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Correlated random-field systems: Dissipative dynamics and phenomenological scaling

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We investigate a $(d+1)$-dimensional ''correlated'' random-field system with d spatial dimensions and one Trotter dimension along which randomness is ''correlated'' or striped. In the sense of universality this model is equivalent to a d-dimensional quantum random-field system. We investigate the dissipative Langevin dynamics of this ''striped'' $(d+1)$-dimensional systems within a replica symmetric framework, employing the perturbative ϵ expansion around the upper critical dimension to explore the effect of an additional dimension (along which the randomness is correlated) on the dynamical scaling. We argue that the ϵ expansion fails to capture the activated nature of dynamics. We also extend the phenomenological renormalization-group calculations to investigate the critical behavior of a $(d+1)$-dimensional correlated random-field Ising systems.

I. INTRODUCTION

The random-field systems (especially the random-field Ising model)[1–3] have been investigated over the last two decades and have provided a sequences of challenges to both the theoreticians and the experimentalists. Even though the resolution of the puzzle concerning the lower critical dimension of the random-field Ising systems[4] confirmed the validity of the ''domain wall'' argument,[1] the nature of the transition is yet to be fully understood. Whether the transition is first order or second order is still questioned[2] and the possibility of an intermediate glassy phase[5,6] has also been reported. The phase diagram is not known for general dimensions and the critical behavior in three dimensions[7] is presently not understood. These models were studied using field-theoretical renormalization-group calculations.[8,9] Very recently the extensive high-temperature series expansion studies[10] have provided important results.

Quantum phase transitions have been attracting a great deal of attention in recent years. Especially the zero-temperature transitions in the transverse Ising model[11,12] and its M-component generalization, the rotor models,[12,13] are being explored extensively. Studies have been carried out with the interaction between the rotors taken to be random and thus elucidating properties of quantum spin glasses.[13–15] The zero-temperature phase transition in a pure d-dimensional quantum system is equivalent to the thermal phase transition in a $(d+1)$-dimensional *anisotropic* classical system[16] with d spatial dimensions and one additional Trotter dimension.[17] The Trotter formalism maps a d-dimensional random quantum system to an equivalent $(d+1)$-dimensional classical system with randomness ''correlated'' in the Trotter direction with the interaction in the Trotter direction being ferromagnetic (see Fig. 1). The equivalent classical system happens to be strongly anisotropic and the dynamical exponent z is different from unity unlike the pure case. The zero-temperature or low-temperature transitions in random/disordered quantum systems (e.g., quantum spin glass and quantum random-field systems) are often characterized by some unique phenomena. Renormalization-group studies of short-range quantum spin-glass rotor systems suggest that quantum fluctuations happen to be ''dangerously'' irrelevant.[13] Dynamical scaling is found to be ''activated'' in the case of disordered quantum Ising systems[18,19] whereas it is argued to be conventional (power law) for rotors ($M \geqslant 2$).[13] Both real-space renormalization-group calculations for one-dimensional random Ising systems[18] and numerical studies of quantum Ising spin glasses[20] emphatically establish that the quantum critical point ($T=0$) is flanked by a ''Griffiths-McCoy'' region (with continuously varying exponents) where disorder wipes out the gap in the energy spectrum (i.e., the correlation length in the Trotter direction diverges) causing a diverging response even away from the critical point.

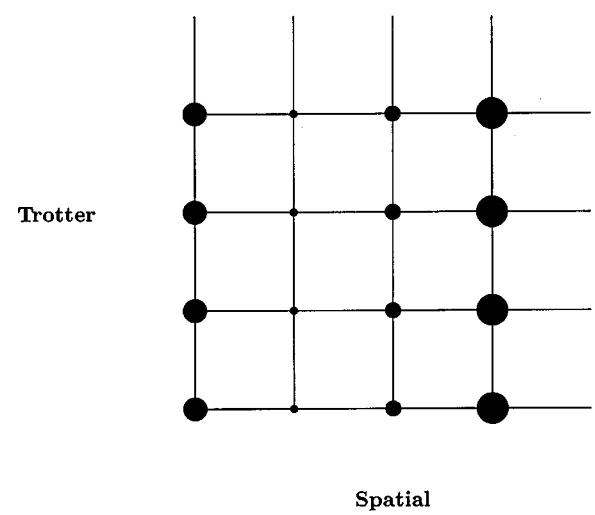

FIG. 1. A typical randomness distribution of a (1+1)-dimensional striped random-field system with one spatial and one trotter dimension. Filled circles of different radii correspond to different values of random field. Note that the randomness is striped (of same value) along the Trotter direction. The interactions in the spatial as well as in the Trotter direction are ferromagnetic.

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In this work, we investigate the dissipative Langevin dynamics and the critical behavior of a $(d+1)$-dimensional random-field classical system with randomness correlated (striped) in the $(d+1)$th (Trotter) direction. This model is derived employing the Suzuki-Trotter (or imaginary time) mapping to a d-dimensional random-field quantum rotor or quantum Ising system. The equivalent classical system thus obtained, happens to be ''correlated'' or striped in the $(d+1)$th (Trotter) dimension. The finite-temperature transition in this $(d+1)$-dimensional model is equivalent in the sense of universality to the quantum phase transition in the quantum d-dimensional random-field system. Aharony, Gefen, and Shapir[21] introduced the Ising version $(M=1)$ of the quantum random-field model with competing quantum fluctuations and random-field fluctuations. Boyanovski and Cardy[22] employed supersymmetric techniques to derive different exponents and investigated the possible ''dimensional reduction.'' In recent years, extensive studies of both the Ising and the rotor version of the model have been made using molecular field theory,[23] spherical limit,[24] and ϵ expansion in the replica-symmetric framework, and the static and dynamic scaling relations have been established.[25,26] Up to now the model has presented plenty of mystery to be unraveled and provides a rich and intriguing field of study.

The experimental motivation behind the study of quantum random-field systems (or the equivalent classical random-field system with correlated randomness; see Fig. 1) is the following: the d-dimensional quantum order-disorder ferroelectrics (potassium dihydrogen phosphte type) with random field at $T=0$ can nicely be modeled by a transverse Ising system in a random longitudinal field.[9] As mentioned already, this is equivalent to a classical Ising system wth randomness correlated along the additional Trotter direction. Moreover, as in the case of classical random-field systems,[27] it has been shown at least in the semiclassical limit that the random-field transverse Ising systems can be mapped to the dilute transverse Ising antiferromagnet in a steady field,[23] which can provide a scope for experimental verification.

The plan of the paper is the following. In Sec. II, we briefly indicate the way to derive a correlated random-field system starting from a quantum random-field Hamiltonian. In Sec. III, we investigate the dissipative Langevin dynamics[28] of a $(d+1)$-dimensional correlated random-field system using ϵ-expansion calculations within the replica symmetric framework.[22] Of course, the dynamics of the classical correlated system does not simulate the relaxational dynamics of the original quantum system (for which one needs to derive the appropriate Langevin equation starting from the original quantum Hamiltonian itself) but the asymptotic $(t\to\infty)$ results should correspond to the quantum random-field transition. We believe some features of quantum dynamics (e.g., activated dynamical scaling, occurrence of the Griffiths-McCoy region) should also show up in the relaxational dynamics of these types of systems as an artifact of correlated randomness. Recently, Stinchcombe and co-workers[29,30] have investigated the random-field Ising model (RFIM) using phenomenological renormalization group and finite-size scaling[31] on a bar geometry. They have addressed the question of marginality breakdown in RFIM due to domain decoration in $d=2$. In Sec. IV, we extend these renormalization arguments to $(d+1)$-dimensional random-field ''Ising'' systems with correlated randomness and provide a zeroth-order theory. Throughout this work, we shall assume that the transition in these models is second order and use a Gaussian distribution of random field so that at least in the mean-field limit the transition is definitely continuous.[32,23]

II. THE DERIVATION OF THE CORRELATED RANDOM-FIELD HAMILTONIAN

The Hamiltonian describing the $O(M)$ quantum rotors is written as[12]

$$H=\frac{g}{2}\sum_i \hat{L}_i^2-\sum_{ij} J_{ij}\hat{x}_i\cdot\hat{x}_j, \quad \hat{x}_i^2=1,$$

where $\hat{x}_i$ is a unit length rotor sitting at the site i with M components $x_{i\mu}$, N the number of sites, and $L_{i\mu\nu}$ $(\nu,\mu=1,2,\ldots,M)$ are the $M(M-1)/2$ components of the angular momentum generator L_i in the rotor space. The first term in the Hamiltonian corresponds to the kinetic energy of the rotors whereas the second term denotes the interaction among the rotors. Noncommutivity of x_i's and $\hat{L}_i$'s introduces quantum fluctuations in the model that result in a zero-temperature quantum phase transition.[12] Here, g denotes the strength of the noncommuting term in the Hamiltonian. In the presence of a static random field h_i (random in space) which couples to the components of the rotors x_i, producing a term $-\vec{h}_i\cdot\hat{x}_i$ in the Hamiltonian, we have

$$H(h_i)=\frac{g}{2}\sum_i \hat{L}_i^2-\sum_{ij} J_{ij}\hat{x}_i\cdot\hat{x}_j-\sum_i \vec{h}_i\cdot\hat{x}_i, \tag{1}$$

where h_i is the site-dependent random field with zero mean and nonzero variance satisfying the Gaussian distribution

$$P(h_i)=\frac{1}{\sqrt{(2\pi\Delta)}}\exp\left(-\frac{h_i^2}{2\Delta^2}\right). \tag{2}$$

The corresponding random-field transverse Ising Hamiltonian $(M=1)$ is written as

$$H=-J\sum_{\langle ij\rangle} S_i^z S_j^z-\Gamma_0\sum_i S_i^x-\sum_i h_i S_i^z, \tag{3}$$

where Γ_0 is the strength of the quantum fluctuation, namely the transverse (tuneling) field.

With the soft-spin consideration, we can work with an effective classical action (obtained via path-integral formalism) for the M-component rotors within the static framework[26]

$$\mathcal{A}=\int_0^\beta d\tau\left(\mathcal{L}_0-\sum_{ij} J_{ij}\hat{x}_i(\tau)\cdot\hat{x}_j(\tau)-\sum_i \vec{h}_i\cdot\hat{x}_i(\tau)\right), \tag{4}$$

where

$$\mathcal{L}_0(\tau)=\frac{1}{2g}\sum_i [\partial_\tau\hat{x}_i(\tau)]^2+\frac{r}{2}\sum_i \hat{x}_i(\tau)^2 + \frac{u}{4}\sum_i [\hat{x}_i(\tau)^2]^2,$$

where $\hat{x}_i$ corresponds to rotors for $M \geqslant 2$ and Ising spins for $M=1$. We will also assume short-ranged interactions among the rotors and the interaction term $-\Sigma J_{ij} x_i x_j$ will contribute $\int (\nabla x_\mu)\cdot(\nabla x_\mu) d^d y$ where $\vec{y}$ denotes the d spatial dimensions. Consequently we arrive at the appropriate continuum action

$$\mathcal{A}=\int_0^\beta d\tau \int d^d y \bigg[\frac{r}{2} x_\mu x_\mu + \frac{1}{2g}[\partial_\tau x_\mu(\tau)]^2 + \frac{1}{2g}(\nabla x_\mu)\cdot(\nabla x_\mu) + \frac{u}{4}(x_\mu x_\mu)^2 - h_\mu x_\mu \bigg]. \tag{5}$$

Here μ denotes the components of the rotor and the repeated index μ implies sum over it from 1 to M. To calculate the the quenched averaged partition function, we introduce replicas at this stage of calculation to obtain the n-replicated classical action (with m_1, m_2 as replica indices) given as

$$\mathcal{A}^{(n)}\int d^d y \bigg[\int_0^\beta d\tau \sum_{m_1=1}^{n} \frac{r}{2} x_\mu^{m_1} x_\mu^{m_1} + \frac{1}{2g}[\partial_\tau x_\mu^{m_1}(\tau)]^2 + \frac{1}{2}(\nabla x_\mu^{m_1})\cdot(\nabla x_\mu^{m_1}) + \frac{u}{4}(x_\mu^{m_1} x_\mu^{m_1})^2 - \Delta/2 \int_0^\beta d\tau_1 \int_0^\beta d\tau_2 \bigg(\sum_{m_1} x_\mu^{m_1}(\tau_1)\bigg)\bigg(\sum_{m_2} x_\mu^{m_2}(\tau_2)\bigg)\bigg]. \tag{6}$$

One is required to take the limit $n\to 0$ at the end of the calculation. It is convenient to rewrite the above action in terms of Fourier components $\phi_\mu^{m_1}(\vec{k},\omega_0)$ (where ω_0 denotes the Matsubara frequencies which are continuous in the $T=0$ limit) of the $x_\mu^{m_1}(\vec{y},\tau)$ fields so that the action now becomes[26]

$$\begin{aligned}\mathcal{A}^{(n)}=&\int \frac{d\omega_0}{2\pi}\int \frac{d^d k}{(2\pi)^d} \sum_m [(r/2+\gamma\omega_0^2/2 + k^2/2)\phi_\mu^m(\vec{k},\omega_0)\phi_\mu^m(-\vec{k},-\omega_0)] \\ &- \frac{\Delta}{2}\sum_{m_1} \int \frac{d\omega_0}{2\pi}\frac{d^d k}{(2\pi)^d} \phi_\mu^{m_1}(\vec{k},\omega_0) \times \sum_{m_2} \phi_\mu^{m_2}(-\vec{k},-\omega_0)\delta(\omega_0) \\ &+ u\int \frac{d\omega_0^1}{2\pi}\cdots\frac{d\omega_0^4}{2\pi}\int \frac{d^d k_1}{(2\pi)^d}\cdots\frac{d^d k_4}{(2\pi)^d} \times \delta^d(\vec{k}_1+\cdots+\vec{k}_4)\delta(\omega_0^1+\cdots+\omega_0^4) \\ &\times \phi_\mu^m(\vec{k}_1,\omega_0^1)\phi_\mu^m(\vec{k}_2,\omega_0^2)\phi_\nu^m(\vec{k}_3,\omega_0^3)\phi_\nu^m(\vec{k}_4\omega_0^4),\end{aligned} \tag{7}$$

where repeated indices (m,μ,ν) denote sum. For $M=1$ this action describes the equivalent classical action for the random-field quantum Ising model. The parameter γ is given by $\gamma=1/g$, where g (or Γ) denotes the strength of quantum fluctuations in Eq. (1) [or Eq. (3)]. Equation (7) shows that the random-field fluctuations couple to the static $(\omega_0=0)$ part of the order parameter. In passing from Eq. (1) to Eq. (7), we have effectively derived an equivalent $(d+1)$-dimensional classical system [described by the action (7)] where the random field is correlated in the $(d+1)$th (Trotter) direction. It has already been shown previously that the upper and lower critical dimensions in the case of quantum random-field systems [or the spatial lower critical dimension of the equivalent $(d+1)$-dimensional classical systems] are the same as those for the isotropic random-field systems, i.e., $d_l^c=2$ for $M=1$ and $=4$ for $M\geqslant 2$, the upper critical dimension being 6 in either case.[26] The free connected and disconnected correlation functions in this terminology and within a replica symmetric framework in the $n\to 0$ limit can be written as[26]

$$G_{\rm con}(k,\omega_0)\,\delta^{\mu\nu}=\overline{\langle \phi_k^\mu(\omega_0)\phi_{-k}^\nu(-\omega_0)\rangle}-\overline{\langle\phi_k^\mu(0)\rangle\langle\phi_{-k}^\nu(0)\rangle} = \frac{1}{k^2+\gamma\omega_0^2+r},$$

$$G_{\rm dis}\delta^{\mu\nu}(k,\omega_0)=\overline{\langle\phi_k^\mu(0)\rangle\langle\phi_{-k}^\nu(0)\rangle}=\frac{\Delta\,\delta(\omega_0)}{(k^2+r)^2}.$$

We wind up this section with the note that we have not included randomness in g or Γ. This helps us to obtain the correlated random system with ferromagnetic interaction in the Trotter direction; otherwise the equivalent classical action will be complicated. However, this does not seem to modify critical behavior drastically. In the coming sections, we will dwell on this correlated classical $(d+1)$-dimensional random-field system and use both the dynamical renormalization as well as the phenomenological renormalization (for the Ising version).

III. THE LANGEVIN DYNAMICS

In this section, we shall consider the dissipative Langevin dynamics of the above correlated random classical system without referring to the original quantum Hamiltonian from which this is derived. Our aim is to explore the effect of the Trotter dimension (along which the randomness is correlated) on the dynamical relaxation of the system. We shall call the Trotter direction the αth direction, and also denote ω_0 as k_α (the Fourier conjugate to this special direction). The action we use is given as (for $M=1$)

$$\begin{aligned}\mathcal{A}=&\mathcal{F} \\ =&\int \frac{dk_\alpha}{2\pi}\frac{d^d k}{(2\pi)^d}[(r/2+\gamma k_\alpha^2/2+k^2/2)\phi(\vec{k},k_\alpha) \times \phi(-\vec{k},-k_\alpha)] - \frac{\Delta}{2}\int \frac{dk_\alpha}{2\pi}\frac{d^d k}{(2\pi)^d} \\ &\times \phi(k,k_\alpha)\phi(-k,-k_\alpha)\delta(k_\alpha)+u\ \ldots\ .\end{aligned} \tag{8}$$

We investigate the dissipative Langevin dynamics (model A dynamics) of the above $(d+1)$-dimensional classical correlated system described by the equation[28]

$$\frac{\partial \phi(k,k_\alpha)}{\partial t} = -\Gamma \frac{\delta \mathcal{F}}{\delta \phi(-k,-k_\alpha)} + \eta(k,k_\alpha,t) \tag{9}$$

with $\eta(k,k_\alpha,t)$ being Gaussian white noise satisfying

$$\langle \eta(\vec{k},k_\alpha,t)\, \eta(\vec{k}',k'_\alpha,t')\rangle = 2\Gamma\, \delta^d(\vec{k}+\vec{k}')\, \delta(k_\alpha+k'_\alpha)\, \delta(t-t'). \tag{10}$$

Conversion of the above equation to the $(\vec{k},\omega)$ representation immediately gives us the ''free'' response function $G_0(k,k_\alpha,\omega)$ [which measures the response of the system, i.e., the variational derivative of the order parameter with respect to an arbitrary magnetic field $h(t)$ computed at $h=0$], and the free correlation function $C_0(k,k_\alpha,\omega)$ given, respectively, as

$$G_0(k,k_\alpha,\omega) = \frac{1}{-iw/\Gamma + k^2 + \gamma k_\alpha^2 + r}, \tag{11}$$

$$C_0(k,k_\alpha\omega) = \overline{\langle \phi(k,k_\alpha,\omega)\, \phi(-k,-k_\alpha,-\omega)\rangle}$$
$$= \frac{2\Gamma + \Delta\, \delta(\omega)\, \delta(k_\alpha)}{|-i\omega/\Gamma + \gamma k_\alpha^2 + k^2 + r|^2}, \tag{12}$$

where the overbar indicates the average over the random-field fluctuations. Clearly, the first term in the above comes from fluctuation dissipation due to noise [Eq. (10)] whereas the second term is due to the random-field fluctuations.

We shall now employ the dynamical renormalization-group calculations, namely the ϵ expansion around the upper critical (spatial) dimension $d_c^u = 6$ (Ref. 26) and look at the dynamical scaling behavior. Static renormalization-group calculations of the classical random-field system[8] and also of the quantum random-field systems[26] clearly show that the random-field fixed point is stable and thermal fluctuations (or quantum fluctuations) are ''dangerously'' irrelevant. It has also been shown that under renormalization the measure of randomness (Δ) grows and tends to take the system beyond the scope of perturbative calculations. The appropriate scaling variable is $W = \Delta u$, which is relevant below the (spatial) upper critical dimension (=6),[8,26] and at the same time u is irrelevant for $d<6$. Thus, we have to look at the flow equations of the scaling variables r, u and W.[26] From the nature of the free correlation function given in Eq. (12), we find that the most infrared divergence $(\sim 1/k^4)$ comes from the random-field fluctuations, so the noise term is altogether neglected[22] and consequently the free correlation function reduces to

$$C_0(k,k_\alpha,\omega) = \frac{\Delta\, \delta(\omega)\, \delta(k_\alpha)}{|-i\omega/\Gamma + \gamma k_\alpha^2 + k^2 + r|^2}. \tag{13}$$

The static critical exponents obtained from perturbative ϵ expansion and the nature of the critical point of the model considered here, are already discussed in the literature.[25,26] We shall here look at the dynamical aspect of this correlated random-field system. We need to calculate the self-energy diagram (Fig. 2), which is the only relevant diagram up to the order ϵ^2,[26] to find its contribution to the self-energy $\Sigma(k,k_\alpha,\omega)$. The line with the thick circle in Fig. 2 denotes the random-field part $C_0(k,k_\alpha\omega)$ with both $\omega,k_\alpha=0$ [Eq. (13)] and the line without the thick circle denotes the free response function G_0 given in Eq. (11). Clearly with two u's and two Δ's the diagram is of the order $W^2(=\epsilon^2)$. We defer the details of the calculation to the Appendix and only quote the results here. We need to introduce an additional exponent z_q such that ξ_α [the correlation length in the $(d+1)$th direction] $\sim \xi^{z_q}$ (where ξ is the correlation length in the d spatial dimensions) in the vicinity of the critical point. Clearly, this exponent corresponds to the dynamical exponent of the d-dimensional random-field quantum model.

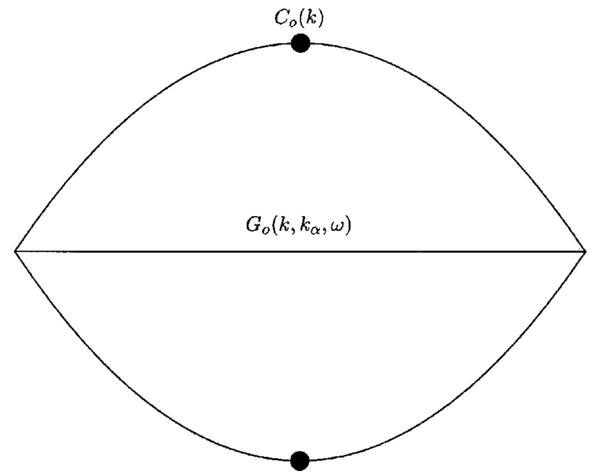

FIG. 2. The self-energy diagram up to the second order in ϵ with two filled circles (Δ) and two u's. The thin line corresponds free propagator $G_0(k,k_\alpha,\omega)$ and the line with a filled circle stands for free correlator $C_0(k)$.

Having calculated the self-energy diagram (Appendix) we look at $\Sigma(k,k_\alpha,\omega)$ in different limits at the cortical point:
$\Sigma(k,0,0)-\Sigma(0,0,0)\sim -k^2\,\eta \ln k$,
$\Sigma(0,k_\alpha,0)-\Sigma(0,0,0)\sim -k_\alpha^2\,\eta_\alpha \ln k_\alpha$
$\Sigma(0,0,\omega)-\Sigma(0,0,0)\sim -i\,\omega\,\eta_t \ln \omega$.
Above two expressions actually define the exponents η_t and η_α.

Under renormalization by a factor of length scale b, Γ and γ scale as

$$\Gamma' = \Gamma b^{2-z-\eta}(1+\eta_t \ln b), \quad \gamma' = \gamma b^{2-2z_q-\eta}(1+\eta_\alpha \ln b). \tag{14}$$

Up to order ϵ^2, one can show that $\eta_\alpha = \eta_t = 3\eta$. Demanding that both γ and the Γ scale to a fixed point, one finds (Appendix)

$$z_q = 1+\eta, \quad z = 2+2\eta. \tag{15}$$

As mentioned earlier, the exponent z_q appearing here is the quantum dynamical exponent which in this particular model is half that of the classical dynamical exponent up to $O(\epsilon^2)$.[22] This is an artifact of the symmetry of the propagator and the dominance of the random-field term.

We have performed this calculation with $M=1$. It is straightforward to extend these calculations to rotors ($M \geqslant 2$) and obtain identical expressions for z and z_q. However, the value of the exponent η will definitely depend upon M.

IV. DYNAMICAL SCALING

The replica symmetric ϵ-expansion studies imply that dynamical scaling in the present model is conventional. One can then propose a scaling form for the dynamical response function given as

$$\chi(k,k_{\alpha},\omega,r)=|r|^{-\gamma}X(k\xi,\omega\xi^{z},k_{\alpha}\xi^{z_q}) \qquad (16)$$

with

$$\omega(k)=k^{z}f(k\xi)=\xi^{-z}f_1(k\xi), \qquad (17)$$

where z is the dynamical exponent for the correlated classical system and z_q is the ''anisotropy exponent'' or the dynamical exponent of the equivalent d-dimensional quantum system and γ is the susceptibility exponent.

Let us now recall the relaxational scaling behavior of the isotropic random-field Ising system as proposed by Fisher[33] (see also Ref. 34). The contribution to the dynamics will essentially come from large, ''rare'' locally ferromagnetic regions having two minima with energy separation almost zero. The barrier height near the criticality increases as ξ^{θ} and the relaxation dynamics is essentially activated with relaxation time diverging as $\tau\sim\exp(a\xi^{\theta})$. ϵ expansion fails to capture this ''activated'' nature of dynamics. From the experimental measurement of King Mydosh, and Jaccarino,[35] it has been found that the variation of the peak height in the ac susceptibility for three-dimensional random-field Ising system ($Fe_{0.46}Zn_{0.54}F_2$) as a function of frequency, is consistent with the activated dynamical scaling as proposed by Fisher. We refer to Ref. 36 for the discussion on various other experimental observations concerning the dynamical scaling behavior of $d=3$ RFIM systems.

One can readily extend the above idea of Fisher to the correlated random-field Ising systems. In this case, the rare regions will be correlated in the Trotter direction, and the dynamics of these types of systems are expected to be more activated. We assume that the barriers in the present case scale as $\xi^{\theta'}$ (note that the effects of the Trotter direction lurk in this modified value θ', which is also seen in ϵ-expansion calculations[26]), which readily leads to the activated scaling form for the dynamical response function given as

$$\chi(\xi,\omega,r)=\xi^{2-\eta}X\left(\frac{\ln\omega}{\xi^{\theta'}}\right). \qquad (18)$$

At criticality ξ disappears from the scaling relation and we have $\chi(\omega)\sim|\ln\omega|^{(2-\eta)/\theta'}$. The relaxational dynamics is thus argued to be activated with an exponent θ' different from that in Ref. 33 and this does not show up in ϵ-expansion studies. Because of the additional ferromagnetically connected Trotter direction, we expect that θ' is larger than θ and the dynamics is slower.

To address the question of replica symmetry breaking (RSB) in classical random-field systems, Mezard and Young[5] considered an M component generalization of the random-field Ising systems and evaluated the exponents η and $\overline{\eta}$ using the nonperturbative self-consistent screening approximation (SCSA), which is exact up to the order $O(1/M)$. They found that the replica symmetric ansatz is unstable with respect to the RSB ansatz. Mezard and Monasson[5] extended this calculation to predict an intermediate glassy phase between the ferro and para phase. Both Mezard and Monasson and also De Dominicis, Orland, and Temessvari[6] determined the temperature $T_{\mathrm{RSB}}>T_c$ where the replica symmetric solution is unstable. In principle, it should be possible to extend these calculations to the correlated random-field model to check the stability of the replica symmetric ansatz. Preliminary calculations based on the SCSA in the present model seem to indicate possible replica symmetry breaking as obtained by Mezard and Young.[5] Let us now recall the ''droplet'' model[37,38] proposed with a view to investigate the spin glass phase of the short-range classical Ising spin glass at very low temperature. This model includes no nontrivial ''ergodicity'' breaking and assumes only a single thermodynamic phase up to an overall flip.[39] From the dynamical point of view, in the low-temperature limit, only the fastest activated processes are considered and eventually the system shows activated dynamics in the glassy phase.[40] If the replica symmetry breaking in the correlated RFIM is of the above type (that is, replica symmetry breaking is trivial, or replica symmetric as in droplet model) it should lead to activated dynamical scaling. But we believe it would be too premature to make any such conjecture at this point. We should also mention here that nonperturbative effects like instantons can break down the ϵ expansion and this could explain that the perturbative computation fails to capture the activated dynamics.

We should also mention here that the above ϵ expansion shows that the correlation in the Trotter direction grows in a power law $\xi_{\alpha}\sim\xi^{z_q}$, i.e., quantum dynamics is conventional, which is possibly not true. The real space renormalization-group calculation of the one-dimensional random quantum Ising chain[18] and also the studies on dilute quantum Ising systems[19] predict that the quantum dynamics is actually activated with the dynamical exponent z_q diverging at the quantum critical point. We should also note in passing that the numerical studies of two- and three-dimensional short-range quantum Ising spin glass[20] seem to indicate a conventional dynamical scaling with a finite dynamical exponent at the critical point. Very recent numerical Monte Carlo simulations, however, predict activated dynamics in $d=2$.[41] In the next section, we shall indicate that the correlation length ξ_{α} of the present model indeed grows much faster than the power law.

We shall now provide a phenomenological argument regarding the dynamical scaling aspect of a general correlated random classical system based on the symmetry of the order parameter. As argued before, the contribution to dynamics will essentially come from the dynamics of ''large rare'' blocks which are locally ordered. Neglecting the coupling to the environment, the fluctuations of this block spin can be described by a one-dimensional [the $(d+1)$th or Trotter dimension along which the rare blocks are correlated] M-component spin chain with ferromagnetic coupling $K_{\alpha}\sim L^{d}$. The relaxation time of this approximate equivalent chain is naively expected to be of the order of correlation length ξ_{α}. Then we find that the relaxation time τ is given as some finite power of the correlation length ξ_{α} of the equivalent one-dimensionl chain,

$$\tau\sim K_{\alpha}\sim L^{d} \quad \text{for} \quad M\geqslant 2, \qquad (19)$$

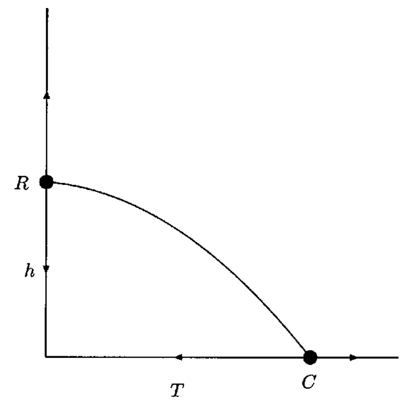

FIG. 3. The schematic phase diagram of the correlated random-field Ising system with spatial dimension $d>2$. **R** denotes the random-field fixed point and **C** corresponds to the pure thermal fixed point. The arrows indicate the renormalization group flows and for any amount randomness, random field fluctuations dominate over the thermal fluctuations and determine the critical behavior.

$$\tau \sim \exp(K_\alpha) \sim \exp(cL^d) \quad \text{for} \quad M=1, \tag{20}$$

where c is constant. The above equations obtained from the phenomenological argument indicate that the relaxational dynamics of correlated random systems is *activated* for $M=1$, but is conventional for $M \geqslant 2$. We thus expect the critical dynamics of short-range Ising spin glass systems with ''striped'' randomness to be activated, even though the critical dynamics of isotropic spin glass systems is conventional.[39]

V. PHENOMENOLOGICAL RENORMALIZATION

In this section, we shall investigate the critical behavior and marginality of a general $(d+1)$-dimensional ''correlated'' (striped) random-field ''Ising'' system, using analytic phenomenological scaling on a bar geometry.[29] The $(d+1)$th dimension, as mentioned earlier, is the Trotter dimension along which the randomness is correlated (see Fig. 1). Before going into the details of this phenomenological scaling, we must briefly discuss the established conjecture concerning the critical behavior of the striped random-field Ising system (with spatial dimension d) or the equivalent quantum random-field Ising systems. The extension of domain wall arguments[1] to the present case implies that that the lower critical (spatial) dimension happens to be 2, i.e., in $(2+\epsilon)$ spatial dimension (with additional Trotter dimension) the correlated RFIM system will sustain long-range order for small randomness and temperature.[26] Secondly, an extension of the Harris criterion[42] in the correlated random-field case suggests that random fluctuations always dominate over the thermal fluctuations and the random-field fixed point determines the nature of criticality. The thermal fluctuations in the correlated classical system actually mimic the quantum fluctuations in the d-dimensional random quantum model. In Fig. 3, we schematically draw the renormalization-group flows and the different fixed points.[43] In this section, we denote the measure of randomness by $h(=\Delta^{1/2})$ where Δ is defined through the distribution given in Eq. (2).

As mentioned previously, Stinchcombe, Moore, and de Queiroz,[29] obtained a scaling description for the d-dimensional random-field Ising model considering domains on a bar geometry. They showed that the wall roughening removes the marginality in $d=2$ and calculated the correlation length for both $d=2$ and $d=2+\epsilon$.[30] We shall here extend the same phenomenological renormalization group ideas to striped random-field Ising systems in $(d+1)$ dimensions. The zeroth-order theory (flat domain walls) for the RFIM (Ref. 29) that provides the asymptotically correct flow equations for the variance of the random-field fluctuations, deals with the ground-state energy of the RFIM at $T=0$. This argument stems from the fact that the random-field fluctuations always dominate over the thermal fluctuations and temperature is irrelevant. The $T=0$ theory discussed in Ref. 29 yields the equilibrium value of the correlation length ξ_L of the finite-width bar, in the asymptotic limit (small h) when the correlation length ξ_L (or the measure of the typical size of the of the domains along a particular direction, arising due to the random-field fluctuations) is much bigger than L, the size of the bar in other $(d-1)$ directions.

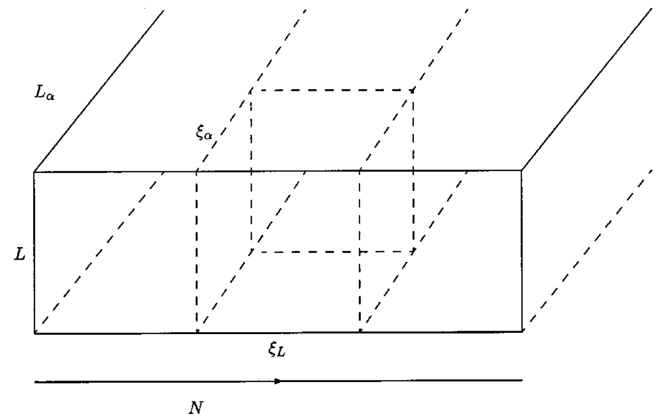

FIG. 4. The correlated random-field Ising system on a bar geometry. We take one spatial direction of size N, the size along the Trotter (αth) direction is L_α, and the transverse size L. The domains are of typical size ξ_L along the direction N and ξ_α along L_α.

The situation is fairly complicated in the present problem (with correlated random field) in comparison to that considered in Ref. 29. In this case as well, the random-field fluctuations slice the system in domains of size ξ_L, but since the randomness is correlated in the Trotter direction this domain will ''percolate'' along that direction causing the correlation length ξ_α to diverge (see Fig. 4). One must then introduce thermal fluctuations at the very outset to slice the domains into finite sizes in the Trotter direction. To be more specific, to formulate the zeroth-order theory for the present problem, we need to consider entropic effects in addition to the assumption of flat domain walls.

With a view to formulate the zeroth-order phenomenological renormalization-group theory in this correlated random-field system, we make a convenient compromise between the random-field fluctuations and the thermal fluctuations. We here study the system on a $(d+1)$-dimensional bar geometry (Fig. 4) with size N in one of the spatial directions and L in the other $(d-1)$ spatial directions. We take the size of the bar along the Trotter direction as L_α. The random-field fluctuations slice the direction N into domains of typical size ξ_L. The random field being relevant in the sense of the renormalization group, we ignore the entropic effect arising due to the arrangements of these walls so that we get back the results of Ref. 29 for the ordinary RFIM. As discussed earlier, we must take into account the thermal fluctuations which create domains of typical size ξ_α in the αth (i.e., Trotter) direction. We consider both the energy and the entropic (finite-temperature) effect of these domain walls in the Trotter direction. We believe that this simple-minded geometry is going to constitute the zeroth-order theory for the striped random-field system, which should be exact in the asymptotic limit (small h, small T), when both ξ_α and ξ_L are much larger than L.

The procedure we employ here is to minimize the free energy $\mathcal{F}=U-TS$ to determine the characteristic length scales ξ and ξ_α via phenomenological scaling.[31] As argued in the preceding paragraph, we work in the low-temperature and low-randomness limit and consider well-separated domains ($\xi,\xi_\alpha \gg L$) as in Ref. 29. In the zeroth-order (flat domain wall) theory, the entropy contribution we consider corresponds to the ways of arranging L_α/ξ_α flat walls with average spacing ξ_α along the Trotter direction of size L_α (Fig. 4), and is given by

$$S(\xi_\alpha)=\ln\binom{L_\alpha}{L_\alpha/\xi_\alpha}. \tag{21}$$

Simplifying the above expression for the entropy using Stirling's approximation one can immediately write the expression for free energy as

$$\mathcal{F}=U-TS$$
$$=\frac{NL_\alpha}{\xi_L\xi_\alpha}[2JL^{d-1}\xi_\alpha-h\xi_\alpha\sqrt{L^{d-1}\xi_L}+2J_0\xi_L L^{d-1}]-TS, \tag{22}$$

where the simplified expression for the entropy is of the form (with $k_B=1$)

$$S=-NL_\alpha[x_\alpha\ln x_\alpha+(1-x_\alpha)\ln(1-x_\alpha)],\quad \text{where}\ \ x_\alpha=\frac{1}{\xi_\alpha}. \tag{23}$$

In the expression for energy U in Eq. (22), the first term is due to the interfacial energy of domain walls along the direction N, the second term originates from the random-field fluctuations (as suggested by the central-limit theorem), and the third one is the interfacial energy due to domain slicing in the Trotter direction. The (ferromagnetic) interaction along the (anisotropic) Trotter direction is denoted by J_0 while interaction along the d spatial directions is J.

Let us now extremalize the above free energy with fixed N and L_α, demanding

$$\frac{\partial\mathcal{F}}{\partial x_\alpha}=0,\quad \frac{\partial\mathcal{F}}{\partial x_L}=0,\quad \text{where}\ \ x_L=1/\xi_L.$$

This gives the characteristic inverse length scales of the problem as

$$x_L=\frac{1}{\xi_L}=\left(\frac{h}{4J}\right)^2 L^{1-d}, \tag{24}$$

$$x_\alpha=\frac{1}{\xi_\alpha}=\frac{1}{\exp(2J_0L^{d-1}/T)+1}. \tag{25}$$

The renormalization-group (RG) transformations of parameters $(h/J,J_0/T)$ under rescaling of L by a factor b are obtained from the phenomenological scaling relations.[31] The scaling relation for h/J is given by

$$\frac{\xi_L((h/J)')}{L}=\frac{\xi_{Lb}(h/J)}{bL}. \tag{26}$$

Using the expression for ξ_L [Eq. (24)] in Eq. (26), one immediately arrives at the recursion relation for h/J, which is given as

$$\left(\frac{h}{J}\right)'=b^{(2-d)/2}\left(\frac{h}{J}\right). \tag{27}$$

This result is identical to that in the case of ordinary random-field systems, as in Refs. 43 and 44. Equation (27) has an unstable fixed point at $h=0$ for $d<2$ and the eigenvalue $b^{(d-2)/2}$ implies that the bulk (spatial) correlation length ξ (for $d<2$) diverges for $h\to 0$ and $T=0$ as

$$\xi\sim h^{-2/(2-d)}.$$

The renormalization is marginal for $d=2$ and one has to consider domain ''decoration'' perturbations of the flat walls to deal with the marginality at $d=2$.[29] The flow Eq. (27) implies that $h=0$ is a stable fixed point for $d>2$ and it is unstable for $d<2$. This suggests that the lower critical (spatial) dimension is 2 in agreement with the existing conjecture.

We shall now apply the corresponding phenomenological finite-size scaling relation to the correlation length ξ_α along the Trotter direction

$$\frac{\xi_\alpha(L_\alpha,(T/J_0)')}{L_\alpha}=\frac{\xi_\alpha(bL_\alpha,(T/J_0))}{bL_\alpha}. \tag{28}$$

Using the expression for ξ_α given in Eq. (24), in the low-temperature limit and ignoring corrections of the order of $\ln b$, we find that the recursion relation for the parameter T/J_0 is given by

$$\left(\frac{J_0}{T}\right)'\sim b^{d-1}\left(\frac{J_0}{T}\right). \tag{29}$$

The recursion relations given in Eqs. (27) and (29) completely describe the phenomenological RG equations for the correlated random field systems in $(d+1)$ dimensions. Above the lower critical (spatial) dimension (i.e., for $d=2+\epsilon$), Eq. (29) shows that temperature scales down to zero confirming that thermal fluctuations (quantum fluctuations in the equivalent quantum model) are irrelevant. Let us now recall the expression for the correlation length ξ_α given in Eq. (25) which shows an exponential growth of the correlation length ξ_α in the Trotter direction as temperature scales down to zero. We identify this as the signature of activated quantum dynamics which was missing the ϵ-expansion calculation of the previous section. It should be noted here that we do not expect to retrieve pure Ising results for $h=0$, because in our zeroth-order theory we have not taken into consideration the complete entropic fluctuations; i.e., we have neglected the entropic fluctuations due to arrangement of domain walls in the direction N.

Just above the lower critical dimension $(d=2+\epsilon)$, the roughening of the domain wall induced by the random fields[45] (see also Refs. 43 and 30) introduces a correction of the order $(h/J)^2$ to the recursion relation (27), given as

$$\left(\frac{h}{J}\right)'=b^{(2-d)/2}\left(\frac{h}{J}\right)+A\ln b\left(\frac{h}{J}\right)^2. \tag{30}$$

Defining $w=h/J$ and $t=T/J_0$ we find [with $b=\exp(l)$] using Eqs. (29) and (30),

$$\frac{\partial w}{\partial l}=-\frac{\epsilon}{2}w+Aw^2, \tag{31}$$

$$\frac{\partial t}{\partial l}=-(1+\epsilon)t. \tag{32}$$

Clearly, the fixed point that determines the criticality is given as $w^*=(\epsilon/2A)^{1/2}$, $t^*=0$ and the corresponding correlation length exponent $\nu=1/\epsilon$ to the order ϵ^2. This immediately leads to the conclusion that the fixed point that determines the criticality in the $(d+1)$-dimensional correlated or equivalent d-dimensional quantum random-field system is the same as that in d-dimensional random-field system and the exponents happen to be the same as the exponents of d-dimensional random field systems. Thus, the phenomenological zeroth-order theory is completely in agreement with the results obtained using the field-theoretic ϵ expansion.

We conclude this section with the note that we have extended the phenomenological renormalization calculations to the correlated random-field Ising systems on a bar geometry. The zeroth order theory we present is completely in agreement with the existing conjecture and provides physically relevant results. Efforts are being made to extend this argument to the systems with $M\geqslant 2$ and also to explore a possible Griffiths singular phase associated with this transition.

VI. CONCLUSION

We have studied $(d+1)$-dimensional striped random-field systems. The finite-temperature transition in this model is equivalent (in the sense of universality) to the zero-temperature transition in quantum random-field systems. The ϵ-expansion studies (within the replica symmetric framework) of the relaxational dynamics of the above model fail to capture the activated dynamical scaling. The dynamical scaling is argued to be activated with a modified exponent θ' which bears the signature of correlated randomness. A phenomenological argument is provided in favor of activated critical dynamics in general random Ising systems with correlated randomness.

We also discuss a zeroth-order phenomenological scaling theory using a bar geometry to investigate the the critical behavior of striped random-field Ising systems. The exponents just above the lower critical dimension [in $(2+\epsilon)$ spatial dimensions] are evaluated and the signature of the activated (quantum) dynamical scaling is clearly indicated.

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APPENDIX: EVALUATION OF EXPONENTS z AND z_q

In this appendix, we shall evaluate the exponents z and z_q up to the second order in ϵ. We shall here consider the self-energy diagram given in Fig. 2, which is the only relevant diagram up to this order of expansion with free propagator and free correlator, given as (at criticality $r=0$)

$$G_0(k,k_\alpha,\omega)=\frac{1}{-iw/\Gamma+k^2+\gamma k_\alpha^2}, \tag{A1}$$

$$C_0(k,k_\alpha,\omega)=\frac{\Delta\,\delta(\omega)\,\delta(q_\alpha)}{|-i\omega/\Gamma+\gamma k_\alpha^2+k^2|^2}. \tag{A2}$$

The contribution of the above diagram to the self-energy can be written as

$$\Sigma(\vec{k},k_\alpha,\omega)=u^2\Delta^2\int\frac{d^Dk_1}{(2\pi)^D}\int\frac{d^Dk_2}{(2\pi)^D}\frac{1}{k_1^4}\times\frac{1}{k_2^4}\frac{1}{[-i\omega/\Gamma+(\vec{k}-\vec{k}_1-\vec{k}_2)^2+\gamma k_\alpha^2]}. \tag{A3}$$

Let us now drop Γ and γ till the end of the calculation. We define exponents η, η_α, and η_t in the following way:

$$\begin{aligned}\Sigma(k,0,0)-\Sigma(0,0,0)&=\eta k^2\ln b\\&=\int_{\Lambda/b}^{\Lambda}\frac{d^Dk_1}{(2\pi)^D}\int_{\Lambda/b}^{\Lambda}\frac{d^Dk_2}{(2\pi)^D}\\&\quad\times\left[\frac{1}{k_1^4}\frac{1}{k_2^4(\vec{k}-\vec{k}_1-\vec{k}_2)^2}-\frac{1}{(\vec{k}_1+\vec{k}_2)^2}\right].\end{aligned} \tag{A4}$$

Similarly,

$$\Sigma(0,k_\alpha,0)-\Sigma(0,0,0)=k_\alpha^2\eta_\alpha\ln b \tag{A5}$$

$$=\int_{k_1}\int_{k_2}\frac{(-k_\alpha^2)}{k_1^4k_2^4(\vec{k}_1+\vec{k}_2)^4}. \tag{A6}$$

to the order k_α^2.

We define identically η_t,

$$\Sigma(\omega,0,0)-\Sigma(0,0,0)=+i\omega\eta_t \ln b \tag{A7}$$

$$=\int_{k_1}\int_{k_2}\frac{i\omega}{k_1^4 k_2^4(\vec{k}_1+\vec{k}2)^4}. \tag{A8}$$

From Eq. (A4), we find

$$\Sigma(k,0,0)-\Sigma(0,0,0)=\int_{k_1}\int_{k_2}\frac{1}{k_1^4}\frac{1}{k_2^4}\frac{1}{(\vec{k}_1+\vec{k}_2)^2}\times\left[\frac{[-k^2+2\vec{k}\cdot(\vec{k}_1+\vec{k}_2)]}{(\vec{k}-\vec{k}_1-\vec{k}_2)^2}\right]. \tag{A9}$$

We have to extract the coefficient of k^2 in the above expression; with a few lines of algebra, in the limit $k\rightarrow 0$, we find

$$\Sigma(k,0,0)-\Sigma(0,0,0)=\int_{k_1}\int_{k_2}\frac{1}{k_1^4}\frac{1}{k_2^4}\frac{k^2(4\cos^2\theta-1)}{(\vec{k}_1+\vec{k}_2)^4}, \tag{A10}$$

where θ is the angle between $\vec{k}$ and $\vec{k}_1+\vec{k}_2$. Inserting the angular average of $\cos^2\theta=1/d$, we find from Eq. (6),

$$\Sigma(k,0,0)-\Sigma(0,0,0)=\int\frac{1}{k_1^4}\frac{1}{k_2^4}\frac{k^2(4/d-1)}{(\vec{k}_1+\vec{k}_2)^4}. \tag{A11}$$

Comparing Eqs. (A11) and (A6), we find

$$\eta_\alpha=\left(\frac{d}{d-4}\right)\eta. \tag{A12}$$

With $d=6$, we find $\eta_\alpha=3\eta$. Similarly comparing Eqs. (A11) and (A8) we find $\eta_t=3\eta$. These immediately lead us to the scaling [using Eq. (9)] of parameters γ and Γ given as

$$\gamma'=\gamma b^{2-2z_q+2\eta},\quad \Gamma'=\Gamma b^{2-z+2\eta}. \tag{A13}$$

Demanding γ and Γ to scale to a fixed point immediately gives us

$$z=2+2\eta \text{ and } z_q=1+\eta. \tag{A14}$$

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