Charge relaxation and dephasing in Coulomb-coupled conductors

Markus Büttiker and Andrew M. Martin
Département de Physique Théorique, Université de Genève, CH-1211 Genève 4, Switzerland
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The dephasing time in coupled mesoscopic conductors is caused by the fluctuations of the dipolar charge permitted by the long-range Coulomb interaction. We relate the phase breaking time to elementary transport coefficients that describe the dynamics of this dipole: the capacitance, an equilibrium charge relaxation resistance, and in the presence of transport through one of the conductors a nonequilibrium charge relaxation resistance. The discussion is illustrated for a quantum point contact in a high magnetic field in proximity to a quantum dot.

Mesoscopic systems coupled only via the long-range Coulomb forces are of importance since one of the systems can be used to perform measurements on the other.[1] Despite the absence of carrier transfer between the two conductors, their proximity affects the dephasing rate. Of particular interest are *which path* detectors that can provide information on the paths of a carrier in an interference experiment.[2–5] It is understood that at very low temperatures the basic processes that limit the time τ_ϕ over which a carrier preserves its quantum-mechanical phase are electron-electron interaction processes.[6,7] The Coulomb interaction ensures overall charge neutrality. Consequently, for the two coupled zero-dimensional conductors of interest here, the basic process is a charge accumulation in one of the conductors accompanied by a charge depletion in the other conductor. The Coulomb coupling of two conductors manifests itself in the formation of a charge dipole, and the fluctuations of this dipole governs the dephasing process. The dynamics of this dipole, and thus the dephasing rate, can be characterized by elementary transport coefficients: In the absence of an external bias, excess charge relaxes, into the leads, towards its equilibrium value with an *RC* time. In mesoscopic conductors[8] the *RC* time is determined by an electrochemical capacitance C_μ and a charge relaxation resistance R_q. In the presence of transport through one of the conductors, shot noise[9] leads to a nonequilibrium charge relaxation resistance[10] R_ν. Below we relate R_q and R_ν to the dephasing rate.

Renewed interest in dephasing was also generated by experiments on metallic diffusive conductors and a suggested role of zero-point fluctuations.[11] We refer to the resulting discussion only with a recent item.[12] More closely related to our work are experiments by Huibers *et al.*[13] in which the dephasing rate in chaotic cavities is measured. At low frequencies such cavities can be treated as zero-dimensional systems.[14]

Consider two mesoscopic conductors coupled by long-range Coulomb interactions. An example of such a system, suggested in Ref. 15, is shown in Fig. 1. In case *A*, a quantum point contact (QPC) in a high magnetic field is close to a quantum dot, and in case *B* the QPC is some distance away from a quantum dot. First, we focus on case *A*. To describe the charge dynamics of such a system, we use two basic elements. First, we characterize the long-range Coulomb interaction with the help of a geometrical capacitance, much as in the literature on the Coulomb blockade. Second, the electron dynamics in each conductor (i) is described with the help of its scattering matrix $s^{(i)}_{\alpha\beta}(E,U_i)$, which relates the amplitudes of incoming currents at contact β to the amplitudes of the outgoing currents at α. The scattering matrix is a function of the energy of the carriers and is a function of the electrostatic potential U_i inside conductor i. In case *A*, the total excess charge on the conductor is of importance. Below we show that in this case the charge dynamics of the mesoscopic conductor can be described with the help of a density-of-states matrix[10,16]

$$\mathcal{N}^{(i)}_{\delta\gamma}=\frac{1}{2\pi i}\sum_\alpha s^{(i)\dagger}_{\alpha\delta}\frac{ds^{(i)}_{\alpha\gamma}}{dE}. \tag{1}$$

Equation (1) can be obtained from the frequency-dependent second-quantization current operator[9,10] $\hat{I}^{(i)}_\alpha(\omega)$ for the total current in lead α of conductor i. We have[10] $\Sigma_\alpha \hat{I}^{(i)}_\alpha(\omega)=i\omega e\hat{\mathcal{N}}_i(\omega)$ where the charge operator $e\hat{\mathcal{N}}_i(\omega)$ for a conductor i is determined by

$$\hat{\mathcal{N}}_i(\omega)=\sum_{\delta,\gamma,\nu}\int_{\Omega_i}d^3\mathbf{r}\int dE\,\hat{a}^\dagger_\delta(E)\mathcal{N}^{(i)}_{\delta\gamma}(\nu,\mathbf{r};E,E+\hbar\omega)\times\hat{a}_\gamma(E+\hbar\omega), \tag{2}$$

where the first integral is over the volume of the conductor (Ω_i), $\hat{a}_\gamma(\hat{a}^\dagger_\gamma)$ annihilates (creates) a carrier in lead γ, and the zero-frequency limit of $\int_{\Omega_i}d^3\mathbf{r}\Sigma_\nu\mathcal{N}^{(i)}_{\delta\gamma}(\nu,\mathbf{r};E,E+\hbar\omega)$ is

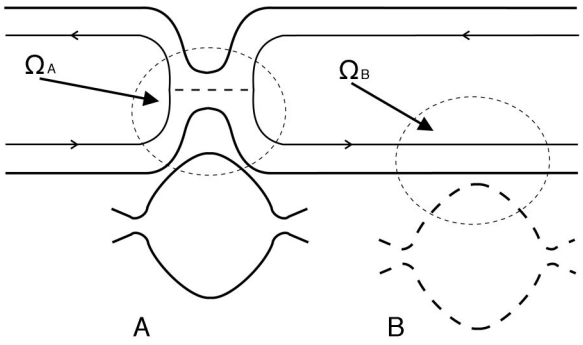

FIG. 1. Quantum point contact coupled to a quantum dot either in position A or B.

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given by Eq. (1). Equation (1) is valid in the WKB limit in which derivatives with regard to the potential can be replaced by an energy derivative.[16] Equation (1) are elements of the Wigner-Smith delay-time matrix.[17,18] Later, we also consider situations in which energy derivatives are not sufficient. The diagonal elements of this matrix determine the density of states of the conductor $N_i=\Sigma_\gamma \mathrm{Tr}(\mathcal{N}^{(i)}_{\gamma\gamma})$; the trace is over all quantum channels. The nondiagonal elements are essential to describe fluctuations.

At equilibrium, if all contacts of conductor i are held at the same potential, the two conductors can be viewed, as the plates of a capacitor holding a dipolar charge distribution with an electrochemical capacitance[8] $C_\mu^{-1}=C^{-1}+D_1^{-1}+D_2^{-1}$, which is the series combination of the geometrical capacitance C of the two conductors and the quantum capacitances $D_i=e^2N_i$ determined by their density of states. An excess charge relaxes with a resistance determined by[8]

$$R_q^{(i)}=\frac{h}{2e^2}\frac{\sum_{\gamma\delta}\mathrm{Tr}(\mathcal{N}^{(i)}_{\gamma\delta}\mathcal{N}^{(i)\dagger}_{\gamma\delta})}{\left[\sum_\gamma \mathrm{Tr}(\mathcal{N}^{(i)}_{\gamma\gamma})\right]^2}. \tag{3}$$

Consider now the total current at all contacts of conductor i, $I^{(i)}_{\mathrm{tot}}=\Sigma_\alpha \hat{I}^{(i)}_\alpha$. The low-frequency current spectral density (at conductor 1 or 2) is determined by $S(\omega)=2\omega^2C_\mu^2Y$, with $Y=R_qkT$ and $R_q=R_q^{(1)}+R_q^{(2)}$ if both conductors are at equilibrium. In the presence of transport through one of the conductors (say 2), the current noise spectrum S exhibits a crossover[10] from the equilibrium value ($e|V|\ll kT$) determined by R_q to a spectrum determined by $Y=R_q^{(1)}kT+R_v^{(2)}e|V|$ for ($e|V|\gg kT$) where[10]

$$R_v^{(i)}=\frac{h}{e^2}\frac{\mathrm{Tr}(\mathcal{N}^{(i)}_{21}\mathcal{N}^{(i)\dagger}_{21})}{\left[\sum_\gamma \mathrm{Tr}(\mathcal{N}^{(i)}_{\gamma\gamma})\right]^2}. \tag{4}$$

Next we relate these resistances to the voltage fluctuations in the two coupled mesoscopic conductors and subsequently to the dephasing time. Our starting point is the Poisson equation for the charge deviations away from their value in the equilibrium state. Since we are interested in fluctuations, we express the Poisson equation in operator form. Assuming that the geometrical capacitance C coupling the two conductors dominates all other capacitances,[19] we can express the charge on the two conductors in two ways: First, with the help of the potential operators $\hat{U}_1$ and $\hat{U}_2$, we have $\hat{Q}=C(\hat{U}_1-\hat{U}_2)$ for the charge on conductor 1 and $-\hat{Q}=C(\hat{U}_1-\hat{U}_2)$ for the charge on conductor 2. Second, we can express these same two charges in terms of the bare charges $e\hat{\mathcal{N}}_i$ [see Eq. (1)] and a screening charge proportional to the average density of states e^2N_i on the conductor times the induced potential $\hat{U}_i$. Thus charge and potential fluctuations are related by

$$\hat{Q}=C(\hat{U}_1-\hat{U}_2)=e\hat{\mathcal{N}}_1-e^2N_1\hat{U}_1, \tag{5}$$

$$-\hat{Q}=C(\hat{U}_2-\hat{U}_1)=e\hat{\mathcal{N}}_2-e^2N_2\hat{U}_2. \tag{6}$$

Note $\hat{Q}$ is the charge operator of the dipole. Equations (5) and (6) ensure charge conservation. Using $D_i\equiv e^2N_i$ for the density of states, we find that we can write the following:

$$\begin{pmatrix}\hat{U}_1\\ \hat{U}_2\end{pmatrix}=\left(\frac{1}{d}\right)\begin{pmatrix}C+D_2 & C\\ C & C+D_1\end{pmatrix}\begin{pmatrix}e\hat{\mathcal{N}}_1\\ e\hat{\mathcal{N}}_2\end{pmatrix}, \tag{7}$$

where $d=(C+D_1)(C+D_2)-C^2$. The potential fluctuation spectra is given by

$$2\pi S_{U_iU_k}(\omega)\delta(\omega+\omega')$$

$$=1/2\langle \hat{U}_i(\omega)\hat{U}_k(\omega')+\hat{U}_k(\omega')\hat{U}_i(\omega)\rangle, \tag{8}$$

where $\langle\,\rangle$ denotes a quantum statistical average over products of four $\hat{a}$ operators. The potential fluctuation spectra is related to the charge fluctuation spectra via $S_{Q_iQ_k}(\omega)=C^2S_{U_iU_k}(\omega)$. Combining Eqs. (1), (2), (7) and (8), we find the potential fluctuation spectra,[8,10]

$$S_{U_iU_k}(\omega)=\left(\frac{C_\mu^2}{C^2}\right)\frac{1}{N_iN_k}\sum_{\gamma,\delta}\int dE F_{\delta\gamma}\,\mathrm{Tr}[\mathcal{N}^{(i)}_{\delta\gamma}(\mathcal{N}^{(k)}_{\delta\gamma})^\dagger], \tag{9}$$

where $F_{\gamma\delta}=f_\gamma(E)[1-f_\delta(E+\hbar\omega)]+f_\delta(E+\hbar\omega)[1-f_\gamma(E)]$ is a combination of Fermi functions, where f_γ is the Fermi function in contact γ with a chemical potential μ_γ. In the low-frequency limit of interest here, the elements of the density-of-states matrix $\mathcal{N}^{(i)}_{\gamma\delta}$ are specified by Eq. (1). From the above we find that at equilibrium the low-frequency fluctuations of the potential in conductor 1 are given by

$$S_{U_1U_1}=2\left(\frac{C_\mu}{C}\right)^2\left[\left(\frac{C+D_2}{D_2}\right)^2R_q^{(1)}+\left(\frac{C}{D_1}\right)^2R_q^{(2)}\right]kT \tag{10}$$

with $R_q^{(i)}$ determined by Eq. (3). Similar results hold for $S_{U_2U_2}$ and the correlation spectrum $S_{U_1U_2}$. If a bias eV is applied, for instance to the conductor 2, we find the same spectrum as above, except that $R_q^{(2)}kT$ is replaced, to first order in $e|V|$, by $R_v^{(2)}e|V|$ for $e|V|>kT$.

To relate the voltage fluctuation spectra to the dephasing rate, we follow Ref. 4. A carrier in conductor 1 moves in the fluctuating potential U_1. As a consequence, the phase of the carrier is not sharp but on the average determined by

$$\langle\exp[i(\hat{\phi}(t)-\hat{\phi}(0))]\rangle=\left\langle \hat{T}\exp\left[i\frac{e}{\hbar}\int_0^t dt'\hat{U}_1(t')\right]\right\rangle \tag{11}$$

$$=\left\langle \hat{T}\exp\left[i\frac{e}{\hbar}\int d\omega\left(\frac{e^{i\omega t}-1}{i\omega}\right)\hat{U}_1(\omega)\right]\right\rangle. \tag{12}$$

Assuming that the fluctuations are Gaussian, this quantum-mechanical average is given by $\exp(-t/\tau_\phi)$ with

$$\tau_\phi^{-1}=(e^2/2\hbar^2)S_{U_1U_1}. \tag{13}$$

Since the voltage fluctuation spectrum [Eq. (10)] consists of two additive terms, we can decompose the dephasing rate into two contributions $(1/\tau_\phi)_{(11)}$ and $(1/\tau_\phi)_{(12)}$, where the

index pair (ik) indicates that we deal with the dephasing rate in conductor i generated by the presence of conductor k.

Before discussing the results, it is useful to clarify the limit in which we are interested. Typically, the Coulomb charging energy $U=e^2/2C$ is large compared to the level spacing Δ in the conductors of interest. Since $\Delta_i=1/N_i$, this has the consequence that any deviations of the electrochemical capacitance from its geometrical value are very small. We can thus take $C_\mu \approx C$ and $(C+D_2)/D_2 \approx 1$ in Eq. (10). Now we are interested in the dephasing rate $(1/\tau_\phi)_{(12)}$ in conductor 1 due to the presence of conductor 2. From $\tau_\phi^{-1}=(e^2/2\hbar^2)S_{U_1U_1}$ we obtain

$$\left(\frac{1}{\tau_\phi}\right)_{(12)}=\frac{e^2}{\hbar^2}\left(\frac{C}{D_1}\right)^2 R_q^{(2)}kT \tag{14}$$

with R_q given by Eq. (3) if conductor 2 is at equilibrium and

$$\left(\frac{1}{\tau_\phi}\right)_{(12)}=\frac{e^2}{\hbar^2}\left(\frac{C}{D_1}\right)^2 R_\nu^{(2)}e|V| \tag{15}$$

with R_ν given by Eq. (4) if it is in a transport state with $e|V|>kT$. Note that for closed 2D conductors, *e-e* scattering leads to a rate[7] proportional to T^2 whereas for open conductors Eq. (14) predicts a rate that is linear in T.

We now specify that transport in conductor 1 is via a single resonant tunneling state. Thus the relevant density of states in conductor 1 is a Breit-Wigner expression. For simplicity, we assume that we are at resonance and hence $N_1=(2/\pi\Gamma)$, where Γ is the half-width of the resonance. The resistance R_q and R_ν for a QPC in the absence of a magnetic field has already been discussed in Refs. 10 and 20. For a quantum point contact with transmission probability $\mathcal{T}_n$ and reflection probability $\mathcal{R}_n$, the relaxation resistance $R_q^{(2)}$ is

$$R_q^{(2)}=\frac{h}{4e^2}\frac{\Sigma_n\left[(d\phi_n/dE)^2+\frac{1}{4\mathcal{T}_n\mathcal{R}_n}(d\mathcal{T}_n/dE)^2\right]}{[\Sigma_n(d\phi_n/dE)]^2}, \tag{16}$$

where ϕ_n is the phase accumulated by carriers in the nth eigenchannel of the QPC traversing the region in which the potential is not screened. Thus in the one-channel limit, the dephasing caused in conductor 1 due to a QPC at equilibrium is given by

$$(1/\tau_\phi)_{(12)}=(\pi^4\Gamma^2)/(hU^2)kT. \tag{17}$$

Next consider the case where a current is driven through the QPC. The nonequilibrium charge relaxation resistance of a QPC with transmission $\mathcal{T}_n$ and reflection probabilities $\mathcal{R}_n$ in the eigenchannels that follows from Eq. (3) is[10]

$$R_\nu^{(2)}=\frac{h}{e^2}\frac{\sum_n \frac{1}{4\mathcal{T}_n\mathcal{R}_n}\left(\frac{d\mathcal{T}_n}{dE}\right)^2}{\left[\sum_n (d\phi_n/dE)\right]^2}. \tag{18}$$

This result depends on the detailed shape of the QPC even in the single-channel limit. The similarity of this result with the one-channel $(n=1)$ result of Buks *et al.*[2] can be seen by identifying the effective variation $\Delta\mathcal{T}$ of the transmission coefficient with $\Delta\mathcal{T}=(d\mathcal{T}_1/dE)(d\phi_1/dE)^{-1}$. Our result provides a complete specification of the dephasing rate in terms of the scattering matrix and geometrical capacitances. We take screening into account and thus can clarify the dependence on the quantum dot properties (via Γ) and the capacitive coupling constant C. R_ν has been evaluated for zero magnetic field in Ref. 10. For the high magnetic-field case, the nonequilibrium charge relaxation resistance for a QPC is shown in Fig. 2. We have used a saddle-point[9] potential $V(x,y)=V_0-(1/2)m\omega_x^2x^2+(1/2)m\omega_y^2y^2$ and, as in Ref. 10 have evaluated the density of states semiclassically. Note the strong suppression of the dephasing rate at threshold of the opening of a new channel. Indeed the experiment of Buks *et al.*[2] shows a *double peak* structure in the visibility of the Aharonov-Bohm oscillations.

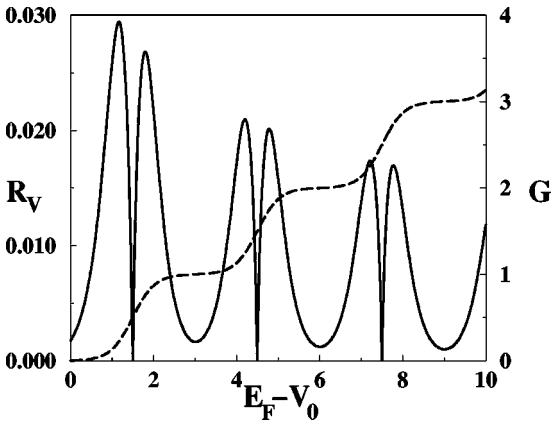

FIG. 2. R_ν (solid line) in units of h/e^2 and G (dashed line) in units of e^2/h as a function of E_F-V_0 for a saddle QPC with $\omega_x/\omega_y=1$ and $\omega_c/\omega_x=4$, where ω_c is the cyclotron frequency $(\omega_c=|eB|/mc)$.

So far we always assumed that the QPC and the dot are located such that the total charge piled up in the QPC matters. Thus the above results involve only the energy derivatives of the scattering matrix of the QPC and the dot. Consider now the situation *B* shown in Fig. 1, where the quantum dot is located away from the QPC downstream along an edge. Clearly, now the predominant interaction effect is due to the charge fluctuations on the edge state adjacent to the quantum dot. The charge that counts is that in a region Ω_B.

Very importantly, the approach introduced above can now be extended to this more general situation. To generalize the above results, we need to find the charge and its fluctuations in region Ω_B. This can be accomplished by taking the derivative of the scattering matrix with respect to a small potential perturbation that extends over the region of interest. Thus in general we arrive at a density-of-states matrix by replacing the energy derivative in Eq. (1) by a functional derivative,[21]

$$d/dE \rightarrow -\int_{V_\Omega} d^3\mathbf{r}\frac{\partial}{\partial eU(\mathbf{r})}. \tag{19}$$

Let us apply this prescription to case *B* of Fig. 1.

First, let us establish the scattering matrix for this system. For the QPC with transmission probability $\mathcal{T}=1-\mathcal{R}$, we chose $r\equiv s_{11}=s_{22}=-i\mathcal{R}^{1/2}$ and $t\equiv s_{21}=s_{12}=\mathcal{T}^{1/2}$. A carrier

traversing the region Ω_B adjacent to the quantum dot acquires a phase $\phi_2(U_2)$ where U_2 characterizes the potential of the edge state in Ω_B. We assume that only the charge pile up in the region Ω_B matters, and consequently, all additional phases in the scattering problem are here without relevance. The total scattering matrix of the QPC and the traversal of region Ω_B is then simply $s_{11}=r$, $s_{12}=t$, $s_{21}=t\exp(i\phi_2)$, and $s_{22}=r\exp(i\phi_2)$.

Consider next the charge operator. We have to evaluate the variation of the scattering matrix with respect to the potential U_2. Only s_{12} and s_{22} depend on this potential. We find $ds_{12}/edU_2=(ds_{12}/d\phi_2)(d\phi_2/edU_2)$. But $(d\phi_2/edU_2)=-2\pi N_2$, where N_2 is the density of states of the edge state in region Ω_B of conductor 2. Simple algebra gives $\mathcal{N}_{11}^{(2)}=\mathcal{T}N_2$,

$$\mathcal{N}_{21}^{(2)}=\mathcal{N}_{12}^{(2)}*=r^*tN_2, \tag{20}$$

and $\mathcal{N}_{22}^{(2)}=\mathcal{R}N_2$. At equilibrium we find $R_q^{(2)}=h/2e^2$ as is typical for an edge state that is perfectly connected to a reservoir.[22] The nonequilibrium resistance is

$$R_\nu^{(2)}=(h/e^2)\mathcal{TR}. \tag{21}$$

Note that in the one-channel case both R_q and R_ν are independent of the density of states N_2. The additional dephasing rate generated by the edge at equilibrium in the quantum dot at resonance is

$$(1/\tau_\phi)_{(12)}=\left(\frac{\pi^4\Gamma^2}{2hU^2}\right)kT. \tag{22}$$

Note that this rate depends on the edge state only through its geometrical capacitance. In the nonequilibrium case, the additional dephasing rate caused by the charge fluctuations on the edge state is

$$(1/\tau_\phi)_{(12)}=\left(\frac{\pi^4\Gamma^2}{hU^2}\right)\mathcal{TR}e|V|. \tag{23}$$

A rate proportional to $\mathcal{TR}$ is also obtained by Buks *et al.*[15] Of interest is the effect of screening: While in the one-channel case, the rate depends on the capacitance of the edge channel only, such a universal result does not apply as soon as additional edge states are present. Thus consider an additional edge state that is transmitted with probability 1. It generates no additional noise and leaves the dc- shot noise invariant.[9] But the additional edge channel contributes to screening. If we take the two edge channels to be close together in the region Ω_B, both edge channels will see the same potential U_2. Now the total density of states of the two edge channels in region Ω_B has a contribution from both the perfectly transmitted edge state (1) and edge state (2) $N_2=N_{21}+N_{22}$. As a consequence, the dephasing rate is now reduced and given by

$$\left(\frac{1}{\tau_\phi}\right)_{(12)}=\frac{\pi^4\Gamma^2}{hU^2}\left(\frac{N_{22}}{N_{21}+N_{22}}\right)^2\mathcal{TR}e|V|. \tag{24}$$

Equation (24) is valid if there is no population equilibration among the two edge channels between the QPC and the dot. If there is equilibration (which can be achieved by placing a voltage probe between the QPC and the dot[15]), we will show elsewhere[23] that the dephasing rate becomes

$$(1/\tau_\phi)_{(12)}=\left(\frac{\pi^4\Gamma^2}{hU^2\nu^2}\right)\mathcal{TR}e|V|, \tag{25}$$

where ν is the number of edge states. In particular, if only one edge state is present, the dephasing rate is unaffected by the presence of a phase randomizing reservoir. This result provides a simple test of the theory presented here.

If the magnetic polarity is reversed, the additional dephasing rate is only due to equilibrium fluctuations, independent of whether or not a bias is applied to the QPC.

In this work we have presented a discussion of the dephasing in Coulomb coupled mesoscopic conductors, which is based on the fluctuations of the dipolar charge that is generated by the long-range Coulomb interaction. This dipole is associated with a capacitance and its dissipative behavior is characterized by charge relaxation resistances R_q and R_ν. These resistances are determined by the low-frequency collective modes of the Coulomb coupled conductors.

Note added. After this paper was submitted, Levinson[24] also addressed the problem of charge fluctuations in edge states and dephasing. In contrast to our work, no attempt to introduce a self-consistent treatment of charge fluctuations is made.

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