Electron-phonon interaction in a very low mobility GaAs/$Ga_{1-x}Al_xAs$ δ-doped gated quantum well

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(Received 12 April 1999)

The energy relaxation rate for hot electrons in a gated GaAs/$Ga_{1-x}Al_xAs$ δ-doped quantum well has been measured over the temperature range 0.3–3 K. At higher temperatures the loss rate varies as T^5 and the magnitude agrees well with that predicted by the standard theory for piezoelectric electron-phonon scattering. At low temperatures the observed dependence changes to T^4, the crossover occuring near $ql \sim 0.35$, where q is the average magnitude of the phonon wave vector and l the electron mean free path. This is in agreement with recent theoretical predictions for piezoelectric scattering in the dirty limit. The theory also predicts that the magnitude of the energy-loss rate should depend inversely on the conductivity of the sample. Good agreement is found at higher conductivities, but the measured values show saturation when the conductivity becomes very low.

I. INTRODUCTION

This paper is concerned with electron-phonon (e-p) scattering at low temperatures in a GaAs/$Ga_{1-x}Al_xAs$ quantum well. Various types of transport measurements provide information on e-p scattering and, to put the present work in perspective, these will be briefly outlined in the following paragraphs. At very low temperatures the various phenomena have simple power-law dependences on the temperature and the measurements are most simply compared with theory in this region. An essential requirement for low-temperature behavior is $q \ll k_F$ where k_F is the magnitude of the Fermi wave vector and q is the average magnitude of the phonon wave vector; this is referred to as the Bloch-Gruneisen limit. In this limit piezoelectric scattering dominates the interaction, and this is the range mainly covered in the present work.

Practically all previous work on piezoelectric e-p scattering has been in the clean limit which corresponds to $ql \gg 1$, where l is the electron impurity mean free path. The phonon contribution to the measured resistivity varies as T^5 at low temperatures and has been observed on a high mobility GaAs based heterojunction.[1] This gives the momentum relaxation time for e-p scattering, but the technique is useful only for the highest mobility material. Very recently it was realized[2,3] that phonon-drag thermopower, which varies as T^4 at low temperatures, also measures the same quantity. This is much more convenient and sensitive over a wide mobility range because it is not affected by electron-impurity (e-i) scattering, providing this is the dominant scattering mechanism. Measurements of phonon drag on material from very high to very low mobilities have been found to be completely consistent with the same e-p interaction.[2,4]

Another method, and the one used in the present paper, is to examine the energy-loss rate for hot electrons. This is related to the energy-loss relaxation time, which varies as T^3 at low temperature and, as with phonon drag, depends only on e-p and not on e-i scattering. Shubnikov–de Haas oscillations in the magnetoresistivity have provided a useful thermometer to measure the electron temperature (e.g., see Ref. 5) at higher mobilities. At low mobilities the correction to the conductivity due to weak localization (WL) and electron-electron (e-e) interaction[6] has been extensively used in the case of Si-based systems, but the only previously published data for a GaAs system seem to be those of Wennberg *et al.*[7] and Chow *et al.*[8] The method relies in the fact that the inelastic scattering time is expected to be dominated by e-e scattering at low temperatures, e-p scattering playing a negligible role. Very recently, one-dimensional thermopower has been used as another probe of the electron-gas temperature.[9]

The energy loss rate should vary as T^5 at low temperatures (see Sec. III). The measurements of Wennberg *et al.*[7] yielded the correct T dependence, but indicated that e-p coupling was two orders larger than expected. Those of Chow *et al.*[8] were mainly aimed at the behavior in the dirty limit (see below), but they also obtained data in the clean limit. In this case their loss rates seem to be a factor of about 2–3 too large. All of the other aforementioned results are consistent with the expected e-p scattering matrix elements and with

TABLE I. Sample parameters.

n (10^{16} m^{-2})	μ (cm^2/V s) [a]	l (nm) [b]	ql/T (K^{-1}) [c]	$k_F l$ [b]
1.86	1060	23	1.05	8.2
1.63	850	17	0.78	5.7
1.40	640	12	0.55	3.7
1.30	540	10	0.44	2.9
1.19	460	8	0.36	2.1
1.11	290	5	0.22	1.3

[a]Using σ measured at 4.2 K and 2.9 T.
[b]Using $\sigma=(e^2/h)k_F l$ with σ measured at 4.2 K and 2.9 T.
[c]Using a velocity of sound $v=3000$ m/s.

static screening of the potential by the electrons.

Very recently, Chow and co-workers[10,8] and Khveshchenko and Reizer[11,12] predicted that when e-i scattering becomes strong enough that $ql \leqslant 1$ then piezoelectric e-p scattering should be enhanced. Even in very low mobility systems this happens only at low temperatures. Chow *et al.*[8] have identified this with a reduction in effective screening of the electrons due to their very low diffusivity. The energy loss rate is predicted to change from T^5 dependence to T^4 in the Bloch-Gruneisen limit. Chow *et al.* presented experimental data on a GaAs/$Ga_{1-x}Al_xAs$ heterojunction which support this prediction for electrons in zero field,[8] and also for electrons under quantum Hall conditions.[10]

The present experiments deal with a GaAs/$Ga_{1-x}Al_xAs$ quantum well into which impurities have been deliberately introduced in the form of a δ-doped layer of Si atoms. This gives very low mobilities at high electron densities n. The latter enables the Bloch-Gruneisen region to be entered at relatively high temperatures. By using a gate, n and l could be reduced in a controlled fashion which enabled us to change the temperature at which $ql \sim 1$. The experiments have clearly revealed the predicted crossover from T^5 to T^4 in approximately the expected location. In the dirty limit, the experiments are in good agreement with predictions for the absolute magnitude when the mobility is not too low. However, at the lowest mobilities the measured rate is too small, though the T^4 dependence is maintained.

II. EXPERIMENTAL TECHNIQUE AND RESULTS

The quantum well has been well characterized and fully described elsewhere.[13] For the present purposes a Ti-Au top gate was added (about 2700 Å from the 2D gas) to vary the electron density n. We have examined two samples from the same wafer. Both give essentially identical results, but only the more accurate data from the second sample will be reproduced here. The gate enables us to decrease n from 2.1×10^{16} m^{-2} to 1.1×10^{16} m^{-2}. The lower limit was determined by the onset of gate leakage. At the highest density the sensitivity in measuring the electron temperature using the WL correction was too low to be useful, so our measurements actually start at $n = 1.86 \times 10^{16}$ m^{-2}. There is strong variation of l (obtained as described below) with n as Table I shows.

Measurements of the energy relaxation rate were carried out at six values of n using a ^{3}He cryostat capable of reaching about 0.26 K. All data were measured by four terminal dc techniques because of the difficulties in analysis that ac techniques introduce.[14] The conductivities σ as a function of temperature T for these six densities are shown in Fig. 1. All exhibit a $\log T$ dependence at higher temperatures as expected for WL and electron-electron interactions.[6] The slopes of these lines are similar, but not identical, at the various densities.

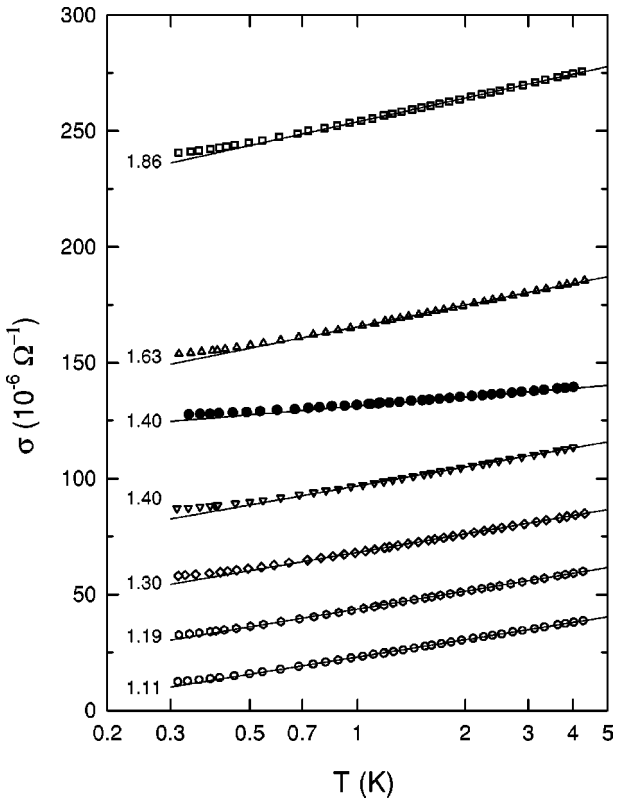

FIG. 1. The measured conductivity σ of the sample as a function of temperature (on a logarithmic scale) at various fixed densities n (in units of 10^{16} m^{-2}). The open symbols are data at zero field, with the straight lines giving $\ln T$ fits at higher temperatures. The solid symbols are an example of data taken in a perpendicular field of 1.4 T to show the suppression of coherent backscattering (WL) effects.

In all cases the data show a tendency to a saturation in conductivity at the lowest temperatures. The simplest explanation of this is that extraneous heating of the 2D gas was causing the temperature of the 2D electron gas to be somewhat higher than that of the bath at the lowest temperatures. This is an important point and, as the following indicates, we have devoted considerable effort to showing that this is very unlikely.

The conductivity is independent of current in the range used to determine these data, typically 0.1–1 nA, so that self-heating was not a problem. All wiring to the cryostat was fully shielded and filtered against rf interference. The current supply and gate voltage supply were exceptionally quiet and were battery operated in heavily shielded enclosures. Disconnecting all leads other than the current and voltage leads, e.g., temperature controllers and various thermometers, had no effect, at the level of $<1\%$, on the measured conductivity at the lowest temperature.

The only remaining possibility was that the voltage detector might be introducing noise into the sample. We used two

commercial voltmeters, an EM N12 and a Keithley 182,[15] for comparison. The latter produces detectable short voltage pulses at the 1-mV level, a few ms wide separated by about 250 ms, at its input terminals (usually referred to as pumpout) which results from autozeroing circuitry in the input amplifier. Although at first sight these pulses appear to be a problem in heating the 2D electron gas, in fact they are not present during an actual voltage measurement and the 2D gas will cool extremely rapidly after the end of a pulse. It is possible to vary the magnitude of the voltage pulses but when this was done there was no visible effect on the results. The EM N12 nanovoltmeter has an ac modulator in the early amplifier stages which has a similar function, but we could not detect any pumpout from this modulator at the input terminals, with better than 10-μV resolution. It is also worth mentioning that this voltmeter was used in a battery operated mode and its output was isolated from subsequent data logging devices. In spite of these many differences, both voltmeters gave identical values for the conductivities to an accuracy of 1% under all conditions.

Finally we mention that the energy loss rates that we calculate using the observed conductivities are completely self-consistent and show no anomalous temperature dependences that would be expected if the conductivity effects were not intrinsic. We conclude from all this that there is no evidence of extraneous heat input and that the saturation appears to be an intrinsic effect in this sample. Similar effects have been noted by many others, e.g., see Ref. 16. Recent theoretical papers have suggested that microwave radiation is the culprit,[17] but experimentally this seems unlikely. Assuming the effect is indeed real, a possible dephasing mechanism is spin scattering. There are many possibilities though the only one that has been clearly observed in GaAs structures seems to be that due to the crystal-field induced splitting.[18] It is found that the inelastic scattering rate $1/\tau_s$ associated with this mechanism $\sim ln^{5/2}$. The expected magnitude is approximately 80 ps for our highest density sample, which should lead to saturation of σ below about 0.5 K. However, for the lower density samples this moves down to about 25 mK and outside our range. Other spin-scattering possibilities also exist[18] and have been suggested as the cause of conductivity saturation in δ-doped layers previously.[19] We note that such scattering typically gives an initial positive magnetoresistance which we do not observe, at least down to 1.25 K.

The energy-loss measurements were made by holding the substrate of the sample at a temperature of about 0.28 K. Currents over a wide range of magnitude provided power input via Joule heating and the temperature of the electron gas T_e was deduced from the conductivity. At high currents the associated voltage drop affected the local gate voltage, but this was reduced to a second-order effect by connecting the gate midway between the potential contacts. The energy relaxation rate is defined as the energy lost per unit time per electron, say P, as a function of the electron temperature T_e. Data are presented in Fig. 2 for the various fixed values of n.

Absolute uncertainties mainly arise from the dimensions of the sample and are at the level of a few percent. The measurement accuracy of the current was typically a few tenths of 1% which caused rather large uncertainties in evaluating the electron temperature for the higher density samples where the relative magnitude of the WL correction

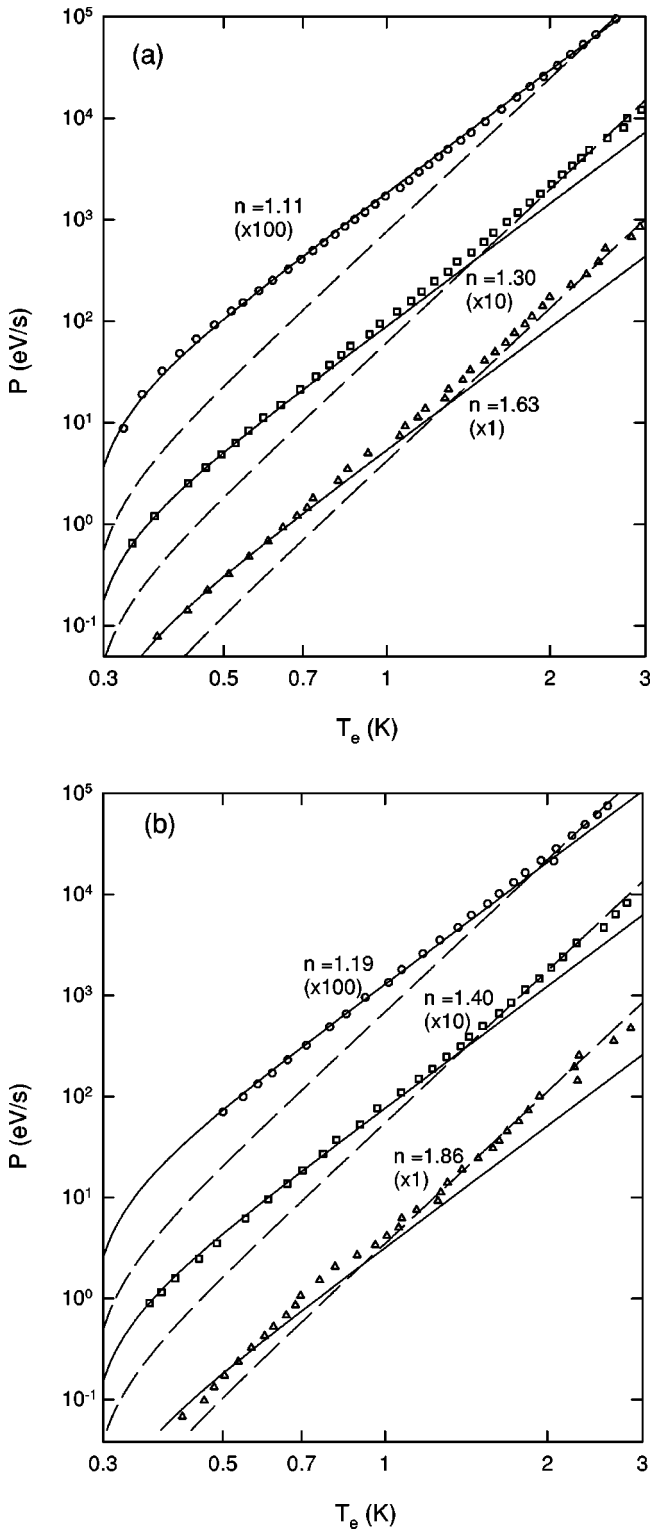

FIG. 2. Both panels (a) and (b) show data on the energy-loss rate P as a function of electron temperature T_e at various fixed densities (in units of 10^{16} m^{-2}). The measured values are represented by open symbols and the results at various densities are offset by the factors shown for clarity. The dashed lines are the results of the standard calculation, Eqs. (1) and (2) with no unknowns, with $F_c(T) \propto T^5$. The solid lines correspond to $F_d(T) = \beta T^4$ where β has been adjusted for the best fits in the low-temperature region.

is small. The scatter of the data in Fig. 2 mainly results from this cause.

To fully characterize the data we also require the impurity mean free path l of the electrons. One could use the conductivity as a crude measure of this but a more accurate determination is obtained when WL effects are suppressed, which can be done either by going to high temperatures or by using a magnetic field. Figure 1 shows an example of σ as a function of T measured at a fixed field of 1.4 T, and one sees that most of the temperature dependence has disappeared as expected for WL. Most of the remaining temperature dependence is presumably due to e-e interaction which is insensitive to relatively low magnetic fields. To minimize the effects of e-e interaction and WL we determined the conductivity at 2.9 T and 4.2 K, and have used these data to calculate l. The results are shown in Table I along with estimates of and $k_F l$ and ql/T (using $q=k_B T/\hbar v$ with v the sound velocity taken as 3000 m/s since transverse phonons dominate piezoelectric scattering).

At low n we note that $k_F l \sim 1$, which is equivalent to the WL correction becoming of the same order of magnitude as the Boltzmann value of the conductivity, as is also clear from Fig. 1. Under these conditions WL theory is no longer expected to be valid in describing the behavior of σ. Nevertheless, we observe the strong suppression of the temperature dependence of σ with a magnetic field at all densities and this is consistent with coherent backscattering as being the major cause of the temperature variation of σ. Thus the use $\sigma(T)$ to measure T_e should remain valid.

III. DISCUSSION

The theory of electron energy loss rate shows that the measured loss rate P is the difference between two terms, which have the same functional form $F(T)$, corresponding to phonon emission at the electron temperature T_e and phonon absorption at the substrate temperature T_s, i.e.,

$$P=F(T_e)-F(T_s). \tag{1}$$

In the clean limit the theory for $F(T)$ [which we will label $F_c(T)$] has been given in various references. The form of the results given by Ma *et al.*[5] is convenient for our purposes. The contribution from piezoelectric e-p scattering at low temperatures can be written

$$F_c(T)=\frac{\zeta(5)}{64\pi}\left(\frac{m^* e h_{12}}{Q_s}\right)^2\frac{(k_B T)^5}{\hbar^7 \rho_d k_F^3}\sum_i \frac{\alpha_i}{v_i^4}. \tag{2}$$

The sum is over the three phonon polarizations i, α_i is a numerical constant equal to 135 for longitudinal and 177/2 for transverse modes, v_i are the velocities of sound, ρ_d is the mass density of GaAs, h_{12} is a piezoelectric coupling constant (1.2×10^9 V/m), m^* is the effective mass, and $Q_s=m^* e^2/(2\pi\epsilon_0\kappa\hbar^2)$ is the screening wave vector with κ the relative dielectric constant (13.2) of GaAs and ϵ_0 the permittivity of free space.

To evaluate Eq. (2) and then Eq. (1), we have used the velocities averaged over crystal direction using the expressions given by Jasiukiewicz and Karpus[20] and the elastic constants and mass density of GaAs ($\rho_d=5335$ kg/m^{-3}) as given by Blakemore[21] for $T\to0$. This procedure yields $v_l=5093$ m/s and $v_t=2971$ m/s. There is also a correction factor[20] of about 0.77 that must be applied to $F(T)$ to account for phonon anisotropy. The final result is $F_c(T)=1.50\times10^6 T^5/n^{3/2}$ J/s (n is in units of m^{-2}) and the dashed curves in Fig. 2 correspond to this result. In all cases the data accurately tend to these curves at higher temperatures.

We should point out that deformation potential scattering should become visible in the upper regions of our temperature range. One finds[5] that the energy relaxation rate for this mechanism is $F(T)=6.1\times10^5 T^7/n^{3/2}$ J/s using a deformation potential of 10 eV, and assuming low-temperature conditions are still appropriate. Thus the contributions from deformation potential and piezoelctric scattering should be the same at about 1.6 K. However, we do not see any upturn in our data corresponding to a T^7 dependence. It is probable that this is just the range where the low-temperature approximations are beginning to break down leading to weaker temperature dependences. Ma *et al.*[5] have made numerical calculations of $F(T)$ over a wide range and indeed found that deformation potential has a transition region where it mimics the low-temperature behavior of piezoelectric scattering both in magnitude and temperature dependence, thus extending the T^5 result to higher temperatures than one would have expected it to be valid. Appleyard *et al.*[9] have suggested the same explanation for a similar feature of their data.

At lower temperatures, the measured energy relaxation rates deviate from, and are larger than, those predicted by the standard theory above. From Table I and Fig. 2 we see that the deviations always begin at a temperature where $ql\sim1$. This is just the region where recent theories[10,8,12] have predicted deviations to occur. In the dirty limit Khveshchenko and Reizer[12] find that Eq. (2) is replaced by

$$F_d(T)=\frac{\pi^2 C_3}{15}\left(\frac{e h_{12}}{Q_s}\right)^2\frac{\nu(k_B T)^4}{nD\hbar^3\rho_d v_l^3}, \tag{3}$$

where ν is the electronic density of states, D the diffusion coefficient $\frac{1}{2}v_F^2\tau$ (with v_F the Fermi velocity and τ the momentum relaxation time) and C_3 is a constant, estimated as 1.35, to take into account the two polarizations of the phonons because the above result is given in terms of the velocity of longitudinal phonons.

The above equation is more conveniently written as

$$F_d(T)=\frac{C_3}{15}\left(\frac{m^* e^2 h_{14}}{Q_s}\right)^2\frac{(k_B T)^4}{n\sigma\hbar^7\rho_d v_l^3}, \tag{4}$$

where $\sigma=ne\mu$. This equation appears to give the same results as that quoted by Chow *et al.*[8] where it is given in a numerical form appropriate to GaAs only. Note that both Eqs. (2) and (4) are actually independent of m^*. For the purposes of analysis, we have taken σ to be independent of T (see below), in which case $F_d(T)\propto T^4$. The solid lines on Fig. 2 are drawn taking $P=\beta(T_e^4-T_s^4)$ but allowing the constant β to be adjusted to obtain the best fit at low temperatures. We see that the T^4 power law is an excellent representation of all our data in this temperature range. Note that we do not have a full theory of how the energy loss varies between the clean and dirty limits so we are unable to provide a theoretical curve covering the whole range.

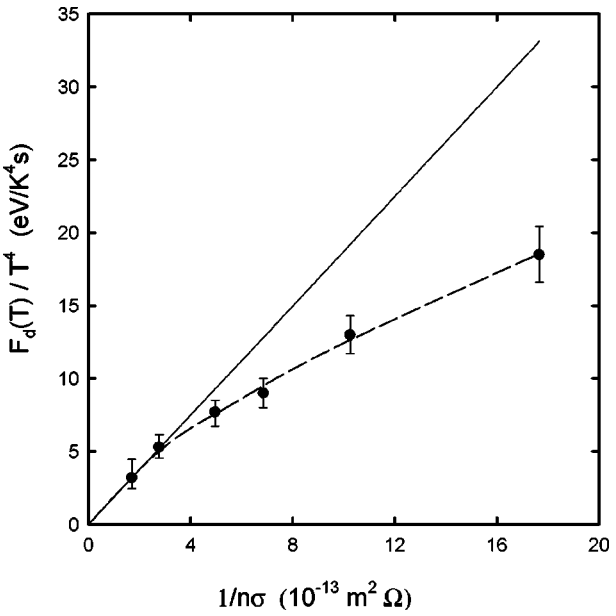

FIG. 3. The solid line is the calculated variation of the coefficient of T^4 according to Eq. (4) as a function of $1/n\sigma$. The closed symbols show the measured values of the coefficient of T^4 (referred to as β in Fig. 2). The dashed line through the measured values is drawn simply to guide the eye.

The parameter β is plotted in Fig. 3 as a function of $(n\sigma)^{-1}$. The error bars were deduced for each data set by simply varying β until the fitted curve became an unreasonable representation of the experimental points in the low-temperature range. In this sense we expect the probable errors to be the maximum possible for β. We have taken σ to be the Boltzmann value $\sigma = ne^2\tau/m^*$ and have estimated it from the measured conductivity at 4.2 K in a magnetic field of 2.9 T (cf. the determination of l described in Sec II). Had we used the measured values of σ as a function of temperature, as Chow *et al.*[8] did in their work, then we would have expected deviations from the T^4 power law arising from the $\ln T$ contribution to σ, particularly in the lowest density samples. For comparison we have also plotted the theoretical result from Eq. (4) which, using the parameter values already mentioned, is a straight line given by $F_d = 3.0 \times 10^{-6} T^4/n\sigma$ J/s.

In Fig. 3 the experimental points appear to lie on a curve which asymptotically tends to the straight line given by Eq. (4) at high $n\sigma$. We conclude that the theory is in good agreement with the data in this limit. However, at lower $n\sigma$ the theory predicts substantially higher loss rates than are observed. If we use the measured σ, which is always lower than the Boltzmann value, as a function of T this discrepancy is aggravated. Being a perturbative result, we expect the theory to be valid only in the regime where $k_F l \gg 1$ and this is why we took σ to be a constant in our analysis above. [Other things being equal, the relative magnitude of the weak localization correction to $\sigma \propto (k_F l)^{-1}$.] We expect the theory to fail when $k_F l \sim 1$ as occurs in our lowest conductivity samples. Even so the T^4 dependence remains an excellent fit to our data. It is also particularly interesting that the value $k_F l \sim 1$ is taken as one of the indicators of the point at which weak localization is changing to strong localization. Very little is known about *e-p* scattering under these conditions and the present work provides the first systematic study in this region.

The value of ql at the point of crossover from T^5 to T^4 varies from about 0.5 at highest mobility to 0.23 at lowest mobility (again using 3000 m/s for the velocity of sound). If the theory had been obeyed for all samples in the dirty limit, then the crossover would have been at $ql \sim 0.5$ in all cases.

Finally we note that Khveshchenko and Reizer[11] also predict a correction to the conductivity due to *e-p* scattering $\propto \ln T$ in the dirty limit, but estimates for our sample show that this is negligible compared to the observed $\ln T$ term. This is consistent with the expectation that the inelastic lifetime of the electrons is dominated by *e-e* scattering in these systems at low temperatures, and *e-p* scattering is negligible, even in the modified form investigated here.

IV. CONCLUSIONS

The experimental data clearly show the transition from the standard low-temperature T^5 dependence of the energy relaxation rate in the clean limit, to the predicted T^4 dependence in the dirty limit. The transition typically occurs near $ql \approx 0.35$ which is consistent with theoretical expectations. The magnitude of the rate in the dirty limit is also in agreement with calculation when the impurity scattering is not too strong. However, when the electronic mean free path becomes very small, so that $k_F l \approx 1$, the calculation overestimates the loss rate.

ACKNOWLEDGMENTS

The support of the Natural Sciences and Engineering Research Council of Canada is gratefully acknowledged. We also thank Jessica Arlett for help in collecting some of the initial data.

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