Delta^2}{[(\zeta^2 - \omega^2)^2 + 4E^2(\omega^2 - \zeta^2) + 4\zeta^2\Delta^2]^2 - 16\omega^2E^2(\zeta^2 - \omega^2)^2} \quad (12)$$

and $\langle F \rangle = \int_0^{2\pi} (d\phi/2\pi) F(\phi)$. From now on, we set the density of state $N(0) \equiv 1$. In all these analyses we neglect $\Omega_L = \omega_L/(1-I)$ for simplicity.

Since the orientation of the order parameter is initially fixed along one of the crystallographic directions, there should be an energy scale Ω_d associated with the tendency to restore the original direction against external perturbations. This ‘‘pinning’’ frequency enters in the correlation between fluctuations of order parameters and leads to

$$\Pi_{xx}(\omega, \mathbf{q}) = g^{-1} - \left\langle \frac{\zeta^2 - \omega^2 + \Omega_d^2}{4\Delta^2} f \right\rangle. \quad (13)$$

Thus we get the following expression for χ_{xx}^0 :

$$\chi_{xx}^0 = \left\langle \frac{1}{\zeta^2 - \omega^2} \left(\zeta^2 + \frac{\omega^2 \Omega_d^2 f}{\omega^2 - \zeta^2 - \Omega_d^2} \right) \right\rangle. \quad (14)$$

In order to find a well-defined excitation, we consider the limit $\omega \gg v_F q$, where the quasiparticles do not generate dissipation which could make collective modes damped. In this case, the above expression can be simplified in the long-wavelength limit ($q \ll k_F$) as

$$\chi_{xx}^0 \approx \left[-\frac{1}{2} \frac{(v_F q)^2}{\omega^2} + \frac{\omega^2 \Omega_d^2 f_d}{\left[\frac{1}{2} (v_F q)^2 - \omega^2 \right] \left[\omega^2 - \frac{1}{2} (v_F q)^2 - \Omega_d^2 \right]} \right]. \quad (15)$$

Finally, in the limit $\omega \gg v_F q$, we get the following full susceptibility after taking into account the exchange interaction:

$$\chi_{xx} \approx \frac{\omega^2 \Omega_d^2 f_d + \frac{1}{2} (v_F q)^2 (\omega^2 - \Omega_d^2)}{\left[\frac{1}{2} (v_F q)^2 - \omega^2 \right] \left[\omega^2 - \frac{1}{2} (v_F q)^2 - \Omega_d^2 \right] - I \left[\frac{1}{2} (v_F q)^2 (\omega^2 - \Omega_d^2) + \omega^2 \Omega_d^2 f_d \right]}, \quad (16)$$

where $f_d = \lim_{\omega \rightarrow 0} \lim_{q \rightarrow 0} \langle f \rangle$ and it is given by

$$f_d = \int_{\Delta}^{\infty} dE \frac{\tanh(E/2T)}{\sqrt{E^2 - \Delta^2}} \frac{\Delta^2}{E^2}. \quad (17)$$

It is called the ‘‘dynamical superfluid density.’’ Now we can read off the dispersion relation of a collective mode from the

pole of the response function. We find that the longitudinal susceptibility supports a spin wave and its dispersion relation is given by

$$\omega^2 = (1 - I f_d) \Omega_d^2 + \frac{1}{2} f_d (v_F q)^2. \quad (18)$$

This is consistent with the similar expression in

superfluid $^3\text{He-A}$ phase.^{14,31} Not surprisingly, p -wave superconductors have the longitudinal spin wave as in superfluid $^3\text{He-A}$.

2. Transverse susceptibility

Now we analyze the transverse susceptibility. The transverse susceptibility $\chi_{+-} \sim \frac{1}{2}\langle [S_y + iS_z, S_y - iS_z] \rangle$ can be obtained from

$$\chi_{+-} = \frac{(1 - I\chi_{-+}^0)\chi_{+-}^0 + I\chi_{++}^0\chi_{--}^0}{(1 - I\chi_{+-}^0)(1 - I\chi_{-+}^0) - I^2\chi_{++}^0\chi_{--}^0}, \quad (19)$$

where $\chi_{(\pm,\pm)}^0$ are the susceptibilities irreducible to I . One can easily show that $\chi_{+-}^0 = \chi_{-+}^0$ and $\chi_{++}^0 = \chi_{--}^0$. Now χ_{+-}^0 and χ_{++}^0 are given by

$$\begin{aligned} \chi_{+-}^0 &= \chi_{+-}^{00} + \sum_{k=y,z} \frac{V_{+k}g\bar{V}_{k-}}{1 - g\Pi_{kk}}, \\ \chi_{++}^0 &= \chi_{++}^{00} + \sum_{k=y,z} \frac{V_{+k}g\bar{V}_{k+}}{1 - g\Pi_{kk}}. \end{aligned} \quad (20)$$

Here $\chi_{+-}^{00} \sim \langle [S_y + iS_z, S_y - iS_z]_0 \rangle$, $\chi_{++}^{00} \sim \langle [S_y + iS_z, S_y + iS_z]_0 \rangle$ are the bare susceptibilities. $V_{\pm k} \sim \langle [S_y \pm iS_z, \delta\Delta_k]_0 \rangle$, $\bar{V}_{k\pm} \sim \langle [\delta\Delta_k, S_y \pm iS_z]_0 \rangle$ and $\Pi_{kk} \sim \langle [\delta\Delta_k, \delta\Delta_k]_0 \rangle$ are spin/order-parameter couplings and the fluctuation propagator of the order parameters, respectively, where $\delta\Delta_k \sim \text{Tr}[\Psi^\dagger \alpha_k \rho_1 \sigma_1 \Psi]$.

When $\omega_L = 0$, we find

$$\begin{aligned} \chi_{+-}^{00} &= \chi_{-+}^{00} = \left\langle \frac{\zeta}{\zeta - \omega} \right\rangle - \left\langle \frac{\zeta + \omega f}{\zeta - \omega} \right\rangle, \quad \chi_{++}^{00} = \chi_{--}^{00} = \left\langle \frac{f}{2} \right\rangle, \\ V_{\pm y} &= \bar{V}_{y\pm} = \frac{1}{\sqrt{2}} \left\langle \frac{\omega}{2\Delta} f \right\rangle, \quad V_{\pm z} = \bar{V}_{\pm z} = 0, \\ \Pi_{yy} &= g^{-1} - \left\langle \frac{\zeta^2 - \omega^2}{4\Delta^2} f \right\rangle, \quad \Pi_{zz} = -\Pi_{yy} + \langle f \rangle. \end{aligned} \quad (21)$$

In the presence of ‘‘pinning’’ frequency, Π_{yy} should be modified to

$$\Pi_{yy} = g^{-1} - \left\langle \frac{\zeta^2 - \omega^2 + \Omega_d^2}{4\Delta^2} f \right\rangle. \quad (22)$$

Now incorporating finite ω_L and, using Eq. (20), we get the following results:

$$\begin{aligned} \chi_{+-}^0 &= \chi_{-+}^0 \\ &= \left[\left\langle \frac{\zeta^2 + \Omega_L^2 - \omega^2 f}{\zeta^2 + \Omega_L^2 - \omega^2} \right\rangle - \frac{\langle f \rangle}{2} - \frac{1}{2} \frac{\omega^2 \langle f \rangle^2}{\langle (\omega^2 - \zeta^2 - \Omega_d^2) f \rangle} \right], \\ \chi_{++}^0 &= \chi_{--}^0 = \left[\frac{\langle f \rangle}{2} - \frac{1}{2} \frac{\omega^2 \langle f \rangle^2}{\langle (\omega^2 - \zeta^2 - \Omega_d^2) f \rangle} \right]. \end{aligned} \quad (23)$$

When $\omega \gg v_F q$, the transverse susceptibilities irreducible to I in the long-wavelength limit ($q \ll k_F$) become

$$\begin{aligned} \chi_{+-}^0 &= \chi_{-+}^0 \\ &\approx N(0) \left[-\frac{1}{2} \frac{(v_F q)^2}{\omega^2} \right. \\ &\quad \left. + \frac{\omega^2 f_d \Omega_d^2}{\left[\frac{1}{2} (v_F q)^2 - \omega^2 \right] \left[\omega^2 - \frac{1}{2} (v_F q)^2 - \Omega_d^2 \right]} \right. \\ &\quad \left. - \frac{f_d}{2} + \frac{1}{2} \frac{\omega^2 f_d}{\omega^2 - \Omega_d^2 - \frac{1}{2} (v_F q)^2} \right], \\ \chi_{++}^0 &= \chi_{--}^0 \approx N(0) \left[\frac{f_d}{2} - \frac{1}{2} \frac{\omega^2 f_d}{\omega^2 - \Omega_d^2 - \frac{1}{2} (v_F q)^2} \right]. \end{aligned} \quad (24)$$

Using the above results and the RPA (random-phase approximation) expression [see Eq. (19)] for the full transverse susceptibility, one finds two poles which correspond to a propagating spin-wave mode and a damped mode. The dispersion relation of the spin wave is the same as the one in the longitudinal susceptibility:

$$\omega^2 = (1 - I f_d) \Omega_d^2 + \omega_L^2 + \frac{1}{2} f_d (v_F q)^2, \quad (25)$$

where ω_L is the Larmor frequency.

B. Magnetic field is parallel to the order parameter: $\mathbf{H} \parallel \hat{\mathbf{z}}$

1. Longitudinal susceptibility

If the magnetic field is applied along the direction of the order parameter, then the longitudinal susceptibility corresponds to $\chi_{zz} \sim \langle [S_z, S_z] \rangle$. As explained in previous sections, the full χ_{zz} can be again obtained from

$$\begin{aligned} \chi_{zz} &= \frac{\chi_{zz}^0}{1 - I\chi_{zz}^0}, \\ \chi_{zz}^0 &= \chi_{zz}^{00} + \frac{V_{zz}g\bar{V}_{zz}}{1 - g\Pi_{zz}}. \end{aligned} \quad (26)$$

Here χ_{zz}^0 is the longitudinal susceptibility irreducible with respect to I . Notice that $\chi_{zz}^{00} \sim \langle [S_z, S_z]_0 \rangle$, $V_{zz} \sim \langle [S_z, \delta\Delta_z]_0 \rangle$, $\bar{V}_{zz} \sim \langle [\delta\Delta_z, S_z]_0 \rangle$, and $\Pi_{zz} \sim \langle [\delta\Delta_z, \delta\Delta_z]_0 \rangle$, where $\delta\Delta_z \sim \Delta \text{Tr}[\Psi^\dagger \alpha_3 \rho_1 \sigma_1 \Psi]$.

In this geometry the superconducting order parameter does not move in the presence of the ac field. So there is no coupling between the spin density and the fluctuation of the order parameter. As a result, we have

$$\chi_{zz} = \frac{\chi_{zz}^0}{1 - I\chi_{zz}^0}, \quad (27)$$

where

$$\chi_{zz}^0 = \chi_{zz}^{00} = \left\langle \frac{\zeta^2(1-f)}{\zeta^2 - \omega^2} \right\rangle. \quad (28)$$

Equation (27) gives a pole which gives the damped mode at

$$\omega = i \frac{1}{\sqrt{2}} v_F |q| \sqrt{I(1-f_d)}. \quad (29)$$

2. Transverse susceptibility

The transverse susceptibility, $\chi_{+-} \sim \frac{1}{2} \langle [S_x + iS_y, S_x - iS_y] \rangle$, can be computed by using RPA expression [Eq. (19)] and the following relation:

$$\begin{aligned} \chi_{+-}^0 &= \chi_{+-}^{00} + \sum_{k=x,y} \frac{V_{+k} g \bar{V}_{k-}}{1 - g \Pi_{kk}}, \\ \chi_{++}^0 &= \chi_{++}^{00} + \sum_{k=x,y} \frac{V_{+k} g \bar{V}_{k+}}{1 - g \Pi_{kk}}. \end{aligned} \quad (30)$$

Here $\chi_{+-}^{00} \sim \langle [S_x + iS_y, S_x - iS_y] \rangle_0$, $\chi_{++}^{00} \sim \langle [S_x + iS_y, S_x + iS_y] \rangle_0$. Also $V_{\pm k} \sim \langle [S_x \pm iS_y, \delta\Delta_k] \rangle_0$, $\bar{V}_{k\pm} \sim \langle [\delta\Delta_k, S_x \pm iS_y] \rangle_0$, and $\Pi_{kk} \sim \langle [\delta\Delta_k, \delta\Delta_k] \rangle_0$ are spin/order-parameter couplings and the fluctuation of the order parameter, respectively. In Appendix B, we show that χ_{+-}^{00} , V_{+-} , and Π_{+-} are the same as χ_{xx}^0 , V_{xx} , and Π_{xx} when the magnetic field is along $\hat{\mathbf{x}}$ direction. It is also shown that $\chi_{\pm\pm} = V_{\pm\pm} = \Pi_{\pm\pm} = 0$. Thus we can use the previous results, Eq. (11), to get

$$\chi_{+-}^0 = \left\langle \frac{1}{\zeta^2 - \omega^2} \left(\zeta^2 + \frac{\omega^2 \Omega_d^2 f}{\omega^2 - \zeta^2 - \Omega_d^2} \right) \right\rangle. \quad (31)$$

This is again exactly the same as χ_{xx}^0 , Eq. (14), when the magnetic field is along $\hat{\mathbf{x}}$ direction. Since $\chi_{++}^0 = \chi_{--}^0 = 0$, the full susceptibility is just given by

$$\chi_{+-} = \frac{\chi_{+-}^0}{1 - I \chi_{+-}^0}. \quad (32)$$

This is also the same as the full longitudinal susceptibility χ_{xx} for the case of $\mathbf{H} \parallel \hat{\mathbf{x}}$.³² Therefore χ_{+-} in this case supports a propagating spin-wave mode (there is no damped mode) and the dispersion relation is given by in Eq. (25). The transverse response χ_{+-} when $\mathbf{H} \parallel \hat{\mathbf{z}}$ is exactly the same as the longitudinal response for the case of $\mathbf{H} \parallel \hat{\mathbf{x}}$.

III. CONCLUSION

In this paper, we study dynamical spin susceptibilities in a two-dimensional p -wave superconductor with $\vec{\Delta} = \hat{d}(\hat{k}_1 \pm i\hat{k}_2)$ symmetry. This order-parameter vector has been a strong candidate for the pairing symmetry of the superconducting Sr_2RuO_4 . Due to the fact that the direction of the order parameter vector is fixed along a crystallographic direction, there are four possible susceptibility measurements: Longitudinal and transverse responses in the cases of the magnetic field parallel and perpendicular to the order param-

eter vector. The existence of spin-wave modes in each case is examined and the dispersion relation is obtained. We found the following three modes; two of them are spin waves:

- (a) $\omega_{\parallel}^2 = (1 - I f_d) \Omega_d^2 + \frac{1}{2} f_d (v_F q)^2$,
- (b) $\omega_{\perp}^2 = \omega_{\parallel}^2 + \omega_L^2$,
- (c) $\omega = i \frac{1}{\sqrt{2}} v_F |q| \sqrt{I(1-f_d)}$.

The most crucial parameter is the pinning frequency.²⁹ It is most likely that the spin wave is observable for electron spin resonance if $\Omega_d \ll \Delta(T)$. We also believe that the experimental determination of $\Omega_d(T)$ will provide an important insight in the pinning mechanism of \hat{d} . Naturally this will provide another test of p -wave superconductivity.

As it is pointed out elsewhere,³³ the superconductivity in Bechgard-like $(\text{TMTSF})_2\text{PF}_6$, $(\text{TMTSF})_2\text{ClO}_4$, etc., is most likely of p -wave character as well. Therefore it is highly desirable to look for the spin wave in the above compounds as well.

Unlike in superfluid ^3He , Sr_2RuO_4 is most likely in the vortex state. Otherwise the magnetic field would penetrate only in the surface. Nevertheless, we believe that we can see the same expression for the spin-wave dispersion, if we reinterpret the superfluid density with the one in the vortex state. We shall postpone the study of the spin wave in the vortex state to the future.

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APPENDIX A

Magnetic field is perpendicular to the order parameter: $\mathbf{H} \parallel \hat{\mathbf{x}}$

a. Longitudinal susceptibility

Each correlation function can be computed from

$$\begin{aligned} \chi_{xx}^{00}(i\omega_\nu, \mathbf{q}) &= T \sum_n \sum_{\mathbf{p}} \text{Tr}[\alpha_1 G(\mathbf{p}, \omega_n) \\ &\quad \times \alpha_1 G(\mathbf{p} - \mathbf{q}, i\omega_n - i\omega_\nu)], \end{aligned}$$

$$\begin{aligned} V_{xx}(i\omega_\nu, \mathbf{q}) &= T \sum_n \sum_{\mathbf{p}} \text{Tr}[\alpha_1 G(\mathbf{p}, \omega_n) \\ &\quad \times \alpha_1 \rho_1 \sigma_1 G(\mathbf{p} - \mathbf{q}, i\omega_n - i\omega_\nu)], \end{aligned}$$

$$\begin{aligned} \Pi_{xx}(i\omega_\nu, \mathbf{q}) &= T \sum_n \sum_{\mathbf{p}} \text{Tr}[\alpha_1 \rho_1 \sigma_1 G(\mathbf{p}, \omega_n) \\ &\quad \times \alpha_1 \rho_1 \sigma_1 G(\mathbf{p} - \mathbf{q}, i\omega_n - i\omega_\nu)]. \quad (\text{A1}) \end{aligned}$$

Summing over \mathbf{p} (circular Fermi surface is assumed) leads to

$$\begin{aligned}\chi_{xx}^{00}(i\omega_\nu, \mathbf{q}) &= \pi TN(0) \sum_n \left(1 - \frac{\omega_n \omega_{n+\nu} + \Delta^2}{\sqrt{\omega_n^2 + \Delta^2} \sqrt{\omega_{n+\nu}^2 + \Delta^2}} \right) \\ &\quad \times \frac{\sqrt{\omega_n^2 + \Delta^2} + \sqrt{\omega_{n+\nu}^2 + \Delta^2}}{(\sqrt{\omega_n^2 + \Delta^2} + \sqrt{\omega_{n+\nu}^2 + \Delta^2})^2 + \zeta^2}, \\ V_{xx}(i\omega_\nu, \mathbf{q}) &= \pi TN(0) \sum_n \left(\frac{-i\omega_\nu \Delta}{\sqrt{\omega_n^2 + \Delta^2} \sqrt{\omega_{n+\nu}^2 + \Delta^2}} \right) \\ &\quad \times \frac{\sqrt{\omega_n^2 + \Delta^2} + \sqrt{\omega_{n+\nu}^2 + \Delta^2}}{(\sqrt{\omega_n^2 + \Delta^2} + \sqrt{\omega_{n+\nu}^2 + \Delta^2})^2 + \zeta^2}, \\ \Pi_{xx}(i\omega_\nu, \mathbf{q}) &= \pi TN(0) \sum_n \left(1 + \frac{\omega_n \omega_{n+\nu} + \Delta^2}{\sqrt{\omega_n^2 + \Delta^2} \sqrt{\omega_{n+\nu}^2 + \Delta^2}} \right) \\ &\quad \times \frac{\sqrt{\omega_n^2 + \Delta^2} + \sqrt{\omega_{n+\nu}^2 + \Delta^2}}{(\sqrt{\omega_n^2 + \Delta^2} + \sqrt{\omega_{n+\nu}^2 + \Delta^2})^2 + \zeta^2},\end{aligned}\quad (\text{A2})$$

where $\zeta = \mathbf{v}_F \cdot \mathbf{q}$ and $N(0) = m/2\pi$ is the two-dimensional density of states. After summing over ω_n and analytic continuation $i\omega_\nu \rightarrow \omega + i\delta$, we get Eq. (11).

b. Transverse susceptibility

Here $\chi_{+-}^{00} \sim \langle [S_y + iS_z, S_y - iS_z] \rangle_0$, $\chi_{++}^{00} \sim \langle [S_y + iS_z, S_y + iS_z] \rangle_0$ are the bare susceptibilities and they can be computed from

$$\begin{aligned}\chi_{+-}^{00}(i\omega_\nu, \mathbf{q}) &= T \sum_n \sum_{\mathbf{p}} \text{Tr}[\alpha_+ G(\mathbf{p}, \omega_n) \\ &\quad \times \alpha_- G(\mathbf{p} - \mathbf{q}, i\omega_n - i\omega_\nu)], \\ \chi_{++}^{00}(i\omega_\nu, \mathbf{q}) &= T \sum_n \sum_{\mathbf{p}} \text{Tr}[\alpha_+ G(\mathbf{p}, \omega_n) \\ &\quad \times \alpha_+ G(\mathbf{p} - \mathbf{q}, i\omega_n - i\omega_\nu)],\end{aligned}\quad (\text{A3})$$

where $\alpha_\pm = (1/\sqrt{2})(\alpha_2 \pm i\alpha_3)$ represents the transverse spin vertices: $(1/\sqrt{2})(S_y \pm iS_z) \sim \text{Tr}[\Psi^\dagger \alpha_\pm \Psi]$. Similarly, we have

$$\begin{aligned}V_{\pm k}(i\omega_\nu, \mathbf{q}) &= T \sum_n \sum_{\mathbf{p}} \text{Tr}[\alpha_\pm G(\mathbf{p}, \omega_n) \\ &\quad \times \alpha_k \rho_1 \sigma_1 G(\mathbf{p} - \mathbf{q}, i\omega_n - i\omega_\nu)], \\ \Pi_{kk}(i\omega_\nu, \mathbf{q}) &= T \sum_n \sum_{\mathbf{p}} \text{Tr}[\alpha_k \rho_1 \sigma_1 G(\mathbf{p}, \omega_n) \\ &\quad \times \alpha_k \rho_1 \sigma_1 G(\mathbf{p} - \mathbf{q}, i\omega_n - i\omega_\nu)].\end{aligned}\quad (\text{A4})$$

It is also useful to represent these correlation functions in terms of $V_{(\pm, \pm)} \sim \langle [S_y \pm iS_z, \delta\Delta_\pm] \rangle_0$ and $\Pi_{(\pm, \pm)} \sim \langle [\delta\Delta_\pm, \delta\Delta_\pm] \rangle_0$, where $\delta\Delta_\pm \sim \text{Tr}[\Psi^\dagger \alpha_\pm \rho_1 \sigma_1 \Psi]$. It is found that

$$\begin{aligned}V_{\pm y} &= \bar{V}_{y\pm} = \frac{1}{\sqrt{2}}(V_{\pm\pm} + V_{\pm\mp}), \\ V_{\pm z} &= \bar{V}_{z\pm} = \frac{1}{\sqrt{2}i}(\pm V_{\pm\pm} \mp V_{\pm\mp}), \\ \Pi_{yy} &= \frac{1}{2}(\Pi_{++} + \Pi_{+-} + \Pi_{-+} + \Pi_{--}), \\ \Pi_{zz} &= -\frac{1}{2}(\Pi_{++} - \Pi_{+-} - \Pi_{-+} + \Pi_{--}),\end{aligned}\quad (\text{A5})$$

where

$$\begin{aligned}V_{(\pm, \pm)}(i\omega_\nu, \mathbf{q}) &= T \sum_n \sum_{\mathbf{p}} \text{Tr}[\alpha_\pm G(\mathbf{p}, \omega_n) \\ &\quad \times \alpha_\pm \rho_1 \sigma_1 G(\mathbf{p} - \mathbf{q}, i\omega_n - i\omega_\nu)], \\ \Pi_{(\pm, \pm)}(i\omega_\nu, \mathbf{q}) &= T \sum_n \sum_{\mathbf{p}} \text{Tr}[\alpha_\pm \rho_1 \sigma_1 G(\mathbf{p}, \omega_n) \\ &\quad \times \alpha_\pm \rho_1 \sigma_1 G(\mathbf{p} - \mathbf{q}, i\omega_n - i\omega_\nu)].\end{aligned}\quad (\text{A6})$$

After summing over \mathbf{p} , we get

$$\begin{aligned}\chi_{+-}^{00} = \chi_{-+}^{00} &= \pi TN(0) \sum_n \left(1 - \frac{\omega_n \omega_{n+\nu}}{\sqrt{\omega_n^2 + \Delta^2} \sqrt{\omega_{n+\nu}^2 + \Delta^2}} \right) \\ &\quad \times \frac{\sqrt{\omega_n^2 + \Delta^2} + \sqrt{\omega_{n+\nu}^2 + \Delta^2}}{(\sqrt{\omega_n^2 + \Delta^2} + \sqrt{\omega_{n+\nu}^2 + \Delta^2})^2 + \zeta^2}, \\ \chi_{++}^{00} = \chi_{--}^{00} &= \pi TN(0) \sum_n \left(\frac{-\Delta^2}{\sqrt{\omega_n^2 + \Delta^2} \sqrt{\omega_{n+\nu}^2 + \Delta^2}} \right) \\ &\quad \times \frac{\sqrt{\omega_n^2 + \Delta^2} + \sqrt{\omega_{n+\nu}^2 + \Delta^2}}{(\sqrt{\omega_n^2 + \Delta^2} + \sqrt{\omega_{n+\nu}^2 + \Delta^2})^2 + \zeta^2}, \\ V_{+-} = V_{-+} = V_{++} = V_{--} &= \pi TN(0) \sum_n \left(\frac{1}{2} \frac{-i\omega_\nu \Delta}{\sqrt{\omega_n^2 + \Delta^2} \sqrt{\omega_{n+\nu}^2 + \Delta^2}} \right) \\ &\quad \times \frac{\sqrt{\omega_n^2 + \Delta^2} + \sqrt{\omega_{n+\nu}^2 + \Delta^2}}{(\sqrt{\omega_n^2 + \Delta^2} + \sqrt{\omega_{n+\nu}^2 + \Delta^2})^2 + \zeta^2}, \\ \Pi_{+-} = \Pi_{-+} &= \pi TN(0) \sum_n \left(\frac{\Delta^2}{\sqrt{\omega_n^2 + \Delta^2} \sqrt{\omega_{n+\nu}^2 + \Delta^2}} \right) \\ &\quad \times \frac{\sqrt{\omega_n^2 + \Delta^2} + \sqrt{\omega_{n+\nu}^2 + \Delta^2}}{(\sqrt{\omega_n^2 + \Delta^2} + \sqrt{\omega_{n+\nu}^2 + \Delta^2})^2 + \zeta^2},\end{aligned}$$

$$\begin{aligned} \Pi_{++} = \Pi_{--} = \pi TN(0) \sum_n \left(1 + \frac{\omega_n \omega_{n+\nu}}{\sqrt{\omega_n^2 + \Delta^2} \sqrt{\omega_{n+\nu}^2 + \Delta^2}} \right) \\ \times \frac{\sqrt{\omega_n^2 + \Delta^2} + \sqrt{\omega_{n+\nu}^2 + \Delta^2}}{(\sqrt{\omega_n^2 + \Delta^2} + \sqrt{\omega_{n+\nu}^2 + \Delta^2})^2 + \zeta^2}. \end{aligned} \quad (\text{A7})$$

Now summing over ω_n and analytic continuation $i\omega_\nu \rightarrow \omega + i\delta$ lead to Eq. (21).

APPENDIX B

Magnetic field is parallel to the order parameter: $\mathbf{H} \parallel \hat{\mathbf{z}}$

a. Longitudinal susceptibility

Notice that $\chi_{zz}^{00} \sim \langle [S_z, S_z] \rangle_0$, $V_{zz} \sim \langle [S_z, \delta\Delta_z] \rangle_0$, $\bar{V}_{zz} \sim \langle [\delta\Delta_z, S_z] \rangle_0$, and $\Pi_{zz} \sim \langle [\delta\Delta_z, \delta\Delta_z] \rangle_0$, where $\delta\Delta_z \sim \Delta \text{Tr}[\Psi^\dagger \alpha_3 \rho_1 \sigma_1 \Psi]$. These correlations functions can be expressed as

$$\begin{aligned} \chi_{zz}^{00}(i\omega_\nu, \mathbf{q}) = T \sum_n \sum_{\mathbf{p}} \text{Tr}[\alpha_3 G(\mathbf{p}, \omega_n) \\ \times \alpha_3 G(\mathbf{p} - \mathbf{q}, i\omega_n - i\omega_\nu)], \end{aligned}$$

$$\begin{aligned} V_{zz}(i\omega_\nu, \mathbf{q}) = T \sum_n \sum_{\mathbf{p}} \text{Tr}[\alpha_3 G(\mathbf{p}, \omega_n) \\ \times \alpha_3 \rho_1 \sigma_1 G(\mathbf{p} - \mathbf{q}, i\omega_n - i\omega_\nu)], \end{aligned}$$

$$\begin{aligned} \Pi_{zz}(i\omega_\nu, \mathbf{q}) = T \sum_n \sum_{\mathbf{p}} \text{Tr}[\alpha_3 \rho_1 \sigma_1 G(\mathbf{p}, \omega_n) \\ \times \alpha_3 \rho_1 \sigma_1 G(\mathbf{p} - \mathbf{q}, i\omega_n - i\omega_\nu)]. \end{aligned} \quad (\text{B1})$$

From the consideration of the matrix elements, we can easily see that $V_{zz} = \bar{V}_{zz} = 0$. Therefore, there is no mixing between spin fluctuations and the order parameter in this case and χ_{zz}^0 is just given by χ_{zz}^{00} . After summing over \mathbf{p} , we get

$$\begin{aligned} \chi_{zz}^{00}(i\omega_\nu, \mathbf{q}) = \pi TN(0) \sum_n \left(1 - \frac{\omega_n \omega_{n+\nu} - \Delta^2}{\sqrt{\omega_n^2 + \Delta^2} \sqrt{\omega_{n+\nu}^2 + \Delta^2}} \right) \\ \times \frac{\sqrt{\omega_n^2 + \Delta^2} + \sqrt{\omega_{n+\nu}^2 + \Delta^2}}{(\sqrt{\omega_n^2 + \Delta^2} + \sqrt{\omega_{n+\nu}^2 + \Delta^2})^2 + \zeta^2}. \end{aligned} \quad (\text{B2})$$

Summing over ω_n and analytic continuation lead to Eq. (28).

b. Transverse susceptibility

Here $\chi_{+-}^{00} \sim \langle [S_x + iS_y, S_x - iS_y] \rangle_0$, $\chi_{++}^{00} \sim \langle [S_x + iS_y, S_x + iS_y] \rangle_0$. Also $V_{\pm k} \sim \langle [S_x \pm iS_y, \delta\Delta_k] \rangle_0$, $\bar{V}_{k\pm} \sim \langle [\delta\Delta_k, S_x \pm iS_y] \rangle_0$, and $\Pi_{kk} \sim \langle [\delta\Delta_k, \delta\Delta_k] \rangle_0$. These correlation functions can be again calculated from

$$\begin{aligned} \chi_{+\mp}^{00}(i\omega_\nu, \mathbf{q}) = T \sum_n \sum_{\mathbf{p}} \text{Tr}[\beta_{\mp} G(\mathbf{p}, \omega_n) \\ \times \beta_{\mp} G(\mathbf{p} - \mathbf{q}, i\omega_n - i\omega_\nu)], \end{aligned}$$

$$\begin{aligned} V_{\pm k}(i\omega_\nu, \mathbf{q}) = T \sum_n \sum_{\mathbf{p}} \text{Tr}[\beta_{\pm} G(\mathbf{p}, \omega_n) \\ \times \alpha_k \rho_1 \sigma_1 G(\mathbf{p} - \mathbf{q}, i\omega_n - i\omega_\nu)], \end{aligned}$$

$$\begin{aligned} \Pi_{kk}(i\omega_\nu, \mathbf{q}) = T \sum_n \sum_{\mathbf{p}} \text{Tr}[\alpha_k \rho_1 \sigma_1 G(\mathbf{p}, \omega_n) \\ \times \alpha_k \rho_1 \sigma_1 G(\mathbf{p} - \mathbf{q}, i\omega_n - i\omega_\nu)], \end{aligned} \quad (\text{B3})$$

where $\beta_{\pm} = (1/\sqrt{2})(\alpha_1 \pm i\alpha_2)$ representing the transverse spin vertices: $(1/\sqrt{2})(S_x \pm iS_y) \sim \text{Tr}[\Psi^\dagger \beta_{\pm} \Psi]$. One can also rewrite these correlation functions in terms of $V_{(\pm, \pm)} \sim \langle [S_x \pm iS_y, \delta\Delta_{\pm}] \rangle_0$, $\Pi_{(\pm, \pm)} \sim \langle [\delta\Delta_{\pm}, \delta\Delta_{\pm}] \rangle_0$, where $\delta\Delta_{\pm} \sim \text{Tr}[\Psi^\dagger \beta_{\pm} \rho_1 \sigma_1 \Psi]$:

$$V_{\pm x} = \bar{V}_{y\pm} = \frac{1}{\sqrt{2}}(V_{\pm\pm} + V_{\pm\mp}),$$

$$V_{\pm y} = \bar{V}_{z\pm} = \frac{1}{\sqrt{2}i}(\pm V_{\pm\pm} \mp V_{\pm\mp}),$$

$$\Pi_{xx} = \frac{1}{2}(\Pi_{++} + \Pi_{+-} + \Pi_{-+} + \Pi_{--}),$$

$$\Pi_{yy} = -\frac{1}{2}(\Pi_{++} - \Pi_{+-} - \Pi_{-+} + \Pi_{--}), \quad (\text{B4})$$

where

$$\begin{aligned} V_{(\pm, \pm)}(i\omega_\nu, \mathbf{q}) = T \sum_n \sum_{\mathbf{p}} \text{Tr}[\beta_{\pm} G(\mathbf{p}, \omega_n) \\ \times \beta_{\pm} \rho_1 \sigma_1 G(\mathbf{p} - \mathbf{q}, i\omega_n - i\omega_\nu)], \end{aligned}$$

$$\begin{aligned} \Pi_{(\pm, \pm)}(i\omega_\nu, \mathbf{q}) = T \sum_n \sum_{\mathbf{p}} \text{Tr}[\beta_{\pm} \rho_1 \sigma_1 G(\mathbf{p}, \omega_n) \\ \times \beta_{\pm} \rho_1 \sigma_1 G(\mathbf{p} - \mathbf{q}, i\omega_n - i\omega_\nu)]. \end{aligned} \quad (\text{B5})$$

From the matrix elements, one can see that

$$\chi_{++}^{00} = \chi_{--}^{00} = 0, \quad V_{++} = V_{--} = 0, \quad \Pi_{++} = \Pi_{--} = 0. \quad (\text{B6})$$

Thus Eq. (30) can be simplified as

$$\begin{aligned} \chi_{+-}^0 = \chi_{-+}^0 = \chi_{+-}^{00} + \frac{V_{+-}g\bar{V}_{+-}}{1-g\Pi_{+-}}, \\ \chi_{++}^0 = \chi_{--}^0 = 0. \end{aligned} \quad (\text{B7})$$

After summing over \mathbf{p} , we obtain the same equation as Eq. (A2). Thus we can use the previous results [Eq. (11)] to get Eq. (31) which is same as Eq. (14).

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