Critical behavior in systems with a long-range correlated frozen-in random field

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The critical behavior of inhomogeneous systems with a frozen-in random field having a nonlocal correlation function decaying according to the power law $|\mathbf{x}-\mathbf{x}'|^{-b}$ is considered. The problem is studied within the context of a model which partially considers interactions of fluctuations and belongs in the same universality class as the spherical model. Depending on the relationship between the power b and the space dimensionality d, a new critical behavior arises.

Critical phenomena at phase transitions are characterized by a variety of different universality classes. Some of the factors that determine the universality class of a system are the dimension d of space, the number m of the components of the order parameter, the symmetry of the Hamiltonian, and the range of interactions among the microscopic degrees of freedom.[1–4] It is well known, for example, how the critical behavior of a system, such as a ferromagnet, is altered substantially when one considers the effect of the long-range interaction due to pairs of magnetic dipoles in addition to the short-range spin-spin interactions.[5–7] In general, the behavior of ferromagnets with this type of long-range interaction differs from that of ferromagnets with only the short-range interactions, according to the number of the components of the order parameter. For example, the critical behavior of a uniaxial Ising ferromagnet in d dimensions with dipole-dipole interactions belongs in the same universality class as a $(d+1)$-dimensional Ising ferromagnet with only short-range interactions.[5]

Another situation where long-range interactions seem to alter the critical behavior is found in random-exchange models. Initial experimental data suggested that the clean-cut phase transitions observed in pure materials were somehow broadened and theoretical considerations[2,8] showed that this need not be the case if the random exchange terms that describe the interaction among impurities are not long range. The fact that the equilibrium behavior of random-exchange systems is qualitatively the same as that of pure systems has been experimentally verified[9] and the earlier smearing out of the transition may indeed be due to macroscopic inhomogeneities in the system.

In another case, random-field d-dimensional systems described by an m-component order parameter $(m\neq 1)$ behave as $(d-2)$-dimensional pure systems as long as the random field is short-ranged correlated.[10–14] For the random-field Ising model, the dimensional reduction does not hold[10,15–17] and its critical behavior depends on the kind of distribution of the random field.[18] Specifically, in four[19] and three[20] dimensions a clearly different critical behavior was found when the random field has a Gaussian or a bimodal distribution. In fact, even for m-component magnets with $m\geqslant 2$ it has been argued that dimensional reduction is likely not valid, at least in $4+\varepsilon$ dimensions, due to weak, short-range random fields and random higher-rank anisotropies.[21]

In this study, it will be shown that in models belonging in the spherical universality class, unlike the case of a system with a random field having short-range correlation, a random field with long-range correlation will significantly alter its critical behavior. Part of the behavior will be the breakdown of dimensional reduction due to the long-range nature of the random field. Long-range random systems are an interesting problem to study theoretically since in experimental systems it is never clear what type of correlations the random field has. An experimental example is a random-field magnet of the type of a dilute Ising antiferromagnet such as $Fe_xZn_{1-x}F_2$.[22] In such a case the distribution and correlation of the random field is not well defined and in general both short- and long-range random fields might influence the critical behavior.[23] Therefore, if a theoretical consideration of a long-range random system derives a distinct behavior from a short-range random system, this perhaps could be used to determine the nature of the random correlations in an experimental system.

The study of such systems will be done by considering a model with reduced interactions of fluctuations, allowing for the exact calculation of the partition function. The model is a considerable improvement over mean-field theory, and has been previously successfully applied to a number of different systems.[14,24–29] The results obtained by the model are in qualitative agreement with those obtained by renormalization group (RG) theory, whenever there are results from both approaches for comparison. Furthermore, through the model, unlike in RG theory, fluctuation interactions can be controlled. Specifically, they can be easily suppressed, thus allowing one to see the crossover to mean-field behavior. As it will be seen below, after a main approximation is applied which will make the model exactly solvable, the model will belong in the spherical universality class.

The system of interest has the Ginzburg-Landau-Wilson functional with a scalar order parameter $S(\mathbf{x})$

$$F[S(\mathbf{x})]=\tfrac{1}{2}\int d^dx[\tau S^2(\mathbf{x})+c(\nabla S(\mathbf{x}))^2+uS^4(\mathbf{x}) -h(\mathbf{x})S(\mathbf{x})-hS(\mathbf{x})], \quad (1)$$

where $\tau=(T-T_c)/T_c$, T_c is a trial critical temperature for the order parameter, h is a constant external conjugate field, and $h(\mathbf{x})$ is a random quenched field. To find the free energy of the quenched system, one must average over all the free energies corresponding to all possible random configurations of the random field $\{h(\mathbf{x})\}$. Therefore,

$$-F=\int [Dh(\mathbf{x})]P\{h(\mathbf{x})\}\ln Z\{h(\mathbf{x})\},$$

where the probability distribution functional $P\{h(\mathbf{x})\}$ for a given configuration of random fields is free of choice and its mathematical characteristics will be defined at a later point.

To proceed with the averaging over all random-field configurations, the replica method[30,31] is used by defining an n-component order parameter $\varphi(\mathbf{x})$. Then,

$$-F=\frac{\partial}{\partial n}\left[\int [D^n\varphi(\mathbf{x})]\exp(-F_{eff}[\varphi(\mathbf{x})])\right]_{n=0}$$

with $F_{eff}[\varphi(\mathbf{x})]$ an effective free-energy functional resulting from the replication of the partition function being given by

$$F_{eff}[\varphi(\mathbf{x})]=\frac{1}{2}\int_{-\infty}^{\infty}d^dx\left[\tau|\varphi(\mathbf{x})|^2+c(\nabla\varphi(\mathbf{x}))^2+\sum_{i=1}^{n}(u\varphi_i^4(\mathbf{x})-h\varphi_i(\mathbf{x}))\right]-G[\varphi(\mathbf{x})] \quad (2)$$

with

$$G[\varphi(\mathbf{x})]=\ln\int [Dh(\mathbf{x})]P\{h(\mathbf{x})\}\times\exp\left(\frac{1}{2}\int d^dx\sum_{i=1}^{n}h(\mathbf{x})\varphi_i(\mathbf{x})\right).$$

It is assumed that a field-field correlation function obeys

$$\langle h(\mathbf{x})h(\mathbf{x}')\rangle=f(\mathbf{x}-\mathbf{x}'), \quad (3)$$

where the averaging is with respect to the distribution $P\{h(\mathbf{x})\}$ and for now $f(\mathbf{x}-\mathbf{x}')$ is still a general function whose choice will be such that the randomness of the system will have characteristics of both short- and long-range correlations. Only for short-range random correlations can $P\{h(\mathbf{x})\}$ in Eq. (3) be decomposed into a product of independent probabilities at the various locations in the system. The effect of only short-range correlations has been studied previously by both RG theory[10–13] and using the exact model.[14] Both approaches produce the same qualitative picture of critical behavior, that is, the dimensional reduction by 2. In this work, the randomness has both short-range and long-range correlations. Therefore the probability function $P\{h(\mathbf{x})\}$ in Eq. (3) cannot be broken down into a product of uncorrelated probabilities in the different points in space inside the system. Then

$$f(\mathbf{x}-\mathbf{x}')=B\delta(\mathbf{x}-\mathbf{x}')+\frac{B_1}{|\mathbf{x}-\mathbf{x}'|^b},$$

where the δ function defines the short-range correlations of the random field. The second term (which is true for $\mathbf{x}\neq\mathbf{x}'$) is a nonlocal function stating that there are isotropic long-range correlations in the quenched impurities. These correlations are chosen to have a power law behavior, with b set as an arbitrary power. The constants B and B_1 are a measure of the strengths of the short- and long-range nature of the random field, respectively. It will be shown below that the value of the exponent b, in relation to the space dimensionality d, will play a crucial role in the critical behavior of the quenched system.

The Fourier transform of the impurity term for small $\mathbf{q}$ is

$$\int_{-\infty}^{\infty}d^dxd^dx'f(\mathbf{x}-\mathbf{x}')=\int_{\mathbf{q}}d^dqB+\int_{\mathbf{q}\neq\mathbf{0}}d^dqB_1q^{(b-d)}, \quad (4)$$

where constants resulting from integrations of d-dimensional Fourier transforms have been absorbed in the strengths of the short- (B) and long-range (B_1) correlations. Equation (4) will be used to express the partition function of the system in momentum space. Note in Eq. (4) that if $b>d$, then in the long-wavelength limit, ($q\ll 1$), the long-range term $\int_{\mathbf{q}\neq 0}d^dqB_1q^{(b-d)}$ vanishes and therefore the random field has only short-range correlations. This means that, in the case of $b>d$, long-range random correlations are irrelevant. Specifically, a system with long-range disorder that falls off with distance faster than $|\mathbf{x}-\mathbf{x}'|^{-d}$ has the same critical behavior as a system with only short-range random correlations. On the other hand, an entirely different behavior is exhibited for the case where $b<d$ as it will be discussed below.

The exact model used here is one which allows for the reduction of all quartic terms in functional (2) as follows:

$$\int d^dx\varphi_i^4(\mathbf{x})\rightarrow\frac{1}{V}a_i^2[\varphi_i(\mathbf{x})], \quad a_i[\varphi_i(\mathbf{x})]\equiv\int d^dx\varphi_i^2(\mathbf{x}), \quad (5)$$

where V is the volume of the system. Such a reduction, proposed by Schneider *et al.*,[32] causes the model to take in to account only the interaction of fluctuations with equal and antiparallel momenta. This can be seen if one rewrites Eq. (2) in the momentum representation. Then reduction (5) becomes equivalent to splitting the δ function, which provides momentum conservation, into the product of two δ-functions:

$$\delta(\mathbf{q}_1+\mathbf{q}_2+\mathbf{q}_3+\mathbf{q}_4)\rightarrow\delta(\mathbf{q}_1+\mathbf{q}_2)\delta(\mathbf{q}_3+\mathbf{q}_4).$$

As a result of approximation (5) the exact model now falls in the same universality class as the spherical model.[33]

Using a transformation analogous to that of Hubbard-Stratonovich, the Boltzmann factor in the partition function becomes bilinear with respect to $\varphi_i(\mathbf{x})$. The consequence of this relative simplification is the introduction of $2n$ new variables, which we call x_i and y_i, with the subscript i ranging from 1 to n. Explicitly, this is done when the Fourier transformation

$$\exp\left[-\frac{V}{2}K\left(\frac{a_i[\varphi_i]}{V}\right)\right]=\frac{1}{(2\pi)^i}\int Dx_iDy_i\exp\left(-\frac{V}{2}K\left(\frac{x_i}{V}\right)+i\sum_{i=1}^{n}(x_iy_i-y_ia_i)\right)$$

is applied to an arbitrary function $K(a_i/V)$. For functional (2), $K(a_i/V)$ is

$$K\left(\frac{a_i}{V}\right)=\sum_{i=1}^{n}\left[\frac{\tau a_i}{V}+\frac{u a_i^2}{V^2}\right].$$

The equation representing the effective free-energy functional (2) then becomes

$$F_{eff}[\varphi(\mathbf{x})]=\frac{V}{2}K\left(\frac{a_i}{V}\right)+\frac{1}{2}\int_{-\infty}^{\infty}d^dx\Bigg[c(\nabla\varphi(\mathbf{x}))^2$$

$$-\sum_{i=1}^{n}h\varphi_i(\mathbf{x})-\frac{1}{4}\int_{-\infty}^{\infty}d^dx'$$

$$\times\sum_{i=1}^{n}\sum_{j=1}^{n}f(\mathbf{x}-\mathbf{x}')\varphi_i(\mathbf{x})\varphi_j(\mathbf{x}')\Bigg]$$

and after the use of Eq. (4), the partition function in momentum space takes the form

$$Z=\int D\varphi_{i\mathbf{q}}Dx_iDy_i\exp\Bigg[-\frac{V}{2}K(x_i,\ldots,x_n)$$

$$+\frac{V}{2}\sum_{i=1}^{n}x_iy_i-\frac{1}{2}\sum_{i=1,\mathbf{q}}^{n}y_i\varphi_{i\mathbf{q}}^2-\frac{c}{2}\sum_{i=1,\mathbf{q}}^{n}q^2\varphi_{i\mathbf{q}}^2$$

$$+\frac{h\sqrt{V}}{2}\sum_{i=1}^{n}\varphi_{i0}+\frac{B}{2}\sum_{\mathbf{q}\neq\mathbf{0}}\left(\sum_{i=1}^{n}\varphi_{i\mathbf{q}}\right)\left(\sum_{j=1}^{n}\varphi_{j-\mathbf{q}}\right)$$

$$+\frac{B_1}{2}\sum_{\mathbf{q}\neq\mathbf{0}}q^{(b-d)}\left(\sum_{i=1}^{n}\varphi_{i\mathbf{q}}\right)\left(\sum_{j=1}^{n}\varphi_{j-\mathbf{q}}\right)\Bigg], \tag{6}$$

where the following substitutions, $x_i/V\rightarrow x_i$, $2iy_i\rightarrow y_i$, and $\varphi_{i\mathbf{q}}/\sqrt{V}\rightarrow\varphi_{i\mathbf{q}}$ have been made.

In order to calculate functional integrals in Eq. (6) the expression in the exponent must be diagonalized with respect to the n components of the order parameter. Note that the nondiagonal terms in the free energy are due to the impurity terms. After diagonalization, it becomes required that $y_i=y_j\equiv y$, since only this choice reproduces the pure φ^4 model upon suppression of the random field. This is so because in the limit of $B\rightarrow 0$, $B_1\rightarrow 0$, the degeneracy of the eigenvalues of every other choice does not reduce to n fold as expected from considerations of the pure φ^4 model treated within the context of the replica method. In the random-field case, there exist two $n\times n$ matrices of interest. One consists of Fourier components with $\mathbf{q}\neq\mathbf{0}$. The other one consists of Fourier components with $\mathbf{q}=\mathbf{0}$. Altogether, there are only four distinct eigenvalues. Two of these correspond to the $n\times n$ matrix with Fourier components having $\mathbf{q}\neq\mathbf{0}$:

$$\lambda_1=-\tfrac{1}{2}(y+cq^2)$$

$$\lambda_2=-\tfrac{1}{2}(y+cq^2-nB-nB_1q^{(b-d)}).$$

The other two correspond to the $n\times n$ matrix with Fourier components having $\mathbf{q}=\mathbf{0}$:

$$\lambda_{10}=-\frac{y}{2},$$

$$\lambda_{20}=-\tfrac{1}{2}(y-nB),$$

where λ_1 and λ_{10} are $(n-1)$-fold degenerate. These eigenvalues can be used to diagonalize the two $n\times n$ matrices with respect to $\varphi_{i\mathbf{q}\neq\mathbf{0}}$ and $\varphi_{i\mathbf{q}=\mathbf{0}}$. Consequently, all functional integrals in Eq. (6) may be calculated to give

$$Z=\int Dx_idy\exp\Bigg(-\frac{V}{2}K(x_i,\ldots,x_n)$$

$$+\frac{V}{2}y\sum_{i=1}^{n}x_i+\frac{(1-n)}{2}\sum_{\mathbf{q}=\mathbf{0}}\ln(y+cq^2)$$

$$-\frac{1}{2}\sum_{\mathbf{q}\neq\mathbf{0}}\ln(y+cq^2-nB-nB_1q^{(b-d)})$$

$$-\frac{1}{2}\ln(y-nB)+\frac{nVh^2}{8(y-nB)}\Bigg).$$

Calculating the summations over momentum, it can be shown that

$$\sum_{\mathbf{q}=\mathbf{0}}\ln(y+cq^2)=V(g_d(y)+y\Theta_d(\Lambda))$$

for which

$$g_d(y)=\frac{S_d}{(2\pi)^d}\begin{cases}\dfrac{\pi y^{d/2}}{dc^{d/2}\sin(\pi d/2)}\equiv\kappa_1(c)y^{d/2} & d\neq\text{even}\\ -\dfrac{1}{d}\left(\dfrac{y}{c}\right)^{d/2}\ln y\equiv\mu_1(c)y^{d/2}\ln y & d=\text{even,}\end{cases}$$

$$\Theta_d(\Lambda)=\frac{S_d}{(2\pi)^d}\begin{cases}\dfrac{\Lambda^{(d-2)}}{c(d-2)} & d\neq 2\\ \dfrac{\ln(c\Lambda^2)}{2c} & d=2,\end{cases}$$

where S_d is the surface area of a d-dimensional unit-radius sphere and Λ is a momentum cutoff.

In order to calculate the summation involving the random-field strengths B and B_1, first, the logarithm must be expanded with respect to small B and B_1. For B the resulting sum is

$$B\sum_{\mathbf{q}=\mathbf{0}}\frac{1}{y+cq^2}=VB(g_{Bd}(y)+y\Theta_{Bd}(\Lambda))$$

for which

$$g_{Bd}(y)=\frac{S_d}{(2\pi)^d}\begin{cases}\dfrac{(-1)^{(d-1)/2}}{c^{d/2}}\dfrac{\pi}{2}y^{(d-2)/2}\equiv\kappa_2(c)y^{(d-2)/2} & d\neq \text{ even}\\ \dfrac{(-1)^{d/2}}{2c^{d/2}}y^{(d-2)/2}\ln y\equiv\mu_2(c)y^{(d-2)/2}\ln y & d=\text{even},\end{cases}$$

$$\Theta_{Bd}(\Lambda)=\frac{S_d}{(2\pi)^d}\begin{cases}\dfrac{-\Lambda^{(d-4)}}{c(d-4)} & d\neq\text{even and}\quad d\geqslant 5\\ \begin{cases}0 & d=2\\ -\dfrac{\ln(c\Lambda^2)}{2c^2} & d=4\\ -\dfrac{\Lambda^{(d-4)}}{(d-4)c^2} & d=6,8,\ldots.\end{cases}\end{cases}$$

For B_1, the resulting sum is

$$B_1\sum_{\mathbf{q}=\mathbf{0}}\frac{q^{(b-d)}}{y+cq^2}=VB_1(g_{B_1b}(y)+y\Theta_{B_1b}(\Lambda)),\tag{7}$$

for which

$$g_{B_1b}(y)=\frac{S_d}{(2\pi)^d}\begin{cases}\dfrac{(-1)^{(b-1)/2}}{c^{b/2}}\dfrac{\pi}{2}y^{(b-2)/2}\equiv\kappa_3(c)y^{(b-2)/2} & b\neq\text{even}\\ \dfrac{(-1)^{b/2}}{2c^{b/2}}y^{(b-2)/2}\ln y\equiv\mu_3(c)y^{(b-2)/2}\ln y & b=\text{even},\end{cases}$$

$$\Theta_{B_1b}(\Lambda)=\frac{S_d}{(2\pi)^d}\begin{cases}\dfrac{-\Lambda^{(b-4)}}{c(b-4)} & b\neq\text{even and}\quad b\geqslant 5\\ \begin{cases}0 & b=2\\ -\dfrac{\ln(c\Lambda^2)}{2c^2} & b=4\\ -\dfrac{\Lambda^{b-4}}{(b-4)c^2} & b=6,8,\ldots.\end{cases}\end{cases}$$

Notice how the summation (7) behaves as a sum in b dimensions. This is the reason for an interesting critical behavior of the system with long-range correlated randomness.

Functions $\Theta_d(\Lambda)$, $\Theta_{Bd}(\Lambda)$, and $\Theta_{B_1b}(\Lambda)$ diverge when $\Lambda\to\infty$ and $d\geqslant 2$, $d\geqslant 4$, $b\geqslant 4$, respectively. However, critical asymptotics do not depend on a particular momentum cutoff and such divergencies are absorbed by renormalizing x_i and τ. This is done by defining $\Theta\equiv\Theta_d(\Lambda)-B\Theta_{Bd}(\Lambda)-B_1\Theta_{B_1b}(\Lambda)$, which will be used for the renormalization of both $x_i\to x_i+\Theta$ and $\tau x_i+2u\Theta x_i\equiv tx_i$. The partition function may now be written

$$Z=\int Dx_i dy\,\exp\left(-\frac{V}{2}F(x_i,y,h)\right)$$

with

$$F(x_i,y,h)=\sum_{i=1}^{n}(tx_i+ux_i^2-yx_i)+ng_d(y)-nBg_{Bd}(y)-nB_1g_{B_1b}(y)-\frac{nh^2}{y-nB},\tag{8}$$

where $h/2\to h$. In the thermodynamic limit $V\to\infty$, the calculation of the partition function becomes exact and can be performed using the steepest descent method. The replicated equilibrium free energy can be calculated by solving for x_i and y in the saddle point equations $\partial F/\partial x_i=0$ and $\partial F/\partial y=0$. It is easy to see that $\partial F/\partial x_i=0$ implies all x_i are equal to one another and therefore $x_i\equiv x$. Taking advantage of this, Eq. (8) for $F(x_i,y,h)$ is simplified and becomes $F(x,y,h)$. From $\partial F/\partial y=0$, it can be shown that, up to order n, the resulting equation for $y(h)$ is independent of n. Hence, the disorder-averaged value of the equilibrium free energy becomes

$$F(h)=\lim_{n\to 0}\frac{F(y(h),h)}{n}=tx+ux^2-yx+g_d(y)$$

$$-Bg_{Bd}(y)-B_1g_{B_1b}(y)-\frac{h^2}{y} \tag{9}$$

and the expression for the equilibrium, averaged order parameter φ is given by

$$\varphi=\lim_{n\to 0}\frac{1}{n}\left[\frac{-\partial F(y,h)}{\partial h}\right]_{y=y(h)}=\frac{h}{y(h)}. \tag{10}$$

Using $\partial F(h)/\partial x=0$, one can eliminate x from Eq. (9). Using $\partial F(h)/\partial y=0$ and the expression for φ [Eq. (10)] an equation for the order parameter is derived. This parameter depends, among other things, on the constant conjugate field h, the dimensionality d of space, the power of the long-range correlations of the random field b, and the scale of microscopic interactions of fluctuations c. One can write these expressions for φ for all possible values of d and b, including nonintegers, as shown below.

$$\frac{d}{2}\kappa_1(c)\left(\frac{h}{\varphi}\right)^{(d-2)/2}-B\kappa_2(c)\frac{(d-2)}{2}\left(\frac{h}{\varphi}\right)^{(d-4)/2}$$

$$-B_1\kappa_3(c)\frac{(b-2)}{2}\left(\frac{h}{\varphi}\right)^{(b-4)/2}+\varphi^2+\frac{t}{2u}-\frac{1}{2u}\frac{h}{\varphi}$$

$$=0\quad\begin{cases}d=\text{noneven}\\ b=\text{noneven},\end{cases} \tag{11}$$

$$\mu_1(c)\left(\frac{d}{2}\left(\frac{h}{\varphi}\right)^{(d-2)/2}\ln\left(\frac{h}{\varphi}\right)+\left(\frac{h}{\varphi}\right)^{(d-2)/2}\right)$$

$$-B\mu_2(c)\left(\frac{(d-2)}{2}\left(\frac{h}{\varphi}\right)^{(d-4)/2}\ln\left(\frac{h}{\varphi}\right)+\left(\frac{h}{\varphi}\right)^{(d-4)/2}\right)$$

$$-B_1\mu_3(c)\left(\frac{(b-2)}{2}\left(\frac{h}{\varphi}\right)^{(b-4)/2}\ln\left(\frac{h}{\varphi}\right)+\left(\frac{h}{\varphi}\right)^{(b-4)/2}\right)$$

$$+\varphi^2+\frac{t}{2u}-\frac{1}{2u}\frac{h}{\varphi}=0\quad\begin{cases}d=\text{even}\\ b=\text{even},\end{cases} \tag{12}$$

$$\frac{d}{2}\kappa_1(c)\left(\frac{h}{\varphi}\right)^{(d-2)/2}-B\kappa_2(c)\frac{(d-2)}{2}\left(\frac{h}{\varphi}\right)^{(d-4)/2}$$

$$-B_1\mu_3(c)\left(\frac{(b-2)}{2}\left(\frac{h}{\varphi}\right)^{(b-4)/2}\ln\left(\frac{h}{\varphi}\right)+\left(\frac{h}{\varphi}\right)^{(b-4)/2}\right)$$

$$+\varphi^2+\frac{t}{2u}-\frac{1}{2u}\frac{h}{\varphi}=0\quad\begin{cases}d=\text{noneven}\\ b=\text{even},\end{cases} \tag{13}$$

$$\mu_1(c)\left(\frac{d}{2}\left(\frac{h}{\varphi}\right)^{(d-2)/2}\ln\left(\frac{h}{\varphi}\right)+\left(\frac{h}{\varphi}\right)^{(d-2)/2}\right)$$

$$-B\mu_2(c)\left(\frac{(d-2)}{2}\left(\frac{h}{\varphi}\right)^{(d-4)/2}\ln\left(\frac{h}{\varphi}\right)+\left(\frac{h}{\varphi}\right)^{(d-4)/2}\right)$$

$$-B_1\kappa_3(c)\frac{(b-2)}{2}\left(\frac{h}{\varphi}\right)^{(b-4)/2}+\varphi^2+\frac{t}{2u}-\frac{1}{2u}\frac{h}{\varphi}$$

$$=0\quad\begin{cases}d=\text{even}\\ b=\text{noneven}.\end{cases} \tag{14}$$

As $h\to 0$, whether φ has a solution and whether the behavior of the system will be critical or mean field, depends on the values of the space dimensionality d and the power b, as well as the scale of microscopic interactions c. As $c\to\infty$, it is easy to see that all results reduce to those of mean-field theory. In the above four equations, it is seen that in the limit $B\to 0$, $B_1\to 0$, as expected, the resulting equations reduce to those of the pure φ^4 model.

The case of only short-range correlated impurities, that is, $B_1=0$ and $B\neq 0$ is now reviewed. First, it is noted that Eqs. (13) and (14) are identical to Eqs. (11) and (12) for noneven and even d respectively. Since $B_1=0$, the critical behavior of the system will basically depend on the space dimensionality d and the scale of microscopic interactions c. Applying the exact model to this case,[14] one finds the dimensional reduction by 2, a result which is in agreement with RG theory analysis.[10–13] From Eqs. (11) and (12) it can be deduced that, as $h\to 0$, there is no solution for φ when $d\leqslant 4$. For $4<d<6$, the dominating terms for small h are the φ^2, the $t/2u$ and the B terms. Therefore Eq. (11) gives the same critical asymptotics as a pure model with the lower dimension $2<d'<4$, where $d'\equiv d-2$. Under such conditions, the critical exponents β and δ take the values $\beta=1/2$ and $\delta=(d'+2)/(d'-2)$. When $d>6$ and $h\to 0$ the last three terms in Eqs. (11) and (12) dominate, and the critical asymptotics are those obtained by the mean-field theory. Finally, for $d=6$, Eq. (12) provides logarithmic corrections in the behavior of the order parameter, $\varphi\propto(h\ln h)^{1/3}$. So, the model explicitly demonstrates the dimensional crossover in the presence of short-range, quenched random fields. It may seem contradictory that the model with the one-component order parameter has a lower critical dimension $d_c=4$. Indeed, the functional (1) corresponds to the random-field Ising model which has $d_c=2$. However, after the reduction (5) the model belongs to the spherical model universality class and, therefore, has the symmetry $O(N=\infty)$.[33]

The picture is substantially altered when long-range correlated random impurities are taken into consideration through the B_1 term. First, it is derived from Eqs. (11), (12), (13), and (14) that regardless of what d is (even for $d>4$, for which a phase transition occurs when only short-range correlations are assumed), when the value of the power b, that describes the long-range correlated impurities, obeys $b\leqslant 4$, then the Eqs. (11), (12), (13), and (14) have no solution in the limit $h\to 0$. This is seen from the divergence of the B_1 term. It seems that the long-range nature of impurities ''overpowers'' the short range for these values of b, and the system prefers to follow the random distribution of the random field, leading to a disordered phase. On the other hand, the long-range nature of the impurities has no effect on the system as long as $b>d$. This means that in spite of the presence of long-range interactions, the system will have the critical behavior dictated by the short-range correlations of the random field. This is derived when, in Eqs. (11), (12), (13), and (14), the limit of $h\to 0$ is taken, as well as by realizing that the B term dominates the B_1 term. This result was anticipated earlier when the long-wavelength limit in Eq. (4) was considered and it was seen how the long-range term B_1 vanishes for $b>d$. For $b=d$, the behavior is qualitatively the same as in the case without the long-range term, with both the short- and long-range terms contributing equally. Basically, these two different nature terms are indistinguishable for $b=d$.

Things are different for $b>4$ and as long as $b<d$. In this case, the various combinations between the values that b and d can take, are examined separately in the following situations: (1) when $4<d<6$ (for which the system exhibits the critical behavior of dimensional reduction as $B_1\to 0$, as discussed above), along with the possibilities for b obeying $4<b<6$. (2) When $d=6$ (for which the system exhibits logarithmic behavior as $B_1\to 0$, as discussed above), along with $4<b<6$. And finally, (3) when $d>6$ (for which the system exhibits mean-field behavior as $B_1\to 0$, as discussed above), along with $4<b<6$, $b=6$, or $b>6$.

As stated above, case (1) is one with $4<d<6$, $4<b<6$, and $b<d$. Then, in the limit $h\to 0$, from Eq. (11) it is derived that the B_1 term dominates the B term. The surviving terms in Eq. (11) are

$$\frac{t}{2u}-B_1\kappa_3(c)\frac{(b-2)}{2}\left(\frac{h}{\varphi}\right)^{(b-4/2)}+\varphi^2=0.$$

It appears as if the system behaves like a b-dimensional one, regardless of the fact that the actual dimension is $4<d<6$. This was anticipated when it was realized that the sum (7) behaves as one of effective space dimensionality b. The critical exponent δ will therefore have the same value as in the case without long-range correlations, but with d replaced by b. The ''dimensional reduction'' will really occur with respect to b, that is $d'=b-2$ and $\delta=(d'+2)/(d'-2)$. It means that a d dimensional $(4<d<6)$ long-range randomly correlated system with $4<b<6$ and $b<d$, behaves as one of effective dimensionality b and has the same critical behavior as a pure d'-dimensional system where $d'=b-2$.

Case (2) deals with $4<b<6$ and $d=6$. In the case of these values of b and d, the short-range correlations are proven irrelevant in the system. In the limit $h\to 0$, it is shown via Eq. (14) that the B_1 term is greater than the B term, and the system has the interesting long-range critical behavior described above. The same result is derived from Eqs. (11) and (14) even for $d>6$, as in case (3). Furthermore, in case (3), for which $d>6$, if $b=6$, the system has a logarithmic behavior, a result strictly due to the long-range correlations, despite the fact that for dimensionality greater than 6, a system with only short-range interactions, had the usual mean-field behavior. The mean-field behavior is restored only if both b and d are greater than 6.

In conclusion, it has been shown that systems belonging in the same universality class as the spherical model, described by long-range correlated random quenched impurities, decaying according to the power law $|\mathbf{x}-\mathbf{x}'|^{-b}$, generate an interesting critical behavior at phase transitions, with b playing the role of an effective dimensionality, as long as $b<d$. Specifically, the long-range random field is the reason for the breakdown of dimensional reduction. As in the case with only short-range correlated impurities, upon suppression of fluctuations with the limit $c\to\infty$, all results reduce to those of mean-field theory, regardless of dimensionality or the presence of impurities.

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