Charged excitons in a low magnetic field in $GaAs/Ga_{1-x}Al_xAs$ and $CdTe/Cd_{1-x}Zn_xTe$ semiconductor quantum wells

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We study the influence of an external magnetic field on the singlet and triplet ground states of negatively and positively charged excitons in $GaAs/Ga_{1-x}Al_xAs$ and $CdTe/Cd_{1-x}Zn_xTe$ semiconductor quantum wells in the low-field limit. The energies are determined using a variational wave function for finite values of the band offsets. We show that there appear additional Landau levels due to the charge of the center of mass. We discuss the influence of the magnetic field on the exciton and charged excitons transition energies.

I. INTRODUCTION

Over the past few years, considerable experimental work has been carried out on charged excitons (or trions) in quasi-two-dimensional (2D) semiconductor structures, mainly because the understanding of their properties is essential in the optical investigation of a 2D electron or hole gas. Two kinds of charged excitons have been observed: the negatively charged exciton (X^-) and the postitively charged exciton (X_2^+) resulting, respectively, from the binding of an exciton (X) and a free electron (e) or a free hole (h). After its first identification[1] in $CdTe/Cd_{1-x}Zn_xTe$ quantum wells (QW), the singlet state of the negatively charged exciton (X_s^-) has subsequently been observed[2–7] in $GaAs/Ga_{1-x}Al_xAs$ QW's. Later, the observations of the negatively charged exciton triplet state (X_t^-) as well as the positively charged exciton singlet ($X_{2,s}^+$) and triplet ($X_{2,t}^+$) states have also been reported. It appears very soon that the use of an external magnetic field may be very useful in the identification of charged exciton lines in CdTe compounds[8–13] as well as in GaAs compounds.[14–21] It results that charged excitons lines and exciton lines exhibit a comparable diamagnetic shift though the energy difference of the lines increases monotonically with the applied field.[17] The X_s^- state may be already observed at zero magnetic field for electron concentrations lower than a critical value. However, even for samples in which this concentration is larger, the application of a magnetic field can enable the observation of X_s^-. On the other hand, the X_t^- state is only observed in a magnetic field. The behaviors of the $X_{2,s}^+$ and the $X_{2,t}^+$ states are quite similar to those of the X_s^- and X_t^- states. Moreover, it has been reported[21] recently that though the ''binding energies'' of X_s^- and $X_{2,s}^+$ are identical at zero field, their spectra are different at high-magnetic field.

There exists little theoretical work concerning the influence of a magnetic field on charged excitons. Recent theoretical studies[22–25] on X_s^- states consider ideal 2D systems in the high-field limit. To our knowledge, the only papers concerning QW's are related to X_s^- and X_t^- states subjected to a strong magnetic field in an infinite[26] or a finite[27] quantum well. A common feature of all these papers is that the wave functions used do not take into account explicitly the interactions between the three particles which are expected to be important because the quantum confinement enhances the Coulomb correlations. The positively charged exciton has not yet been studied. The theoretical prediction[27] that at high magnetic fields and in narrow wells the X_s^- state becomes less stable than the X_t^- state contradicts recent experimental observations.[17] We have previously shown in the case of bulk materials[28] and 2D semiconductors[29] that in the low magnetic field limit the energy of charged excitons should split into several Landau levels due to the charge of the center of mass.

In the present paper we study the influence of a perpendicular magnetic field in the low-field region on the negatively and positively charged excitons singlet and triplet states in finite quantum wells. We restrict our discussion to $GaAs/Ga_{1-x}Al_xAs$ and $CdTe/Cd_{1-x}Zn_xTe$ semiconductor QW's. To this purpose we extend our previous theories of X_s^- in 3D and 2D semiconductors in a magnetic field,[28,29] and our study in a QW at zero magnetic field.[30]

II. THEORY

We present explicitly the theory of the negatively charged exciton X^- (e,e,h), the case of the positively charged exciton X_2^+ (h,h,e) being quite analogous by interchanging the electrons (e) and the holes (h) with isotropic effective-masses m_e^* and m_h^*. Within the envelope function approximation, the effective-mass Hamiltonian of the system reads

$$H=\frac{1}{2m_e^*}\left(-i\hbar\nabla_1+\frac{e}{c}\mathbf{A}_1\right)^2+\frac{1}{2m_e^*}\left(-i\hbar\nabla_2+\frac{e}{c}\mathbf{A}_2\right)^2+\frac{1}{2m_h^*}\left(-i\hbar\nabla_h-\frac{e}{c}\mathbf{A}_h\right)^2+V_c+V_w. \quad (1)$$

When taking into account the detailed band structure, the hole kinetic energy term may become more complicated and depends on the valence-band Luttinger parameters. In this case, strictly speaking, the symmetry between the X^- and X_2^+ theories is no longer present, because of the different wave functions of the electrons and the holes. However, in our numerical calculations, we will, in the framework of the effective-mass approximation, use mean hole masses ob-

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tained from the experiment or using the Luttinger parameters. This approximation is expected to lead to reasonable qualitative results. The potential vectors $\mathbf{A}_i$ of the electron and hole are expressed as a function of the uniform magnetic field $\mathcal{H}$ in the Coulomb gauge by $\mathbf{A}(\mathbf{r}_i)=\frac{1}{2}\mathcal{H}\times\mathbf{r}_i$ $(i=e,h)$. In the following we assume that the magnetic field is directed along the growth axis. The interaction between the three particles is modeled by a Coulombic potential V_c screened by the dielectric constant κ:

$$V_c=\frac{e^2}{\kappa}\left(\frac{1}{r_{12}}-\frac{1}{r_{1h}}-\frac{1}{r_{2h}}\right). \tag{2}$$

The total well potential V_w arising from the band offsets is described by the sum of three square-well potentials. Using the in-plane relative and in-plane center-of-mass coordinates $\mathbf{r}$, $\mathbf{R}$, and $\mathbf{R}_0$ related to the in-plane coordinates ρ_i $(i=1,2,h)$ by

$$\mathbf{r}=\rho_1-\rho_2, \quad \mathbf{R}=\frac{\rho_1+\rho_2}{2}-\rho_h,$$

$$\mathbf{R}_0=\frac{m_e^*(\rho_1+\rho_2)+m_h^*\rho_h}{2m_e^*+m_h^*}, \tag{3}$$

the Hamiltonian can be expressed as

$$H=H_0+H_1+H_2, \tag{4}$$

where

$$H_0=-\frac{\hbar^2}{2\mu}\Delta_{\mathbf{r}}-\frac{\hbar^2}{2M}\Delta_{\mathbf{R}}-\frac{\hbar^2}{2M_0}\Delta_{\mathbf{R}_0}-\frac{\hbar^2}{2m_e^*}(\Delta_{z_1}+\Delta_{z_2}) -\frac{\hbar^2}{2m_h^*}\Delta_{z_h}+V_c+V_w, \tag{5}$$

$$H_1=-\frac{ie\hbar}{cm_e^*}\Bigg[\mathbf{A}(\mathbf{r})\cdot\nabla_{\mathbf{r}}+\frac{1-2\sigma^2}{1+2\sigma}\mathbf{A}(\mathbf{R})\cdot\nabla_{\mathbf{R}} +\frac{2\sigma(1+\sigma)}{(1+2\sigma)^2}\mathbf{A}(\mathbf{R})\cdot\nabla_{\mathbf{R}_0}+(1+\sigma)\mathbf{A}(\mathbf{R}_0)\cdot\nabla_{\mathbf{R}} +\frac{\sigma}{1+2\sigma}\mathbf{A}(\mathbf{R}_0)\cdot\nabla_{\mathbf{R}_0}\Bigg], \tag{6}$$

$$H_2=\frac{e^2}{2c^2m_e^*}\Bigg[\frac{1}{2}\mathbf{A}^2(\mathbf{r})+2\frac{1+2\sigma^3}{(1+2\sigma)^2}\mathbf{A}^2(\mathbf{R})+(2+\sigma)\mathbf{A}^2(\mathbf{R}_0) +4\frac{1-\sigma^2}{1+2\sigma}\mathbf{A}(\mathbf{R})\cdot\mathbf{A}(\mathbf{R}_0)\Bigg] \tag{7}$$

with

$$\mu=\frac{m_e^*}{2}, \quad M=\frac{2m_e^*m_h^*}{2m_e^*+m_h^*}, \quad M_0=2m_e^*+m_h^*. \tag{8}$$

We remark that the Hamiltonian H commutes with the projection L_z of the angular momentum along the z axis, but does not commute with the 3D momentum operator $\mathbf{P}_0$. However, it appears that the components of the in-plane operator $\pi(\pi_x,\pi_y)$,

$$\pi=\mathbf{p}_1+\mathbf{p}_2+\mathbf{p}_h-\frac{e}{c}[\mathbf{A}(\rho_1)+\mathbf{A}(\rho_2)-\mathbf{A}(\rho_h)] =-i\hbar\partial_{\mathbf{R}_0}-\frac{e}{c}\left[2\frac{1+\sigma}{1+2\sigma}\mathbf{A}(\mathbf{R})+\mathbf{A}(\mathbf{R}_0)\right], \tag{9}$$

where $\mathbf{p}_i$ are the in-plane free-particle momentum operators, commute with H but do not commute with each other. We choose that the component π_x and the Hamiltonian may be simultaneously diagonalized. Thus we can transform the wave function Ψ by eliminating the in-plane coordinate X_0 of the center of mass:

$$\Psi(z_1,z_2,z_h,\mathbf{r},\mathbf{R},\mathbf{R}_0)=U\Phi(z_1,z_2,z_h,\mathbf{r},\mathbf{R},Y_0), \tag{10}$$

where the unitary operator U is expressed as

$$U=\exp i\left[\left\{\mathbf{K}+\frac{2e}{c\hbar}\frac{1+\sigma}{1+2\sigma}\mathbf{A}(\mathbf{R})\right\}\cdot\mathbf{R}_0-\frac{e\mathcal{H}}{2c\hbar}X_0Y_0\right]. \tag{11}$$

We remark that the function Φ is independent of X_0. We have introduced the in-plane vector $\mathbf{K}\equiv(K_x,0)$, which must not be confused with the 3D wave vector $\mathbf{K}_0$ of the center of mass without the magnetic field. The transformed Hamiltonian H' reads

$$H'=U^{-1}HU=H_1'+H_2'+H_3'+H_4'+H_5', \tag{12}$$

where

$$H_1'=-\frac{\hbar^2}{2\mu}\Delta_{\mathbf{r}}-\frac{\hbar^2}{2M}\Delta_{\mathbf{R}}-\frac{\hbar^2}{2m_e^*}(\Delta_{z_1}+\Delta_{z_2})-\frac{\hbar^2}{2m_h^*}\Delta_{z_h}+V_c +V_w^e(z_1)+V_w^e(z_2)+V_w^h(z_h) \tag{13}$$

is the zero field contribution. The second term is the contribution due to the linear Zeeman effect:

$$H_2'=-\frac{ie\hbar}{cm_e^*}\left[\mathbf{A}(\mathbf{r})\cdot\nabla_{\mathbf{r}}+\frac{1-2\sigma^2}{1+2\sigma}\mathbf{A}(\mathbf{R})\cdot\nabla_{\mathbf{R}}\right]. \tag{14}$$

The contribution H_3' represents the quadratic diamagnetic effect:

$$H_3'=\frac{e^2}{c^2m_e^*}\left[\frac{1}{4}\mathbf{A}^2(\mathbf{r})+\lambda(\sigma)\mathbf{A}^2(\mathbf{R})\right], \tag{15}$$

where

$$\lambda(\sigma)=\frac{1+4\sigma(1+\sigma)(2+\sigma+\sigma^2)}{(1+2\sigma)^3}, \tag{16}$$

with $\sigma=m_e^*/m_h^*$. The fourth term H_4' describes the action of the magnetic field on the motion of the charged center of mass:

$$H_4'=-\frac{\hbar^2}{2M_0}\partial_{Y_0}^2+\frac{M_0\omega_{\mathrm{CM}}^2}{2}\left(Y_0-\frac{\hbar c}{e\mathcal{H}}K_x\right)^2. \tag{17}$$

It is analogous to the Hamiltonian of a harmonic oscillator of mass M_0 and circular frequency $\omega_{\rm CM}=e\mathcal{H}/M_0c$ oscillating around the point $Y_0^0=(\hbar c/e\mathcal{H})K_x$. Its contribution will lead to a splitting of the energy levels into Landau levels. Finally the fifth term

$$H_5'=2\hbar\omega_{\rm CM}\frac{1+\sigma}{1+2\sigma}\left[-iX\partial_{Y_0}+\frac{M_0\omega_{\rm CM}}{\hbar}Y\left(Y_0-\frac{\hbar c}{e\mathcal{H}}K_x\right)\right] \tag{18}$$

arises from the coupling between the relative and center of mass motions.

Due to the occurrence of the coupling term H_5', the in-plane relative and center-of-mass motions remain coupled. We have previously shown[28,29] that at low enough magnetic fields, the influence of this term may be neglected so that the in-plane motion of the center of mass may be separated from the relative motion. In this approximation, we can write the envelope wave function as the product of the wave function of the relative motion and the in-plane motion of the center-of-mass:

$$\Phi(z_1,z_2,z_h,\mathbf{r},\mathbf{R},Y_0)\simeq\psi(z_1,z_2,z_h,\mathbf{r},\mathbf{R})\Phi_{\rm CM}(Y_0). \tag{19}$$

The relative wave function ψ results as a solution of the following equation:

$$H^{\rm rel}\psi=(H_1'+H_2'+H_3')\psi=E^{\rm rel}\psi. \tag{20}$$

The wave function $\Phi_{\rm CM}$ describing the oscillatory motion of the center of mass satisfies the equation

$$H_4'\Phi_{\rm CM}=E_L^N\Phi_{\rm CM}. \tag{21}$$

In this paper we concentrate essentially on the diamagnetic effect. Thus we neglect the orbital Zeeman contribution as well as all spin effects. Thus the total energy reads

$$E^{\rm tot}=E^{\rm rel}+E_L^N, \tag{22}$$

where $E^{\rm rel}$ denotes the relative energy and where

$$E_L^N=\hbar\omega_{\rm CM}\left(N+\frac{1}{2}\right),\quad N=0,1,2\ \ldots \tag{23}$$

stands for the Landau energies corresponding to the in-plane motion of the center of mass. We remark that the coupling term H_5' does not give rise to any contribution in our approximation because $\langle\psi|X|\psi\rangle=\langle\psi|Y|\psi\rangle=0$ with relative wave functions of cylindrical symmetry, and we can also verify that $\langle\Phi_{\rm CM}|\partial/\partial Y_0|\Phi_{\rm CM}\rangle=\langle\Phi_{\rm CM}|Y_0-(\hbar c/e\mathcal{H})K_x|\Phi_{\rm CM}\rangle=0$.

We have determined the relative ground-state energy $E^{\rm rel}$ within the variational method using the same kind of the relative wave function that we used previously[30] in our study of X^- in a quantum well at zero magnetic field. It is expected that this function remains well adapted to the present case because of its axial symmetry. It reads

$$\psi(s,t,u,z_1,z_2,z_h)=\sum_{lmnpqr}c_{lmnpqr}\phi_{lmn}(s,t,u)\times\theta_{pqr}(z_1,z_2,z_h), \tag{24}$$

where l,m,n,p,q,r are positive integers or zero. The basis functions are chosen as products of functions ϕ_{lmn} depending only on the in-plane coordinates and functions θ_{pqr} depending only on the z coordinates. The in-plane functions

$$\phi_{lmn}(s,t,u)=e^{-k(s/2)}s^lu^mt^n \tag{25}$$

are Hylleraas-type basis wave functions. The z-dependent part of the wave function is chosen as follows:

$$\theta_{pqr}(z_1,z_2,z_h)=f_e(z_1)f_e(z_2)f_h(z_h)\times e^{-\alpha_e(z_1^2+z_e^2)-\alpha_h z_h^2}(z_1^pz_2^q+z_1^qz_2^p)z_h^r, \tag{26}$$

where f_e and f_h are the ground-state eigenfunctions of the electron and hole in a quantum well. For the singlet ground state, ψ has to be symmetrical with respect to the interchange of the two electrons. In this case n must be even. On the other hand, in the case of the triplet states, ψ has to be antisymmetric with respect to the interchange of the two electrons, and n must be odd. Further, because of the inversion symmetry in the z direction, our wave function must have definite parity in all cases. It is expected that for the ground state the even function will lead to the lowest energy. Thus we choose $p+q+r$ to be even. The linear parameters c_{lmnpqr} as well as the positive nonlinear variational parameters k, α_e, and α_h will be determined using the Ritz variational method. We have calculated the energies $E^{\rm tot}$ of the singlet (triplet) ground state of the charged excitons using a symmetrical (antisymmetrical) 66-term (39-term) wave function defined by $l+m+n\leqslant 4$ and $\{pqr\}=(000,101,110)$.

The case of the X_2^+ charged exciton is quite analogous. All of the above discussion remains valid in this case by interchanging the electrons and the holes. For the sake of simplicity, we choose an analogous wave function for the X_2^+ charged exciton. However, it is expected that this atomiclike wave function becomes less well adapted in the case of very heavy holes, i.e., $m_e^*/m_h^*\lesssim 0.1$.

In order to estimate the influence of both the quantum confinement as well as the magnetic field on the Coulombic correlations, we have defined the ''correlations energies,'' $E_{X^-}^c$, $E_{X_2^+}^c$, and E_X^c of the charged excitons and the exciton

$$\begin{aligned} E_{X^-}^c&=E_{X^-}^{\rm rel}-2E_{e,L}^0-E_{h,L}^0,\\ E_{X_2^+}^c&=E_{X_2^+}^{rel}-2E_{h,L}^0-E_{e,L}^0,\\ E_X^c&=E_X^{\rm rel}-E_{e,L}^0-E_{h,L}^0, \end{aligned} \tag{27}$$

where $E_{e,L}^0$ and $E_{h,L}^0$ are the electron and hole fundamental Landau energies in a quantum well. It may be verified that the conditions of stability against dissociation into an exciton and a free electron or hole may be written as

$$\begin{aligned} W_{X^-}&=E_{X^-}^c-E_X^c+\frac{\sigma}{1+2\sigma}\frac{\hbar\omega_e}{2}\leqslant 0,\\ W_{X_2^+}&=E_{X_2^+}^c-E_X^c+\frac{\sigma}{2+\sigma}\frac{\hbar\omega_e}{2}\leqslant 0, \end{aligned} \tag{28}$$

where $-W_{X^-}$ and $-W_{X_2^+}$ are the X^- and X_2^+ binding energies. We remark that contrary to what happens at zero magnetic field, the binding energy is no longer equal to the difference of the exciton and charged exciton correlation energies. In order to verify these stability conditions, it is necessary to know the values of the excitonic relative energy E_X^{rel} with an accuracy comparable to what we get in the cases of charged excitons. To this purpose we have performed a variational calculation for the excitonic ground-state energy using a similar wave function:

$$\psi_X = f_e(z_e) f_h(z_h) e^{-\beta\rho-\beta_e z_e^2-\beta_h z_h^2} \sum_{pr} c_{pr} z_e^p z_h^r, \tag{29}$$

where β, β_e, β_h, and c_{pr} are variational parameters, and p and r are positive integers chosen such that $p+r$ is even. It was found that the use of a nine-term wave function, defined by the condition $p+r \leqslant 4$, yields to the same accuracy that we obtained for the trions.

We have shown previously[31] that in the 2D case, the magnetoabsorptions between initial "free" electron or hole states and a final X^- or X_2^+ state reduce to Dirac peaks. It may be shown that the same behavior is expected in the case of semiconductor QW's and that the corresponding transition energies are given by

$$h\nu_{X^-} = \epsilon_g + E_{X^-}^{\mathrm{rel}} - E_e - \frac{1+\sigma}{1+2\sigma}\left(N+\frac{1}{2}\right)\hbar\omega_e, \tag{30}$$

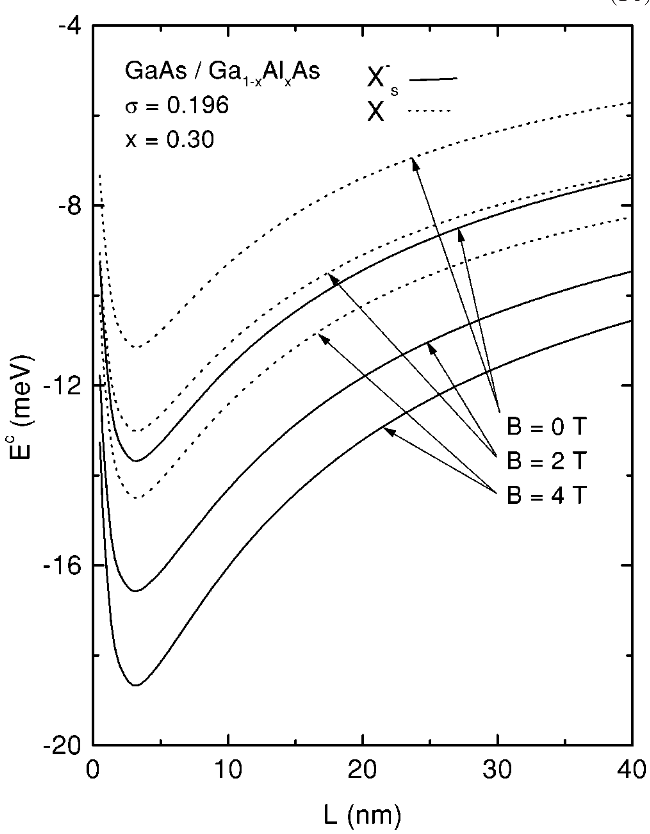

FIG. 1. Plot of the correlation energies of the exciton and the lowest Landau X_s^- singlet state against the well width L for different values of the magnetic field in a GaAs/$Ga_{1-x}Al_xAs$ QW.

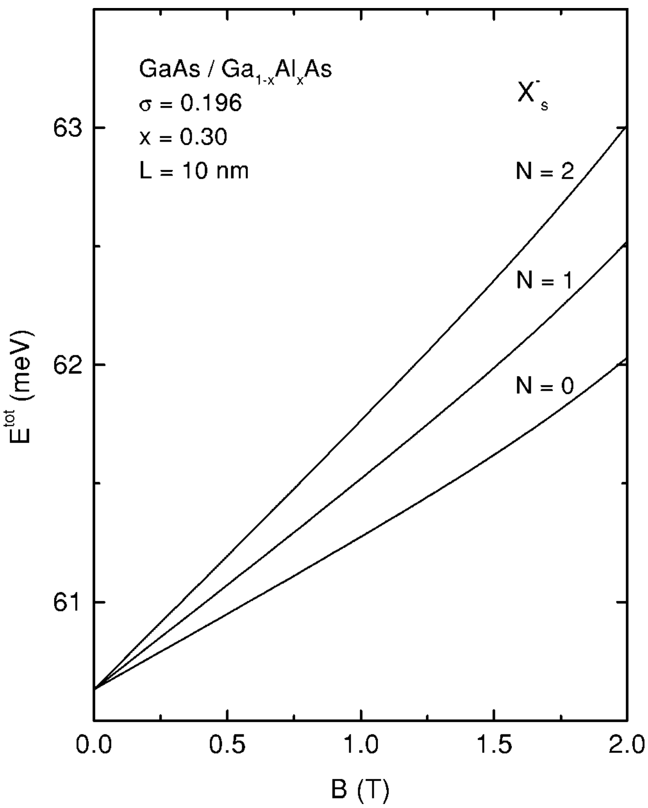

FIG. 2. Variations of the total energies of the first Landau levels of X_s^- as a function of the magnetic field in a GaAs/$Ga_{1-x}Al_xAs$ QW.

$$h\nu_{X_2^+} = \epsilon_g + E_{X_2^+}^{\mathrm{rel}} - E_h - \sigma\frac{1+\sigma}{2+\sigma}\left(N+\frac{1}{2}\right)\hbar\omega_e,$$

where ϵ_g is the zero-field band gap of the well material. E_e and E_h denote the zero-field electron and hole energies in a quantum well. $\omega_e = e\mathcal{H}/m_e^* c$ corresponds to the electron cyclotron frequency. The corresponding exciton transition energy is given by

$$h\nu_X = \epsilon_g + E_X^{\mathrm{rel}} + \frac{\hbar^2 K_p^2}{2M} \tag{31}$$

while $\mathbf{K}_p$ is the in-plane exciton center-of-mass wave vector, which vanishes for transitions at $T=0$ K in direct gap materials. M is the total exciton mass. So it appears that the charged exciton "localization" energies corresponding to the lowest $(N=0)$ Landau levels are given by

$$\Delta h\nu = h\nu - h\nu_X = -W + \frac{\hbar^2 K_p^2}{2M}. \tag{32}$$

It is worth pointing out that the localization energy, which may be deduced from experimental observations, is not equal to the difference $E_X^c - E_{X^-}^c$ of the exciton and charged exciton correlation energies. Indeed, we get, for instance, in the case of the X^- trion,

$$\Delta h\nu_{X^-} = E_X^c - E_{X^-}^c - \frac{\sigma}{1+2\sigma}\frac{\hbar\omega_e}{2} + \frac{\hbar^2 K_p^2}{2M}. \tag{33}$$

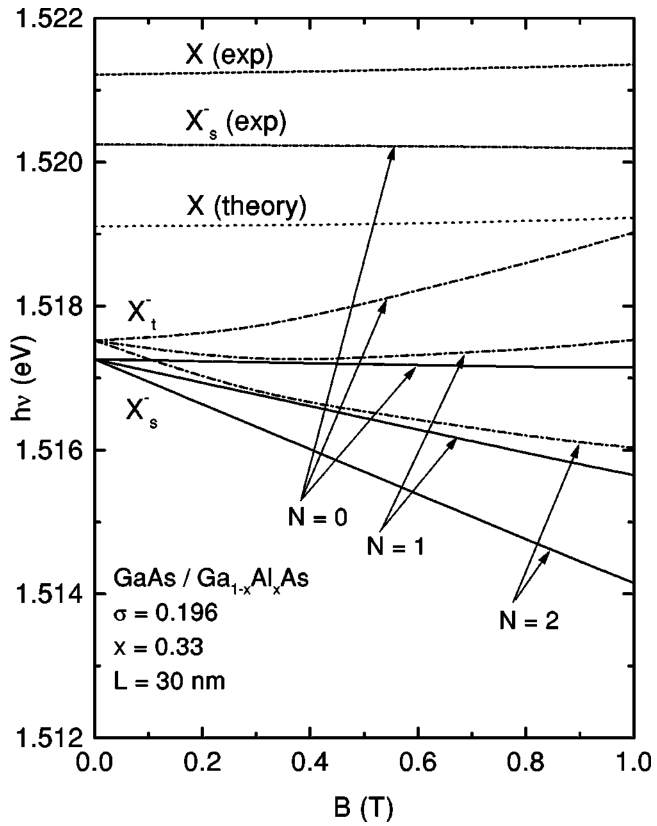

FIG. 3. Comparison between the theoretical and experimental transition energies of the exciton and singlet and triplet states of negatively charged excitons as functions of the magnetic field in a GaAs/$Ga_{1-x}Al_xAs$ QW.

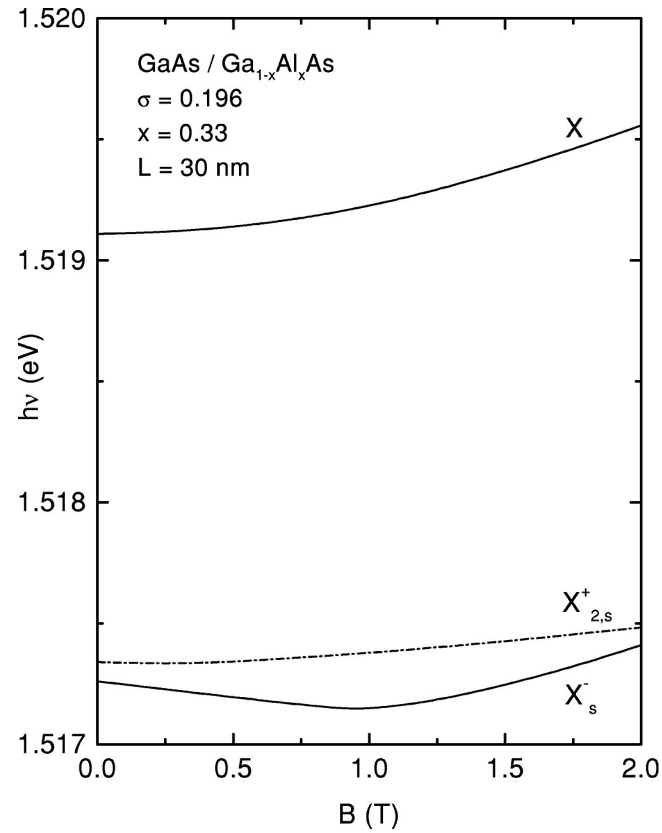

FIG. 4. Comparison between the transition energies of the exciton and the singlet negatively and positively charged excitons as functions of the magnetic field in a GaAs/$Ga_{1-x}Al_xAs$ QW.

III. RESULTS AND DISCUSSION

In the case of GaAs/$Ga_{1-x}Al_xAs$ QW's, we use the following material data:[32] $m_e^*/m_0=0.0665$ for the electron mass and $m_{\rm hh}^*/m_0=0.34$ for the heavy-hole mass. Thus, the value of the electron-to-hole effective-mass ratio amounts to $\sigma=0.196$. The bands offsets are given by $V_e=Q_e\Delta\epsilon_g$ and $V_h=Q_h\Delta\epsilon_g$, where $Q_e=0.57=1-Q_h$. Further we assume that the band gap difference $\Delta\epsilon_g$ and the aluminum concentration x are related by[33] $\Delta\epsilon_g=1.155x+0.37x^2$ eV and we use the value $\kappa=12.5$ for the dielectric constant.[34] In the case of CdTe/$Cd_{1-x}Zn_xTe$, we use the electron mass $m_e^*/m_0=0.096$, resulting from cyclotron resonance measurements[35] in CdTe, and the in-plane heavy-hole mass, $m_{\rm hh}^*/m_0=0.19$, deduced from the Luttinger parameters $\gamma_1=4.11$, $\gamma_2=1.08$, and $\gamma_3=1.95$, obtained[36] by two-photon magnetoabsorption in CdTe. The effective electron-to-hole mass ratio is thus given by $\sigma=0.505$. For $x=0.16$, the conduction- and valence-band offsets are[37] $V_e=71.4$ meV and $V_h=24.1$ meV. Figure 1 shows the variations versus L of the X and X_s^- correlation energies for three different values of the magnetic field in a GaAs/$Ga_{1-x}Al_xAs$ QW. It appears, as expected, that the quantum confinement as well as the magnetic field increase the Coulomb correlation. We remark also that the exciton and charged exciton energies behave quite analogously and that the energy difference remains quite constant. Further, the X_s^- state remains always stable for the reported values of L and B. Indeed, although the binding energy W is not equal to the difference of the exciton and charged exciton correlation energies, it can be verified that in our case the last term in Eq. (28) amounts to 0.1 meV/T, and can therefore be neglected. In Fig. 2 we report the variations of the total energies, Eq. (22), of X_s^- as a function of the magnetic field in a GaAs/$Ga_{1-x}Al_xAs$ QW. We observe the expected quadratic behavior of the diamagnetic contribution as well as a splitting due to the in-plane motion of the charged center of mass. This result is quite analogous to what we obtained previously[28,29] in the case of 3D and 2D semiconductors. The possibility of the existence of charged exciton Landau levels has never been reported previously. In Fig. 3, we report the theoretical and the experimental[6] transition energies for X, X_s^-, and X_t^- states in a GaAs/$Ga_{1-x}Al_xAs$ QW with heavy holes and with $x=0.33$ and $L=30$ nm. Our theoretical values for the X_s^- states are lower than the observed energies, even at zero magnetic field. We discussed this point in our previous paper.[30] The discrepancy may be due to the material parameters we used, but also to the fact that we neglected the electron-hole exchange interaction, which we expect to be more important in QW's than in 3D materials. However, we observe a qualitative agreement between the slope of the theoretical curve corresponding to the X_s^- $(N=0)$ state and the X_s^- experimental curve. In particular, we get a minimum near 1 T, which appears also in the experimental curve. This behavior may be explained by the fact that when the magnetic field goes to zero, the transition energy, Eq. (30), behaves like the last linear term which decreases when the magnetic field increases. However, at higher magnetic fields,

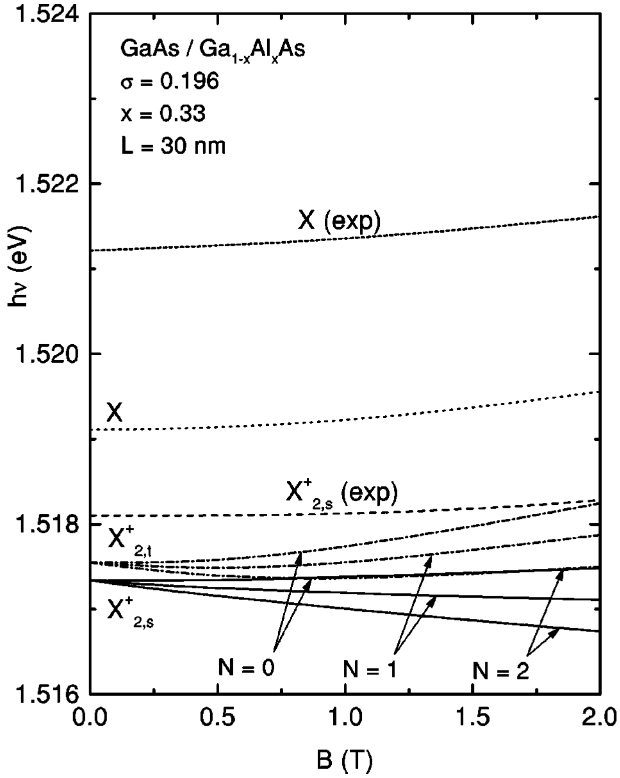

FIG. 5. Comparison between the theoretical and experimental transition energies of the exciton and singlet and triplet states of positively charged excitons as functions of the magnetic field in a $GaAs/Ga_{1-x}Al_xAs$ QW.

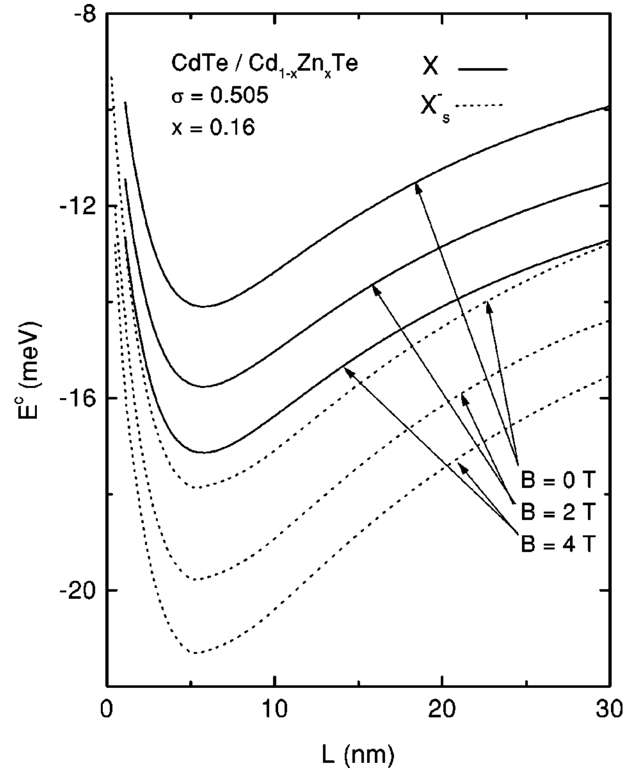

FIG. 6. Plot of the correlation energies of the exciton and the lowest Landau X_s^- singlet state against the well width L for different values of the magnetic field in a $CdTe/Cd_{1-y}Zn_yTe$ QW.

the quadratic positive second term becomes more important, and leads to an increase of the transition energy when the field increases. In the case of X_t^-, the contribution of the second term is more important because the triplet state is less bound than the singlet state. Indeed, the quadratic term is proportional to the spatial extension of the wave function so that it is more important for X_t^- than for X_s^- states for a given value of the magnetic field. Nevertheless, the experimental results reported up to now do not display our predicted Landau splitting, though shake-up processes have been reported[18] in the luminescence spectra of X_s^- states at moderate magnetic fields. But the most important result is that the X_t^- state is stable even at zero magnetic field. This result contrasts with the fact that, up to now, X_t^- lines have only been observed in a magnetic field. In Fig. 4 it appears that the X_s^- $(N=0)$ and $X_{2,s}^+$ $(N=0)$ transition energies are very close in a $GaAs/Ga_{1-x}Al_xAs$ QW regardless of the values of the magnetic field, and they become quite identical at zero field. This result is in agreement with recent observations.[21] It may be explained by the fact that the quantum confinement localizes more the X_2^+ states than the X^- states because the holes are more heavy than the electrons. Thus the Coulomb repulsion is more important for the holes than for the electrons. This explains why the X_2^+ state becomes less stable in a QW than in 3D materials. We observe also that the X_s^- curve passes through a minimum which is not the case for the $X_{2,s}^+$ state. This is due to the fact that the quadratic diamagnetic term in the transition energy is more important for the $X_{2,s}^+$ state. In Fig. 5, we report the theoret-

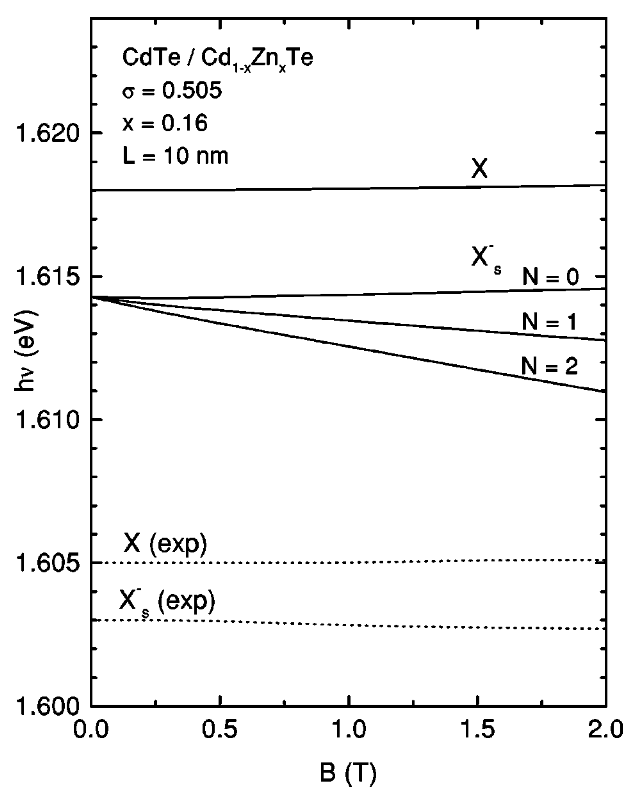

FIG. 7. Comparison between the theoretical and experimental transition energies of the exciton and the singlet negatively charged excitons as functions of the magnetic field in a $CdTe/Cd_{1-y}Zn_yTe$ QW.

ical and the experimental[2] transition energies for X, $X_{2,s}^{+}$, and $X_{2,t}^{+}$ states in a GaAs/Ga$_{1-x}$Al$_x$As QW with heavy holes and with $x=0.33$ and $L=30$ nm. The same discussion holds as for Fig. 3. In particular, it appears also that the $X_{2,t}^{+}$ state is stable even at zero magnetic field, though its observation has not yet been reported at low magnetic fields. In Figs. 6 and 7 we report our results obtained in the case of a CdTe/Cd$_{1-x}$Zn$_x$Te QW for heavy holes and with $x=0.16$ and $L=10$ nm. Our results are essentially the same as those reported in Figs. 1 and 3 in the case of a GaAs/Ga$_{1-x}$Al$_x$As QW. However, we remark that our theoretical transition energies are higher than the observed energies,[9] even at zero magnetic field. This discrepancy may be due to the material parameters we used. We observe a qualitative agreement between the slope of the $X_s^-(N=0)$ theoretical and experimental curves.

We must remark that all our above results have been obtained neglecting the spins as well as the Zeeman effect. Therefore, our transition energies have to interpreted as approximations of the mean values of the transitions energies corresponding to the different Zeeman components. On the other hand, it must be stressed that our results are only valid in the low-field limit. At higher magnetic fields, the comparative behavior of the singlet and triplet states may become different. Nevertheless, our main result is that we predict the existence of a Landau splitting for negatively and positively charged excitons in the low-field regime. Further, our calculations show that the triplet states X_t^- and $X_{2,t}^{+}$ are always stable at zero fields and in the low-field limit.

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